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Econophysics and sociophysics: their milestones & challenges

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Abstract

In this review article we present some of achievements of econophysics and sociophysics which appear to us the most significant. We briefly explain what their roles are in building of econo- and sociophysics research fields. We point to milestons of econophysics and sociophysics facing to challenges and open problems.

1. Introduction

As the name suggests, econophysics and sociophysics are hybrid fields that can roughly be defined as quantitative approaches using ideas, models, conceptual and computational methods of statistical physics applied to socio-economic phenomena. The idea of a *social physics* is old since it dates back to the first part of the 19th century – this term occurred for the first time in Saint-Simon's book (1803) [2] in which the author describes society through the laws of physics and biology. This approach has been popularized later by Adolphe Quetelet (1835) [3] and August Comte (1856) [4].

In contemporary terms, this idea of social physics led to the emergence of sociophysics and partially to econophysics. While the former dates back to the 1970s (papers of Weidlich in 1971 [5] and Callen with Shapiro in 1974 [6]), the latter has been coined more than twenty years ago by physicists (H. Eugene Stanley et. al) [7]. Although sociophysics roots might be traced back to Majorana (1942) [8] with his paper on the use of statistical physics to describe social phenomena, the major works in sociophysics mainly appeared in the 1970s

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and 1980s with an increasing number of publications applying statistical physics to model large scale social phenomena (see [9] for review). Among others, the popular themes modelled by sociophysicists are behavioral dissemination, opinion formation, cultural dynamics, crowd behavior, social contagion and rumors, conflicts, and evolution of language.

It is worth mentioning that this increasing interest of physicists in social sciences is mainly due to two factors: (i) the *Golden Age* of condensed matter physics thanks to the success of the modern theory of phase transitions based on the renormalization group techniques that is, an ϵ-expansion of Wilson and Kogut (the Nobel prize winners) [10] (the application of real renormalization group in sociology at the turn of the centuries is due to Serge Galam [11, 12, 13]) and (ii) the growing computerization (or digitization) of society that paved the way to new perspectives by offering a very high number of data (or observations). This computerization process also concerned financial markets by recording every single transaction or changes in financial prices offering therefore huge database (made in time lag even so short as miliseconds) for scholars to be statistically investigated. That was the original purpose of econophysics.

The influence of physics on economics is an old story [14, 15, 16]. However, in contrast to previous works importing models from physics in socio-economics, socio- and econo-physics refer to a new trend since scholars involved in these fields are not economists who take their inspiration from the work of physicists to develop their discipline but rather physicists who are moving beyond their disciplinary boundaries. Financial markets, or speaking much more generally, socio-economic life should be considered in the wider sense of complex systems displaying emergent behaviors – creating new properties, phenomena, and processes, e.g., self-organized criticality (SOC) [17, 18] or spontaneous log-periodicity – the former is the prominent example of a multiscale avalanching paradigm, while the latter resulting from discrete translational invariance without the need for a pre-existing hierarchy [19, 20, 21]. From this point of view, the link between the micro- and macroscales is a constant challenge and well motivated interest. In this context, much debate and many questions about the ability of financial economists to deal with financial reality were generated. The time has come to reflect on the way of describing and understanding our contemporary societies.

2. Birth of modern econophysics

The origin of modern econophysics dates back to when it became possible to publish economically oriented papers in physical journal (see ref. [22, 23] for details). Presumably, one of the first papers belonging to this stream to appear in Physica A in year 1991 was *Lévy walks and enhanced diffusion in Milan Stock-Exchange* by Rosario Nunzio Mantegna [24] (student of H. Eugene Stanley) who published a pioneering paper by discovering the breaking of the central limit theorem on the stock market. He replaced it with the Lévy-Khinchine generalization of the central limit theorem. That is, he noticed that a stable Lévy pdf rules the stock market in any time scale. This discovery means that the world entered an age of significantly increasing risk of financial market investments, where not only huge losses but also colossal profits are possible. This created in turn the basis of moral hazard on markets, which has now grown on an unprecedented scale leading to destructive social tensions.

The Mantegna discovery has opened the eyes of the physics community to non-Gaussian processes on financial markets, in particular, on the multiscale and scale-free properties of complex systems such as financial markets. This has been inspiringly confirmed and expanded at canonical work of Rosario N. Mantegna and H. Eugene Stanley [25] and summarized in their book *An Introduction to Econophysics Correlations and Complexity in Finance* [26]. Crowning this series of papers is article [27]. It shows that the central limit theorem is present in the financial market away from a crash, while the theorem is not applicable for time series containing the crash. Instead, in the latter case a scale invariance or data collapse is observed, because the Gaussian statistics was replaced there by the scale-free distribution, i.e. the power law. Apparently, the beginning of modern econophysics is directly connected with physical analysis of financial markets focused on the non-Brownian or non-Wiener random walks.

We would like to suggest a general point – more than one of the biggest success/contribution of econophysics up to now has been in the data analysis (both empirical and analytical). That is, it has been in the identification of empirical regularities and stylized facts – see for details book [28], review papers [29, 30], and paper concerning new stylized facts [31]. These references also consider the best mathematical models and tools for dealing with such vast amount of data. In particular, the high-frequency data become, for a variety of reasons, a way for understanding the market microstructure.

The actual birth of econophysics should be, however, dated back to the mid-nineties of the last century. Interestingly, this new trend coincided with the opening of high-tech opportunities for risky investing in the financial markets on a massive scale. Fortunately, a number of renowned physicists had an instrumental role at that time in getting approved econophysics by editorial boards of such significant physical journals as Physica A, The European Physical Journal B, and the International Journal of Modern Physics C. Currently, almost all major physical journals already accept econophysical works. It was during this period that an avalanche of econophysical publications set off.

