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# Dynamic Status Effects, Savings, and Income Inequality\*

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and Ronald Wendner†

**Abstract.** This paper advances the hypothesis that the intensity of status preferences depends negatively on the average wealth of society (endogenous dynamic status effect), in accordance with empirical evidence. Our theory replicates the contradictory historical facts of an increasing saving rate along with declining returns to capital over time. By affecting the dynamics of the saving rate, the dynamic status effect raises inequality, thereby providing a behavioral mechanism for the observed diverse dynamics of income inequality across countries. In countries in which the dynamic status effect is strong (weak) inequality rises (declines) over time in response to a positive productivity shock.

**JEL classification:** *D11, D31, O11.*

**Keywords:** Status preferences, dynamic status, saving rate, growth, inequality

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## 1. INTRODUCTION

It is well documented that individuals are concerned with social comparisons and status, particularly as it pertains to consumption. This paper advances the hypothesis that the degree to which individuals have this concern is determined by their society's stage of development, which we proxy by its average level of wealth. Social comparisons in terms of consumption seem to be more important during the early stages of development rather than in later stages, due to, among other factors, the evolution of institutions (education), culture, and social norms that are opposed to, or at least discourage, conspicuous consumption activities. To address this process, we endogenize the degree of status concern, by relating it to average national wealth, and incorporating the fact that over time, as a country develops, this degree (with respect to consumption) declines. We refer to this mechanism as "*endogenous dynamic status preferences*". While this is a natural departure from standard assumptions, *dynamic* status preferences lead to dramatic changes in the implications of the basic neoclassical growth model, as shown below.

The idea that individuals are often motivated in their behavior by a quest for social status has been a recurring theme in a diverse range of endeavors long before the birth of economics.<sup>1</sup> While economic theory has focused on the implication of status preferences for economic outcomes and policy, little work has been done on the bi-directional interaction of status preferences and economic development.<sup>2</sup> The novelty of our framework is that status concerns affect not only the level of economic development (as has been extensively studied) but also the converse applies, namely that the level of development feeds back onto the evolution of agents' status concerns.

Introducing endogenous dynamic status preferences enables us to explain important phenomena that are not satisfactorily addressed by the standard neoclassical growth model. These include: (i) the historical and contemporary evolution of the saving rate, together with that of the real return on capital; and (ii) the historical and contemporary dynamics of wealth and income inequality.

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<sup>1</sup> Dubey and Geanakoplos (2017) provide a comprehensive introduction to the importance of status in motivating and shaping individual behavior.

<sup>2</sup> Examples include early 'modern' models and applications like Pigouvian taxation, Buchanan and Stubblebine's (1962) treatment of externalities, Becker's (1971) analysis of discrimination, Becker's (1974) theory of social interaction, and Frank's (1985) model of positional goods.

In this paper we consider the following stylized facts pertaining to the transitional dynamics of the saving rate and income inequality that the standard growth model, augmented by endogenous dynamic status preferences, can readily replicate. (i) Historical data show that from the dawn of the modern world the saving rate increases (along with declining returns to capital), a fact that cannot be reproduced by the standard neoclassical growth model, for reasonable calibrations (*Fact 1*, poor countries save less, Dynan et al. 2004). (ii) World income inequality decreases from the 1900s until the 1970s (*Fact 2*, illustrated in Figures 1 and 2). (iii) However, after the 1970s, for one group of countries income inequality remains approximately constant, yielding an L-shaped pattern (*Fact 3*, illustrated in Figure 1). (iv) In contrast, for another group of countries inequality increases sharply after the 1970s reaching the level of income inequality of 1900s, yielding a U-shaped pattern (*Fact 4*, illustrated in Figure 2). While both groups of countries have similar levels of economic development (income per capita), their respective developments in income inequality after 1970s diverge sharply. In addition, using historical data from 1913 to 2013, Saez and Zucman (2016) confirms a U-shaped path for wealth inequality in US, similar to that for income inequality illustrated in Figure 2.

Our key mechanism enabling reconciliation with Facts 1 to 4 – endogenous dynamic status preferences – operates through the transitional dynamics of the saving rate, which in turn affects the development of income inequality, as discussed below. This mechanism relies on behavioral changes that occur during the development process. As already noted, it is well documented that people derive utility not only from their own consumption but also from their relative social position (Easterlin, 2001). As long as consumption is visible (Heffetz, 2011, 2012), the social position of individuals can largely be inferred from their own consumption relative to the average consumption of others.<sup>3</sup> Thus, by consuming more, people increase their own relative position, and in turn, their utility. However, the pursuit of one's own status likely initiates a race with others, which results in excessively high equilibrium consumption that strains savings and intertemporal utility. We argue that during the development process, increases in average wealth lead to the formation of educational institutions, cultures, and social norms that discourage such conspicuous consumption. As a result, the increase in

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<sup>3</sup> A commodity is visible if, in the cultural context in which it is consumed, society has direct means to correctly assess the expenditures involved (Heffetz, 2011).

average wealth induces behavioral changes that lead to a lower degree of status concern, which tends to reduce the initial level of the saving rate followed by a rising saving rate along subsequent transitional paths. We show that this latter effect dominates over long periods during (the stages of) development, so that the saving rate is observed to increase over an extended period of time.

The hypothesis of a declining degree of status concern during development is supported by a number of empirical studies. Clark and Senik (2010), using a large European survey, demonstrate that comparisons are mostly in an upward direction. In this respect, there is much more scope for upward comparisons for the poor countries than exists for the rich countries. Moreover, the poor tend to care more about status with respect to relative consumption.<sup>4</sup> In line with our hypothesis, Figure 3 demonstrates that citizens of rich European countries find it less important to compare their income with that of others (Clark and Senik, 2010). In the figure, the mean importance of income comparisons is monotonically increasing, while the trend in income per capita is uniformly decreasing. Heffetz (2011) estimates income elasticities for the consumption of “status” goods and confirms the negative relationship between the degree of status concern and income. Moav and Neeman (2012) provide examples where the consumption basket of poor countries includes many goods that do not appear to alleviate poverty.

Our explanation of the long-run development of income inequality is based on the interplay between the dynamics of the saving rate, on the one hand, and the dynamics of the return to capital, on the other, during the development process. While there is an extensive literature that examines the effect of capital returns on income inequality (most notably, Piketty 2014), we highlight how their interaction with the savings rate is impacted by the evolution of the dynamic status preferences. In a standard neoclassical world, as the capital stock increases, the rate of return to capital declines. This “*return-to-capital effect*” disadvantages the rich, who hold more capital, than it does the poor. In contrast, the additional mechanism being emphasized here – the endogenous dynamic status effect – impacts both the *level* and the *rate of change* of the saving rate. This effect initially reduces the level

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<sup>4</sup> Importantly, literature in psychology states that individuals seem to care about their ranking and the esteem of others, even if they derive no clear economic benefits, and are willing to pay respect to others and to modify their behavior accordingly, without receiving any direct benefit (cf. Heffetz and Frank, 2011).

of the saving rate, while during the development process, as the economy accumulates capital, people increase their saving rate due to a reduction of the degree of status concerns. The lower *level* of the saving rate implies a lower rate of capital accumulation, causing the rate of interest also to decline at a slower pace. This latter effect benefits the wealthy relative to the poor households. Hence, in a society with a heterogeneous wealth distribution, the dynamic status effect contributes to more unequal wealth and income distributions. Overall then, the strength of the endogenous dynamic status effect relative to that of the standard return-to-capital effect governs the evolution of income inequality.

We characterize analytically, and simulate numerically, the effects of a positive technology shock on savings and income inequality. Our results show how the interaction between the return-to-capital and the endogenous dynamic status effects can play an important role in reconciling the implications of the augmented neoclassical growth model with the empirical evidence illustrated in Figures 1 and 2. Starting in 1900, when all economies were relatively undeveloped, the return-to-capital was strong and clearly dominated the status-effect; accordingly inequality declined in response to a positive productivity shock. Over the period 1900-1970 as economies developed, the strength of the dynamic status effect increased relative to the return-to-capital effect, and the rate of decline in income inequality decreased. After around 1970, with the diverse rates of economic development, for the slower developing countries cultural developments occur slowly, so that the two effects are roughly in balance and inequality remains roughly constant, yielding the L-shaped curve as in Figure 1. For other economies where the dynamic status effect is stronger and continues to increase, it begins to dominate the return-to-capital effect. Income inequality starts to increase, eventually yielding the U-shaped curve illustrated in Figure 2.

Following this introduction, Section 2 relates our contribution to the relevant prior literature. Section 3 sets out the model and provides further empirical evidence for status concerns to decline in average wealth over time. Section 4 solves the optimization problem of households and firms, and studies the impact of the dynamic status effect on the transitional dynamics of the saving rate. Section 5 analyzes the dynamics on wealth inequality. Section 6 provides a brief discussion on further research directions. Finally, Section 7 concludes, while technical details are relegated to the Appendix.

## 2. RELATED LITERATURE AND CONTRIBUTION

In formalizing and analyzing the hypothesis relating the declining subjective evaluation of status preferences to the level of development, the paper contributes to three bodies of literature. These include: (i) implications of positional goods in utility; (ii) studies of the dynamics of the saving rate; (iii) the dynamics of wealth and income inequality.

### 2.1. *Degree of Positionality*

The proposition that people derive utility not only from their own consumption but also from their relative consumption level can be traced back to Smith (1759) and Veblen (1889). Veblen's observation has been empirically justified by Easterlin's (1974) paradox, who found that increases in income of all individuals had a negligible effect on their happiness. This finding was confirmed in empirical studies by Clark and Oswald (1996) and Frank (1997) and subsequent authors. The consequences of positional preferences have been extensively investigated in a number of areas. These include their effects on capital accumulation and growth (Carroll et al., 1997, Alvarez-Cuadrado et al., 2004, Liu and Turnovsky, 2005, Wendner, 2010), on asset pricing (Abel 1999, Campbell and Cochrane 1999, Dupor and Liu 2003), on optimal tax policy over the business cycle (Ljungqvist and Uhlig 2000) and on public good provision (Micheletto, 2011, Wendner and Goulder, 2008, Wendner, 2014). But in all those applications the strength of positional preferences is exogenous and remains constant over time, rendering these models incapable of deriving the non-monotonic evolution of savings and the distribution of wealth we observe in the historical and contemporary empirical data.

Our hypothesis is based on two elements regarding the formation of households' preferences. First, the evolution of preferences for status is negatively related to the level of average wealth, which we shall show has important consequences for the dynamics of savings. Second, the dependence of status preferences on average wealth varies across countries, due to different cultural and institutional characteristics. This helps explain how cultural differences in the evolution of the concern for status can account for the divergence in wealth and income inequality across otherwise similar economies.

Empirical studies provide support for both elements of the determinants of status preferences. Bloch et al. (2004), Banerjee and Duflo (2007), and Heffetz (2011) support empirically the ideas that people rely on relative consumption to raise their perceived status and that average income or wealth

plays an important role in shaping the strength of status preferences. Using international data, Banerjee and Duflo (2007) and Clark and Senik (2010), show that in poor countries, people care more about status than they do in advanced economies. Moav and Neeman (2012) suggest that if human capital is visible (e.g. an academic title), then in more developed countries, the signaling of status (unobservable income) is pursued more with human capital than with consumption.

## **2.2. *Dynamics of the Saving Rate and the Real Return to Capital***

The first implication of our hypothesis relates to the determination of the saving rate. According to the standard neoclassical growth model, for empirically plausible parameter values, more capital (wealth) leads to a lower rate of return on capital and, in turn, to a lower saving rate (see Barro and Sala-i-Martin 2004, pp.109f, pp.135ff). However, empirical evidence indicates that saving rates are higher for richer countries (Loayza, Schmidt-Hebbel, and Serven, 2000). Also, examining historical data for the US, Saez and Zucman (2016) find that the saving rates tend to rise with wealth.

