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When Extremes Meet: Redistribution in a Multiparty Model with Differentiated Parties[*]

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Abstract

In this paper we consider a multi-party electoral competition model in which parties –which care both about implemented policy and their electoral performance– strategically promise a redistribution scheme while their social ideologies are considered to be known and fixed (differentiated parties). Voters, who differ both in income and in social ideologies, vote sincerely for the party that they cumulatively like most (that is, taking into account both the redistribution scheme proposals and parties' social ideologies). Formal analysis of this game uncovers a *moderates-vs-extremists* equilibrium: parties with moderate social ideologies tend to favor generous redistribution in order to capture the votes of the poor majority, while parties with extremist social ideologies are more likely to be non-competitive in the economic dimension by proposing policies that do not reflect the interests of the poor. An implication of this result is that, ceteris paribus, an increase in income inequality should lead to an increase in the cumulative vote share of moderate parties and, hence, in a decrease in party-system fragmentation.

Keywords: redistributive politics, taxation, differentiated candidates, policy motives, social polarization, multi-party elections.

JEL classification: D72, H20

1 Introduction

The relationship between social ideologies and redistributive outcomes has recently received renewed attention mostly in the context of two-party elections (e.g., Krasa and Polborn 2012, 2014). The main idea in this literature is that if parties are less flexible in determining their platforms as far as social issues are concerned compared to their redistribution promises, then voters' preferences on social issues should be relevant in determining which redistribution schemes parties promise.

Lindbeck and Weilbull (1987), Dziubinski and Roy (2011), Krasa and Polborn (2012, 2014) and Matakos and Xefteris (2016) consider two-party models –such that parties have fixed positions on the *social ideology* dimension and promise redistributive schemes (or public goods) in order to improve their prospects of success in forthcoming elections– and show that indeed equilibrium redistributive schemes are sensitive to changes in the distribution of voters' social ideologies. In these models parties generally end up promising identical redistribution schemes[1] and these results are perfectly robust to considering that parties may be partially policy motivated. This is so because since Calvert (1985) we know that when only two parties compete, a party may affect the implemented policy to its satisfaction only by increasing her vote share: office and policy motives are aligned to a great extent.

On the contrary very little is known regarding how redistribution promises should look like when many differentiated parties take part in electoral competition. Does it still hold that all parties will want to offer the same redistribution scheme as in two-party systems? Do equilibrium redistribution promises depend on whether parties have policy preferences or not? In case parties offer in equilibrium distinct redistribution schemes, which parties should be anticipated to promise more generous redistribution? Which parties should be expected to favor less generous redistribution schemes? Moreover, since equilibrium existence and uniqueness arguments are not always straightforward in such *differentiated candidates* games –even in the two party case (Matakos and Xefteris 2016; Xefteris 2015)– what happens when more than two parties (or candidates) compete? For instance, is a pure strategy Nash equilibrium likely to exist?

In this paper we try to answer these questions by studying a model of electoral competition under the simplest possible electoral rule (plurality) among four parties which have fixed and known social ideologies and which strategically promise a redistribution scheme in order to affect both policy outcomes and their vote share. Each voter has well-defined preferences both regarding parties' social ideologies and about parties' redistributive schemes and votes for the party which offers the bundle that the voter likes best, exactly like in the papers that we already discussed.

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[1] Krasa and Polborn (2014) show that if each party is characterized by a distinct public good generation technology, then parties' promises need not converge. In the case of identical technologies though their model predicts convergence to the same platform as the rest of the literature.

That is, the main difference of our approach compared to the previous ones is that we consider a larger number of parties and more general objective functions.

Considering that parties' social ideologies are symmetrically distributed about the center of the policy space we uncover a strong *moderates-vs-extremists* result: the two socially extremist parties meet in promising no redistribution at all while the two moderate parties promise generous redistribution. That is, in multiparty systems, *socially moderate parties tend to offer economic policies that benefit the poor and socially extremist parties tend to promise economic policies that do not reflect the interests of the poor.* The intuition behind our main result is as follows. Since Dixit and Londregan (1995, 1996) we know that, in the context of two-party electoral competition, parties have an incentive to woo the poor voters by promising generous redistribution because their votes are relatively cheaper to "buy." But in the context of bidimensional multiparty electoral competition, where parties also care about the implemented social ideology, a new dynamic arises: socially extremist parties have strong incentives to behave strategically in order to bring the implemented social ideology closer to their own. As a result, an extremist party promises –in equilibrium– a less generous redistribution scheme –compared to the socially moderate ones– in order to avoid cannibalizing the vote share of the moderate party with which it has *closer ideological affinity on the non-economic dimension.*

The prediction that socially extremist parties propose economic policies that do not reflect the interests of the poor majority is arguably intriguing and should be treated with caution in order to avoid misinterpretations. When the majority of voters is relatively socially moderate and socially moderate parties propose popular and similar economic platforms, socially extremist parties know that: a) they cannot win, independently of the economic platform that they propose, and b) their strategy can influence only the implemented social policy by affecting which of the two moderate parties ranks first in the election. Hence, they choose unpopular economic platforms in order to bring the implemented social policy as close as possible to their ideal one. In a sense, socially extremist parties have incentives to be non-competitive in the economic issue and, hence,

they endogenously become niche parties.[2] Indeed, one could have a more elaborate model in which parties choose both economic policy and assign different weights to different issues, but the addition of an extra strategic dimension would only complicate the analysis without interfering with the existing dynamics: extremist parties would still wish to be non-competitive in the economic dimension.[3]

This result has several implications at least as far as European parliamentary politics are concerned. Most European democracies have multiparty systems with two large moderate parties and a number of smaller –and usually niche– parties in the periphery of the political spectrum that focus on specific policies such as the environment, or the relationship with the EU. Thus our model helps explain some features of multi-party European politics. For instance, in the 2015 parliamentary election in the UK, in a typical example of a niche party behavior, the strongly euroskeptic UKIP –if one treats parties' position towards the EU as the second dimension–[4] run on a platform of very low taxes and redistribution, while the mildly euroskeptic Conservatives, under the leadership of David Cameron, run under the platform of one-nation conservatism which involves considerable redistribution towards the poorer segments of society. Another example of this type of non-monotonicity in proposed tax policies, predicted by our model, comes from the Greek 2015 parliamentary election where the two extreme parties (on the pro- and anti-EU dimension), the populist right-wing *Independent Greeks* party and the liberal *To Potami* party, run on a low taxes platform –focusing mainly on issues of European identity– while the more moderate ones (on the EU dimension), such as *New Democracy* and *Syriza*, proposed relatively more redistribution (and taxation) and focused primarily on the economy.

Moreover, the qualitative part of this finding –socially moderate parties tend to offer more generous redistribution than socially extremist ones– is robust to voters' and parties' social ideologies

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[2]Following Meguid's (2005) seminal analysis of niche parties' differences to mainstream ones, Wagner (2012) described niche parties "as parties that compete primarily on a small number of non-economic issues."

[3]Notice that our results align with the empirical regularity of niche party existence (that is, of parties that do not compete in the economic dimension) without assuming that these parties actually care about economic policy less than mainstream parties do. This observation is of independent interest since it establishes a correlation between a party's extremity in non-economic issues and whether it is a niche party or not.

[4]It is worth noting that UKIP's discourse about the EU was centered around the concepts of identity and sovereignty versus the "authoritarian" and "non-elected" Brussels bureaucracy.

not being absolutely symmetrically distributed about the center of the policy space, to the two dimensions being correlated,[5] to parties caring to any arbitrarily small degree about the implemented social policy,[6] and to alternative electoral systems (proportional representation). What changes, of course, is that in equilibrium the two moderate parties win elections with different probabilities: if, for example, the socially liberal extremist offers more redistribution –but still strictly less than both moderate parties– then this will give an electoral advantage to the moderate socially conservative party. As a result, extending the results to cases with such asymmetries allows our model to provide some insight into electoral patterns observed in many European party-systems. For example, if extreme socially liberal parties have a floor on their redistribution proposals –perhaps because the two dimensions are correlated– then our model can explain the right-wing electoral advantage in Germany (few left-wing chancellors in recent years), France (only two left-wing presidents during the Fifth Republic) and Italy (dominance of the right until the recent crisis). As a result, our theoretical framework can be useful both in helping us understand the mechanisms behind the emergence (and electoral success) of niche parties –a feature that is quite common in many European parliamentary democracies– and also in shedding some light into some particular features of European politics (e.g., the recent electoral advantage of the right).

Since, in equilibrium, both in the symmetric and the asymmetric case, different parties offer different redistribution schemes, and, since different voters have different preferences regarding redistribution (which originate form the fact that they might have different incomes), it is straightforward that two voters with the same social ideology need not be voting for the same party: the share of poor voters that vote for the socially moderate parties is larger than the share of rich voters who vote for these parties. This observation dictates that the cumulative vote share of socially moderate parties –which, in most cases, negatively relates to the fragmentation of the party system– is correlated with the exact degree of income inequality of a given society. Following the argument laid by Piketty (2014), take the case of a society where inequality is rising as the income

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[5]In terms of modelling, as we show in the appendix, assuming that the two dimensions are correlated is similar to parties positioning asymmetrically in the social ideology dimension.

[6]In the extreme case in which parties do not care at all about the implemented policy then, in equilibrium, all parties offer the same platform. Since this convergent equilibrium collapses once one introduces policy motives of any arbitrarily small degree, we do not find it necessary to formally investigate this case.

of the very rich is rapidly increasing relative to that of the rest –for simplicity call them the *poor*– and consider our equilibrium, where moderate parties offer more generous redistribution schemes than the extremist ones. Then our model makes a direct prediction on the exact relationship between income inequality and the fragmentation of the party-system: party-system fragmentation decreases with inequality. In the last part of our paper we present some rough empirical evidence which back up the identified relationship.

The remaining of the paper is organized in the following way: In section 2 we introduce our theoretical model, in section 3 we present the results of the formal analysis, in section 4 we discuss empirical implications of our findings and, finally, in section 5 we conclude.

2 Theoretical model

We consider a model of electoral competition among four parties, taking place in a two-dimensional policy space. We name those dimensions as *social ideology* and *economic policy*, respectively (see also Stokes 1992; Groseclose 2007; Krasa and Polborn 2012).[7] Following the literature on differentiated candidates (Krasa and Polborn 2010, 2012, 2014; Dziubinski and Roy 2011, Matakos and Xefteris 2016, Xefteris 2015) we assume a framework, where the four parties differ in their *fixed* social ideology position, while in the second dimension (economic policy) they *strategically* choose the tax rate –and the implied level of redistributive spending– in order to maximize their utility. Moreover, in our set-up, parties have mixed –office and policy– motives while the preferences of the voters in both dimensions are *heterogeneous*. Finally, both dimensions are continuous.

2.1 Political parties

We formally define the parties' *social ideology* positions in the $[0, 1]$ space as follows:

$$\mathcal{P} = \{l, L, A, a\} \subseteq [0, 1] \text{ such that } l < L < A < a$$

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[7]A typical dichotomy in the dimension of social ideology can be, for instance, libertarian vs. authoritarian or socially liberal vs. socially conservative (e.g., Groseclose 2007). Examples of the first dimension (social ideology) may include issues such as: abortion, gun legislation, or same-sex marriage. The second dimension may include policies such as: redistribution, taxation and government spending.

where l is the position of the extreme libertarian party, L is the position of the moderate socially liberal party, A is the position of the moderate socially conservative party and a is the position of the extreme authoritarian party. These positions are fixed and we will henceforth refer to each party by its *social ideology* position $p \in \mathcal{P}$. In order to give more structure to our model we consider the symmetric case.