At the beginning of the 21st century Hideki Takayasu undertook the task of reviewing the state of econophysics and its actual and potential uses by publishing materials from international conferences organized by him in the Nikkei Institute in Tokyo [32, 33]. Thanks to this he made the whole world aware of what econophysics is and what its possibilities, tasks, and challenges are.

Much attention attracted that time statistical systems that are described by power-law distributions and scale-invariant correlations – see [34] for details and refs. therein. More specifically, the challenge is to understand the dynamics of markets manifesting long-range nonlinear correlations.

One of the attractive possibilities of insight into this type of phenomenon is offered by the self-organized criticality (SOC). The SOC introduces dynamics by separation of time scales that is, assuming that the increasing instability is slow (slow mode), while relaxation is fast (fast mode). This fast mode leads to avalanche-like, bursty event release on a broad range of scales. The dynamics of an avalanche is fundamentally multiscale, it occurs by coupling across many spatial scales in the system. As is the case for critical phenomena, the dynamics is insensitive to details of the instability, thus in a socio-economical life containing the finance systems [35, 36, 37], where series of instabilities and routes to instability are possible, one expects to see some universality, that is a robust emergent behavior. Apparently, one can find SOC paradigm in multiscale avalanching, which is sufficient to provide a new, insightful framework for explanation or at least the proper ordering the observations [17].

3. Scale invariance

The second half of the nineties was dominated by the subject of crises and bursts/crashes in the financial markets, as the risks and uncertainties were associated with it, and attempts to forecast extreme events. The logo of these works can be seen as the discovery of log-periodic oscillations on the stock exchanges presented in papers [20, 38, 39]. This discovery itself, its origin, and consequences were summarized in 2003 in book *Why Stock Market Crash* by Didier Sornette [40]. The discovery of log-periodic oscillations was an inspiration for many authors for almost a decade – see review paper *Physical approach to complex systems* by Jarosław Kwapień and Stanisław Drożdż [41].

The log-periodic correction to scaling is a hallmark of discrete scale invariance as defined only for specific choices of characteristic lengths. A solution of the corresponding discrete scaling relation, it is thus represented by a power-law function modulated by oscillations that are periodic in the logarithm of explanatory variable. In other words, the discrete scale invariance leads to complex critical exponents or dimensions - indeed, to log-periodicity as a correction to scaling, which can appear even spontaneously – see *Discrete-Scale Invariance and Complex Dimensions* by Didier Sornette [42]. This spontaneity is, yet, an immanent endogeneous feature of financial markets, which is why its role for econophysics is hard to overestimate.

Loosely speaking, going from continuous scale invariance to discrete scale invariance can thus be compared with going from the fluid state to the solid state in condensed matter physics. The symmetry group is limited to those translations which are multiple of a basic discrete generator. This is true for endogeneous causes, in particular, when a system is not in equilibrium and is further forced out. It can be said that in the frame of econophysics, both critical phenomena are investigated, including, e.g., self-organized criticality, described by means of pure power-laws, as well as structures hidden in discrete-scale invariance. The existence of these structures results from the existence of characteristic length scales forced by underlying mechanisms and resulting, indeed, in log-periodic oscillations. In particular, very interesting is the sandpile model of Marcel Ausloos et al. where they pointed to the origin of log periodic oscillations [43].

The approach above is an example of so called global analysis. Its aim is to observe well defined, repeatable structure in financial time series before the phase transition point t_c (the crash point) occurs.

Other global approaches to periodicity in finances have also been developed. It is especially worth to mention, e.g., those based on analogy with properties of viscoelastic materials [44]. The periodic evolution of a stock index before and immediately after the crash is described within this approach by Mittag-Leffler generalized exponential function superposed with various types of oscillations.

Although the global approach seems to be interesting and encouraging, the main difficulty in its application lies in the fractal structure of financial time series. In fact we are never sure, due to this fractal nature of time series, whether oscillations or even the leading shape of the price index are connected with the main bubble (i.e., the specific structure of time series being formed from the beginning of increasing trend till the crash point t_c) or with some mini-bubbles appearing as second or higher order corrections to solutions of equations of price evolution. Usually, it is difficult to separate data connected with the main bubble and its mini-bubble corrections before an extreme event (crash) happens and this distinction becomes explicitly clear only after the event already had happened.

Therefore, the other approach based on complex phenomena applied to finances has been developed to study the scaling properties of financial time series in order to distinguish whether the involved stochastic process can be long-memory correlated or not. Several techniques have been proposed in literature to attack this problem. Their common aim is to calculate the Hurst exponent H [45] of the system.

Among various techniques to do so the accurate and fast algorithm enabling to extract H from given time series is served by Detrended Fluctuation Analysis (DFA) [46, 47, 48].

The DFA can be used as the basis of so called 'local DFA' applied for the first time in analysis of financial crashes in [49] and then extended in other publications [50, 51]. The local DFA is nothing else but DFA applied to small subseries of a given set of data. This way it characterizes the local fractal pattern of time series instead of its global properties in large time horizon. Therefore the latter approach is an example of local analysis contrary to previous global attempt like log-periodic oscillations.