To address the reality that the saving rate increases with wealth over time, the literature mainly considers technological factors that increase the return to capital over time and, in turn, the saving rate. But by associating the increased saving rate with increasing returns to capital, this explanation contradicts recent evidence provided by Boppart (2014) and Ledesma and Moro (2016), suggesting that the return to capital is decreasing over time.

On the preference side, Strulik (2012) shows that as wealth increases, the pure rate of time preference declines, which tends to raise the saving rate. In this respect, by arguing that as a country develops people are less concerned with relative consumption, we provide an alternative mechanism also based on preferences. According to this mechanism individuals reduce their consumption growth rate over time, that is, they increase their saving rate, while the return on capital simultaneously declines, consistent with empirical evidence.

## **2.3 *Savings and Inequality***

An emerging literature attributes the contemporary increase in income- and wealth inequality to differences in the saving rates across individuals. The main assumption of this literature is ex-post heterogeneity, and its theoretical underpinning is Bewley (1977), which features an incomplete market environment, in which people save to self-insure against idiosyncratic earnings shocks.



These models are compelling and useful for capturing quantitatively the increase in wealth inequality in the US after 1970s. But they do not aim to explain: (a) why those factors (the richer model structure) were less crucial during the decline in wealth and income inequality that we observe from the 1900s (Saez and Zucman, 2016); (b) why the saving rate of rich individuals is higher but still declining in wealth (as capital accumulates) over time – something that we do not observe before the 1970s and in many countries even not after the 1970s; (c) why wealth inequality develops differently in the US and similar developed countries; (d) the transitional dynamics of the wealth- or income distribution (but rather focus on contemporary data).

Our theory is consistent with the results of the previously discussed literature, but in addition, it also provides explanations of the aforementioned points (a) to (d). To accomplish this, we follow a different methodological approach. First, we depart from the incomplete markets assumption and from stochastic environments by assuming *ex-ante* rather than *ex-post* heterogeneity in individual wealth endowments. Second, we emphasize a behavioral mechanism according to which the saving rate is not only determined by the rate of return to capital, but also by a change in status preferences over time. This preference-based mechanism enables us to explain the contemporary differentials in wealth- and income inequality across developed countries when they are hit by the identical aggregate shock.

This approach follows, among others, Caselli and Ventura (2000) and García-Peñalosa and Turnovsky (2015), who assume *ex-ante* heterogeneity in wealth and/or abilities. Caselli and Ventura (2000) show that a technology bias (differences in the elasticity of substitution in production) is able to capture the contemporary increase in inequality under a positive productivity shock. In particular, such a shock benefits the holders of capital, if the production function is, or becomes, more intensive in capital. This mechanism is also in line with Piketty's (2014) empirical observation of an increasing capital share in production as economies develop. However, these frameworks do not aim to explain the differentials in savings behavior of rich relative to poor countries, as recent evidence suggests (Dyan, Skinner and Zeldes, 2004, and De Nardi, French and Jones, 2010). Moreover, as technologies in developed countries seem to converge (e.g. according to Caselli and Feyrer, 2007, the marginal product of capital is very similar across countries), we need additional structure to explain why inequality evolves differently in countries having similar factor shares in production. To this end, our

framework complements this literature by providing a preference-based mechanism that operates through the strength in status preferences (implying differential behavior of savings) whose development is captured by cultural characteristics (Acemoglu and Robinson, 2015).

Finally, since the objective of our paper is to focus on the dynamic status effect, which clearly operates primarily through the evolution of wealth, we choose to minimize the role of labor income, by assuming a uniform wage rate and inelastic labor supply. We view our focus on the wealth accumulation aspects very much in the spirit of the pioneering work of Piketty (2014), Saez and Zucman (2016), and others. To this end our endogenous status dynamics on income inequality derives from changes in wealth inequality, and via our framework the evolution of wealth and income inequality will be proportional (as in the historical data of Saez and Zucman, 2016).

### 3. THE MODEL

We modify the standard neoclassical growth model with heterogeneous agents to allow for interdependence in consumption and *endogenous dynamic status* preferences, the strength of which decline as the country develops.

#### 3.1. Households

The economy is populated by a continuum of individuals (households) of mass one, each of whom is endowed with one unit of labor that it supplies inelastically. Units are identical in all respects except for their initial endowment of capital (wealth),  $K_{i,0}$ .<sup>5</sup> At each instant,  $k_i(t) \equiv K_i(t)/K(t)$  is household  $i$ 's share of total wealth.<sup>6</sup> Heterogeneity in wealth shares is summarized by the cumulative distribution function,  $H_i(k_i(t))$  with the standard deviation [coefficient of variation of  $K_i(t)$ ] denoted by  $\sigma_k(t)$ . The initial distribution  $H_0(k_{i,0})$  is exogenous, with standard deviation  $\sigma_{k,0}$ .

**3.1.1. Endogenous Status Preferences.** An individual's utility depends both on his own consumption level,  $C_i(t)$  as well as his consumption relative to some comparison group,  $S_i(C_i(t), \bar{c}(t))$ , where  $\bar{c}(t)$  represents a consumption reference level. The status function,  $S_i(t)$  is

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<sup>5</sup> Restricting labor supply to be inelastic and assuming that individuals are homogeneous in terms of their productivity, has the advantage of sharpening the discussion (and intuition) of the impact of *endogenous dynamic status preferences*. A natural and straightforward extension would allow labor, with heterogeneous skills, to be endogenously supplied; see García-Peñalosa and Turnovsky (2015).

<sup>6</sup> Capital is assumed to be the only asset so that the aggregate capital stock,  $K(t)$  constitutes total wealth.

increasing in  $C_i(t)$ , decreasing in  $\bar{c}(t)$ , and is specified by  $S_i(t) \equiv C_i(t)/\bar{c}(t)$ .<sup>7</sup> We represent the consumption reference level by average consumption, i.e.  $\bar{c}(t) = \int_0^1 C_i(t) di$  where the bar indicates that individual households view the consumption reference level as exogenously given.<sup>8</sup> A preference for relative consumption is frequently termed “positional or status preference”. Our theory of endogenous dynamic status preferences focuses on how intensely  $S_i(t)$  is valued in a given country and how the valuation of  $S_i(t)$  relative to own individual consumption evolves over time, as a country develops, as measured by the *average* capital stock  $k(t)$ . We specify this process by a development-dependent variable,  $\varepsilon(k(t))$ , which measures the relative strength of status preferences, the properties of which are discussed below (for a graphical illustration of our hypothesis see Clark et al., 2008).

Thus, instantaneous individual utility is specified by

$$(1) \quad U(C_i(t), S_i(t), \varepsilon(k(t))) = U\left(C_i(t), \left(\frac{C_i(t)}{\bar{c}(t)}\right), \varepsilon(k(t))\right).$$

Instantaneous utility increases in both individual and relative consumption ( $U_{C_i(t)} > 0, U_{S_i(t)} > 0$ ) and follows the usual concavity conditions in  $C_i(t)$  and  $S_i(t)$ .

To capture the weight that is being applied to the absolute and relative consumption levels, we introduce the notion of the *degree of positionality* (DOP). The DOP, as defined by Johansson-Stenman et al. (2002), reflects the proportion of the total marginal utility of individual consumption that can be attributed to its impact on the increase in relative consumption. Formally, we specify this by

$$(2) \quad DOP_i(t) = \frac{(\partial U / \partial S_i(t))(\partial S_i(t) / \partial C_i(t))}{(\partial U / \partial S_i(t))(\partial S_i(t) / \partial C_i(t)) + \partial U / \partial C_i(t)}.$$

Thus, if  $DOP_i(t) = 0.4$ , then 40% of marginal utility of consumption arises from an increase in relative consumption, and 60% of marginal utility of consumption arises from an increase in own *absolute* consumption (holding  $S_i(t)$  fixed). To render our analysis tractable, we introduce

**Assumption 1.** *The instantaneous utility function  $U(C_i(t), S_i(t), \varepsilon(k(t)))$  is homogeneous of degree*

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<sup>7</sup> This specification of status preferences in relative terms – by  $C_i(t)/\bar{c}(t)$  – is prevalent throughout the literature; see. e.g. Galí (1994). Formulating it as a difference,  $S_i(t) = C_i(t) - \bar{c}(t)$  is also possible and yields essentially equivalent results.

<sup>8</sup> Clearly, the consumption reference level might differ from  $\bar{c}(t)$ . In this paper, however, we focus on the endogeneity of status preferences and would otherwise like to keep the setup as simple as possible.

$R$  in  $C_i(t)$ . Specifically,  $U(C_i(t), S_i(t), \varepsilon(k(t))) = C_i(t)^R V(\bar{c}(t), \varepsilon(k(t)))$ , where  $V_{\bar{c}(t)} < 0$ , the elasticity  $V_{\varepsilon(k(t))} \varepsilon(k(t)) / V > 0$ , and subscripts to the function  $V$  denote partial derivatives.

Adopting Assumption 1, the utility from status,  $V(\bar{c}(t), \varepsilon(k(t)))$ , is decreasing in the consumption reference level and increasing in the strength of status concerns. Also, as shown in Appendix A.1, the degree of positionality becomes

$$(3) \quad DOP(\bar{c}(t), k(t)) = -\frac{V_{\bar{c}(t)}(\bar{c}(t), \varepsilon(k(t)))\bar{c}(t)}{RV(\bar{c}(t), \varepsilon(k(t)))},$$

implying that the DOP is identical for all individuals. As seen in (3), the DOP is a function of both the reference consumption level,  $\bar{c}$ , and the average stock of capital,  $k$ , and we incorporate the fact that the DOP declines with average wealth by endogenizing  $\varepsilon(k(t))$  in accordance with:

**Assumption 2.** *The properties of  $\varepsilon(t) \equiv \varepsilon(k(t))$  are:*

- (i).  $\varepsilon(t) > 0$  is strictly positive and continuous;
- (ii).  $\varepsilon'(t) \equiv \partial \varepsilon(t) / \partial k(t) < 0$ ;
- (iii).  $\lim_{k(t) \rightarrow 0} \varepsilon(t) = \bar{\varepsilon} > 0$  and  $0 < \lim_{k(t) \rightarrow \infty} \varepsilon(t) = \underline{\varepsilon} < 1$ , with  $\bar{\varepsilon} > \underline{\varepsilon}$ .

Assumptions (2.i) and (2.iii) characterize the concern for status (positional preferences). Households do not choose their individual DOP to display status. Rather, the strength of the status preference is socially determined by the society's wealth (proxied by average wealth), which individuals treat as given. Assumption (2.ii) asserts that the strength of status concerns declines with wealth (income), as suggested by Figure 3, and the empirical evidence summarized in Section 2.<sup>9</sup> That is, agents are more concerned with status in a low-wealth society than in a high-wealth society.

**3.1.2. Household Optimization.** The household's optimization problem is to choose a consumption stream,  $C_i(t)$ , and to accumulate capital,  $K_i(t)$ , so as to maximize intertemporal utility

$$(4) \quad \int_0^{\infty} U(C_i(t), S_i(t), \varepsilon(k(t))) e^{-\beta t} dt, \beta > 0,$$

subject to the flow budget constraint:

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<sup>9</sup> Specifying status as a function of average income,  $y$ , rather than average wealth is an alternative hypothesis, which in our case yields equivalent results. The equivalence is immediately apparent by recognizing the form of the production function,  $y = f(k)$ , (see (8) below), the inelastic labor supply, and that capital is the only source of wealth.

$$(5) \quad \dot{K}_i(t) = r(t)K_i(t) + w(t) - C_i(t) ,$$

the initial endowment of capital,  $K_{i,0}$ , the transversality condition, and taking  $\bar{c}(t)$  and  $k(t)$  as given. In (4) and (5),  $\beta$  is the constant pure rate of time preference,  $r(t)$  is the real return on asset (capital) and  $w(t)$  is the wage rate, also all taken as given.