Condition 1 *(Symmetry)* *Parties l and a are positioned in the extremes of the social ideology space, that is at $l = 0$ and $a = 1$. Parties L and A are symmetrically positioned at distance ϵ around the median. That is, at $L = 1/2 - \epsilon$ and $A = 1/2 + \epsilon$.*

Each party's social ideology position is assumed to be public knowledge. Furthermore, each party will propose a tax rate which uniquely identifies a specific redistribution scheme –the budget must be balanced and, hence, the total amount of redistributive transfers should always equal the total revenues raised through taxation. Formally, each party p proposes a tax rate t_p such that $t_p \in [0, \tau]$ and $\tau \in (0, 1]$; τ is fixed and depends on the degree of institutional constraints (e.g., fiscal and monetary policy rules, or central bank autonomy in the case of inflation taxes).

We assume that parties care both about their vote share (office motives)[8] and also about the social ideology of the winner (policy motives).[9] Formally, their utility function takes the following form:

$$V_p(\omega, v_p) = -|\omega - p| + v_p, \ p \in \mathcal{P}$$

where v_p is the vote share of party $p \in \mathcal{P}$, and ω is the social ideology related policy that gets implemented by the winner (or winners), once the electoral result has been realized. To conclude the discussion, we note that each party's *social ideology* position p, together with its *tax rate* proposal t_p constitute party p's *political platform*, upon which citizens vote.

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[8] One possible justification why parties, especially extremist ones, care about their vote shares is that parties might receive state subsidies or free broadcasting time in public media both of which are tied to their electoral performance. An alternative justification, which becomes very relevant in the extension of our model in section 3.5, is that a party's vote share determines its bargaining power in the allocation of cabinet portfolios in the event of having to form a coalition government.

[9] We prefer not to consider that the parties have preferences regarding the implemented economic policy at this point in order to make analytical arguments as easy to follow as possible. After the presentation of our formal results, though, we argue that our equilibrium is robust to considering that parties care about the implemented economic policy as well.

2.2 The voters

Each voter is characterized by her social ideology, $x \in [0,1]$ (she has symmetric and single-peaked preferences on social issues), and her income $y \in \{m, M\}$, with $m \in [0, M)$ and $M < \left(\frac{1}{2} - \epsilon\right)^2$.[10] We consider: a) a continuum of voters; b) a mass of $q \in (0,1)$ voters each having income m and a mass of $(1-q)$ voters each having income M; and c) within each income group the distribution of voters' social ideologies is uniform on $[0,1]$. We name the voters with income m *poor* and those with income M *rich*.

We employ a standard balanced-budget redistribution scheme (Meltzer and Richard 1981). That is, under a flat tax rate $t_p \in [0, \tau]$, the average revenue raised is:

$$T(t_p) = t_p[qm + (1-q)M], \, t_p \in [0, \tau] \text{ such that } \tau \in (0,1]$$

This is identical to the individual redistributive transfer. Therefore, the utility of an (x, y) voter that votes for party p is given by:

$$U_{x,y}(p, t_p) = -|p - x| + \sqrt{y(1-t_p) + T(t_p)},$$

where $T(t_p)$ is the transfer proposed by party p. The first component of this expression, $-|p-x|$, is the disutility that voter x experiences when party p does not share the same social ideology with her. The second component, $\sqrt{y(1-t_p) + T(t_p)}$, is the utility that voter (x, y) receives from her disposable income –after all taxes and transfers– given the proposed tax rate t_p.[11]

Then, the resulting utility of income for a poor voter, given transfer $T(t_p)$, is given by:

$$\sqrt{m(1-t_p) + qmt_p + (1-q)Mt_p} = \sqrt{m + (1-q)(M-m)t_p} > \sqrt{m}, \text{ for } t_p > 0$$

whereas, for a rich one it is:

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[10] This assumption only guarantees that the social ideology dimension plays a sufficiently significant role in voters' utilities and, thus no voter votes on the basis of economic policy alone. Since we have assumed a bounded social ideology space $[0,1]$ if we place no upper bounds on M, then for very large values of M the significance of social ideology relative to economic policy decreases and, hence, the *moderates-vs-extremists* result might fade. But this assumption comes at no additional cost as in our modelling decision we could have assumed instead an unbounded social ideology space $(-\infty, +\infty)$ or introduce a parameter λ in the utility function that measures the relative importance of social ideology issues vis-a-vis economic ones.

[11] We can show that our results can be generalized if one replaces the square root with any twice continuously differentiable function $f(\cdot)$ such that $f'(\cdot) > 0$, $f''(\cdot) < 0$, and $f(0) = 0$. Those additional results are available by the authors upon request.

$$\sqrt{M(1-t_p) + qmt_p + (1-q)Mt_p} = \sqrt{M - q(M-m)t_p} < \sqrt{M}, \text{ for } t_p > 0$$

Since the LHS of both inequalities is the utility of income after redistribution, while the RHS is the utility of income when redistribution is zero, we can deduce that *all* poor voters prefer the highest possible tax rate τ since redistribution takes place in their favour. On the contrary, rich voters have no preference for redistribution and strictly prefer zero taxes.[12]

2.3 The voting game

We consider a voting game with three stages. All information is publicly available and known *ex ante* to all agents. The solution concept we employ is Nash equilibrium. The three stages of the game are as follows:

Stage 1: Parties announce simultaneously their complete *political platforms* $\{p, t_p\}$. Since t_p is the only strategic choice made by parties, we can rewrite their maximization problem as follows:[13]

$$\max_{t_p} V_p(t_p, t_{-p}) = -|\omega(\boldsymbol{v}(t_p, t_{-p})) - p| + v_p(t_p, t_{-p})$$

The winning party is denoted by ω. Clearly, the winner depends on the allocation of vote shares among parties, which in turn depends on their tax rate proposals. Hence, we can express the winner of the electoral game –and the implemented social ideology– as $\omega(\boldsymbol{v}(\boldsymbol{t}))$. Formally, we have $\omega(\boldsymbol{v}(\boldsymbol{t})) \in [0,1]$, where $\boldsymbol{v}$ is the vector $\{v_p\}_{p \in \mathcal{P}}$ and $\boldsymbol{t}$ is the vector $\{t_p\}_{p \in \mathcal{P}}$.

Stage 2: Voters vote *sincerely* for their most preferred platform, given parties' tax rate announcements.[14] Formally, sincere voting in this setup means that each voter $\{x, y\}$ solves the following maximization problem:

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[12] This formulation of preferences, within each group, is exactly equivalent to Groseclose's (2007) "one-and-a-half dimensional" preferences where "alternatives are described by two characteristics: their position in a spatial dimension, and their position in a good-bad [high-low tax rate] dimension, over which voters [of the same group] have identical preferences." It is also related to Aragonès and Xefteris (2013, 2014) who consider elections between two candidates whose non-policy characteristics are heterogeneously valued by two distinct voters' blocks.

[13] Given that in the social ideology dimension the position of each party p is fixed, we can save in notation by omitting p from $\{p, t_p\}$.

[14] In our context sincere voting merely implies that we rule out *misaligned* voting (i.e., voting for a candidate other than the most preferred one) as in Kawai and Watanabe (2013). In fact, as evidence suggests only a tiny fraction of voters vote for a candidate other than the one they most prefer (Kawai and Watanabe 2013; Fisher and Myatt 2014) as voters are found to have a very strong sincerity bias (e.g., Spenkuch 2015). As a result, the assumption that voters vote for their most preferred platform is consistent with empirical evidence.

$$\max_{p \in \mathcal{P}} U_{x,y}(p, t_p) = -|p - x| + \sqrt{y(1 - t_p) + T(t_p)}.$$

Stage 3: Given voters' choices at *Stage 2*, each party receives its vote share $v_p \in [0, 1]$ such that $\sum_{p \in \mathcal{P}} v_p = 1$, and the voting outcome is realized. The party that collects most votes wins (plurality rule) and implements its political platform. In case of ties, parties do so with equal probability. We assume commitment. That is, the winner fully implements her announced platform.

3 Equilibrium analysis

3.1 Social ideology dominates

In this section, we analyze the case where social ideology is dominant, that is, we assume that $M < \min \left\{ \left(\frac{1}{2} - \epsilon \right)^2; 4\epsilon^2 \right\}$ which, in turn, implies that ϵ is sufficiently large and the two moderate parties are sufficiently apart from each other –as far as social ideology is concerned– and *relatively* closer to the extremist ones. That is, from the perspective of an extremist party, there are substantial ideological differences between the two moderate ones –this is what we mean by social ideology being dominant. First, through Lemma 1, we characterize how parties' vote shares vary with q and the chosen tax rates. Then, we establish existence and characterize the *unique* pure-strategy Nash equilibrium of the game by highlighting the strategic behavior of the extremist parties.

Before presenting our formal results, it is useful to define the function that measures the maximum gain in votes for a party, as a function of q and of its tax rate differential (mark-up) with respect to one of its neighboring parties –this will help us pin down the voters who are indifferent between any two parties. First, define the tax rate differential for a party p as $\hat{t}_p \equiv t_p - t_{\hat{p}}$, for some neighboring $\hat{p} \neq p$. Also, to spare on notation, define $\delta \equiv (1 - q)(M - m)$ and $\gamma \equiv q(M - m)$, and let $\hat{m} \equiv m + \delta t_{\hat{p}}$ and $\hat{M} \equiv M - \gamma t_{\hat{p}}$. Then, for every $q, \hat{t}_p \in (0, 1]$ and every m and M, such that $m \in [0, M)$ define:

$$z(q, \hat{t}_p) \equiv q \underbrace{\left[\sqrt{\hat{m} + \delta \hat{t}_p} - \sqrt{\hat{m}} \right]}_{\text{vote gains from poor}} - (1 - q) \underbrace{\left[\sqrt{\hat{M}} - \sqrt{\hat{M} - \gamma \hat{t}_p} \right]}_{\text{vote losses from rich}}.$$

The first component is the gain in votes from the poor voters for a party proposing excess taxation $\hat{t}_p$, compared to its opponent's proposal. The second part captures the loss in votes from the rich voters.[15] Now we prove the following helpful Lemma that characterizes $z(q, \hat{t}_p)$.

Lemma 1 *The following are true: (i) $z(q, \hat{t}_p)$ is continuous in $[0, 1]$ and differentiable in $(0, 1)$ for every q, m, t_p; (ii) $\partial z(\cdot, \hat{t}_p)/\partial t_p > 0$ for all t_p and $\forall q, m$; and (iii) $z(q, \hat{t}_p)$ is positive if and only if $\hat{t}_p > 0$ (i.e. $t_p > t_{\hat{p}}$) $\forall \, p, \hat{p}$ and $t_p, t_{\hat{p}} \in (0, 1]$*

Function $z(\cdot)$ can be interpreted as the net gain in votes for party p as a function of its tax rate mark-up $\hat{t}_p$. Clearly, $z(\cdot)$ is strictly increasing with respect to t_p, and positive whenever party p proposes higher taxes than its opponent (that is, $t_p > t_{\hat{p}}$). Hence, Lemma 1 highlights the incentives that parties have to target the poor voters and go for full redistribution. But then, do *all* parties (moderate and extremist) propose the *same* redistribution schemes? It turns out that extreme parties can never win, in equilibrium, even if they propose maximum redistribution. If they propose more tax they can increase their vote shares but, since they can never win, the implemented social ideology will never coincide with their ideal one. In fact, by proposing more tax they could be shooting themselves on the foot by causing the victory of the most ideologically remote (to them) moderate party. As a result, they have incentives to behave strategically. Lemma 2 summarizes this result.

Lemma 2 *Proposing tax rate $t_p = \tau$ is a strictly dominant strategy for $p \in \{L, A\}$ (i.e., for both moderate parties) and, hence, extreme parties (l and a) can never win, in equilibrium.*

We can now characterize in Proposition 1, with the help of those two results, the equilibrium in the case where social ideology is dominant.[16]

Proposition 1 *Let $M < \min\left\{\left(\frac{1}{2} - \epsilon\right)^2; 4\epsilon^2\right\}$. Then, for every $q \in (0, 1)$, every $m \in [0, M)$ and every $\tau \in (0, 1]$ the following vector $\boldsymbol{t}^* = (t_l^*, t_L^*, t_A^*, t_a^*) = (0, \tau, \tau, 0)$ constitutes the unique Nash equilibrium of the electoral game in pure strategies.*

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[15] Clearly $\delta \hat{t}_p$ measures the net income transfer to a poor voter, due to taxation, whereas $-\gamma \hat{t}_p$ measures the net income transfer to a rich one.