One expects positive autocorrelations in time series if financial system relaxes (i.e., just after the critical moment t_c). Thus, the local Hurst exponent $H(t)$ should reach the value $H > 1/2$ corresponding to persistent (long-range autocorrelated) signal. It means however, that for some time before the crash ($t < t_c$) the system is antipersistent in order to reproduce the observed mean Hurst exponent value $\langle H \rangle \simeq 1/2$ for large time limit. In this way, clear trends in local values of H are formed; these should be carefully translated into repeatable scheme revealing the major forthcoming events like, e.g., crashes, rupture points, beginning of bullish periods, etc., which are particularly interesting for investors. It seems there exists a strong connection between trends in local values of H and phase transitions (crashes or rupture points) on the market caused by the intrinsic organization of the financial market as a complex system.

The method proposed in [49, 50, 51] was successfully applied by many authors and well checked for European and non-European capital markets (see, e.g., [52, 53, 54, 55, 56, 57, 58]). Beside providing some intrinsic explanation of such major features of financial markets, the local DFA can be also used in a practical way, suggesting short term investment strategies to agents following some stocks far from a $H = 1/2$ values in order to optimize profits [59]. In a similar way the case of correlated fluctuations between foreign currencies exchange rates, whence suggesting strategies can be demonstrated [60, 61].

Challenges are based on empirical data deriving from rapidly changing reality. This rapid variability has not only an increasing amplitude, but abounds in extreme events (the so-called swans) and superextreme ones (the so-called dragon kings, see [62] for details).

4. Multiscaling and multifractality

The concept of extended scale invariance, that is multifractality, with its coupled scales, becomes today a routine methodology (derived from statistical physics) [63] for study both complex systems [64, 41, 65, 66] as well as non-linear low degree of freedom dynamical ones [67]. Generally speaking, this is an inspiring rapidly evolving approach of nonlinear science in many different fields even outside the traditional physics [68, 69, 70, 71, 72, 73, 74, 75]. Multifractals are fractal objects and/or signals with heterogeneously distributed measure. Therefore, the description of multifractals requires, in general, an infinite family of fractal dimensions that is, spectrum of dimensions. Apparently, their scaling properties are defined only locally.

There are several well-functioning techniques [65, 66] (some of them have been initiated and inspired by particularly popular Multifractal Detrended Fluctuation Analysis [64]) that allow not only the construction of spectrum of dimensions for stationary but also non-stationary series. By the way, these techniques allow to obtain other important characteristics of multifractality. Intensive research is in progress to classify the market states using the spectrum of dimensions. Generally speaking, the wider this spectrum as a function of Hölder's exponent, the more collectivized and more nervous (fluctuating) market is. In addition, the magnitude of the asymmetry of the spectrum allows us to say what fluctuations dominate the market. It must be said, however that the identification of multifractal time series (signals) is technically difficult due to the significant number of sources of apparent multifractality [76, 77]. The list of known sources of (true) multifractality is presumably incomplete. On the possible origin of multifractality in finance – see for details papers of Marcel Ausloos and coauthors [78, 79, 80, 81].

The research on this apparent multifractality, indicated already in [76], is the main goal of recent activity in formal study of multifractal observable phenomena caused entirely by nonlinear correlations. The article [82] has shown quantitatively how multifractal effects may arise from the finite sizes (lengths) of data and (or) from linear autocorrelations involved in time series. This kind of spurious multifractality should be clearly separated from the real multifractality caused by memory effects dependent on the time scale and thus leading to different scaling properties at various scales. The ready to use semi-analytic formulas have been found [82, 83]. They are general enough to be applied also to real data analysis in other areas (e.g., medicine, physiology, geology, etc.) in order to distinguish if and how their observed multifractal properties have real multifractal origin. The similar semi-analytic study of the influence of broad data distribution on multifractal phenomena is under search now [84].

5. Continuous-time random walk on financial markets

At the very beginning of the present century very flexible continuous-time random walk (CTRW) formalism was adopted by Masoliver, Montero, and Weiss to the systematic description of the financial market evolution [85, 86, 87, 88]. They proposed a dependent model in which large return increments are infrequent. This model predicts that the volatility should behave in an anomalous diffusive way at short times, something that is seen in some markets. The possibility of using CTRW formalism to describe empirical data coming from some financial markets was also suggested in refs. [89, 90] on example of Lévy walks with varying velocity of the walker. The results obtained under this latter model are complementary to the results obtained under the former one.

The CTRW formalism assumes the interevent-times continuous and fluctuating; ('interevent time' appears in literature under such names as 'pausing time', 'waiting time', 'inter-transaction time', 'intratrade time', and 'interoccurrence time'). It must be noted that term 'walk' in the name 'continuous-time random walk' is commonly used in the generic sense comprising two concepts: namely, both the walk (associated with finite displacement velocity of the process) and flight (associated with an instantaneous single-step displacement/increment of the process). Thus, we have to specify in a detailed way what kind of process we are considering. Apparently, not only the process increments but also interevent times can be considered as stochastic variables. These variables are characterized by distributions creating the stochastic process base, quite often the broaden non-Gaussian ones and/or long-term correlated, giving a fundamentally new description of stochastic processes, e.g., favoring extreme value theory and multiscaling insight into the process activity.

Thus, the variance of the stochastic process is no longer sufficient to identify the dynamics of the process. The non-ergodic or weak ergodicity behavior of the system isssociated with new description. The ergodicity breaking effects are essential in understanding fluctuation-generated phenomena, in particular fluctuation-dissipation relations and linear response. The understanding of mechanisms generating consistent statistics has therefore become a central issue. It so happens that the mentioned above properties of interevent times are also an immanent feature of financial markets' tick data studied in recent decade [91, 92, 93, 94, 95, 96, 97]. Their distinct real (and not spurious) multiscaling and multifractality were found. Thus, not only stock quotation and currency quotation but (what is even more significant) also inter-event times have these properties.