Solving the intertemporal maximization problem, the individual's equilibrium consumption growth rate is given by (see Appendix A.2):

$$(6) \quad \frac{\dot{C}_i(t)}{C_i(t)} = \frac{1}{1 - R(1 - DOP(t))} \left[ r(t) - \beta + \left( \frac{V_{\varepsilon(t)} \varepsilon'(k(t))}{V(t)} \right) \dot{k}(t) \right].$$

Equation (6) represents the usual Euler equation, modified by the dynamic status effect. The individual's consumption growth depends positively on the difference between the return on assets and the pure rate of time preference (return-to-capital effect). In the absence of positional preferences ( $DOP = 0 = \varepsilon'(k)$ ), the optimal consumption growth rate (6) reduces to that of the standard neoclassical growth model. Since the right-hand side of (6) are aggregates, observed by all agents, individuals therefore experience a common consumption growth rate, which also equals the average growth rate,  $\dot{c}(t)/c(t)$ . Accordingly,  $C_i(t)/c(t) = \theta_i$ , where  $\theta_i$  is constant.

Positional preferences modify the optimal consumption growth rate in two ways. First, they impact the intertemporal elasticity of substitution (IES), which is now given by<sup>10</sup>

$$(7) \quad IES(c(t), k(t)) = \frac{1}{1 - R(1 - DOP(c(t), k(t)))} > 0 .$$

If  $R < 0$ , as empirical evidence overwhelmingly suggests, positionality raises the IES, relative to that of the standard neoclassical growth model,  $(1 - R)^{-1}$ .<sup>11</sup> For a given interest rate, individuals raise the optimal consumption growth rate, as documented by, among others, Liu and Turnovsky (2005).

Second, positional preferences introduce a dynamic status effect. If  $\dot{k} > 0$ , under Assumption 2(ii), the status effect causes the optimal consumption growth rate to decline as a country develops. The intertemporal consumption decision is affected by the degree to which people evaluate their social

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<sup>10</sup> By taking into account the impact of the consumption externality on the agent's intertemporal substitution, (7) can be interpreted as measuring the "social intertemporal elasticity of substitution".

<sup>11</sup> See e.g. Guvenen (2006) for extensive empirical evidence on the IES, and indirectly on  $R$ .

status over time. The more agents evaluate their *relative* position, the more they consume in order to raise their respective relative position. However, as the economy accumulates capital, the degree of positionality declines. That is, the marginal utility derived from relative consumption decreases over time. As a consequence, consumption is shifted from the future to the present, and the optimal consumption growth rate declines. The latter has an impact on both the level of the saving rate and its subsequent evolution. As discussed below, the level of the saving rate is reduced, and its rate of change becomes positive along transitional paths. It is this effect of positional preferences that we emphasize.

### 3.2. Production

There is a single representative firm, which produces aggregate output,  $Y(t)$ , in accordance with the Cobb-Douglas production function

$$(8) \quad Y(t) = F(K(t), L(t), A) = AK(t)^\alpha L(t)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where  $K(t)$ ,  $L(t)$  denote capital and labor inputs, and  $A$  represents total factor productivity (TFP). Dividing through by  $L(t)$  this can be expressed in the usual per capita form  $y(t) \equiv f(k(t)) = Ak(t)^\alpha$  where  $y(t) \equiv Y(t)/L(t)$ ,  $k(t) \equiv K(t)/L(t)$ .

Labor endowment is normalized to unity, and we assume no population growth. The representative firm maximizes profit,  $\pi(t) = Y(t) - w(t)L(t) - (r(t) + \delta)K(t)$  where  $\delta \geq 0$  is the depreciation rate of capital, yielding the standard first-order optimality conditions:

$$(9) \quad \begin{aligned} r(t) &= \alpha AL(t)^{1-\alpha} K(t)^{\alpha-1} - \delta = \alpha k(t)^{\alpha-1} - \delta \\ w(t) &= (1-\alpha)AL(t)^{-\alpha} K(t)^\alpha = (1-\alpha)Ak(t)^\alpha. \end{aligned}$$

## 4. EQUILIBRIUM AND THE DYNAMICS OF SAVINGS

In this section we solve for a competitive equilibrium and analyze its properties.

**Definition 1.** A competitive equilibrium is a price vector  $(r(t), w(t))$  and an attainable allocation for  $t \geq 0$  such that:

- (i) Individuals choose  $C_i(t)$  and  $K_i(t)$  to maximize their intertemporal utility function, given factor prices, initial wealth endowments, aggregate capital, and the consumption reference level.
- (ii) Firms choose  $K(t)$  and  $L(t)$  to maximize profits, given the factor prices.
- (iii) All markets clear. Capital market clearing implies  $k(t) = K(t)$  (total assets held by agents equal

the firms capital stock). Labor market clearing implies  $L(t) = 1$ .

(iv) Aggregation:  $K(t) = \int_0^1 K_i(t) di = k(t)$ , and  $C(t) = \int_0^1 C_i(t) di = c(t)$ .

(v) Consumption reference level:  $\bar{c}(t) = c(t)$ ,  $c(t) \equiv C(t)/L(t)$  denotes per capita consumption.

From (iii), we observe that the mean individual to total wealth ratio equals unity:  $\int_0^1 k_i(t) di = \int_0^1 K_i(t)/K(t) di = 1$ . While individual households take the consumption reference level,  $\bar{c}(t)$ , as given, in equilibrium we assume that the consumption reference level is determined by the economy-wide average consumption level, in accordance with (v).

Combining equations (5)-(9), and assuming market clearing (and aggregation), we obtain the equilibrium dynamics of the aggregate (average) economy-wide variables:

$$(10a) \quad \dot{k}(t) = f(k(t)) - c(t) - \delta k(t)$$

$$(10b) \quad \dot{c}(t) = \frac{c(t)}{1 - R(1 - DOP(t))} \left[ f'(k(t)) - (\delta + \beta) + \left( \frac{V_\varepsilon(t) \varepsilon'(k(t))}{V(t)} \right) \dot{k}(t) \right].$$

where  $DOP(t)$  is defined by (3),  $f(k(t)) \equiv Ak(t)^\alpha$ ,  $f'(k(t)) \equiv \alpha Ak(t)^{\alpha-1}$ . Defining the elasticity of status-utility with respect to  $k$  by  $E(c, k) \equiv V_k k / V = V_\varepsilon \varepsilon'(k) k / V \leq 0$ , we can conveniently rewrite the dynamic system to include (10a) and

$$(10b') \quad \dot{c}(t) = c(t) IES(c(t), k(t)) \left[ f'(k(t)) - (\delta + \beta) + \left( \frac{E(c(t), k(t))}{k(t)} \right) \dot{k}(t) \right].$$

In the absence of the endogenous dynamic status effect  $\varepsilon'(t) = 0 = E(c, k)$ , while in its presence  $\varepsilon'(t) < 0 \Rightarrow E(c, k) < 0$ , in which case there is a bi-directional relation between the average capital stock and status preference. That is, not only do status concerns affect the equilibrium dynamics, but the level of economic development, as reflected by  $k(t)$ , feeds back onto the formation of status preferences. Finally, we define the saving rate by

$$s(t) = 1 - \frac{c(t)}{f(k(t))} = 1 - \frac{c(t)}{Ak(t)^\alpha}.$$

Setting  $\dot{c}(t) = \dot{k}(t) = 0$  in (10a, 10b'), the steady-state per capita capital and consumption,  $(k^*, c^*)$ , are

$$(11a) \quad k^* = \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{1}{1-\alpha}} > 0, \quad c^* = \beta \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{1}{1-\alpha}} + A(1-\alpha) \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{\alpha}{1-\alpha}} > 0,$$

which further yield the long-run capital-output and consumption-output ratios and saving rate

$$(11b) \quad \frac{k^*}{y^*} = \frac{\alpha}{\beta + \delta}, \quad \frac{c^*}{y^*} = \frac{\beta + \delta(1-\alpha)}{\beta + \delta}, \quad s^* = \frac{\alpha\delta}{\beta + \delta}.$$

The steady-state quantities are unique and positive, with the saving rate lying in the range,  $0 < s^* < \alpha$ . They are also independent of the (dynamic) status preferences and therefore identical to those of the standard neoclassical growth model. This is because the strength of status preferences does not affect the steady-state production process, which is the driving force behind the long-run equilibrium.

Linearizing the dynamic system (10a, 10b') around the steady state, one can show that the determinant of the Jacobian matrix of coefficients of the linearized system is negative implying that the unique steady state is a saddle point. From (10a) the stable saddlepath can be expressed as

$$c(t) - c^* = (\beta - \mu^*)(k(t) - k^*),$$

where  $\mu^* < 0$  is the stable eigenvalue. By impacting  $\mu^*$ , the dynamic status effect does affect the transitional dynamics and the distribution of income and wealth, as the change in the intensity of status matters for agents' intertemporal decisions.<sup>12</sup>

The impact of the changing status on savings behavior is summarized by the proposition:

**Proposition 1.** *Given (a) Assumptions 1 and 2, and, in particular, the endogenous dynamic status effect,  $\varepsilon'(t) < 0$ , (b) the Cobb-Douglas production technology, and (c) assume  $k_0 < k^*$ . During the transition associated with an increasing capital stock:*

(i) *The dynamics of the saving rate are characterized as follows:*

$$(12) \quad \text{If } s^* \geq \frac{IES(c(t), k(t))}{\xi(c(t), k(t))} \text{ for all } k(t) \in [k_0, k^*], \text{ then}$$

$$s(t) \geq 0 \text{ for a range of } k(t) \text{ in } [k_0, k^*], \quad \text{where}$$

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<sup>12</sup> This characteristic is identical to the conventional model where status preferences are exogenously fixed; see Liu and Turnovsky (2005). As in that model, status preferences have only long-run effects if labor supply is elastic. As we show below, isolating any long-run productive effects of status is quite helpful in facilitating comparisons between economies with similar income per capita while having different levels of income inequality (e.g. US vs. Europe).



$$(13) \quad \xi(c(t), k(t)) \equiv 1 - \frac{IES(c(t), k(t)) E(c(t), k(t))}{\alpha} \geq 1,$$

and  $E(c(t), k(t))$  denotes the elasticity of status with respect to  $k$ .

The saving rate may therefore be increasing, decreasing, or non-monotonic.

(ii) The interest rate is always declining monotonically.

**Proof.** See Appendix A.3.

Proposition 1 provides a general characterization of the equilibrium dynamics of saving and the rate of return, as capital is accumulated monotonically. Conditions under which the saving rate may be associated with non-monotonic behavior are identified in Corollary 1. This has been observed to be relevant for the US as well as other OECD countries (see Antràs, 2001), and examples illustrating this possibility are provided in Appendix A.4.

**Corollary 1.** Consider the above assumptions (a), (b), and (c) applicable to Proposition 1.

(i) During the transition associated with an increasing capital stock the dynamics of the saving rate are characterized by a non-monotonic (an inverse U-shaped) path if:

$$\frac{IES(c(0), k(0))}{\xi(c(0), k(0))} < s^* < \frac{IES(c^*, k^*)}{\xi(c^*, k^*)}.$$

The saving rate increases for low  $k$  and decreases for high  $k$ . Specifically, the saving rate increases for  $k < \tilde{k}$  and decreases for  $k > \tilde{k}$ , where  $\tilde{k}$  is implicitly determined by:

$$f'(\tilde{k}) \left[ \xi(\tilde{k}) s(\tilde{k}) - IES(\tilde{k}) \right] = (\beta + \delta) \left[ \xi(\tilde{k}) s^* - IES(\tilde{k}) \right].$$

(ii) Depending on the transitional behavior of  $IES(c(t), k(t)) / \xi(c(t), k(t))$  other forms of non-monotonicities are possible.