[16] Throughout this paper the term *symmetric* strategy profile is meant to imply a profile such that the two extremist parties play the same strategy while the same is true for the two moderate ones (i.e., $t_a = t_l$ and $t_A = t_L$ in such a profile).

This result simply says that when the social issue is sufficiently important for voters –that is, $M < \min\left\{\left(\frac{1}{2} - \epsilon\right)^2; 4\epsilon^2\right\}$– then there always exists a unique Nash equilibrium in pure strategies such that the two moderate parties will propose the highest *permitted* tax rate τ while, the two extremists "meet" in proposing zero tax. That is, in our *moderates-vs-extremists* equilibrium, the two extremist parties –despite the fact that they espouse radically opposite social ideologies– propose identical economic policies in stark contrast with the proposed economic policies of the more moderate parties which favor the less well-off voters.

The intuition behind this result is as follows. First, due to the diminishing marginal utility of income, poor voters are more responsive to generous redistribution (the marginal rate of substitution of income for social ideology is relatively higher for a poor voter). Second, a very high nominal tax rate does not always imply very aggressive redistribution. In fact, whenever the share of poor voters q is very low, even for extremely high values of t_p, due to the proportional redistribution scheme, the total redistributive transfer from the rich towards the poor will be extremely mild (the term $q(M - m)t_p$ will be close to zero).[17] Hence, the trade-off always works in favor of those parties that target the poor by proposing more redistribution, irrespective of the size of that group. In a sense, our result echoes that one by Myerson (1993) on the incentives to exploit inequalities among voters by making campaign promises that favor weaker groups.

But then, why *all* parties do not make *identical* tax proposals –in equilibrium– in order to target the poor as Lemma 1 would seem to imply? Lemma 2 provides the answer: even if it proposes maximum redistribution, an extremist party can never win the election. Instead, it cannibalizes the vote share of the moderate party which is closer to its social ideology, thus causing the victory of the *more remote* –in social ideology terms– moderate party which, in turn, brings the implemented social ideology farther away from its ideal point. That is, extreme parties face a trade-off between their office motives (vote-maximization pushes towards high redistribution promises) and their ideological (policy) motives which trigger their strategic behavior in an attempt to manipulate the electoral outcome and the implemented social ideology (policy motives push

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[17]To see this, check that for $q = 0$ or for $q = 1$ the total redistribution transfer will always be zero, even if $t = 1$.

towards low redistribution promises). Here the assumption about M comes into play: by making the social ideology dimension more pronounced, it guarantees that any gains –in terms of vote shares– that extremist parties score by proposing more redistribution cannot be offset by the loss –in social ideology terms– caused by the victory of a more distant moderate party. As a result, when the social ideology dimension is sufficiently important –that is, when M satisfies the above condition– and the two moderate parties are expected to be very close as far as their winning prospects are concerned, extremist parties know that by proposing a higher tax they can only increase their vote share by a little –social ideology matters a lot to the voters. At the same time though, they are actually helping the moderate party they like the least to win. Given that in our framework parties also care about the social ideology of the winner (policy motives), extremist parties are better off proposing no redistribution at all.

Further notice that the equilibrium of Proposition 1 is robust to considering that parties not only care about social ideology but also about the implemented economic policy. In the pure strategy equilibrium that we have characterized above an extremist party –whichever its platform choice is– can never win –the winner of the election is always on of the two moderate parties– and, hence, it cannot change the implemented redistribution in equilibrium. At the same time, if a moderate party changes its redistribution promise, then it can only cause the other moderate party to win with certainty. As a result, neither a moderate nor an extremist party can single-handedly alter the implemented redistribution in equilibrium. In other words, given profile $\boldsymbol{t}^*$, changes in redistribution promises by any party only can influence its vote share or the identity of the winner but *not* the implemented economic policy (redistribution). Therefore, we conclude that the equilibrium of Proposition 1 is robust to parties having preferences for the implemented economic policy as well.

While this result appears to be robust and offers a clear picture of the forces that are in operation in this model, it raises a question. It is often the case that in actual elections not only extremist parties do not abstain from making generous redistribution promises, but quite the contrary they try to compete with the more moderate parties in this dimension. Obviously, the special case

presented above cannot capture this element of electoral competition. Nevertheless, the general case presented below certainly does so in a more realistic manner.

3.2   General case

One complication of considering the more general case –where social ideology still matters but is not dominant– in a game with discontinuous payoffs, is that an equilibrium in pure strategies need not exist. As social ideology becomes less important for voters compared to redistribution (and taxes), the incentives of extreme parties to behave strategically diminish. Proposition 2 generalizes our equilibrium characterization for the case of mixed strategies and shows existence of a *symmetric* mixed-strategy Nash equilibrium for values of M such that $M < \left(\frac{1}{2} - \epsilon\right)^2$.

Proposition 2 *For every $q \in (0,1)$, every $m \in [0, M)$ with $M < \left(\frac{1}{2} - \epsilon\right)^2$, every $\tau \in (0,1]$, and every $\epsilon \in (0, \frac{1}{2})$, $\exists \tau_\epsilon < \tau$ such that the following vector $\boldsymbol{\sigma}_\epsilon^*(\boldsymbol{t}) = (\sigma_l^*(t_l) = \sigma_a^*(t_a); \sigma_L^*(t_L) = \sigma_A^*(t_A))$ constitutes a symmetric equilibrium of the electoral game in mixed strategies, where σ_l^* and σ_a^* are such that $E[\sigma_l^*(t_l)] = E[\sigma_a^*(t_a)] = \tau_\epsilon < \tau$, whereas σ_L^* and σ_A^* are degenerate strategies that assign probability one to choosing τ.*

In an equilibrium where the two extremists employ proper mixed strategies, their tax rate proposals are, *in expectation*, strictly less than those of the two moderate ones.[18] Nevertheless, there is still a *strictly positive* probability that the two extreme parties propose a tax rate other than zero. In fact, the mixed-strategy equilibrium offers a very intuitive interpretation in the framework of repeated elections. It can be seen as the frequency with which extreme parties propose some (non-zero) redistribution.[19] When the social issue becomes of little importance (this is equivalent to M being large enough) the incentives of all parties to make generous redistribution promises dominate and one can trivially show that, in equilibrium, all parties assign probability

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[18]Notice that, by employing arguments similar to the ones used in the proof of Lemma 2, we can show that parties L and A still have the same strictly dominant strategies, and, hence, the two extremist parties never expect to win the election.

[19]Another interpretation of our mixed strategy equilibrium can be that candidate lists of extremist parties are strategically let to be heterogeneous –as far as candidates' positions on economic issues and redistribution are concerned– in order to commit not to tax too much, while moderate parties have homogeneous party lists, which is consistent with Kernell (2015).

mass almost equal to one arbitrarily close to τ.[20] That is, in certain realizations, extreme parties might even propose the same tax rate as the moderate ones. Nevertheless, since we still have that extremist parties are proposing, in expectation, *strictly* less redistribution (lower taxation) than the moderate ones, the qualitative features of the pure strategy equilibrium and the *moderates-vs-extremists* nature of our result carry through, even in the more general case.

Finally, let us make two brief comments here. The first one is related to the assumption that there are only two classes in our model: rich and poor voters. While this assumption is made for expositional simplicity, one can see that our general result (equilibrium characterization in Proposition 2) does not depend on the discrete income distribution that we have employed. By inspecting the proof of Proposition 2, it becomes clear that our formal argument does not depend, at all, on the assumption that the income distribution is discrete. As a result, it can be generalized for a broader class of continuous income distributions.[21]

The second one relates to the number of competing parties. One might worry that our result is sensitive on the number of parties that we have assumed. While this is a reasonable worry, our mixed strategy equilibrium allows us a closer look. Consider the case where $\epsilon \to 0$. Then, clearly, the condition on M outlined in Proposition 1 is not satisfied and, hence, the game admits an equilibrium in mixed strategies like the one characterized in Proposition 2, where moderates propose full redistribution while extremists mix. On the limit, as ϵ approaches 0, the two moderate parties are effectively becoming one. What this tells us about how the equilibrium would look like in the case of three parties? If there is only one moderate party –limit case when $\epsilon \to 0$– then, in equilibrium, the limit probability that extremists will assign to strategy τ will be equal to one ($\lim_{\epsilon \to 0} \sigma_l^*(\tau) = \lim_{\epsilon \to 0} \sigma_a^*(\tau) = 1$). In such a case, all parties will be proposing the same redistribution scheme. In fact, this observation helps us unravel a necessary condition for the emergence of niche parties –parties that focus solely on social ideology: it must be the case that moderate parties are sufficiently differentiated as far as social ideology is concerned in order for such

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[20] Formal results which provide a more detailed characterization of the mixed strategy equilibrium in Proposition 2 and back up this claim are available from the authors upon request. Solely for economy of space we refrain from presenting such arguments here.

[21] For a more detailed discussion of this technical point see also Xefteris (2015) and Matakos and Xefteris (2016).

parties to exist. Otherwise, all parties choose to compete in all dimensions, including redistribution.

3.3   The asymmetric case

So far, we have studied the symmetric case. That is, we have assumed that parties occupy symmetric positions –over the social ideology dimension– and that the distributions of the two groups of voters are also symmetric. Yet, one would like to known whether the main qualitative features of the equilibrium that we have characterized, namely that extremist parties propose less redistribution than the moderate ones, are robust to introducing such asymmetries. In this section, we briefly elaborate on three possible sources of asymmetry: the case in which the poor are more socially liberal (conservative) than the rich –what we call correlated dimensions–, the case where there are more socially liberal (conservative) parties at the one extreme, and the case where some parties face asymmetric restrictions in the economic dimension and the amount of redistribution they can propose. Here, we will argue that, as long as such asymmetries are *mild* –and by mild we mean that $t_p = \tau$ is still a *strictly dominant* strategy for the two moderate parties and, hence, Lemma 2 still holds– then, the qualitative implications of our result will not change in any such asymmetric equilibrium.

Despite recent evidence that both (socially) liberal and conservative voters reward parties for increased government spending and redistribution (e.g., Kriner and Reeves 2012), it can be the case that social ideology conditions voters' preferences for redistribution. For instance, socially conservative voters might be relatively poorer and, hence, they might prefer more redistribution. One can try to model this social ideology-dependent (and fixed) component of redistribution, by assuming that the distributions of the rich and poor voters are not symmetric in the $[0, 1]$ interval. To fix ideas further, assume that the distribution of voters is mildly asymmetric. An example is when the poor are represented by a uniform distribution on $[-\lambda, 1 - \lambda]$ and the rich by a uniform distribution on $[\lambda, 1+\lambda]$. Then, depending on the shape of the two distributions (that is, depending on whether λ is positive or negative), one of the two moderate parties would have an advantage compared to the symmetric case. This would induce a non-symmetric equilibrium outcome.[22]

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[22]In the appendix, we explore this scenario in greater detail and we show how an asymmetric equilibrium might look like

Nevertheless, when λ is sufficiently close to zero, this will not affect the strategic behavior of the moderate parties, and, as a result, it will not upset the main qualitative feature of the equilibrium that we have characterized.