The results obtained in paper [95] also suggest something more. Even the statistical dependence of time steps is insufficient to describe the autocorrelation of absolute price changes. It is necessary to take into account the long-term dependence of the inter-event times as well. This long-term relationship is one of the most important sources of multifractality of interevent time series. What has been said above, forces the use of CTRW formalism describing market processes that are not renewal. It is a pressing, open issue.

It is worth to mention the threshold phenomena both in physical and social sciences. The chemical reactions starting at over-threshold concentrations of reagents, phenomena of decays and escapes, including photoelectric effect above some threshold are typical examples. Coming back to the financial markets, there is a lot of empirical data and publications on this subject. The threshold phenomena were analyzed with very effective tools of CTRW formalism (see, e.g., [97] and refs. therein). More specifically, the statistics of interevent times for excessive losses (those below some negative fixed threshold) and excessive profits (those greater than some positive threshold) can be explained by the same CTRW formalism.

6. Complex networks

Important tools to describe and understand the collective behavior of financial time series (based on correlated graphs) include the minimal spanning tree (MST) [98]. This was applied to finance for the first time by Rosario Mantegna [26], opening a new, extremely prolific chapter in econophysics and recently to sociophysics.

The MST (is a connected graph) that allows only such unique paths connecting nodes of a complete graph, which minimizes the sum of edge distances [99]. In this way, MST extracts the most important relevant informations in financial time series [100] and numerous applications [101] (e.g., in seismic, meteorological, cardiological, and neurological time series).

The analysis of cluster hierarchy deserves special attention within MST. It well reproduces the sectorial nature of stock exchange. It must be said, however, that the MST is not robust in a sense that by removing one data one gets another (topologically non-equivalent) tree. Only the proper family of MST trees enables to give a sufficiently robust result [102, 103].

The MST based work [104] details numerical and empirical evidence for dynamical, structural and topological phase transitions on the Frankfurt Stock Exchange (FSE) in the temporal vicinity of the worldwide financial crash 2007/8. Indeed, using the MST technique, two typical transitions of the topology of a complex network representing the FSE were found. The first transition is from a hierarchical Abergel scale-free MST representing the stock market before the recent worldwide financial crash, to a superstar-like MST decorated by a scale-free hierarchy of trees. The latter one represents the market's state for the period containing the crash. Subsequently, a transition is observed from this transient, (meta)stable state of the crash to a hierarchical scale-free MST decorated by several star-like trees after the worldwide financial crash.

Another method, called Planar Maximally Filtered Graphs (PMFG), is a powerful tool to study complex datasets [105, 106, 107]. It has been shown that by making use of the 3-clique structure of the PMFG a clustering can be extracted allowing dimensionality reduction. This keeps both local information and global hierarchy in a deterministic manner without the use of any prior information [108]. Filtered graphs can also be used to diversify financial risk by building a well-diversified portfolio that effectively reduces investment risk. This is done by investing in stocks that occupy peripheral, poorly connected regions in the financial filtered networks [109, 110, 111].

However, the algorithm so far proposed to construct the PMFG is numerically costly with $O(N^3)$ computational complexity and cannot be applied to large-scale data. There is a challenge therefore to search for novel algorithms that can provide, in a numerically efficient way, such a reduction to planar filtered graphs.

A new algorithm, called the TMFG (Triangulated Maximally Filtered Graph), was introduced to efficiently extracts a planar subgraph, which optimizes an objective function. The method is scalable to very large datasets and it can take advantage of parallel and GPUs computing. The method is adaptable allowing online updating and learning with continuous insertion and deletion of new data as well changes in the strength of the similarity measure [112].

Network filtering procedures are also allowing to construct probabilistic sparse modeling for financial systems that can be used for forecasting, stress testing and risk allocation [113, 114, 115].

The problem of studying the economic growth patterns across countries is actually a subject of great attention to economists and econophysicists [116, 117]. Cluster analysis methods allow for a comparative study of countries through basic macroeconomic indicator fluctuations. Statistical (or correlation) distances between 15 EU countries are first calculated for various moving time windows. The decrease in time of the mean correlation distance is observed as an empirical evidence of globalization. Besides, the most strongly correlated countries can be partitioned into stable clusters. The Moving Average Minimal Length Path algorithm indicates the existence of cluster-like structures both in the hierarchical organization of countries and their relative movements inside the hierarchy.

All mentioned above methods enabled effective exploration of any complex networks, opening new, extremely interesting research fields and triggering a real flood of not only econophysical and sociophysical works but also far beyond these research areas (e.g., in biology, ecology, climatology, medicine, telecommunications).

7. Systemic risk and network dynamics.

This type of risk has spread widely culminating in the subprime crisis of 2007/08. The analysis and control of systemic risk has therefore become an extremely important social and economical challenge. This challenge was taken up by economics, finance, and also by econophysics. It was found that the role of the financial institutions' network was crucial in the dissemination of the financial crisis of 2007/08. The greater the degree of cross-linking, the greater the risk of system crash. This was thoroughly considered in review entitled: *Econophysics of Systemic Risk and Network Dynamics* edited in 2013 by the Abergel, Chakrabarti, Chakraborti, and Ghosh [118].