**Proof.** See Appendix A.4.

Proposition 1 shows that the transitional dynamics of the saving rate need not follow the steady decline implied by the standard neoclassical growth model for plausible parameterization. To see the intuition for this result consider first the (standard) neoclassical growth model. An increase in the capital stock reduces its rate of return, which imposes both a substitution effect and an income effect. According to the former, the price of future consumption rises relative to that of present consumption.

Consequently, current consumption increases, thereby reducing the saving rate. In the case of the latter, the lower return to capital reduces income for both present and future consumption. Accordingly, individuals tend to reduce current consumption, thereby raising the saving rate. For standard parameterization, the substitution effect dominates the income effect and thus the neoclassical growth model predicts a declining saving rate as capital increases (Barro and Sala-i-Martin 2004, p.136).<sup>13</sup>

Consider now our augmented neoclassical growth model. The dynamic endogenous status effect introduces a third channel, whereby an increase in the capital stock impinges on the intertemporal consumption-savings decision. This effect tends to increase the saving rate over time (as capital is accumulated). As the capital stock increases, agents choose a lower rate of consumption growth, together with an initially higher level of consumption, in comparison to the standard neoclassical growth model. This is evident from (10b') due to the fact that  $E(c, k) < 0$ . The higher initial consumption level necessitates a lower *initial* saving rate, compared to the standard neoclassical model. Recall that the steady-state saving rate is unaffected by status preferences. Consequently, the presence of dynamic endogenous status preferences implies either a lower rate of decline of the saving rate or an increasing saving rate along the transitional path toward its steady state. In particular, if the dynamic status effect is sufficiently strong – that is, the absolute value of  $E(c, k)$  is sufficiently large – then the consumption growth rate is lower than the output growth rate, and the saving rate increases along its transitional path.<sup>14</sup> In other words, even when the substitution effect exceeds the income effect, our extended model can produce an increasing saving rate under plausible calibrations.

More formally, in the absence of dynamic status preferences (i.e.  $E(c, k) = 0 \Leftrightarrow \xi(c, k) = 1$ ), condition (i) of Proposition 1 reduces to Barro and Sala-i-Martin's (2004) familiar condition,  $\dot{s} \leq 0 \Leftrightarrow s^* \leq IES(c, k)$ .<sup>15</sup> However, in the presence of the endogenous dynamic status effect,  $\xi(c, k) > 1$ , and is unconstrained by any upper bound. For this reason, if  $\xi(c, k)$  is large enough,  $\dot{s} > 0$  during transition. This holds true even when  $s^* < IES(c, k)$ , i.e. the substitution effect exceeds the income

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<sup>13</sup> Assuming that the long-run savings rate  $s^* < IES$  is the plausible case, Barro and Sala-i-Martin (2004) show that this implies  $\dot{s}(t) < 0$ .

<sup>14</sup> Notice that  $s = 1 - c / f(k)$ . Clearly, the proposition allows for a third pattern according to which the saving rate first increases, overshoots its steady-state level, to which it eventually declines.

<sup>15</sup> Unless needed for clarity, we omit time indexes in what follows.

effect, following empirical evidence (among many others, see Barro and Sala-i-Martin, 2004). In this latter case, though, for large  $k$ ,  $E(c, k)$  becomes close to zero, thus  $\xi \approx 1$ , and the saving rate eventually declines.

To summarize: On the one hand, the increase in capital reduces the return to capital. This tends to lower the saving rate, which, empirically, dominates the income effect. On the other hand, the increase in capital reduces the consumption growth rate via the endogenous dynamic status effect ( $\varepsilon'(k) < 0$ ). The lower consumption growth rate tends to raise the saving rate. As long as the dynamic status effect dominates the return-to-capital effect, the saving rate increases during transition.

#### 4.1. *An Example of Endogenous Dynamic Status Preferences*

In this subsection, we employ numerical simulations to provide an example of our analytical results and to illustrate the performance of our model with respect to historical data. Preferences are specified by the CES utility function which satisfies our assumptions:

$$(14) \quad U = \frac{1}{R} \left( [1 - \varepsilon(k)] C_i^\rho + \varepsilon(k) \left( \frac{C_i}{\bar{c}} \right)^\rho \right)^{\frac{R}{\rho}} = \frac{C_i^R}{R} \left( [1 - \varepsilon(k)] + \varepsilon(k) \bar{c}^{-\rho} \right)^{\frac{R}{\rho}}.$$

The degree of homogeneity of  $U$  is  $R$  and the corresponding degree of positionality is

$$DOP = \frac{\varepsilon(k) \bar{c}^{-\rho}}{1 - \varepsilon(k) + \varepsilon(k) \bar{c}^{-\rho}}.$$

Letting  $\rho \rightarrow 0$  yields the Cobb-Douglas case, and  $DOP = \varepsilon(k)$ . Technology remains specified by the Cobb-Douglas function (8). For the evolution of the dynamic status preferences, we use an explicit function that satisfies Assumption 2:

$$(15) \quad \varepsilon(k) = \bar{\varepsilon} - (\bar{\varepsilon} - \underline{\varepsilon}) \left( \frac{k - k_0 + \Delta}{k^* - k_0 + \Delta} \right)^\kappa, \quad \bar{\varepsilon} > \underline{\varepsilon} \geq 0, \quad \kappa \geq 0, \quad \Delta > 0 \text{ (and small)}.$$

In (15)  $\kappa$  denotes the strength of the dynamic status effect (the sensitivity of  $\varepsilon(k)$  with respect to a change in  $k$ ). The limiting case  $\kappa = 0$  corresponds to the static status assumption, in which case  $\varepsilon(k) = \underline{\varepsilon}$  for all time (for all  $k$ ). The case  $\kappa > 0$  corresponds to the dynamic status model, in which

$\varepsilon(k)$  declines from (close to)  $\bar{\varepsilon}$  to  $\underline{\varepsilon}$ .<sup>16</sup> While both specifications (static, dynamic status) share the same steady-state value of  $\varepsilon(k) = \underline{\varepsilon}$ , the model with dynamic status starts with a higher value of  $\varepsilon(k)$  (arbitrarily) close to  $\bar{\varepsilon}$ . Consequently,  $\varepsilon(k)$  is declining from (close to)  $\bar{\varepsilon}$  to  $\underline{\varepsilon}$ . Also, parameter  $\kappa$  determines the speed at which the degree of positionality declines. If  $\kappa < 1$  ( $\kappa > 1$ ) then  $\varepsilon(k)$  declines most intensively initially (asymptotically) as  $k(t)$  is accumulated from  $k_0$  to  $k^*$ .

The parameterization is standard in the growth literature, and is based on a time unit of one year. The technology parameters are assigned the following values:  $\alpha = 0.36$ ,  $A = 2$  and  $\delta = 0.05$ . The preference parameters assume the following values:  $\beta = 0.04$ ,  $R = -2.5$ , implying an IES less than unity. Finally, the status parameters for our base case are  $\bar{\varepsilon} = 0.9$ ,  $\underline{\varepsilon} = 0.2$ , with  $\kappa = 0.4$  for the dynamic status case and  $\kappa = 0$  for the static status case. For this parameterization, the steady-state capital stock  $k^* = 25.77$ . We consider the transitional dynamics when the economy starts with a capital stock  $k_0 = (1/4)k^*$ , that is clearly well below its steady-state level.<sup>17</sup>

Figure 4 displays the transitional paths of (i) the saving rate, (ii) the rate of interest, and (iii) the degree of positionality  $\varepsilon$ . The solid lines display the transitional dynamics in the presence of dynamic status effects. Specifically, as analyzed above (as well as in the Appendix), in early stages of development savings increase, and after a threshold level of the capital stock is reached, savings level out. Thus, our model, augmented to include dynamic status, is able to capture both the joint historical dynamics of the savings and real interest rates (Fact 1).<sup>18</sup> The dashed lines in Figure 4 display the transitional dynamics of the saving rate in the absence of the dynamic status effect  $\varepsilon' = 0$ , thus  $\varepsilon(t) = \underline{\varepsilon} > 0$  which is constant). Without the dynamic status effect, the saving rate always decreases (due to the return-to-capital effect). Specifically, we have  $0.2 = s^* < \text{IES} = [1 - R(1 - \varepsilon)]^{-1} = 0.33$ , in which case, as shown by Barro and Sala-i-Martin (p.136, 2004),  $\dot{s} < 0$ . By contrast, in the presence of the

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<sup>16</sup> Thus  $\varepsilon(k_0) = \bar{\varepsilon} - (\bar{\varepsilon} - \underline{\varepsilon})(\Delta / (k^* - k_0 + \Delta))^\kappa$ . The reason for setting  $k_0$  in this manner is simply to be consistent with Assumption 2(iii) which defines  $\bar{\varepsilon} = \lim_{k \rightarrow 0} \varepsilon(k)$ .

<sup>17</sup> We simulate the dynamic system using the Relaxation Algorithm, described in Trimborn et al. (2008). Since this a global technique, the fact that the initial capital stock is well below its steady-state equilibrium does not cause any problem.

<sup>18</sup> Corollary 1 allows for an increasing saving rate for a low level of capital stock while a decreasing rate at later stages of development. Although for many countries the saving rate increases historically and contemporarily, Corollary 1 (Figure A.1) captures the inverse U-Shaped dynamic behavior of the saving rate in US as noticed by, among others, Antràs (2001).

dynamic status effect, we have  $0.2 = s^* > (IES(c, k)/\xi(c, k))$  for all  $k \in [k_0, k^*]$ .<sup>19</sup> From Proposition 1 (and as depicted in Figure 4), in this case it follows that  $\dot{s} > 0$  for some  $k$ .

Several observations merit comment. First, *in contrast to the standard neoclassical growth model*, the positive correlation between the saving rate and the level of development in our model helps to explain the cross-country evidence where the saving rate increases historically as presented in Fact 1 and Section 2.2. Second, Figure 4 also shows that the rate of return declines at a slower pace when  $\varepsilon' < 0$ , as a result of the lower level of the saving rate in the presence of the dynamic status effect. This, in turn impacts the development of inequality: inequality increases (decreases) in the presence (absence) of the dynamic status effect, thereby providing a new mechanism to capture the aforementioned empirical facts (Facts 2 and 3). Section 5 formally proves and explains this behavior.

Finally, the level and development of the saving rate across time play a crucial role with respect to the speed of convergence to the long-run equilibrium. This becomes even more important in a heterogeneous agent world in which people differ in their initial wealth endowments. In Section 5 below, we show how the interplay of the endogenous dynamic status- and return-to-capital effects, by affecting the speed of convergence, helps to explain the behavior of income inequality qualitatively. Considering the transitional dynamics of the rates of return to capital in Figure 4, we see that the dynamic status effect slows the decline in the rate of return (the solid line is located weakly above the dashed line). This indicates a negative impact of the dynamic status effect on the (average) speed of convergence. The following table quantifies this impact.

Table 1 shows the average speed of convergence (ASOC) for static ( $\kappa = 0$ ) and dynamic ( $\kappa = 0.4, 0.7, 1.2$ ) status.<sup>20</sup> The ASOC of  $k$  measures the rate of decline of the distance of  $k$  to its steady-state value at a given point in time, during specified time intervals. Consider a fraction  $\tau \in (0, 1)$  of the initial distance from the steady state:  $\tau |k_0 - k^*|$ . For a fixed fraction,  $\tau$ , the ASOC of  $k$  during the time interval needed for the fraction  $(1 - \tau)$  of the initial distance from the steady-state value to be completed is defined by  $\eta$  satisfying

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<sup>19</sup> For this simulation,  $(IES(c, k)/\xi(c, k))$  increases from 0.01 to 0.14 during the transitional path.