To see why this is the case, notice that it will still be a strictly dominant strategy for both moderate parties to promise more redistribution in order to increase their vote shares. Similarly, the two extremist parties will face the same dilemma as before: how many votes will they be willing to sacrifice in order to strategically manipulate the outcome by boosting the chances of electoral success of the most proximal moderate party? The trade-off faced will be the same in nature. Yet, this time it is not identical in magnitude for the two extremist parties. As a result, just like in Proposition 2, the game admits *no pure strategy* equilibrium. Rather, we will have a *non-symmetric* equilibrium in *mixed strategies* which will involve the two moderate parties choosing pure strategy τ, while the two extremists mix –still offering, in expected terms, lower redistribution than the moderate ones.[23]

In general, the key point to note when considering any asymmetric case is that, in the spirit of Lemma 2, our results hold qualitatively if the two sufficiently differentiated in the social ideology dimension moderate parties –notice, though, that ϵ can be arbitrarily small– have a strictly dominant strategy to propose maximum redistribution. To see this, consider the case where there might be many extremist parties on the one side of the social ideology spectrum, but only one on the other (party position asymmetries). Then, moderate parties will always play their dominant strategy, while some –perhaps all– extremist parties, which cannot win in equilibrium, will still have incentives to behave strategically and propose less than maximum redistribution. Of course, in this case, it is quite possible that some of the *non-winning* extremists (on the side that there are many of them) would rather propose full redistribution, as their actions *might not* affect the implemented social policy; the actions of some extremist parties will not have an impact on the vote share –and the chances of winning– of the moderate parties. Nevertheless, in equilibrium, it would

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in a discretized version of our game.

[23] For a proof of this claim, see the existence proof of the symmetric general case (Proposition 2) and notice that it does not depend on the symmetry of the distributions of social ideology –it only requires that the two moderate parties have a strictly dominant strategy (in the spirit of Lemma 2) to propose τ.

be still the case that moderate parties propose full redistribution while, on average, extremist parties propose strictly less than full redistribution.

Finally, consider the case in which different parties face different (i.e., asymmetric) constraints regarding their redistribution promises. For example, some parties might be constrained –by their founding charters or principles– to have a "floor" in their redistribution proposals, and, hence those parties might be inclined to propose more generous redistribution than the others. Without loss of generality, consider the case that socially liberal parties, which tend to represent the poor voters more often, are constrained by their founding principles to always propose a certain level of redistribution: that is, there is a "floor", $\phi_p \geq 0$, on their redistribution promises.[24] As a result, poor voters might be more inclined to vote for such parties, while the opposite will be true for the rich ones. Then, following once more a line of arguments similar to the ones presented above, this will induce a non-symmetric equilibrium in mixed strategies, in which extremists still offer, in expected terms, strictly less redistribution than the moderate parties.[25] In general, we can conclude that the qualitative features of our *moderates-vs-extremists* result are robust to mildly asymmetric distributions of voters' social ideologies, to the existence of multiple extremist parties, and to parties facing different (asymmetric) constraints regarding their redistribution promises.

3.4 Comments on the assumptions of the objective functions

In this section, we briefly discuss how the assumptions we have made about the smoothness of the two –the policy and the vote share– components of the utility function affect our results. First, note that both components of a player's payoff function can be discontinuous in own strategy for certain parameter values and strategies of the other players. Consider the following examples that illustrate this point. When the two moderate parties propose τ, the payoff of extremist party l –in its policy component– is discontinuous when it offers the same redistribution as the rightist

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[24] Assume that apart from the strategic choice t_p that each party is making prior to the elections, there is a fixed component ϕ_l or ϕ_a which measures this constraint and is asymmetric; that is, it is different across parties with different social ideology (e.g., $\phi_l > \phi_a > 0$). As a result, we have $t_l \in [\phi_l, \tau]$ and $t_a \in [\phi_a, \tau]$, $\tau > 0$.

[25] Note that the asymmetry in the equilibrium is in terms of the strategies that the two extremists play and, of course, in the winning probabilities of the two moderate parties (but moderate parties still play their strictly dominant strategy to propose τ).

extremist party. When the two moderates parties are very similar (ϵ is very small) and extremists are expected to propose the same tax rate, then the payoff of each moderate party is discontinuous both in its policy and in its vote share component. The latter is true because, for example, starting from a situation in which both moderate parties propose zero redistribution, when moderate party L increases the proposed tax rate, is preferred by more poor voters and by less rich ones. When the social ideal policy of the indifferent poor voter reaches the social ideal policy of moderate party A, the vote share of L exhibits a jump since, from that point on, no poor voter votes for moderate party A (something equivalent occurs with the rich voters, but the other way round).

Of course, one should stress that not both of these possible discontinuities bring along the same amount of complications in our formal analysis. Moderate parties have a dominant strategy (τ) and in every profile in which they both use it, the payoff functions of the extremist parties can be discontinuous only in their policy component. Indeed, when moderate parties propose the highest possible redistribution it is impossible for an extremist party to win, not to mention the impossibility of making any type of supporters of moderate parties vanish. Hence, for these particular strategy profiles –which are undoubtedly the most relevant ones– the discontinuity in a payoff function might arise only in the policy component, while the measure of electoral success is quite smooth.

A natural question is, how would our analysis look like if we had instead a less smooth measure of electoral success: if, for example, parties cared about policy in the same manner they do now and *only the winner* of elections was rewarded with some additional utility. In such a case, the equilibrium of Proposition 1 would be intact: moderate parties would still stick to proposing the highest possible redistribution since by doing that each has probability $\frac{1}{2}$ to win –and get the extra utility–, and also brings the implemented social policy as close as possible to its ideal one; if one deviates one gets to lose in both fronts. Extremists too have no incentives to propose anything larger than zero since by doing that: a) they cannot be elected with positive probability (see Lemma 2), and b) they reduce the election probability of its nearest moderate party, thus, making

the implementation of a worse social ideology more probable.[26]

In fact, considering that parties care about vote share rather than just winning makes our result stronger in three ways. First, since extremists parties can never win when moderates propose popular policies, a partially policy motivated extremist party has no incentives to propose a popular redistribution policy when it is partially win motivated. In contrast, when it cares about vote share, it has certain gains when it does that. Showing that, despite these non-negligible gains in electoral performance, the extremist party is still (overall) better-off by being non-competitive in the economic dimension, is arguably more solid than just showing the same result for the case in which an extremist party has no gains at all from proposing a more popular economic platform. Secondly, it aligns with institutional characteristics in most multiparty systems: parties usually receive state subsidies, broadcasting time in public media and other gains depending on their electoral performance. Hence, it is natural to assume that in such frameworks parties do not care crudely about winning or not but have incentives to be voted by as many voters as possible, even if they cannot affect their win prospects. Finally, when moderate parties become "too similar", then it is natural to expect that extremist parties should not want to sacrifice their electoral performance to bring the implemented social policy only a little bit closer to their ideal one. This is well captured by Proposition 2 and would not hold if parties cared crudely about winning: for any distance, even an infinitesimal one, between the two moderate parties, the strategy profile of Proposition 1 would constitute an equilibrium making our argument arguably less realistic and relevant for real world politics.

3.5   Robustness to different electoral institutions: Proportional rules and coalitions

So far in the analysis we have assumed that the party that wins most votes implements its platform (social ideology and tax proposal). That is, we have implicitly assumed simple plurality (or the

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[26]Notice that this equilibrium would still exist if, on top of assuming that parties care only about winning, we assumed that the implemented social policy moves closer to the winner's ideal policy in a smooth manner. This is so because: a) if an extremist deviates from proposing zero, and thus breaks the tie between the moderate parties in favor of the rightist one, then it gains no probability of winning while it moves the implemented social ideology, from the midpoint between the two moderate parties, to a location farther away from its ideal one; and b) if a moderate party deviates from proposing the highest possible taxation it loses both in terms of election probability and it moves the implemented social policy farther away from its ideal one.

FPTP rule) and, hence, we have implicitly excluded the possibility of a coalition government being formed. Instead, we have only briefly mentioned in the introduction that our equilibrium characterization –and thus comparative statics results as well– do not depend on the institutional architecture (the type of electoral rule) because the incentives for strategic behavior and the trade-off between votes and ideology outcomes –that the extreme parties face– are also present in any other institutional set-up.

In this section, we address these issues directly by examining what happens when we introduce the *proportional rule* which, in turn, allows for coalition governments. Before presenting the result, we need to comment that we will only provide a characterization result that is similar to the one of Proposition 1. We shall not address issues of uniqueness of equilibrium here, since the aim is to demonstrate that the equilibrium we have characterized in Proposition 1 can arise even if we vary the institutional set-up. Then, the comparative statics predictions that we will derive from that equilibrium will be robust to considering different electoral rules. To fix ideas further, we make some extra assumptions on the institutions and the process of coalition formation which are in line with the literature that studies proportional representation systems (see e.g., Austen-Smith and Banks 1988; Baron and Diermeier 2001; Gamson 1961; Kalandrakis 2014; Laver and Shepsle 1990; Laver 1998; Riker 1962; Troumpounis and Xefteris 2015).

Assumption 1 *If no party succeeds in winning at least half of the votes (that is if $v_p < 1/2$ for all $p \in \mathcal{P}$) a coalition of parties $C \subset \mathcal{P}$ is formed such that $\sum_{p \in C} v_p \geq \frac{1}{2}$ and the policy implemented is the weighted average of their most preferred policies. That is, $\omega(C) \equiv \frac{\sum_{p \in C}(v_p * p)}{\sum_{p \in C} v_p}$, for $p \in \{l, L, A, a\} \subseteq [0, 1]$.*

Assumption 2 *The formateur $\dot{p}$ (the party with most votes such that $v_{\dot{p}} > v_{p'}$ for all $p' \neq \dot{p}$) will always have to participate in the coalition C. That is, $\dot{p} \in C$.*

Assumption 3 *Let $\mathcal{C}$ be the set of all possible coalitions C that satisfy the above conditions. Then, the formateur $\dot{p}$ will always form the coalition that gives her the highest utility.*

The first assumption is straightforward and very common in the literature of coalition formation.

The remaining two are also fairly intuitive. As it is commonly the case, the *plurality* winner becomes the formateur and is responsible of forming a coalition (for the electoral benefits of the most voted party in PR elections see Ansolabehere et al. 2005; Snyder et al. 2005; Matakos and Xefteris 2015). Also since in our model parties care not only about winning or being part of a coalition but also about social ideology, it is normal to assume that they care about the outcome that each potential coalition will implement. Hence, the formateur will strictly prefer to form the coalition that minimizes the distance between its ideal point and the policy of the coalition. This means that its most likely coalition partners are its spatial neighbors (in the social policy dimension). In particular, note that, for sufficiently large values of ϵ –we provide an exact expression in the appendix– the coalition C that minimizes, in equilibrium, the distance $d(\dot{p}, \omega(C))$ for the formateur $\dot{p}$ is the one that includes the (moderate) formateur and its neighboring extremist party. We can now state the main result.

Proposition 3 *Let Assumptions 1-3 hold and $M < \min\left\{\left(\frac{1}{2} - \epsilon\right)^2; 4\epsilon^2\right\}$. Then, there exists $\tilde{\epsilon}$ such that for every $\epsilon > \tilde{\epsilon}$, the following vector $\boldsymbol{t}^* = (t_l^*, t_L^*, t_A^*, t_a^*) = (0, \tau, \tau, 0)$ constitutes a Nash equilibrium of the electoral game inducing a coalition C^* with the median social ideology outcome being implemented, in expected terms.*

Two brief comments with respect to the equilibrium characterization of Proposition 3 are now in order. Firstly, we note that this equilibrium need not be *unique*. In fact, non uniqueness is not a problem in the following sense: the purpose of this section is to demonstrate that the identified equilibrium feature (moderates-vs-extremists) is *robust* to alterations of the institutional architecture (from plurality to proportionality and coalitions). Since our purpose is to show that our equilibrium characterization –where moderate parties always propose more redistribution than the extreme ones– and the resulting comparative statics analysis do *not* depend on the choice of particular electoral institutions, it suffices to show that an equilibrium with those characteristics exists under proportionality. That is exactly what we have shown in Proposition 3. Issues related with the full characterization of the complete set of NE for every possible institutional arrangement (and coalition formation process) are beyond the scope of our analysis. Hence, we defer them for

future work.