7.1. Financial market risk and the first-passage time problem.

The uncertainty and risk are inextricably linked to the activity of financial markets [119, 120]. One has approached the very promising issue of risk evaluation and control as a first-passage time (FPT) problem. The mean first-passage time (MFPT) was used as a basis for the assumption of stochastic volatility (expoited within the Heston model) [121]. One significant result is the evidence of extreme deviations – which implies a high risk of default – when the strength of the volatility fluctuations increases. This approach may provide an effective tool for risk control, which can be readily applicable to real financial markets both for portfolio management and trading strategies. Analysis of extreme times considered in [122] (also as a significant quantity of FPT) is closely related to at least two challenging problems which are of great practical interest: the American option pricing and the issue of default times and credit risk. Both problems require the knowledge of first-passage times to certain thresholds. It was found that the MFPT versus the threshold level can be represented as a power law. Thus the usefulness of FPT approach to financial times series analysis has been proven.

7.2. Agent-based modelling

Agent-based modelling (ABM) opens the possibility for describing the phenomena and processes occurring on financial markets (and not only) at ab initio level. In general, the market modelling is one of the challenges of modern econophysics [29, 123, 124, 125, 126, 127]. The main purpose of market modelling is to reveal the laws and underlying processes of market behavior supplying (as one of the results) some signatures or warnings of upcoming extreme events or crashes.

Agent-based models, also called computational economic models, are widely exploited, for instance, in economics (Ausloos et al., 2015 [128]; Farmer and Foley, 2009 [129]), sociology (Macy and Willer, 2002 [130]) and in the environmental sciences (Billari et al., 2006 [131]). A thorough review was made from the econophysics point of view in 2014 year in the collective review publication entitled: *Econophysics of Agent-Based Models* edited by Abergel, Aoyama, Chakrabarti, Chakraborti, and Ghosh [132].

The hallmark of ABMs is the coupling of individual and collective degrees of freedom of the analyzed system that is, its micro- and macroscales. The former is represented by individual agents, while the latter one by the system as a whole (or its macroparts). Frequently, agents are divided into two completely different groups: stabilizing (e.g., fundamentalists or rebalancers) and destabilizing market activity (e.g., chartists, noise traders or portfolio insurers). The competition between them can be a source of long-range and long-term nonlinear correlations, critical phenomena and fat-tailed distributions.

Firstly, a few inspiring canonical models belonging to the field of portfolio analysis are presented. The pioneering Kim-Markowitz (KM) agent-based model [133, 134] was inspired by the stock market crash of 19th October 1987, when DJIA decreased by more than 20% per day. This model confirmed by numerical simulation a common observation that strategies of portfolio insurers (and not that of rebalancers) destabilize financial markets. This model has raised hopes for the promising agent-based modelling capabilities.

Besides, the Levy-Levy-Solomon (LLS) model [135] was developed to consider the risk-averse investors having arbitrary long memory. The LLS model describes the spontaneous periodicity of the market, its booms and crashes. Although the results obtained depend significantly on the initial conditions assumed, the model has demonstrated (by numerical simulation) that the wealth available on the market (in the form of shares and bonds) will, after sufficiently long time, be taken over by a group of investors equipped with a long memory (one hundred steps back in simulation). This outcome is in line with expectations.

An extremely popular model describing the evolution of the market, going beyond the aforementioned portfolio analysis category is the Lux-Marchesi (LM) model [68]. It is able to correctly describe many stylized facts, for example: volatility clustering, power-law distribution of returns, and long-term autocorrelation of absolute returns. This model is based on the concept of mutual exchange and interaction between different groups of investors (i.e. chartists and fundamentalists) and on the process of price adjustments with a demand-supply imbalance. Additionally, chartists are divided into optimists and pessimists - the competition between them as well as with fundamentalists create an effective opinion of agents leading to strong interconnection of chartists amount with the price amplitude. This interconnection is responsible for the observed large market fluctuations. A similar influence of portfolio insurers is observed within the Kim-Markowitz model. The technical disadvantage of the LM model is the large number of free parameters in the model involved.

A very important category of models describing the behavior of financial markets, and inspired by models drawn from physics, are primarily Ising-like on complex networks, whose prominent example is the Iori numeric model [136]. The agent is represented here by three-state spin vector, where state +1 means buying a stock, -1 selling, while 0 means inactive state. Obviously, the agent activity is limited by amount of his capital however, his activity has still a probabilistic character with threshold. Besides, the market maker is present guarding the liquidity of the market. The price in this model depends not only on the ratio of the supply of securities to their demand but also on the available securities volume. This multiparameter model managed to describe all the stylized facts (i.e. volatility clustering of returns, the positive correlation between volatility and trading volume, the power-law decay of autocorrelation).

The above models inspired the econophysicists in a significant way. The first model that grew out of this society and was characterized by a small number of parameters was the Cont-Bouchaud (CB) model [137] based on a discrete percolation phenomenon – a phenomenon previously analyzed in the field of chemistry and statistical physics, condensed matter physics and mathematics. A year later, Dietrich Stauffer also used percolations to model the behavior of financial markets [138].

As a part of the CB model, neighboring network nodes form a cluster making collectively investment decisions in a probabilistic manner. Therefore, it can be said that this model is based on the so-called lattice-gas model isomorphic with canonic Ising model. The market price is (as usual) a function (here exponential) of the difference between demand and supply. This type of approach is very flexible, generating (depending on the input probability) either Gaussian distributions or various types of power-laws distributions – both observed on financial markets.

The next interesting ABM is the Bornholdt spin model [139, 140] primarily designed to recreate the price dynamics in short time horizons. Similarly to the KM and LM models, it assumes that there are two types of investors on the market: fundamentalists and noisy traders. The fundamentalists only respond to price changes, making the market price as close as possible to the fundamental value of stock. The mutually interacting noisy traders take the probabilistic decisions to buy or sell the stocks depending on the market situation. This situation is described by the local, time-dependent threshold function of influence having a threshold character. The size of this threshold is connected linearly with the volume. In this model, the interacting traders are responsible for non-Gaussian behavior of the market. The Bornholdt model describes a lot of stylized facts: power-law return distributions, volatility clustering, positive correlation between volatility and volume, and self-similarity between volatilities on various time scales. Unfortunately, the shape of the absolute-returns autocorrelation function is not a power law herein.