<sup>20</sup> The *asymptotic* speed of convergence need not generally be informative, as the *dynamic* status effect disappears asymptotically. Hence, in the numerical simulations, we focus on the ASOC.

$$(16) \quad |k(t_\tau) - k^*| = |k_0 - k^*| e^{-\eta t_\tau},$$

where  $t_\tau$  is the minimum real number such that  $|k(t) - k^*| < \tau |k(0) - k^*|$  for all  $t > t_\tau$ .<sup>21</sup>

Table 1 provides the following two key insights. First, the ASOC varies greatly along the transitional paths and is lower initially, when the dynamic status effect is stronger, than when close to the steady state (where the dynamic status effect tends to disappear). Second, in all cases, the ASOC is substantially higher - often twice as high - in the absence of the dynamic status effect ( $\kappa = 0$ ) than when it is present ( $\kappa > 0$ ). These differences can be traced back to the saving rate behavior, as analyzed above. A lower *level* of the saving rate reduces the ASOC - thereby slowing the pace at which the rate of interest declines along transitional paths. The latter has major implications for the transitional dynamics of inequality, as analyzed in Section 5.

## 5. WEALTH (AND INCOME) INEQUALITY

It is well known that under our assumptions of (i) inelastic labor supply and (ii) Cobb-Douglas production functions, income inequality, as measured by the coefficient of variation (CV), is strictly proportional to wealth inequality; see e.g. Turnovsky and García-Peñalosa (2008). The two measures therefore move identically enabling us to refer to them interchangeably, or simply as inequality. We first characterize analytically the main mechanism driving the evolution of inequality. We then examine the comparative inequality dynamics across countries that experience the identical productivity shock, but differ in the intensities of their respective status preferences responses to the productivity shock-induced change in  $k$ . Our analytical results are illustrated by numerical examples.

### 5.1. *The Dynamics of Inequality*

We first determine the equilibrium dynamics of individual  $i$ 's share of total capital,  $k_i(t)$ . To do so, we combine the individual wealth accumulation equation (5), together with the corresponding aggregate accumulation relationship,  $\dot{K}(t) = r(t)K(t) + w(t) - C(t)$ , to yield:

$$(17) \quad \dot{k}_i(t) = \frac{w(t)}{k(t)}(1 - k_i(t)) + \frac{c(t)}{k(t)}(-\theta_i + k_i(t)),$$

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<sup>21</sup> For calculating the ASOC, we consider  $|k(t_\tau) - k^*| = \tau |k(0) - k^*|$  in (16). Hence,  $\eta = -\ln \tau / t_\tau$ .

where, recalling (6),  $\theta_i \equiv C_i(t)/c(t)$  is constant, and is obtained by considering the steady state of (17), viz.  $\theta_i = k_i^* + (w^*/c^*)(1 - k_i^*)$ . Following the identical procedure described by García-Peñalosa and Turnovsky (2008, p. 463ff) the unique bounded solution for  $k_i(t)$  is<sup>22</sup>

$$(18) \quad k_i(t) = k_i^* + h(k^*)(k_i^* - 1) \left( \frac{k(t) - k^*}{k^*} \right)^{\beta - \mu^*},$$

where variables with an asterisk are *final* steady-state values,  $h(k^*) = -f''(k^*) - v_1^* w^*/c^*$ ,  $f''(k^*) = A\alpha(\alpha - 1)(k^*)^{\alpha - 2}$ ,  $\mu^*$  is the negative eigenvalue associated with the dynamic system (10a, 10b'), and  $\beta - \mu^*$  will be recalled is the slope of the stable saddlepath.

As the sign of  $h(k^*)$  plays a key role for the shock-induced development of income inequality, we need to investigate this term further. First,  $h(k^*)$  depends only on average economy-wide characteristics and therefore impacts all agents identically. This is a reflection of the underlying assumption of homogeneity. Second, for the Cobb-Douglas technology we are assuming,  $\text{sgn}[h(k^*)]$  is shown in Appendix A.5 to be:

$$\text{sgn}[h(k^*)] = \text{sgn}[(1 - \alpha)\delta + \mu^*],$$

which in general is ambiguous, and involves a tradeoff between the productivity characteristics, reflected by  $\alpha, \delta$ , and the asymptotic speed of convergence as determined by  $\mu^*$ . If status preferences are exogenous ( $\varepsilon'(t) = 0$ )  $\mu^*$  dominates and  $h(k^*) < 0$ . However, in the present case,  $\text{sgn}[h(k^*)]$  also depends on the change of the intensity of status concerns,  $\varepsilon'(t)$ , via its impact on the negative eigenvalue,  $\mu^*$ . If  $\varepsilon'(t) < 0$  and large enough (in absolute terms), then  $h(k^*)$  becomes positive, and dominates the transitional dynamics of inequality induced by shocks.<sup>23</sup>

Integrating (18) across agents, García-Peñalosa and Turnovsky (2008) show that the dynamics of the coefficient of variation of wealth across agents (treated as a measure of inequality) are given by

$$(19) \quad \sigma_k(t) = \frac{\zeta(t)}{\zeta_0} \sigma_{k,0},$$

<sup>22</sup> The unbounded solutions are rejected since they are associated with degenerate wealth distributions.

<sup>23</sup> Other aspects of the technology are also involved in the tradeoff determining  $\text{sgn}[h(k^*)]$ . For example, García-Peñalosa and Turnovsky (2008) obtain a tradeoff in terms of the elasticity of substitution in production, showing that while  $h(k^*) < 0$  for the Cobb-Douglas technology,  $h(k^*) > 0$  is possible if the elasticity is sufficiently small.

where  $\zeta(t) \equiv 1 + \frac{h(k^*)}{\beta - \mu^*} \frac{k(t) - k^*}{k^*}$  and  $\zeta_0 \equiv 1 + \frac{h(k^*)}{\beta - \mu^*} \frac{k_0 - k^*}{k^*}$ .

Since the solution (18) is only local, equation (19) serves as a measure of the transitional development of inequality, as average wealth accumulates, *close to a steady state*. It is seen from (19) that for  $k_0 < k(t) < k^*$ ,  $\sigma_k(t) \geq \sigma_{k,0}$  according as  $h(k^*) \geq 0$ , and depends upon only average wealth.<sup>24</sup>

It is well known that aggregate measures of inequality, such as  $\sigma_k(t)$  may conceal the impact of structural changes on different parts of the wealth/income distribution. Using (19) we can rewrite (18) as  $k_i(t) - 1 = (k_i^* - 1)\zeta(t)$  and taking the time derivative we obtain

$$(20) \quad \frac{\dot{k}_i(t)}{k_i(t) - 1} = \frac{\dot{\zeta}(t)}{\zeta(t)} = \left( \frac{h(k^*)}{(\beta - \mu^*)k^*} \right) \frac{\dot{k}(t)}{\zeta(t)}.$$

From (20), and recalling that  $\dot{k}(t) > 0$ , we infer the following. If  $h(k^*) > 0$  so that inequality is increasing, agents whose wealth is above the mean ( $k_i(t) > 1$ ), will increase their relative wealth ( $\dot{k}_i(t) > 0$ ), while the relative wealth of agents having below average wealth will decline. In other words, the rich will get relatively richer and the poor, relatively poorer. If  $h(k^*) < 0$  the opposite will apply, and the aggregate wealth distribution will become more compressed.<sup>25</sup>

In the previous section, we have shown that the endogenous dynamic status effect influences the transitional dynamics of the interest rate (see Figure 4), causing it to decline at a slower pace. This, in turn, impinges on the development of inequality – both during transition and in steady state – and leads us to the following condition determining  $h(k^*)$  and the resulting response of inequality.

**Proposition 2.** *Consider the above assumptions (a), (b) and (c) applicable to Proposition 1. For any initial distribution and standard deviation of wealth, in the neighborhood of the steady state where  $k_0 < k^*$ , inequality rises (falls), if  $h(k^*) > 0$  (if  $h(k^*) < 0$ ):*

$$(21) \quad h(k^*) \geq 0 \Leftrightarrow -\frac{E(c^*, k^*)}{\alpha} \geq \left[ \frac{1}{s^*} - \frac{1}{IES(c^*, k^*)} \right].$$

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<sup>24</sup> In making this statement we are assuming that  $k_0$  is sufficiently close to  $k^*$  to ensure that  $\zeta_0 > 0$  and therefore  $\zeta(t) > 0$  during the transition. This condition is easily met for our simulations for inequality, where we assume  $k_0 = (3/4)k^*$ .

<sup>25</sup> This is a reflection of the “representative agent theory of distribution” that we are employing, and as Caselli and Ventura (2000) have named it.



**Proof.** See Appendix A.5.

Consider first the case of no dynamic status effect,  $E(c^*, k^*) = 0$ . If the substitution effect is sufficiently strong, then  $s^* < IES(c^*, k^*)$ , as empirical evidence suggests. Condition (21) then implies  $h(k^*) < 0$ , so that inequality declines. Intuitively, the saving rate is high and declining toward its steady-state value, in accordance with Proposition 1. As a result, the rate of capital accumulation (and speed of convergence) is high as well. This causes the return to capital to decline rapidly, which disadvantages the wealthy households more than it does the poor. As a consequence, inequality declines (Fact 2).

Now, consider the impact of a dynamic status effect,  $E(c^*, k^*) < 0$ . Once this effect becomes sufficiently strong  $E(c^*, k^*) < 0$  and  $h(k^*) > 0$ , in (21). In this case, the dynamic status effect induces households to reduce their consumption growth rate, *ceteris paribus*. In conjunction with the lower consumption growth rate, households initially raise their consumption level and reduce their saving rate. As capital increases, the saving rate rises toward its new steady-state level. Since during transition the level of the saving rate is less than when  $E(c^*, k^*) = 0$ , capital is being accumulated at a slower rate (and the speed of convergence is lower). Therefore, the rate of interest declines at a slower pace. This benefits the wealthy households, whose share of income from capital is large, more than it does the poor. As a result, wealth inequality increases along the transition (Fact 3).

To shed additional light on Proposition 2, we consider the following corollary as well as providing some numerical simulations.

**Corollary 2.** *In a neighborhood of the steady state, conditions (21) in Proposition 2 and (12) in Proposition 1 are equivalent.*

**Proof.** *Considering  $\xi(c^*, k^*) \equiv 1 - IES(c^*, k^*)E(c^*, k^*)/\alpha$  in (12) and rearranging terms immediately yields the right-hand side of the equivalence in (21). As the right-hand sides of the equivalences in (12) and (21) are identical, the left hand sides are identical as well. Thus,  $h(k^*) \geq 0 \Leftrightarrow \dot{s} \geq 0$ .*

Corollary 2 states that the saving rate behavior and the development of inequality are closely

linked. In particular, the presence of a strong dynamic status effect can explain the joint occurrence of increasing savings, together with income inequality, even when the substitution effect is high, that is when  $s^* < IES(c^*, k^*)$ , as found by Saez and Zucman (2016). In the presence of a sufficiently responsive dynamic status effect (high  $\xi$ ), for any initial capital stock the saving rate is initially lower than in its absence, and, in turn, increases toward the steady state. The interest rate declines and generates a substitution effect that tends to reduce savings, *ceteris paribus*. At the same time, the dynamic status effect induces a behavioral change against conspicuous consumption, inducing an increase in the saving rate. The lower level of the saving rate prolongs the transition to the steady state. As a consequence, agents that hold proportionally more capital benefit from the longer period.<sup>26</sup>

To illustrate further Proposition 2 relating to the evolution of inequality, we perform numerical simulations. Our parameterization is identical to that of Section 4, with the exception that because the solution (18) involves linearization, the starting point,  $k_0$  is set much closer to the ultimate steady state. Figure 5 displays the transitional dynamics of wealth inequality for both cases: presence and absence of the endogenous dynamic status effect. The vertical axis of the figure shows the growth factor of the standard deviation of wealth inequality, as given by (19), with  $\sigma_{k,0} \equiv 1$ . With labor supply being exogenous, this can also be interpreted as representing the growth factor of income inequality.