Secondly, the reason that those alterations in the institutional architecture do not affect our equilibrium characterization is that the main drivers of our result remain intact: the trade-off between getting more votes (office motivation) and altering the policy outcome (policy motivation) and the strategic behavior of extreme parties that this trade-off triggers. Much like in the standard case, extreme parties would be again shooting themselves on the foot (by causing a disfavored change in the policy outcome) if they over-compete with moderate parties on the economic (redistribution) dimension. Of course, Assumption 2 is critical in this respect as it guarantees that the plurality winner of the election should participate in any post-electoral coalition that is formed. As a result, extreme parties, once more, act strategically in order to manipulate the policy outcome and bring it closer to their ideal point (by attempting to alter the winning coalition). Moreover, the strategic behavior of extreme parties can have an additional justification: it is not only ideological (policy) motivations –by inducing the formation of a more favorable to them coalition– but also office motivations –they can themselves participate in government via the coalition– that drive their strategic behavior. Consequently, the trade-off and our point are strengthened further.

4   Empirical and welfare implications

In the previous section we characterized our *moderates-vs-extremists* result. But what are the implications of this result on political competition, party-system stability and welfare? Conventional wisdom suggests that an increase in income inequality might lead to greater party-system fragmentation and increased electoral support for extremist parties. Nevertheless, our formal arguments presented above seem to contrast this view, especially in the context of multi-party electoral competition.

In order to see this point better, in this section we perform a comparative statics analysis of our equilibrium. That is, we will examine how electoral fragmentation (or else party-system fragmentation) varies with changes in the distribution of wealth and the level of inequality (proxied

by M in our model)[27] within a particular society. First, following Rae (1968) and Laakso and Taagepera (1979), we need to define electoral fragmentation in our context. Given $v_p \in [0,1]$, for every $p \in \mathcal{P}$, we define electoral fragmentation as:

$$F(\boldsymbol{v}) = 1 - \sum_{p \in \mathcal{P}} (v_p)^2.$$

Then, we calculate the electoral fragmentation index, $F(\boldsymbol{v}(\boldsymbol{t}^*))$, as a function of the vote share allocation that corresponds to the symmetric equilibrium of Proposition 1. Recall that the induced vote share allocation $\boldsymbol{v}(\boldsymbol{t}^*) = (v_l^*, v_L^*, v_A^*, v_a^*)$ takes the following form:

$$v_L^*(\boldsymbol{t}^*) = v_A^*(\boldsymbol{t}^*) = \tfrac{1}{4} + \tfrac{\epsilon}{2} + \tfrac{1}{2}z(q,\tau) \text{ and } v_l^*(\boldsymbol{t}^*) = v_a^*(\boldsymbol{t}^*) = \tfrac{1}{4} - \tfrac{\epsilon}{2} - \tfrac{1}{2}z(q,\tau).^{[28]}$$

Hence, we can compute:

$$F(\boldsymbol{v}) = 1 - 2[v_L^*(\boldsymbol{t}^*)^2 + v_l^*(\boldsymbol{t}^*)^2].$$

We can, then, rewrite F as a function of q and τ:

$$F(q,\tau) = 1 - 2\left\{ \left[(\tfrac{1}{4} + \tfrac{\epsilon}{2}) + \tfrac{1}{2}z(q,\tau) \right]^2 + \left[(\tfrac{1}{4} - \tfrac{\epsilon}{2}) - \tfrac{1}{2}z(q,\tau) \right]^2 \right\} = 1 - z^2(q,\tau) - 2\epsilon z(q,\tau) - 2C,$$

where $C \equiv (\tfrac{1}{4} + \tfrac{\epsilon}{2})^2 + (\tfrac{1}{4} - \tfrac{\epsilon}{2})^2$, a constant.

Proposition 4 *Let conditions of Proposition 1 hold. Then, electoral fragmentation is strictly decreasing in economic inequality (that is, $\frac{\partial F(q,\tau)}{\partial M} < 0$).*

The intuition behind this result is clear. Since, in equilibrium, both parties L and A offer the same tax (redistribution) scheme, it follows that the identity of the (poor or rich) voter who is indifferent between those two parties (denoted by $i_m(t_L, t_A)$ and $i_M(t_L, t_A)$ respectively) does not vary with parties' tax proposals t_L and t_A. For this reason, any changes in the index of

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[27] Following the argument of Piketty (2014), discussions on inequality have centered around the incomes of the rich (top incomes) as the main driver behind the rise in income inequality observed over the last four decades. That is why, focusing on M makes sense; an increase in M, ceteris paribus, makes the income gap between the rich and the poor larger, and, hence, makes the distribution of total income more unequal. Indeed, one could employ more elaborate measures of inequality but they would complicate analysis without providing additional insights.

[28] By symmetry of equilibrium note that $\hat{t}_L = t_L - t_l = t_A - t_a = \hat{t}_A = \tau$ and, hence, $z(q, \hat{t}_L) = z(q, \hat{t}_A) = z(q, \tau)$. Again, by symmetry $z(q, \hat{t}_l) = z(q, \hat{t}_a) = -z(q, \hat{t}_L) = -z(q, \hat{t}_A) = -z(q, \tau)$

electoral fragmentation as a result of changes in the income distribution (M) must come through the shift of votes between the extremist (l and a) and the centrist (L and A) parties –in equilibrium extremist parties propose strictly less taxes than the centrist ones and poor voters with initial income m strictly prefer more taxes. When inequality is high (M takes large values) aggressive redistribution is most effective and, hence, centrist parties can increase their vote shares at the expense of extremist ones –which offer little or no redistribution– by getting the vote of almost all poor voters –who are relatively more responsive to redistribution (see also Dixit and Londregan 1996)– at the cost of losing very few rich voters.

Moreover, without making the claim that we provide a comprehensive test of our theoretical prediction, note that our results also seem to be supported by empirical evidence (Figures 1 and 2). In Figures 1 and 2, we plot the relationship between income inequality (measured by the popular Gini coefficient)[29] and electoral fragmentation (measured by the Rae (1968) index as detailed above) or, alternatively, electoral support for moderate parties.[30] As one can see, there is a clear negative relationship between inequality and electoral fragmentation –a positive one when we use the vote shares of moderate parties instead. Thus, our findings clearly bring into question the often popular –yet not fully backed-up by evidence– view that high income inequality is associated with high electoral support for extreme (right or left) parties and thus more fragmented and polarized party-systems (Esteban and Ray 2011).

5 Final remarks

In this paper, we have studied a model of multi-party electoral competition in two dimensions (economic policy and social ideology) where parties are differentiated. Our equilibrium analysis revealed a very strong *moderates-vs-extremists* result which was found to be robust to various

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[29] When there are two income groups, and for any given value of q, the Gini coefficient takes the following expression: $G = M(1-q)/[M(1-q) + mq] - (1-q)$. Then, it is straightforward to check that the Gini is strictly and monotonically increasing in M.

[30] Data on the vote shares of all parties, which are also used to compute the electoral fragmentation index, in 22 OECD democracies for the period from 1960 to 2007 are taken from the Comparative Political Data Set 1 (Armingeon et al. 2009). The 22 countries include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, New Zealand, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom.

alternative specifications considered. The political implications of our findings are twofold. One the one hand, our results highlight the fact that socially moderate parties are more likely to propose more generous redistribution and economic policies that mostly favor poor voters. On the other hand, our model provided a prediction regarding the relationship between income inequality and party-system (electoral) fragmentation that starkly contrasts the current narrative that has extremist parties benefiting the most from high income inequality. That is, our equilibrium prediction, which is not refuted by the data, suggests that it is the centrist and more moderate parties that tend to benefit from increased inequality.

The latter finding, arguably, casts a shadow of doubt to the widely accepted assertion that the recent rise of extreme right or left parties in many European countries can be attributed solely to high levels of income inequality experienced recently. This, in turn, implies that additional factors –which might have been overlooked– also can be partially responsible for the increased electoral success of such extreme parties in many recent elections across Europe. Perhaps reasons related to some intrinsic characteristics of those party-systems can explain better this phenomenon instead of rising levels of income inequality. In fact, our model suggests that income inequality certainly is not the "silver bullet" when it comes to providing an explanation of this recent success of many extreme parties –if anything, our results point to the opposite direction. As a result, we are in need of alternative explanations that can account for this phenomenon.

Of course, this is not to imply that income inequality is irrelevant to this debate. Rather its importance might hinge on particular aspects of the party-system such as –to name only one– the ability of opposition parties or special interest groups to place limits on taxation (e.g., Wolton 2014). Consider, for example, the case where constitutionally inscribed fiscal rules, independent fiscal authorities or special interest groups can limit the ability of political parties to choose their desired redistributive schemes. Then it may be no longer the case that centrist parties can attract the votes of the poor by proposing generous redistribution, as was the case in our model. Ultimately, the overall effect of income inequality on the structure of the party-system and the chances of electoral success of extreme parties seems to be ambiguous and, perhaps, depends on a series

of other parameters as well as the structural characteristics of the party-system. In light of these findings, our work calls for a better understanding of the role of special interest groups in the case of multi-party elections.

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6 Appendix

6.1 Correlated preferences

In the first part of the appendix, we examine more carefully the possibility that economic and social preferences are correlated, and we compactly argue that our moderates-vs-extremists result is still valid qualitatively (extremists propose lower redistribution compared to moderates). That is, we assume that poor voters are on average more socially liberal as far as social issues are concerned and rich voters lean more towards social conservatism (this "pairing" is obviously without any loss of generality). Unless stated differently, every assumption made in the main part of the paper holds in this section too.

Formally, we consider that the social ideal policies of poor voters are distributed uniformly on $[-\lambda, 1 - \lambda]$ and the social ideal policies of rich voters are distributed uniformly on $[\lambda, 1 + \lambda]$, where $\lambda < 0$. To simplify formal arguments we consider a finite approximation of our game in which parties are allowed to choose a redistribution policy from the set $\{t^l, t^h\}$, where $0 = t^l < t^h < 1$. Namely, we consider that parties can either propose low redistribution (t^l) or high redistribution (t^h). To avoid technical complications and existence of many similar sub-cases, we finally assume that the parameter restrictions of Proposition 1 hold, that $q > \frac{1}{2}$, and that $\lambda \to 0$. That is, all parameter values are such that each party is strictly preferred by a positive measure of poor and rich voters to any other party in every strategy profile and such that extremist parties can never win just like in the main part of our analysis (Lemma 2), while type-symmetric strategy profiles (profiles such that moderates use the same strategy and extremists use the same strategy), generically, result in a certain election of a moderate party and do not involve ties.

We first notice that $t_A = t^h$ (and $t_L = t^h$ respectively) is strictly dominant for party A (and L respectively) here as well. This is so, because the arguments in the proof of Lemma 2 do not depend on the fact that social and economic preferences are uncorrelated. They merely depend on the fact that the office and ideological objectives of the moderate parties are always aligned: a more popular redistribution policy only brings the implemented social policy (weakly) closer to a moderate's party ideal one. So, in every equilibrium, we have that both moderate parties propose

t^h. But what about the extremist ones? Is it true that they they both propose the lowest possible level of redistribution (t^l), as in the no correlation case $(\lambda = 0)$ that we have examined in detail in the paper? The answer is, generically, no. If both extremist parties propose t^l, then we have that party A wins with certainty and hence party l has incentives to propose t^h: in such a way it increases its vote share while the implemented social policy remains unchanged. If l chooses t^h then party a also has incentives to propose t^h since, by doing that, it only increases its vote share while not interfering with the implemented social policy. But if party a proposes t^h then party l has strong incentives to propose t^l: in this way it sacrifices some votes in exchange for making party L win and, thus, moving the social policy substantially closer to its ideal policy. Finally, if l chooses t^l, then a has incentives to do the same in order to bring the implemented social ideology closer to its ideal one at the expense of losing a few votes.