Although the ABMs circumscribed above are valuable and useful, none of them were used to model the interevent-time statistics so much significant in a study of correlations on financial markets. In 2014 the model of so-called cunning agents was developed [141], which reproduces not only stylized facts but also empirical statistics of interevent times.

One can say that we are dealing with a cunning agent if he accepts a position, for example, a long one indicating the willingness to buy additional items and informs his neighbors about it, but in fact, simultaneously sells the possessed assets. The situation is similar in the short and neutral position. Recently, a model appeared [142], which starting from the level of stochastic dynamic equations, was able to reproduce mentioned above the empirical statistics of interevent times.

The interesting extension of the Geometrical Brownian Motion was made by Dhesi and Ausloos [143] who introduced so-called the Irrational Fractional Brownian Motion model. They re-examined agent behaviour reacting to time dependent news on the log-returns thereby modifying a financial market evolution. Authors specifically discuss the role of financial news or economic information as a positive or negative feedback of such irrational (or contrarian) agents upon the price evolution. A kink-like effect reminiscent of soliton behaviour was observed, suggesting how forecasts uncertainty induces stock prices. This way they proposed a measure of irrational force in a market, which seems to be a very significant for understanding the dynamics of stock market.

It should be emphasized that agent-based models, along with network models, have gained immense popularity not only in the society of econophysicists but also sociophysicists.

8. Phase transitions, catastrophic and critical phenomena

Phase transitions, catastrophic and critical phenomena have long been studied both in the framework of econo- and sociophysics (see, for instance, [20, 144]). However, phase transition of the global financial system observed at the end of 2008 deserves the special attention. This is because it was just after the bankruptcy of Lehman Brother [145]. The signature of this transition is a sharp increase in the susceptibility/sensitivity of the system to the negative global shock with an initially well-defined epicenter focused on mortgage backed securities. This shock was the source of the observed cascade of defaults or a succession of problems associated with the most prominent global institutions (belonging to the banking, insurance and mortgage sectors). This cascade caused crash on the stock market and the subsequent panic among economical institutions from the global ('too-big-too-fall') to the local ones – leading many of the latter to bankruptcy.

The model developed in paper [145] is, in essence, a simplified discrete correlated random walk of walkers (or firms) on the ladder consisting of the effective credit rating grades (ECRGs), where the firm either remains at a given ECRG or change its value by one (with blocking boundary condition at top and the bottom of the ladder). By using the statistical-mechanic partition function based on the Ising-like sociological influence function, the conditional single step probability for each firm is constructing in the exponential form. This partition function contains the field of panic taking into account the firm's bankruptcy. For simplicity, the direct coupling between firms is a random variable drawn from the Gaussian distribution. This model exhibits a critical behaviour that is, the second-order phase transition at well-defined critical point. Besides, the phenomenon of spontaneous symmetry breaking is observed (by the increasing the number of bankruptcies) due to the nonvanishing of the panic field. The model offers the phase diagrams and enables the system time evolution. This is the first so complete model in the field although earlier more sociophysical oriented models by Schweitzer et al. were published [146].

One should also mention works that still raise controversy regarding the presence of bifurcation on the stock exchange or, more generally, phase transformations of the first order. The related issue of the critical and catastrophic slowing down phenomenon are the most refined indicators of whether a system is approaching a critical point or a tipping point – the latter being a synonym for the catastrophic threshold located at a catastrophic bifurcation transition. The still open problem raised by Scheffer et al. [147] is whether early-warning signals in the form of a critical or catastrophic slowing down phenomena (such as those observed in multiple physical systems) are present on financial market. The possibility of existence of the above-mentioned early-warning signals was highlighted in publication of Kozłowska et al. [148] and refs. therein. A specially created page that accompanies this work (posted at address cited in [149]) allows the reader to look for bifurcation on various stock markets by using himself the indicators presented in the publication [148].

A microscopic approach to macroeconomic features has always been a challenge [150] and refs therein. A birth-death lattice gas model for macroeconomic behavior under heterogeneous spatial economic conditions takes into account the influence of an economic environment on the fitness and concentration evolution of the economic entities. The reaction-diffusion model can be also mapped onto a high order logistic map. The role of the selection pressure along various dynamics (with entity diffusion on a square symmetry lattice) has been studied by Monte-Carlo simulation. The model leads to a sort of phase transition for the fitness gap as a function of the selection pressure and to cycles. The scalar control parameter is a sort of a "business plan". The business plan(s) allows for spin-offs or merging and enterprise survival evolution law(s), once bifurcations, cycles and chaotic behavior are taken into account.

The problem whether a power law or an exponential law describes better the distribution of occurrences of economic recession periods is significant not only for econo- and sociophysics but primarily for socio-economical science and life. In order to clarify the controversy a different set of GDP data were examined in [151] for example. The conclusion about a power law distribution of recession periods seems to be more reliable though the matter is not entirely settled. The case of prosperity duration is also studied and it is found to follow also a power law. Considering that the economy is basically a bistable system (recession/prosperity) a characteristic (de)stabilization time is posssible to quantitatively derive.