The solid line in Figure 5 displays the evolution of inequality in the presence of the endogenous dynamic status effect ( $\varepsilon' < 0$ ) when  $\xi$  is sufficiently large so that  $h(k^*) > 0$ . In that case, following our analytical results, inequality increases as the dynamic status effect dominates the substitution effect. The increase in inequality is consistent with (the rising part of) the U-shaped dynamics of inequality, displayed in Figure 2 (Fact 4). The dashed line displays the case of exogenous dynamic status preferences ( $\varepsilon' = 0$ ), where  $h(k^*) < 0$  and inequality falls slightly. In this case, the substitution effect roughly balances (slightly exceeds) the dynamic status effect. This is in line with the roughly constant part of the L-shaped dynamics of inequality, displayed in Figure 1.

Table 2, Panel A indicates the impact of the presence/absence of dynamic status effects on steady-state inequality (as displayed in Figure 5) in response to an increase in  $k$  (development). In the

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<sup>26</sup> In other words, consider the area under the interest rate curves in Figure 4. The area is larger for the case  $\varepsilon' < 0$  than for the case  $\varepsilon' = 0$ . The larger the area the more beneficial it is for wealthy households relative to poor ones.

absence of the dynamic status effect,  $\kappa = 0$ , long-run inequality declines, consistent with García-Peñalosa and Turnovsky (2008). As the dynamic status effect increases in strength (i.e. as  $\kappa$  increases first to 0.4 and then to 1.2), long-run inequality increases to 1.09 and to 1.15.

Three remarks are in order. First, the dynamic status effect on the aggregate economy is only transitory, in that it does not impact the aggregate steady-state level of wealth. In contrast, the dynamic status effect impacts the wealth distribution both during the transition *and* in the steady state. In fact, steady-state inequality is higher in the presence of the dynamic status effect than in its absence (see Figure 4). The higher inequality during the transition carries over to the new steady state, making the long-run response path-dependent.<sup>27</sup> This enables us to capture the empirical evidence according to which countries at approximately the same level of economic development (steady state) may nevertheless have substantial differences in their respective wealth distributions. These may reflect cultural differences with respect to the responsiveness of status preferences to the accumulation of wealth as they have developed.

Second, as mentioned above, this result accords with García-Peñalosa and Turnovsky (2008), who show that the presence of exogenous status preferences ( $\varepsilon > 0, \varepsilon' = 0$ ) contribute to a *lower* steady-state wealth inequality. However, while we compare an economy with *endogenous* dynamic status ( $\varepsilon > 0, \varepsilon' < 0$ ) to one with *exogenous* (static) status ( $\varepsilon > 0, \varepsilon' = 0$ ), García-Peñalosa and Turnovsky (2008) compare an economy *without* status ( $\varepsilon = 0$ ) to one with *exogenous* (static) status ( $\varepsilon > 0, \varepsilon' = 0$ ). They show that the presence of status raises the IES (in eq. (10b') DOP becomes positive, and for  $R < 0$ , the IES increases). As a consequence, households desire, for any given  $k$ , a higher consumption growth rate, which is compatible only with an initially lower consumption level, or, equivalently, an initially higher saving rate and *declining saving rates during the transition*. The initially higher saving rate raises the ASOC (see Table 1), thus, it *reduces wealth inequality* relative to a model without status. In contrast, with endogenous dynamic status,  $s$  is initially reduced compared to a model without status (or with static status preferences) and *increases over time* (see Proposition 1). As a consequence, the speed of convergence is lower (see Table 1) and as saving increases over

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<sup>27</sup> The issue of the path dependence of long-run wealth and income inequality in response to structural changes is a general phenomenon and is discussed in detail by Atolia et al. (2012).

time, rich individuals that hold more capital benefit more than do the poor. Consequently, the presence of a strong dynamic status effect (through its differential impact on savings dynamics relative to the return on capital) causes wealth inequality to increase. To that end, our framework with endogenous status differs significantly from prior models with exogenous or no status, due to novel implications on the savings dynamics which feeds back to the distribution of wealth.

Third, for typical plausible empirical parameter values ( $s^* < IES(c^*, k^*)$ ), the neoclassical growth model predicts a decline in income inequality as the economy develops (as capital increases). Thus, it fails to explain the increase in contemporary income inequality. In contrast, in our model – in spite of a strong substitution effect – income inequality can increase or decrease, depending on the strength of the dynamic status effect. Despite the fact that the decline in the return to capital during the development process (due to diminishing returns) tends to reduce savings, the behavioral changes mitigating the consumption race for status tend to increase savings. That is, our behavioral mechanism is rich enough to account for the evolution of income inequality, as a reflection of the strength of the dynamic status effect in the process of economic development.

## **5.2. Evolution of Inequality under Universal Productivity Shocks**

To illustrate how differences in status preferences between countries can account for the varying dynamic paths of income inequality as observed in contemporary data, we consider a universal productivity shock. In doing so, differences in status preferences are reflected in different values of  $E(c, k) \leq 0$ , viewed as proxying national *cultural differences*. Specifically, the smaller (the more negative)  $E(c, k)$  the more responsive are a country's status concerns to an increase in its aggregate capital,  $k$ . From Proposition 2, we know that the impact of the shock on inequality depends on the strength of the dynamic status effect via its influence on the saving rate. Indeed, we find that the impact of the productivity increase on wealth inequality is closely related to condition (21) in Proposition 2.

We shall demonstrate that the key mechanism explaining the impact of a positive technology shock on the development of inequality relies on the initial response of the saving rate (a jump variable) to the shock. This response, in turn, depends on whether the propensity to consume out of wealth,  $c/k$ , is greater than or less than the slope of the saddle path,  $(\beta - \mu)$ , which characterizes the aggregate dynamics. This, in turn, is closely related to  $\text{sgn}[h(k^*)]$ , which is the key determinant of

inequality, which we summarize in the following proposition.

**Proposition 3** *An increase in productivity,  $\Delta A > 0$ , impacts both the transitional dynamics and the steady state of income and wealth inequality. The strength of the dynamic status effect is key in determining whether inequality rises or falls following such a shock. In particular, we show:*

$$\left( \beta - \frac{\mu^*}{s^*} \right) \geq \frac{c^*}{k^*} \Leftrightarrow h(k^*) \leq 0.$$

*If  $h(k^*) < 0$ , so that the country has no (or only a weak) dynamic status effect, inequality declines in response to a positive productivity shock. If  $h(k^*) > 0$ , so that the country has a sufficiently strong dynamic status effect, inequality increases following a positive productivity shock.*

**Proof.** See Appendix A.6.

Proposition 3 provides an explanation for the cross sectional variation in income inequality, attributing it to a cultural factor, the responsiveness of status preferences as the economy develops. The main mechanism is via the initial response of the saving rate following the technology shock, and its dependence on the strength of status preferences. We identify two alternative scenarios.

In the first, the saving rate jumps *up* initially, and declines monotonically thereafter to its steady-state value (that is unaffected by the enhanced level of productivity). In this case,  $h(k^*) < 0$  (which corresponds to no or weak dynamic status preferences) and inequality declines during the transition to a post-shock steady state that is below its initial equilibrium. The high(er) saving rate implies a rapid rate of capital accumulation and a fast decline in the rate of interest. This fast decline disadvantages wealthy households, who derive a large share of income from capital, more so than it does poor households. Consequently, inequality declines.

In the other case, the saving rate jumps *down* initially, and during the subsequent transition increases toward its steady-state value. This occurs if  $h(k^*) > 0$ , that is, under strong dynamic status preferences. The low saving rate reduces both the rate of capital accumulation, and the pace at which the rate of interest declines. This benefits the wealthy households more than the poor, and so inequality rises during transition and is higher in the post-shock steady state than in the initial steady state.

Proposition 3 delivers a second result. It presents the precise conditions under which  $h(k^*) < 0$

(or  $h(k^*) > 0$ ). It implies that the initial response of the saving rate  $s(0)$  (positive or negative jump) to the technology shock depends on the initial response of consumption  $c(0)$ . As the capital stock  $k_0$  is fixed instantaneously, the enhanced level of technology allows for more output for given  $k_0$ . Thus, if  $c(0)$  jumps down initially, then  $s(0)$  must jump up. In the other case, when  $c(0)$  jumps up initially, whether  $s(0)$  declines or increases initially depends on the magnitude of the jump in  $c(0)$ . For a “small” (“large”) upward jump of  $c(0)$  the technology effect dominates (is dominated by) the consumption change, and the saving rate  $s(0)$  jumps up (down) initially. As long as  $h(k^*) < 0$ ,  $s(0)$  jumps up; when  $h(k^*) > 0$ ,  $s(0)$  jumps down.

Under what conditions does  $c(0)$  jump up (down) initially? Intuition is gained by considering the phase diagram representing the aggregate dynamic system (10a, 10b’); see Figure 6. Three observations are pertinent. First, a productivity increase,  $\Delta A > 0$ , while raising both  $c$  and  $k$ , does not affect the steady-state  $c^*/k^*$  ratio. That is, both the pre-shock (SS0) and the post-shock (SS1) steady states are located on a ray through the origin, with slope  $c^*/k^*$ . The post-shock steady state, though, is located to the north-east of the pre-shock steady state. Second, the response of initial  $c(0)$  to the technology shock (given the initial capital stock  $k_0$ , depends on whether or not the saddle path is steeper or flatter than the  $c^*/k^*$  ray. In the first (second) case, the saddle path shifts downwards (upwards), implying a downward (an upward) jump of  $c(0)$ . Third, the flatter the saddle path, the stronger the upward jump of  $c(0)$ . We illustrate the argument in Figure 6 where, following the positive technology shock, the saddle path shifts from the dotted to the solid line.

As is easily seen, whenever  $c(0)$  jumps down, as in the left pane of Figure 6, then  $s(0)$  jumps up, implying a decline in income inequality (due to a high rate of capital accumulation and a rapid decline in the interest rate). In contrast, if  $c(0)$  jumps up, as in the right pane of Figure 6, whether the saving rate initially jumps up or down is ambiguous. Initially,  $s(0)$  jumps up (down) when  $c(0)$  jumps up by little (jumps up substantially – i.e., when the saddle path is flat enough).

As long as  $s(0)$  jumps up,  $h(k^*) < 0$ , and inequality decreases following the productivity increase. Similarly, when  $s(0)$  jumps down,  $h(k^*) > 0$ , and inequality increases. Formally, Proposition 3 provides the necessary and sufficient condition for  $c(0)$  to jump up sufficiently, so that  $s(0)$  jumps down initially (cf. the proof in the Appendix).

To add intuition, consider equation (10b'). The dynamic status effect reduces the optimal consumption growth rate (as  $E(c, k) < 0$ ). In comparison, with no dynamic status preferences, households choose a lower rate of consumption growth, together with an initially higher level of consumption. The higher initial consumption level necessitates a *lower* initial saving rate. If the dynamic status effect is strong enough, initial consumption jumps up so much that the initial saving rate jumps down.

To illustrate Proposition 3 further, we provide a simple numerical example to explain cross-sectional variations in income inequality due to cultural differences captured by parameter  $\kappa$ , with the intensity of response to changes in aggregate wealth increasing with  $\kappa$ . All other functional forms and parameters are as employed in the previous simulations.