In other words, there is no equilibrium in pure strategies and in every equilibrium it must be the case that moderate parties propose t^h while extremists place strictly positive weight to both t^l and t^h and, hence, they both propose (in expected terms) *lower redistribution compared to the moderate parties.* In specific, in the mixed strategy equilibrium, party l places probability $\pi = \frac{4\epsilon - z(q, t^h)}{4\epsilon}$ to t^h and $1 - \pi$ to t^l while party a places probability $1 - \pi$ to t^h and π to t^l. That is, unlike the mixed strategy equilibrium that we characterized in Proposition 2, this time the probabilities that the two extremists assign to a specific strategy are *not identical* and, hence, the equilibrium is asymmetric: the socially liberal party l proposes (in expected terms) higher redistribution than party a –it assigns higher probability on t^h than t^l compared to party a– yet still strictly lower than *both* moderate parties that propose t^h.[31] In turn, this would imply that, in equilibrium, the electoral outcome will also be asymmetric: because one of the two extremists will propose more redistribution, in expected terms, one of the two moderate parties (in this particular example party L) will have an electoral disadvantage (in expected terms) compared to the other one. Thus, our results can also speak to recent patterns of electoral disadvantage that were recently observed in many European countries (e.g., Germany).

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[31] It is easy to check that, under the assumptions of Proposition 1, $z(q, t^h) < 2\epsilon$ and, hence, $\pi > \frac{1}{2}$.

Finally, notice that an equivalent mathematical representation of the problem presented above would have been to assume that the two extremist parties (l and a) occupy asymmetric positions in the social ideology space such that $p_l \neq 1 - p_a$, while we still have $p_l < p_L < \frac{1}{2} < p_A < p_a$ –recall that, so far, we have assumed that $p_l = 1 - p_a$ throughout the paper. Again, if both extremists propose t^l, then one of the two moderate parties wins with certainty –which one depends on the type of the asymmetry– and, hence, one extremist has incentives to propose t^h. That is, we are again in the same situation as above and the argument developed there still applies. As result, we conclude that the qualitative features of our moderates-vs-extremists equilibrium –moderate parties propose, in expected terms, higher redistribution– are robust to such asymmetries.

6.2   Proofs

Proof of Lemma 1.   Continuity and differentiability of $z(\cdot)$ are straightforward given that $M < \min\left\{\left(\frac{1}{2} - \epsilon\right)^2; 4\epsilon^2\right\}$.

For (ii) we first compute: $\frac{\partial z(\cdot)}{\partial t_p} = \frac{q\delta}{2\sqrt{m+\delta t_p}} - \frac{(1-q)\gamma}{2\sqrt{M-\gamma t_p}}$.[32] Further, observe that $\partial z(\cdot)/\partial t_p > 0$ is equivalent to $\frac{q\delta}{2\sqrt{m+\delta t_p}} > \frac{(1-q)\gamma}{2\sqrt{M-\gamma t_p}}$. Since $q\delta = (1-q)\gamma$ this, in turn, is equivalent to $m + \delta t_p < M - \gamma t_p \iff (\delta + \gamma)t_p < M - m \iff (M - m)t_p < M - m$. The latter is always true for all $t_p \in [0, 1)$.

For (iii) notice that when $t_p = t_{\hat{p}}$ (and, hence, $\hat{t}_p = 0$) we have that $z(q, 0) = 0$ for every $q \in (0, 1)$. Given (ii), it follows immediately that $z(q, \hat{t}_p)$ is positive *iff* $\hat{t}_p > 0 \iff t_p > t_{\hat{p}}$. This completes the proof of the lemma. ∎

Proof of Lemma 2.   The proof follows three steps. *Step 1* is to recall that, by Lemma 1, vote shares are increasing in tax rates, that is $\partial v_p/\partial t_p > 0$ for $p \in \{L, A\}$. *Step 2* is to show that whenever a moderate party A (or L) chooses $t_A = \tau$ (or $t_L = \tau$), then the extremists (l and a) can *never win,* even if they offer *maximum* redistribution. Finally, *Step 3* entails showing that for all $\epsilon > 0$ and every t_{-A} (t_{-L}), strategy $t_A = \tau$ ($t_L = \tau$) is *strictly dominant* for party A (L); that is, in equilibrium, parties A (and L) will always choose $t_A = \tau$ (and $t_L = \tau$), and, as a result, extremists can never win.

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[32] Note that $\hat{m}$ and $\hat{M}$ do not depend on t_p, whereas $\hat{t}$ is a linear function of t_p.

To prove the statement of *Step 2*, w.l.o.g. fix $t_A = \tau$ and compute the minimum vote share that party A can get. Then we compare it against the maximum vote share that party l can get for any t_l. We will show that $v_A^{\min}(t_A = \tau; t_{-A}) > v_l^{\max}(t_l, t_{-l})$ for any t_{-A} and every t_l, t_{-l}. But first, we need to show formally that the indifferent voter always lies between two ideologically adjacent parties.

Consider the case that the indifferent voter lies between parties l and A; that is, party L receives no votes. Since we have fixed $t_A = \tau$ and $v_l^{\max}$ implies $t_l = \tau$, the economic dimension is cancelled-out and parties A and l will split the votes in the interval $[0, \frac{1}{2} + \epsilon]$ (the indifferent voter is the equidistant voter at $i_{A,l} = \frac{1}{4} + \frac{\epsilon}{2}$). Hence, at most $v_l^{\max} \leq \frac{1}{4} + \frac{\epsilon}{2}$. But, notice that, even if L plays $t_L = 0$ against $t_A = t_l = \tau$, we have $v_L = \frac{1}{4} + \frac{\epsilon}{2} - \frac{z(q, \hat{t}_l = \tau)}{2} - \frac{z(q, \hat{t}_A = \tau)}{2} = \frac{1}{4} + \frac{\epsilon}{2} - z(q, \tau)$. But we know that $z(q, \tau) \leq \sqrt{M} < \min\left\{\left(\frac{1}{2} - \epsilon\right); 2\epsilon)\right\}$ and, hence, $v_L > 0$; a contradiction. Thus, the indifferent voter always lies between to adjacent parties which, in turn, implies that $v_l^{\max}(t_l = \tau, t_{-l}) < \frac{1}{4} + \frac{\epsilon}{2}$ (strict inequality). Moreover, notice that, when all parties propose the same tax rate τ the economic dimension is cancelled out, and, hence, we have $v_A^{\min}(t_A = \tau; t_{-A} = \tau) = \frac{1}{4} + \frac{\epsilon}{2}$.[33] Taken together they imply that $v_A^{\min} > v_l^{\max}$. A directly analogous argument can be constructed for parties L and a. As a result, the two extremist parties can never win, when parties A and L propose tax rate τ. That is, either party A is the sure winner or party L, or they both win with probability $1/2$.

In *Step 3* we need to show that party A (and L) have no incentive to deviate from $t_A = \tau$ ($t_L = \tau$). This entails an exhaustive case by case analysis when party A (L) chooses $t_A < \tau$ ($t_L < \tau$). By symmetry, we only work with party A. Let $t_A < \tau$. Then, consider the following cases.

Case 1: Party A is the sure winner. Then, switching to $t_A = \tau$ results in an increase in its utility (same outcome but more votes, since by Step 1 we have shown that $\partial v_p / \partial t_p > 0$ for $p \in \{L, A\}$)

Case 2: Party L is the sure winner. Then, switching to $t_A = \tau$ results in an increase in its utility (more votes by Step 1 and same or better social ideology outcome, since by Step 2 the

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[33] The indifferent voter between parties A and a is at $\frac{3}{4} + \frac{\epsilon}{2}$ while the indifferent voter between parties L and A is at $\frac{1}{2}$.

only possible outcomes are: either party A is the sure winner or party L, or they both win with probability $1/2$).

Case 3: Parties A and L tie in first place. Then, switching to $t_A = \tau$ results in an increase in its utility (more votes by Step 1 and *strictly* better outcome, since now by Step 2, A *must* be the sure winner).

Case 4: An extremist party (either l or a) is the sure winner. Then, switching to $t_A = \tau$ causes A to win with *certainty* (by Step 2 that is the only possible outcome out of the three since for an extremist to win in the first place we *must* have had that both $t_{L,A} < \tau$). Hence, this results in an increase in its utility (more votes by Step 1 and *strictly* better outcome).

Case 5: The two extremists (l and a) tie in first place. This implies that party L offers a t_L that is *strictly less* than t_l. Then, switching to $t_A = \tau$ causes A to be the *sure winner* (since by Step 2 that is the only possible outcome). Hence, this results in more utility for party A (more votes by Step 1 and better outcome).

Case 6: There is a tie between a centrist (L or A) and an extremist (l or a) party. There are 4 sub-cases: i) $\{A, a\}$; ii) $\{A, l\}$; iii) $\{L, a\}$ and iv) $\{L, l\}$. Then, switching to $t_A = \tau$ results in an increase in its utility. In all sub-cases A gets more votes by Step 1 and at the same time it wins by Step 2 (better outcome). To verify this recall that for L to tie with l (or a) it implies that $t_L < \tau$, hence switching to $t_A = \tau$ causes A to win with *certainty*.

Case 7: There is a tie among any *three* parties. If A is among the winners, then, switching to $t_A = \tau$ results in more utility (more votes by Step 1 and better outcome, *sure winner* by Step 2). If A is *not* among the winners, then $t_A = \tau$ results in more utility (more votes and better outcome in expected terms) since it wins with *certainty* (recall that by Step 2 a tie between L and one or more extremists implies that $t_L < \tau$).

Case 8: *All* four parties tie. Then, switching to $t_A = \tau$ causes A to win with *certainty* (Step 1). So, it increases its utility (more votes and better outcome).

Hence, we conclude that strategy $t_A = \tau$ ($t_L = \tau$) is *strictly dominant* for parties A (and L). As a result, we can eliminate all other strategies and conclude that, in equilibrium, it must be that

both A and L play $t_L^* = t_A^* = \tau$. But then, the two extremist parties can never win (see *Step 2*). This completes the proof of Lemma 2. ∎

Proof of Proposition 1. First, we will show that the proposed equilibrium is the unique symmetric NE of the game. Then we argue why the game does not admit any asymmetric equilibrium. To prove that the proposed equilibrium is indeed a symmetric NE, we need to show that no party has an incentive to deviate unilaterally from its equilibrium strategy, that is:

$$\forall p \in \mathcal{P},\ V_p(t_p^*, t_{-p}^*) > V_p(t_p', t_{-p}^*),\ \forall\ t_p'.$$

First, we calculate the vote share $v_p(t_p, t_{-p})$ that each party receives as a function of its strategy (tax rate proposal) t_p, for every t_p, t_{-p}. To do so, we have to identify the voter who is indifferent between voting for party a or A, A or L and L or l respectively. Then, we can compute the vote share for each party. Recalling (Lemma 2) that the indifferent voter always lies between two ideologically adjacent parties, define as $i_m(t_A, t_a)$ the position of the poor voter (with income m) who is indifferent between parties a and A when they propose tax rates t_a and t_A respectively. Then, the following equality must hold:

$$-|1-i_m(t_A,t_a)|+\sqrt{m+(1-q)(M-m)t_a} = -|1/2+\epsilon-i_m(t_A,t_a)|+\sqrt{m+(1-q)(M-m)t_A} \iff$$

$$i_m(t_A,t_a) = \tfrac{3}{4}+\tfrac{\epsilon}{2}+\tfrac{1}{2}\left(\sqrt{m+\delta t_A}-\sqrt{m+\delta t_a}\right) = \tfrac{3}{4}+\tfrac{\epsilon}{2}+\tfrac{1}{2}\left(\sqrt{\hat{m}+\delta\hat{t}_A}-\sqrt{\hat{m}}\right).^{34}$$