9. Significant elements of global economy

The global economy has its source in important connections (dependences, interactions, influences, etc) between countries and regions [152]. An international trade is a glaring example of this. Obviously, the globalization is one of the central processes of our age. The common perception of such process is that, due to declining communication and transport costs, distance becomes less and less important. However, the distance coefficient in the economical gravity model of trade [153] (which grows in time) indicates paradoxically that the role of distance becomes a more important. In the paper [152] it was shown that the fractality of the international trade system (ITS) provides a simple solution for this globalization puzzle. It was argued that the distance coefficient corresponds to the fractal dimension of ITS and not to the Cartesian distance.

The world economic conditions evolve and are quite varied on different time and space scales. This evolution forces developing of macroeconomic entities within a geographical type of framework [154, 155]. For the firm fitness evolution a constraint is taken into account such that the disappearance of a firm modifies the fitness of nearest neighboring ones (as in Bak-Sneppen population fitness evolution model [156]). The concentration of firms, the averaged fitness, the regional distribution of firms, and fitness for different time moments, the number of collapsed, merged and new firms as a function of time have been recorded and are discussed. A power law dependence, signature of self-critical organization, is seen in the firms' birth and collapse asymptotic values for a high selection pressure (control parameter) only. A lack of self-organization is also seen at region borders. The research and market modeling of companies is still one of the main goals of econophysics.

10. Contemporary sociophysics

The systematic research on society that gives rise to the modern sociology is mainly due to the work of Quetelet [157] (see also [3]). Today it is clear that only a comprehensive approach to economic phenomena and processes, including both psychology, social psychology and sociology, enables the description and understanding of the mechanisms governing socio-economic life (including also financial markets). This was shown convincingly in 2006 in the collective work [158]. We are increasingly attempting to understand the emotional nature of human activity and activity of human communities. This emotional component can be seen particularly clearly in cyberspace – this has been well presented in the collective work entitled: *Cyberemotions. Collective Emotions in Cyberspace*, edited by Janusz A. Hołyst [159]. This type of interdisciplinary approach to the complex socio-economic reality is extremely inspiring, stimulating and promising. In this context, we should say about the role of the Sznajd model ('united we stand, divided we fall' – USDF model) [160, 161]. It has become credible thanks to its success in predicting the result of elections in Brazil, opening the way for contemporary sociophysics. The Sznajd model easily introduces the possibility of obtaining a consensus by exchanging opinions between members of a given community. It is based on the Ising model with characteristic social interaction – it is by far the most exploited by sociophysicists toy model with the cluster-like ever-growing number of different variants. A complementary, important model that should also be mentioned here is the Bonabeau model [18] showing how hierarchies are created in a given community. Let us add that currently the study of various hierarchical structures, cascades, and networks is fashionable and very advanced [162, 163].

The social impact is one of the most important and the most common social phenomena. The dynamical theory of this impact proposed in 1990 [164] gave rise to a huge stream of works. The sociophysicists have made a significant contribution to the development of this trend. Today, this type of modeling is a canonical component of the sociophysics without which one cannot imagine an advanced analysis of the societies' behavior.

The attempts made by physicists to understand so-called social "forces" have lasted at least since the mid-1970s [165]. Quite interestingly, the source of social force is attributed to technological innovation made by competing goods and new population. Another view about quantifying social forces (found in [166]) pretends that they result as coupling to some external fields.

The role of emotions in opinion dynamics mentioned above was used in a variant of the ABM complementary to the Sznajd model. The combination of information and emotions interplay was used successfully to predict the results of Polish election in 2015 [167, 168]. This is the prominent evidence of the practical use of sociophysical modeling.

Let us add that the collective work entitled: *Why Society is a Complex Matter* edited by Philip Ball in 2012 [169] also played a prominent role in the development of contemporary sociophysics. This collective work pointed to sociophysics as a new kind of science. There the Helbing's work [170] (see also [171]) has shown a crucial role of information and communication technology for society.

It should be noted that in the last decade issues related to the evolution of cultures (including linguistics) have been continuing to represent an attractive, intriguing course of research [172, 173, 174, 175, 176]. A key tool for modeling this evolution is the Axelrod model and its various variants [172].

The Axelrod model [177] is defined by stochastic process which, similarly to the voter model, contains a social interaction between nodes of a network, but unlike the voter model also accounts for homophily. The aim of the model is to describe and explain macroscopic observations in real-world social networks, based on simple microscopic rules. These microscopic rules are also inspired by empirical observations or concluded from sociology or psychology. Every node of the network is described, in the frame of the model, by a vector of traits representing internal degrees of freedom. The idea behind the model was simple – to explain cultural diversity observed in societies, despite the fact that people become more alike within a face to face interaction. Therefore, Axelrod asked why eventually all differences do not disappear? In his model the vector of traits describes culture of an individual (regional society or nation) in a sense of habits, beliefs, religion, language, hobbies, views, etc. During the evolution two individuals become more similar to each other, unless they stay different. This is a crucial observation leading to an interesting result, because only that one can obtain frozen (or equilibrium) states. Depending on the initial conditions, simulations can end in one of the states: in a homogeneous state with a monoculture or heterogeneous with many small subcultures, called 'domains'. The coexistence of these many different subcultures is a main result, confirming the possibility of existence of heterogeneous societies, despite people become more and more similar.

The model gained interest among physicists a few years later [178] along with the discovery of the phase transitions between homogeneous and heterogeneous states (continuous or discontinuous types). To make the model more realistic, it was extended to complex networks with very different topologies [179] as well as to dynamic complex networks. Moreover, this latter issue was addressed in [180], where different rewiring mechanisms were analyzed. It was then possible to obtain real-world features, like power-law degree distribution or high values of clustering coefficient. Besides, it was shown that a key to the proper scaling of the number of languages is triadic closure – type of rewiring proved to be very important in social networks [181].