Consider two countries, I and II, having an identical technology and initial income distribution, but differing in cultural parameters as manifested in different status preferences. For both countries, we consider  $\underline{\varepsilon} = 0.2$ . While status concerns respond to the development of wealth in country I ( $\kappa = 0.7, \bar{\varepsilon} = 0.9$ ), they are static in country II ( $\kappa = 0, \varepsilon = 0.2$ ). All other parameter values are identical to those employed in the previous section. Figure 7 illustrates the dynamics of income inequality and other economic variables following a positive productivity shock, specified by an increase in  $A$  is increased from 2 to 3. In the figure, solid (dashed) lines refer to the presence (absence) of the dynamic status effect. The figure shows that in the economy where status preferences are responsive to changes in wealth (Country I, solid line), inequality increases, while for the economy where status is not responsive to a rise in wealth (Country II, dashed line), inequality declines in response to the same positive technology shock. Panel B of Table 2 summarizes the sensitivity of the change in inequality to the strength of the dynamic status effect following the productivity increase. The intuition follows closely the mechanism involving the convergence speed described above.

For the simulation displayed in Figure 7, parameters were chosen to produce opposite effects regarding the impact of the productivity shock on the transitional dynamics of inequality. More generally, whether inequality rises or falls following a positive productivity shock depends on the respective strengths of the return-on-capital- and dynamic status effects, as implied by Proposition 3.

Two remarks merit comment. First, and more important, our mechanism whereby productivity

shocks generate inequality contrasts sharply with those proposed by previous authors. In Caselli and Ventura (2000), the productivity shock has a positive effect on income inequality if the positive technological increase is biased towards capital returns relative to labor wages. Atolia et al. (2012) show that the impact of a neutral increase in TFP, such as is being considered here, on inequality depends upon the speed with which it is implemented. In our approach, the differential dynamics of income inequality in response to a productivity shock operate through the evolution of agents' behavior, and specifically the sensitivity of status concerns with respect to wealth. Accordingly, this result complements the literature by providing an alternative explanation for why countries that share the same production technology (no technology bias in the factors of production) and have the same income in the long-run (the case of many advanced countries) can nevertheless end up with a very different distribution of income after a technology- or policy shock.

Second, following Proposition 1, cultural differences in status concerns (as proxied by differences in  $\kappa$  in our numerical example) do not affect production and, in turn, do not have any long-run impact on aggregate income. This is important because the differentials in income inequality come through the dynamics of the economy rather than the long-run level of economic development. This way we provide a framework to analyze the behavior of income distribution under a productivity shock in countries at the same stage of economic development (see for example the case of advanced countries in Figures 1 and 2).

## **6. DISCUSSION AND RESEARCH DIRECTIONS**

In this section we further discuss our assumptions, as well as suggesting future research directions based on our theory of endogenous status preferences.

### **6.1. *Positionality in Wealth***

Our model is based on the assumption that agents are positional with respect to consumption. While most empirical and theoretical studies (going back to the classics, Smith, Veblen, Duesenberry, etc.) focus on consumption positionality there is a strand of literature that assumes that people are positional in terms of their wealth. According to this literature (originated by Weber (1930) and formalized by Zou (1994)) individuals have a direct preference for thriftiness (keeping wealth) for



themselves and relative to the others. Such an assumption would give similar results in our framework. The main drawback of such an approach is the lack of empirical support, as wealth – particularly that of the reference group – is clearly not visible. In contrast, conspicuous consumption is the device of signaling wealth as Moav and Neuman (2012) (among many others) formalize. We believe that further empirical evidence on wealth positionality and its determinants, and utilizing our theory of dynamic status preferences may be a fruitful area for future research.

## **6.2. *Wealth versus Income Inequality: Endogenous Labor Supply***

In general, income inequality is quantitatively distinct from wealth inequality, although the positive correlation between the two inequality measures is well documented; see e.g. Hendricks (2007). Under our assumptions (i. Cobb-Douglas production function; ii. inelastic labor supply; iii. uniform labor skills and therefore a common wage rate), income inequality, as measured by the coefficient of variation, is strictly proportional to wealth inequality, namely  $\sigma_y(t) = \alpha \sigma_k(t)$ . But as Turnovsky and García-Peñalosa (2008) show, if one endogenizes labor supply and/or generalizes the production function beyond the Cobb-Douglas technology, the relationship between income inequality and wealth inequality generalizes to  $\sigma_y(t) = \lambda(t) \sigma_k(t)$  where  $\lambda(t)$  reflects the impact of changing factor returns (the wage rate and the returns to capital) as the economy evolves over time. This generates a secondary channel whereby dynamic status effects impact income inequality, over and above their direct impact via wealth inequality, so that the strict proportionality between the two inequality measures ceases to hold.

To endogenize labor is straightforward. It introduces an additional intratemporal decision for agents that does not have any effect on the Euler equation. Thus, the savings dynamics, which are the crucial driving force of our mechanism, would remain largely unaffected. The implications for wealth inequality would remain also unaffected, although for the reasons just noted, the dynamics of income inequality would be impacted. An alternative, potentially more interesting, hypothesis might be to treat individuals' leisure time as the signal to display status. Endogenizing the strength of status using leisure time may provide an alternative explanation for the negative slope of labor supply we observe for high levels of income, offering possible new insights for policy making.

### **6.3. *Redistribution and other Policies***

We believe that our theoretical framework can provide the basis for addressing a range of interesting questions pertaining to inequality. For example, our framework suggests that distributive policies financed through the taxation of luxury/status goods may increase, rather than decrease, inequality. This is because poor individuals care more about status and their consumption will be inelastic to taxes on status goods. Moreover, the conventional effects of taxation on the time path of savings may be reversed if individual concerns for status are sufficiently strong. Finally, recent evidence shows that status anxiety increases with inequality. This introduces another channel for (further) endogenizing the degree of positionality.

## **7. CONCLUSIONS**

This paper advances the hypothesis that the intensity of status preferences depends negatively on the average wealth of society. Within an otherwise standard neoclassical growth model, we provide a new mechanism to explain saving rate dynamics (a rising- or inversely U-shaped transitional path) and the comparative development of income inequality across countries. We advance our knowledge of the evolution of income and wealth inequality, and thereby complementing previous work by introducing a dynamic behavioral factor, as opposed to relying on a technological factor. The advantage of our behavioral mechanism is that it can explain the development of income inequality even after economies have converged technologically (e.g. US vs. Europe, cf. Figure 1 and Figure 2). In particular, we showed that differentials in the strength of the dynamic status effect can propagate variation of income inequality that is attributed to the differential response of agents to productivity shocks rather than a technological bias on the factor of production as in Piketty (2014). As a policy implication, our theory suggests that policies that target productivity advancement towards increasing the income of the poor countries may not be sufficient to reduce income inequality when poor countries direct their income to “unproductive” uses such as status goods consumption. In addition, such policies may need to be supplemented with investment in institutions that support behavioral changes (like educational institutions) that discourage conspicuous consumption, in order to be truly effective in terms of both raising income and alleviating inequality.

**Table 1. Average Speed of Convergence**

<b>ASOC</b>	$\kappa = 0$	$\kappa = 0.4$	$\kappa = 0.7$	$\kappa = 1.2$
$\tau = 0.7$	0.66	0.43	0.46	0.54
$\tau = 0.5$	0.84	0.62	0.62	0.64
$\tau = 0.3$	1.11	0.86	0.85	0.84

*Notes.*  $k_0 = 1/4k^*$ . The time unit is one year.  $\bar{\varepsilon} = 0.9$ ,  $\underline{\varepsilon} = 0.2$ ,  $R = -2.5$ . ASOC refers to the average speed of convergence of  $k$  during the time interval needed for the fraction  $(1 - \tau)$  of the initial distance from the steady-state value to be completed.

**Table 2. Steady-state Inequality for Static ( $\kappa = 0$ ) and Dynamic ( $\kappa > 0$ ) Status Preferences**

	$\kappa = 0$	$\kappa = 0.4$	$\kappa = 0.7$	$\kappa = 1.2$
<b>Panel A. <math>k</math> increases from <math>k_0 = 3/4 k^*</math> to <math>k^*</math></b>				
$\sigma_k$	0.97	1.09	1.12	1.15
<b>Panel B. <math>A</math> increases from 2 to 3</b>				
$\sigma_k$	0.95	1.15	1.22	1.28

*Notes.*  $\sigma_k = 1$  in the pre-shock steady state. In the table,  $\sigma_k$  refers to the post-shock steady state value of inequality. A value greater (lower) 1 indicates an increase (a decline) in inequality.  $A$  is a technology parameter. The time unit is one year.  $\bar{\varepsilon} = 0.9$ ,  $\underline{\varepsilon} = 0.2$ ,  $R = -2.5$ .

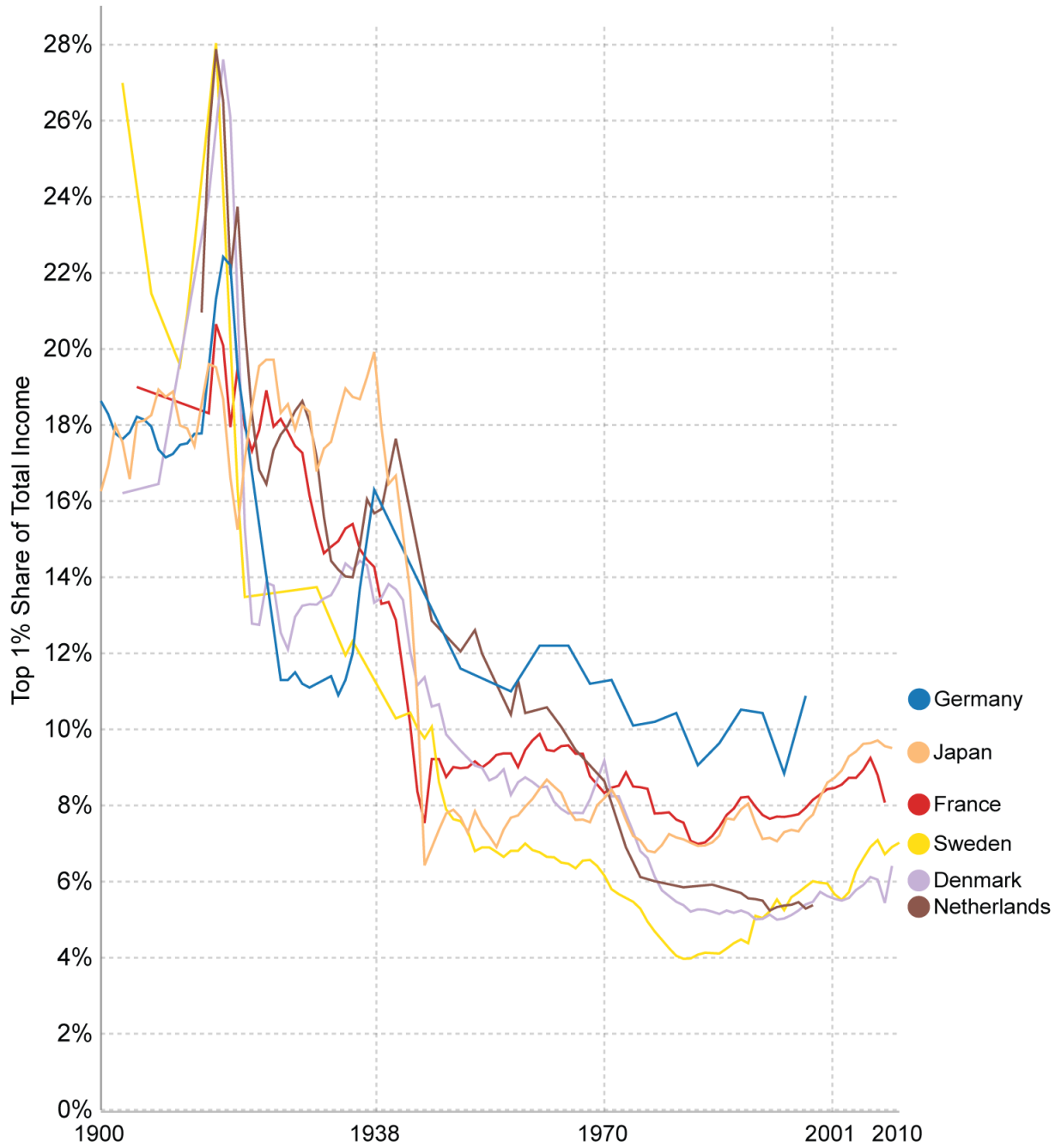


Figure 1. Top 1 % share of total income - Europe and Japan (L-shaped) 1900 – 2011.