By analogy, we denote the indifferent rich voter (with income M) as $i_M(t_A, t_a)$ and the condition becomes:

$$i_M(t_A,t_a) = \tfrac{3}{4}+\tfrac{\epsilon}{2}-\tfrac{1}{2}\left(\sqrt{M-\gamma t_a}-\sqrt{M-\gamma t_A}\right) = \tfrac{3}{4}+\tfrac{\epsilon}{2}-\tfrac{1}{2}\left(\sqrt{\hat{M}}-\sqrt{\hat{M}-\gamma\hat{t}_A}\right).$$

Given that a fraction q of the electorate has income m and the remaining $1-q$ has M, and given that the two continua of voters are identical in all other respects, we can then compute the *"aggregate"* indifferent voter:

$$i(t_A,t_a) = \tfrac{3}{4}+\tfrac{\epsilon}{2}+\tfrac{1}{2}\left[q\left(\sqrt{\hat{m}+\delta\hat{t}_A}-\sqrt{\hat{m}}\right)-(1-q)\left(\sqrt{\hat{M}}-\sqrt{\hat{M}-\gamma\hat{t}_A}\right)\right] = \tfrac{3}{4}+\tfrac{\epsilon}{2}+\tfrac{1}{2}z(q,\hat{t}_A)$$

Then, all voters to the right of $i(t_A, t_a)$ will vote for party a whereas all voters to the left of $i(t_A, t_a)$ –and till voter $i(t_L, t_A)$– will voter for party A. By a symmetric argument a similar

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[34] Here, w.l.o.g. let $t_p = t_A$ and $t_{\hat{p}} = t_a$, and hence, $\hat{t}_p \equiv \hat{t}_A \equiv t_A - t_a$.

analysis applies when we compare the indifferent voter between parties L and l. We now compute the voter who is indifferent between parties A and L, which we denote by $i(t_A, t_L)$. By letting w.l.o.g. $t_p = t_L$, $t_{\hat{p}} = t_A$ and $\hat{t}_L = t_L - t_A$ an analogous computation as above yields:

$$i(t_L, t_A) = \tfrac{1}{2} + \tfrac{1}{2}\left[q\left(\sqrt{\hat{m} + \delta\hat{t}_L} - \sqrt{\hat{m}} \right) - (1-q)\left(\sqrt{\hat{M}} - \sqrt{\hat{M} - \gamma\hat{t}_L} \right) \right] = \tfrac{1}{2} + \tfrac{1}{2}z(q, \hat{t}_L).$$

The above implies that $i(t_L, t_A) > \tfrac{1}{2}$ iff $t_L > t_A$, $i(t_L, t_A) < \tfrac{1}{2}$ iff $t_L < t_A$ and $i(t_L, t_A) = \tfrac{1}{2}$ iff $t_L = t_A$. In a symmetric equilibrium, by definition, we have $t_L = t_A$ and $\hat{t}_L = 0$ which, in turn, implies that the indifferent voter $i(t_L, t_A) = \tfrac{1}{2}$.[35] Then, we can compute the vote share allocation for each party as a function of its strategy choice (by symmetry it suffices to do so for parties A and a):

$$v_A(t_A) = i(t_A, t_a) - i(t_L, t_A) = \left\{ \tfrac{3}{4} + \tfrac{\epsilon}{2} + \tfrac{1}{2}z(q, \hat{t}_A) \right\} - \left\{ \tfrac{1}{2} + \tfrac{1}{2}z(q, \hat{t}_L) \right\} = \tfrac{1}{4} + \tfrac{\epsilon}{2} + \tfrac{1}{2}\left\{ z(q, \hat{t}_A) - z(q, \hat{t}_L) \right\}$$

and, by analogy, $v_a(t_a) = 1 - i(t_A, t_a) = \tfrac{1}{4} - \tfrac{\epsilon}{2} - \tfrac{1}{2}z(q, \hat{t}_A)$.[36]

Then observe that the vote share of each party is strictly increasing in its own strategy (tax rate proposal). That is, $\frac{\partial v_p}{\partial t_p} > 0$ iff $\partial z(\cdot)/\partial t_p > 0$, which by Lemma 1 is always true for all p and $\forall q, m$ such that $t_p \in [0, \tau]$ with $\tau \in (0, 1]$. Hence, the vote share v_p for *every* party $p \in \mathcal{P}$ is *strictly increasing* in t_p. By Lemma 2 we know that $t_A^* = t_L^* = \tau$ (dominant strategy for A and L). Assume for a moment that, since $\frac{\partial v_p}{\partial t_p} > 0$ for all p, parties a and l also choose $t_a' = t_l' = \tau$. Again by Lemma 2, we know that, in equilibrium: i) parties a and l can never win, and ii) parties L and A tie in first place. Then, any of the extremist parties (say a) has an *incentive to undercut* l (that is, to propose $t_a'' < t_a' = t_l' = \tau$) and cause party A to win with certainty *iff*:

$$\underbrace{\epsilon}_{\substack{\text{gain from causing a shift in policy outcome}}} \geq \underbrace{\max\left\{ v_a(t_a' = \tau) - v_a(t_a'' < \tau) \right\}}_{\text{maximum vote loss of a deviation from } t_a' \text{ to } t_a''}$$

But given $t_A^* = t_L^* = \tau$, we know that $\forall t_a, t_{-a}$ the maximum loss in votes is equal to the following expression:

$$\max\{v_a(t_a' = \tau) - v_a(t_a'' = 0)\} = \tfrac{1}{4} - \tfrac{\epsilon}{2} - \left[\tfrac{1}{4} - \tfrac{\epsilon}{2} + \tfrac{z(q, \hat{t}_a)}{2} \right] = -\tfrac{z(q, \hat{t}_a)}{2} = \tfrac{z(q, \hat{t}_A)}{2} = \tfrac{1}{2}z(q, \tau).[37]$$

Further notice that $\tfrac{1}{2}z(q, \tau) < \tfrac{1}{2}\sqrt{M} < \tfrac{1}{2}\min\left\{ \left(\tfrac{1}{2} - \epsilon \right); 2\epsilon \right) \right\} \leq \epsilon$ and, hence, the above constraint is *always* satisfied with *strict* inequality. Thus undercutting is *always* profitable (i.e. it

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[35] When $t_L = t_A$ we have $T(t_L) = T(t_A)$ and, hence, $\sqrt{y + T(t_L)} = \sqrt{y + T(t_A)}$ for every q and all $y \in \{m, M\}$. That is, the second part of the utility function is always cancelled out.

[36] Note that, by definition, we have $\hat{t}_A \equiv t_A - t_a = -(t_a - t_A) \equiv -\hat{t}_a$

[37] Recall that $\hat{t}_a = t_a'' - t_A^* = -\tau$ and $\hat{t}_A = t_A^* - t_a'' = \tau$. Hence, the last two equalities follow.

is a *strictly dominant* strategy for party a) That is, it is true that $\epsilon > \frac{1}{2}z(q, \tau) \geq \max\{v_a(t'_a = \tau) - v_a(t''_a = 0)\}$. Since by symmetry, the same also is always true for party l, we conclude that in equilibrium we must have $t^*_a = t^*_l = \min\{t \mid t \in [0, \tau]\} = 0$.

Consider now an equilibrium strategy profile such that $t_a < t_l$. Given that $t_L = t_A$ in every equilibrium (by strict dominance), it follows that party party A wins with certainty. Hence, party a has a profitable deviation: it can increase t_a to $t_a + \varepsilon$, for some positive $\varepsilon < t_l - t_a$, and thus increase her vote share without affecting the implemented social policy. That is, such an asymmetric strategy profile cannot be an equilibrium of the game. By symmetry, equilibria with $t_a > t_l$ are also ruled out, and, hence, the game admits no asymmetric equilibria at all. Thus, we conclude that the *unique* Nash equilibrium strategy profile is $\boldsymbol{t}^* = (t^*_l, t^*_L, t^*_A, t^*_a) = (0, \tau, \tau, 0)$. ∎

Proof of Proposition 2. Notice that, by employing arguments similar to the ones used in the proof of Lemma 2, we can establish that parties L and A still have the same strictly dominant strategies, namely $t^*_L = t^*_A = \tau$. Moreover, w.l.o.g. we can fix $t^*_L = t^*_A = \tau$ and restrict attention to the reduced two-player game $G = (\mathcal{T}_p, V_p)_{p \in \{l,a\}}$, where $\mathcal{T}_p = [0, \tau]$. Then, we have a standard game with discontinuous payoffs. Observe that in this general case, M need not be smaller than $4\epsilon^2$. That is, the constraint $v_a(t'_a = \tau) - v_a(t''_a) \leq \epsilon$ is not always satisfied. As a result, undercutting is *not* always profitable. That is, there exist values of ϵ such that the gain in implemented ideology (ϵ) does not suffice to offset the incurred loss in the vote share. Hence, the game need not have a pure-strategy equilibrium.[38]

Yet, there exist a symmetric equilibrium in mixed strategies. First, note that game $G = (\mathcal{T}_p, V_p)_{p \in \{l,a\}}$ is a *symmetric* in pure strategies, *compact, Hausdorff* game, since $\mathcal{T}_p$ is a compact *Hausdorff* space. In order to show that game G possesses a symmetric mixed strategy Nash equilibrium we need only show that its mixed extension $\bar{G}$ is *better-reply secure* along the diagonal, since *quasi-symmetry* of $\bar{G}$ follows from the symmetry of G (Corollary 1.3; Reny 1999). Consider the mixed extension of the game $\bar{G} = (\Sigma_p, V_p)_{p \in \{l,a\}}$, where we extend each V_p to $\boldsymbol{\Sigma} = \Sigma_l \times \Sigma_a$ by

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[38] Clearly, $t_a = t_l = \tau$ cannot be an equilibrium because a Bertrand-style induced competition will eventually lead to $t_a = t_l = 0$. But this cannot be an equilibrium either because a (or l) can go all the way and promise $t_a = \tau$ since the gain in vote share $\max\{v_l(t'_l = \tau) - v_l(t''_l = 0)\}$ might exceed the loss in utility ϵ that is now incurred by the fact that party L wins with certainty.

defining $V_p(\sigma_l, \sigma_a) = \int_0^\tau \int_0^\tau V_p(t_l, t_a) d\sigma_l d\sigma_a$ for all $(\sigma_l, \sigma_a) \in \Sigma$.

Then, in turn, better–reply security of $\bar{G}$ implies two conditions: (i) *reciprocal upper semi-continuity* and (ii) *payoff security* along the diagonal.[39] For (i) we only need to verify that the sum of the payoffs of the two parties $\sum_p V_p(t)$ is u.s.c. in t on $\mathcal{T}$. Then, by Proposition 5.1 (Reny 1999) $\sum_p \int_{\mathcal{T}} V_p(t).d\sigma$ is also u.s.c. in σ on Σ and the mixed extension game $\bar{G}$ is reciprocally u.s.c. The payoff function for party a (and by symmetry l) is as follows:

$$V_a(t_a, t_l) = \begin{cases} -\frac{1}{2} + \epsilon + v_a(t_a, t_l), & \text{if } t_a < t_l \\ -\frac{1}{2} + v_a(t_a, t_l), & \text{if } t_a = t_l \\ -\frac{1}{2} - \epsilon + v_a(t_a, t_l), & \text{if } t_a > t_l \end{cases}$$

Then, by continuity of $v_p(t_p, t_{-p})$ for all t_p, t_{-p} and $\forall p \in \{l, a\}$, it is clear that $\sum_p V_p(t) = -1 + \sum_p v_p(t)$ is continuous in t on $\mathcal{T}$. As a result, condition (i) is trivially satisfied. For diagonal payoff security in mixed strategies we need to show that:

$$\forall p, \forall \varepsilon > 0, \forall \sigma \in \Sigma, \exists \hat{\sigma}_p \in \Sigma_p \text{ s.t. } V_p(\hat{\sigma}_p, \sigma'_{-p}) \geq V_p(\sigma) - \varepsilon, \forall \sigma'_{-p} \text{ in some open neighborhood}$$

of σ_{-p}.