A "degree of freedom" in a population is also the religion adhesion. The pioneering work on such adhesion aspect, in fact similar to market/company growth and market share influence, was published almost a decade ago [182]. The observed features and some intuitive interpretations point to opinion based models with vector like agent rather than scalar ones (many degrees of freedom instead of one). This supports the assumption of the Axelrod approach.

It is worth to mention also the works from the borderline of econo- and sociophysics regarding household incomes (especially in the European Union and the United States). The approach based on the stationary solution of the reinterpreted Fokker-Planck equation turned out to be particularly useful [183, 184]. This approach allowed to describe the distribution of income of all three social classes: low income, medium and high income well reproducing the Pareto laws (with different Pareto exponents) for the last two classes.

Concerning the wealth distribution, one of the most interesting outputs is the generic existence of a phase transition, separating a phase where the total wealth of a very large population is concentrated in the hands of a finite number of individuals (condensation phenomenon) from a phase where it is shared by a finite fraction of the population [185]. The rich phase diagram was examined in [186], in which both open and closed Pareto macroeconomics were studied. The wealth condensation takes place in the social phases both for closed (with the fixed total wealth) and open (with the fixed mean wealth) macroeconomy. The wealth condensation takes place also in the liberal phase for super-open macroeconomy (it was proved, indeed, in [185]). It was found that in the first two cases of macroeconomy, the condensation is related to the mechanism known from the balls-in-boxes model, while in the last case, to the fat tails of the Pareto distribution. Besides, for a closed macroeconomy in the social phase, the emergence of a "corruption" phenomenon was pointed out. A sizeable fraction of the total wealth is always amassed by a single individual. In publications cited above the dependence of Pareto exponents on microscopic parameters of the model was found. This is an achievement useful both for theoreticians and practitioners in social sciences.

Recently, several studies were published [187] (and refs. therein) which have given better insight into how birth is affected by exogenous factors. Especially, the adverse conditions (e.g. famines, epidemics, earthquakes, droughts, floods, etc.) temporarily affect the conception capacity of populations, thus producing birth rate troughs nine months after mortality waves. The challenge here is the discovery of the birth rate patterns and their interpretation. A promising step in this direction was made in paper [187], where several important patterns were found and discussed.

11. Challenges and warnings

It is already known that the analysis should take into account the feedback between econonophysics and sociophysics (including socio-psychology and even psychology of leaders and the policy of the state). Even roughly approximated modelling of reality should take into account the rivalry of the rational multicomponent with irrational one. The interdependence and networking of elements of socio-economical complex systems constitute (within econo- and sociophysics) the basis for the research even if the available empirical data is dirty and uncertain. The researchers realize that they are affecting the problems generated by complex systems. This complexity is the source of emergent phenomena and processes, including catastrophic and critical ones (on a macroscale). This may result in a dichotomy of descriptions within the micro- and macroscales. It is understand that, for example, breaking the principle of ergodicity may lead to the impassable barrier creating a dichotomy in the statistical description of socio-economical reality. That is, phenomena and processes in the macro scale mainly result from the properties of the system as a whole (especially when the system stays in a critical state) and not only from the behavior and properties of individual objects forming the system in the microscale. The understanding the role of dependency or correlation, causality, and coevolution or adaptation in markets on the complexity of markets and emerging phenomena and processes, become one of the greatest challenges for modern research of a socio-economical reality [188, 189, 190]. However, the econophysicists discoveries has miserable impact on the main stream works of financial economy (see Jovanovic and Schinckus [191]).

Finally, we must say about an event that put a shadow on mathematics and financial physics as a great warning and a lesson for all of us. The portfolio analysis in the nineties of the previous century was based, in fact, on the canonical option pricing formula of Black-Scholes-Merton (BSM) derived in the canonical paper [192]. The BSM formula was derived mainly assuming that the prices of basic financial instruments, on which options were issued, are subject to the geometrical Brownian motion while considered options are risk-neutral. As for the trend, its constant growth would be driven by investors constantly seeking arbitrage opportunities. Based on this theoretical approach, the hedge fund Long-Term Capital Management (LTCM) was created in year 1994; the key people behind LTCM were Myron S. Scholes and Robert C. Merton – the Nobel Prize winners.

Although initially successful (for three consecutive years) with annualized return of over 20% netto, from August to September 1998 (short after the Asian financial crisis in 1997 and 1998 Russian financial crisis) LTCM lost, however, about 4.5 miliard (US billion) dollars severely disrupting global markets for several months. This was the consequence of violating the key assumptions of the theory in new market circumstances and neglecting the constant verification of these assumptions. Besides, used by LTCM leverage of portfolio composition has reached an unbearable ratio of debt-to-equity as 25:1. An in-depth systematic econophysical analysis of this subject, and especially issues related to market risks, was provided in year 2001 by Jean-Philippe Bouchaud and Marc Potters in the book *Theory of Financial Risks. From Statistical Physics to Risk Management* [193].

It must be clearly stated that we live in an increasingly risky society which is particularly vulnerable to extreme types of risk – both market and systemic [194]. Concerning the financial sector, among all possible extreme phenomena, indeed crashes are presumably the most striking events with an impact and frequency that has been increasing in the last two decades increasing the risk of market activity extremely. Understanding what is happening as well as risk control and management is an urgent challenge for investors and researchers alike.

The collective effort of many communities is likely to be more effective thanks to the Econophysics Network [195] (founded in Leicester by Schinckus, Jovanovic, Haven, Sozzo, Di Matteo, and Ausloos).

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