Clearly, this is always true. To see this pick any profile $\sigma = (\sigma_l, \sigma_a)$ such that $\sigma_l = \sigma_a$ and consider party a playing strategy $\hat{\sigma}_a$ that assigns larger probability to $t_a = 0$ such that $\hat{\sigma}_a(t_a = 0) > \sigma_a(t_a = 0)$. Then, for small perturbations of σ'_l, close enough to σ_l, the condition is always satisfied since there is at most a small loss in expected vote share that can be offset by a positive change in the expected outcome. As a result, since both conditions are satisfied we conclude that mixed extension game $\bar{G}$ is better-reply secure. Hence, $\forall \epsilon$ the reduced game G possesses a symmetric Nash equilibrium in mixed strategies $\sigma^*(t) = (\sigma_l^*(t_l) = \sigma_a^*(t_a); \sigma_L^*(t_L) = \sigma_A^*(t_A))$, such that σ_l^* and σ_a^* have finite support on $[0, \tau]$ with $E[\sigma_l^*(t_l)] = E[\sigma_a^*(t_a)] = \check{\tau}_\epsilon < \tau$, whereas σ_L^*, σ_A^* are the degenerate strategies with $\sigma_L^*(\tau) = \sigma_A^*(\tau) = 1$. This completes the proof. ∎

Proof of Proposition 3.   First note that the vector t^* can induce a coalition that satisfies Assumptions 1 to 3. Clearly, as we have shown in Proposition 1 the resulting vote share allocation $v(t^*)$ is symmetric, such that the two centrist parties L and A share the same amount of votes and

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[39] It is easily checked that the condition of payoff security is satisfied even off the diagonal. Hence, the same argument can be extended to show existence of non-symmetric mixed strategy equilibria.

tie in the first place.[40] Then, they each become the formateur with probability one-half. Hence, and for large enough values of ϵ, with probability one-half the coalition (satisfying Assumptions 1-3) between l and L is formed (call it C_L^*). Otherwise, a coalition between a and A is formed (call it C_A^*). Since $\omega(C_L^*)$ and $\omega(C_A^*)$ are symmetric, the expected social policy outcome that is implemented is the median policy $(1/2)$. We need only show that no party has an incentive to deviate unilaterally.

First consider unilateral deviations by parties L and A (by symmetry we need only examine L). Observe that any $t_L' \neq t_L^*$ will cause it to lose votes and cease being a formateur. Then, party A becomes the formateur with certainty and, for large enough values of ϵ s.t. the condition $\epsilon > \left(\frac{1}{2} - \epsilon\right) - \left(\frac{1}{4} + \frac{\epsilon}{2} + \frac{z(\cdot)}{2}\right)\left(\frac{1}{2} - \epsilon\right)$ is satisfied, the C_A^* coalition occurs with probability one.[41] Clearly, this is not a profitable deviation, since $d_L(p_L, 1/2) < d_L(p_L, \omega(C_A^*))$ implies strictly lower utility (less votes and worse policy outcome in expected terms). Now, consider an extremist party (w.l.o.g. take l) and its incentives to deviate from $t_l^* = 0$. Any $t_l' > 0$ will increase its vote share but it will deprive (as shown in the proof of Proposition 1) party L from some votes. Moreover, note that the allocation of votes between parties A and a will remain unchanged. As a result, party A is the sure winner and becomes again the *formateur* with probability one. This induces coalition C_A^*, satisfying Assumptions 1-3. As a result, deviating from t_l^* to t_l' cannot cause party l to enter a more favorable coalition (l can never become the formateur). In fact, it gives rise to a strictly worse coalition (expected social policy outcome of new coalition C_A^* is to the right of the median and hence, further away from l's ideal policy point 0). Yet, this deviation does give party l some extra votes. To show that it is not profitable we have to compare the maximum gain in utility from increasing its vote share with the loss of inducing a strictly worse policy outcome. From Proposition 1 we have: $\max\{v_l(t_l' = \tau)\} = \frac{1}{2}z(q, \tau)$

Then, the deviation is *not* profitable if and only if the following is satisfied:

$\frac{z(q,\tau)}{2} < d_l(p_l = 0, \omega(C_A^*)) - d(p_l = 0, 1/2) \iff$

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[40] Recall that we have computed that $v_L = v_A = 1/4 + \epsilon/2 + 1/2[z(q,\tau)]$ and $v_l = v_a = 1/4 - \epsilon/2 - 1/2[z(q,\tau)]$

[41] The condition on ϵ is derived as follows: the grand coalition between A and L induces policy outcome $\frac{1}{2}$, and, hence, it is ϵ away from each moderate party's ideal position. If a moderate party coalesces with an extremist one (C_L^* or C_A^* coalitions) then $\omega(C_L^*) = v_l p_l + v_L p_L$ ($\omega(C_A^*) = v_a p_a + v_A p_A$). Then a moderate party would rather coalesce with an extremist one if and only if ϵ is larger than the distance between p_L and $\omega(C_L^*)$ (p_A and $\omega(C_A^*)$) –the RHS of the inequality.

$$\frac{z(q,\tau)}{2} < d_l(0,\omega(C_A^*)) - \underbrace{1/2}_{d(p_l,1/2)} \iff$$

$$z(q,\tau)/2 < \underbrace{3/4 - z(q,\tau)/2 + \epsilon^2 + \epsilon z(q,\tau)}_{d(0,\omega(C_A^*))} - 1/2 \iff z(q,\tau) < 1/4 + \epsilon^2 + \epsilon z(q,\tau)$$

Since $z(q,\tau) < \min\left\{\left(\frac{1}{2} - \epsilon\right); 2\epsilon\right\}$, we can always find an $\tilde{\epsilon} > 0$ such that the above condition is always satisfied for any $\epsilon > \tilde{\epsilon}$. Hence, the deviation is never profitable and t^* constitutes a Nash equilibrium of the game. This completes the proof. $\blacksquare$

Proof of Proposition 4. Recall that $F(q,\tau) = 1 - 2C - z(q,\tau)^2 - 2\epsilon z(q,\tau)$. Then, we compute:

$$\frac{\partial F(q,\tau)}{\partial M} = \frac{\partial F(q,\tau)}{\partial z(q,\tau)}\frac{\partial z(q,\tau)}{\partial M} = -2[z(q,\tau)+\epsilon]\frac{1}{2}(1-q)\left(-\frac{1}{\sqrt{M}} + \frac{q\tau}{\sqrt{m+(m-M)(-1+q)\tau}} + \frac{1-q\tau}{\sqrt{M+mq\tau-Mq\tau}}\right).$$

Notice that $M \geq M + mq\tau - Mq\tau \geq m + (m-M)(-1+q)\tau$ for any admissible values. Hence,

$$\frac{1}{\sqrt{M}} \leq \frac{1}{\sqrt{M+mq\tau-Mq\tau}} \leq \frac{1}{\sqrt{m+(m-M)(-1+q)\tau}} \text{ and } \frac{1}{\sqrt{M}} \leq \zeta\frac{1}{\sqrt{M+mq\tau-Mq\tau}} + (1-\zeta)\frac{1}{\sqrt{m+(m-M)(-1+q)\tau}}$$

for every $\zeta \in [0,1]$, including $\zeta = q\tau$, and all generic parameter values. Moreover, we have shown (Lemma 1) that $z(q,\tau)$ is positive, and, hence, the above derivative is generically negative. That is, fragmentation is strictly decreasing in income inequality. $\blacksquare$
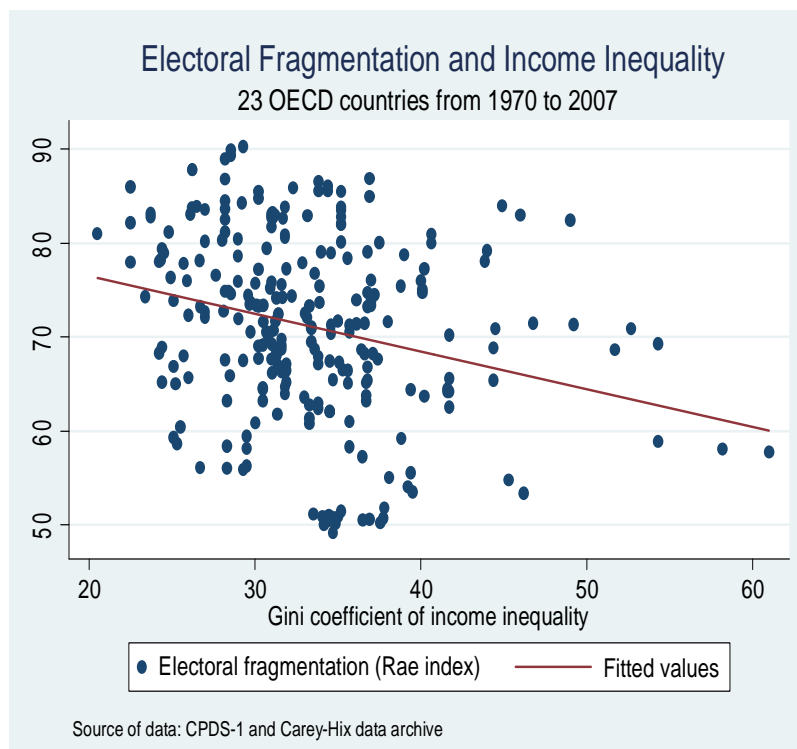
6.3 Figures

Figure 1. The relationship between electoral fragmentation (measured by the Rae index) and income inequality (measured by the Gini coefficient) in 23 Western OECD democracies (1970-2007).
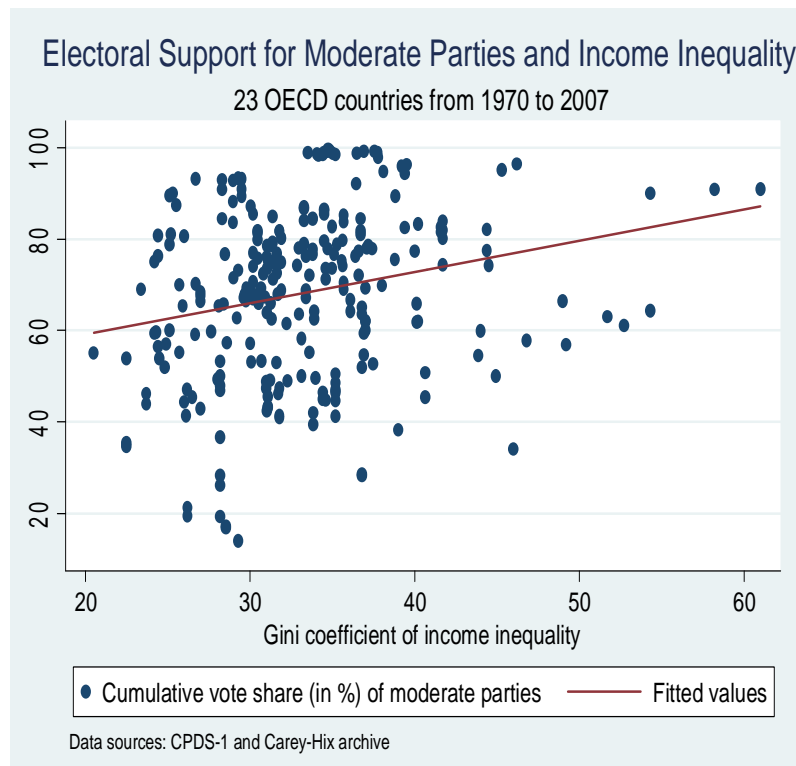

Figure 2. The relationship between electoral support (in %) for moderate parties and income inequality (measured by the Gini coefficient) in 23 Western OECD democracies (1970-2007).