Source: Roser and Ortiz-Ospina (2017).

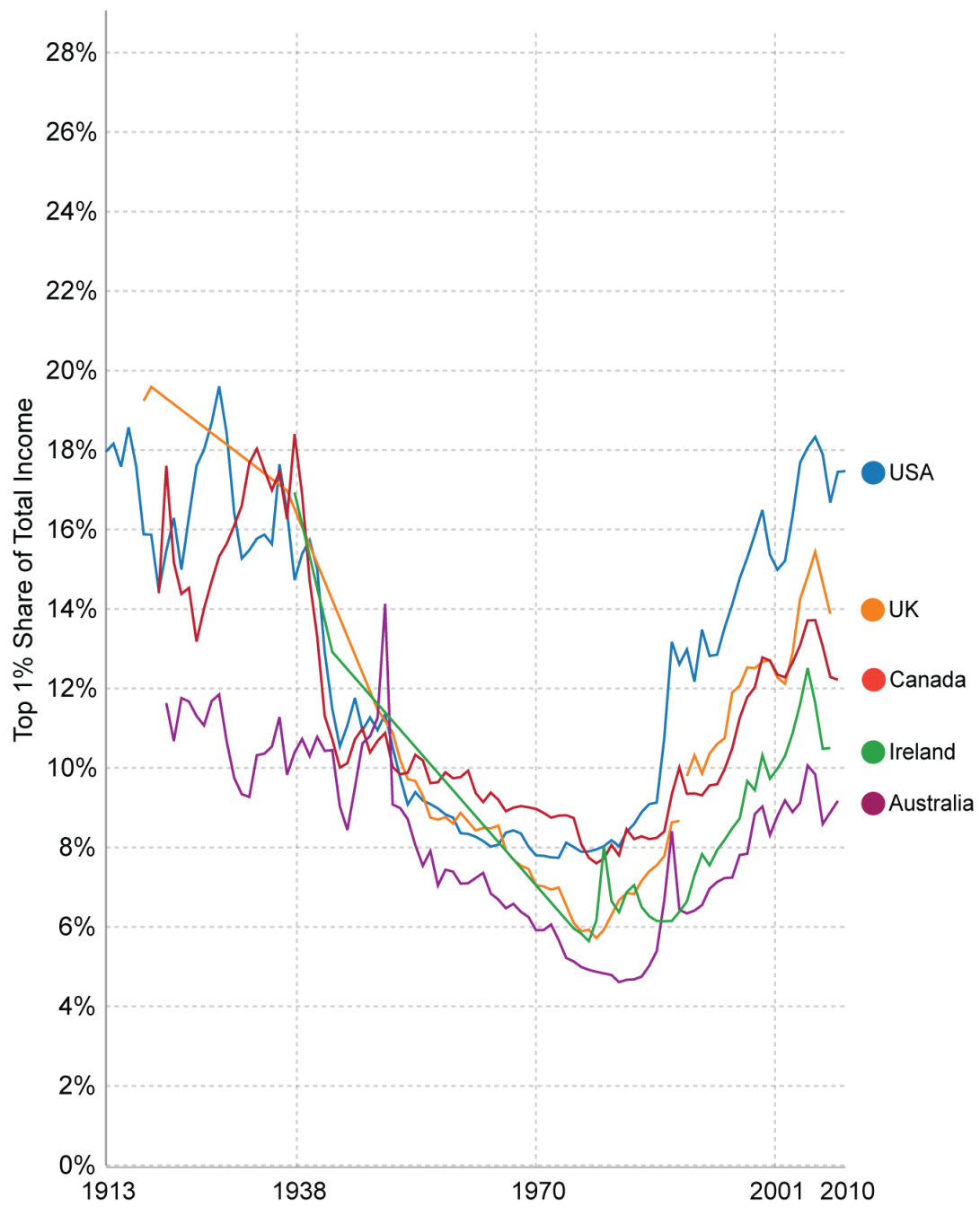


Figure 2. Top 1 % share of total income - English speaking countries (U-shaped) 1900 – 2011. *Source: Roser and Ortiz-Ospina (2017).*

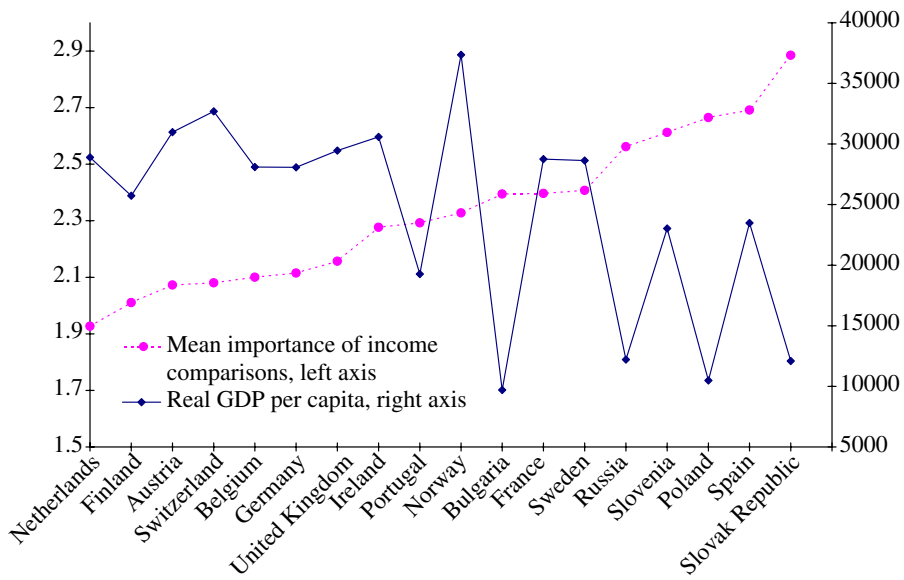
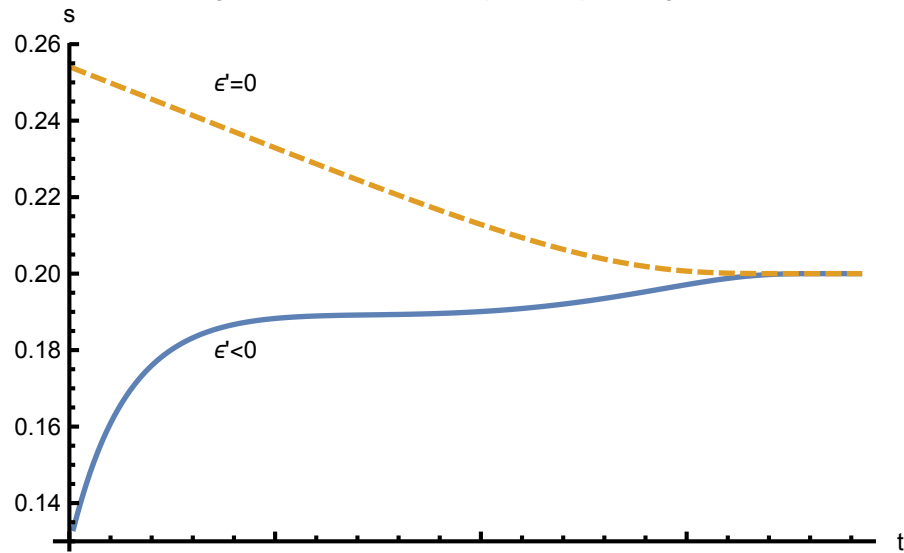


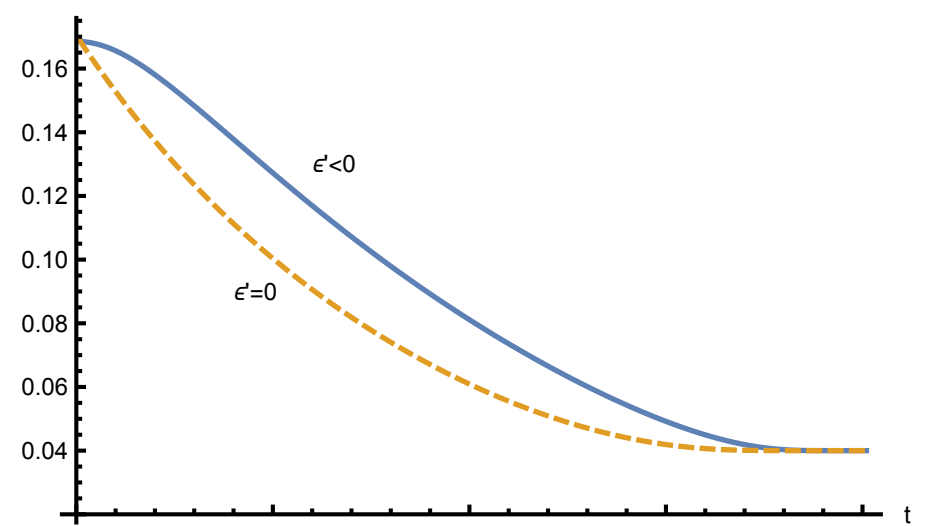
Figure 3. How important is it to you to compare your income with other people's incomes? *Source: Clark and Senik (2010, p.580).*

**Figure 4.** Dynamics of saving, rate of return, and degree of positionality

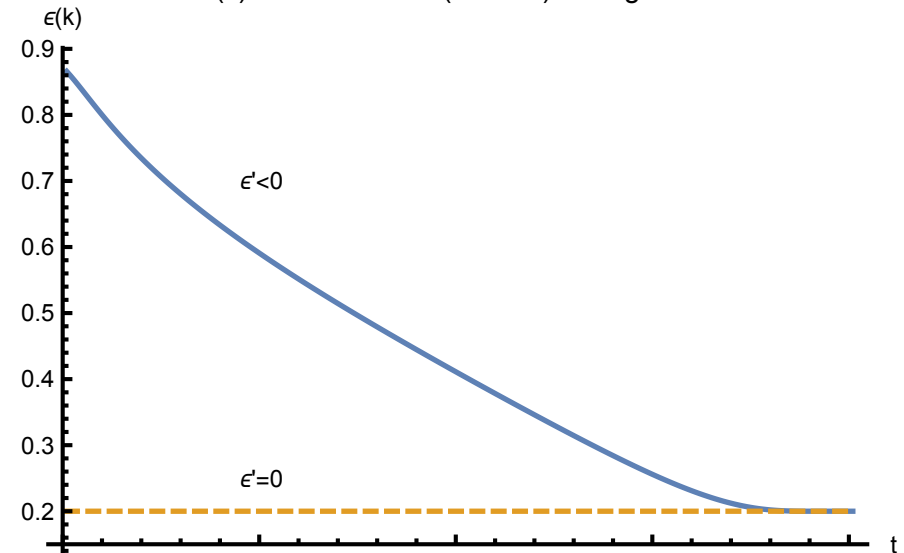
The saving rate over time with (without) endogenous status



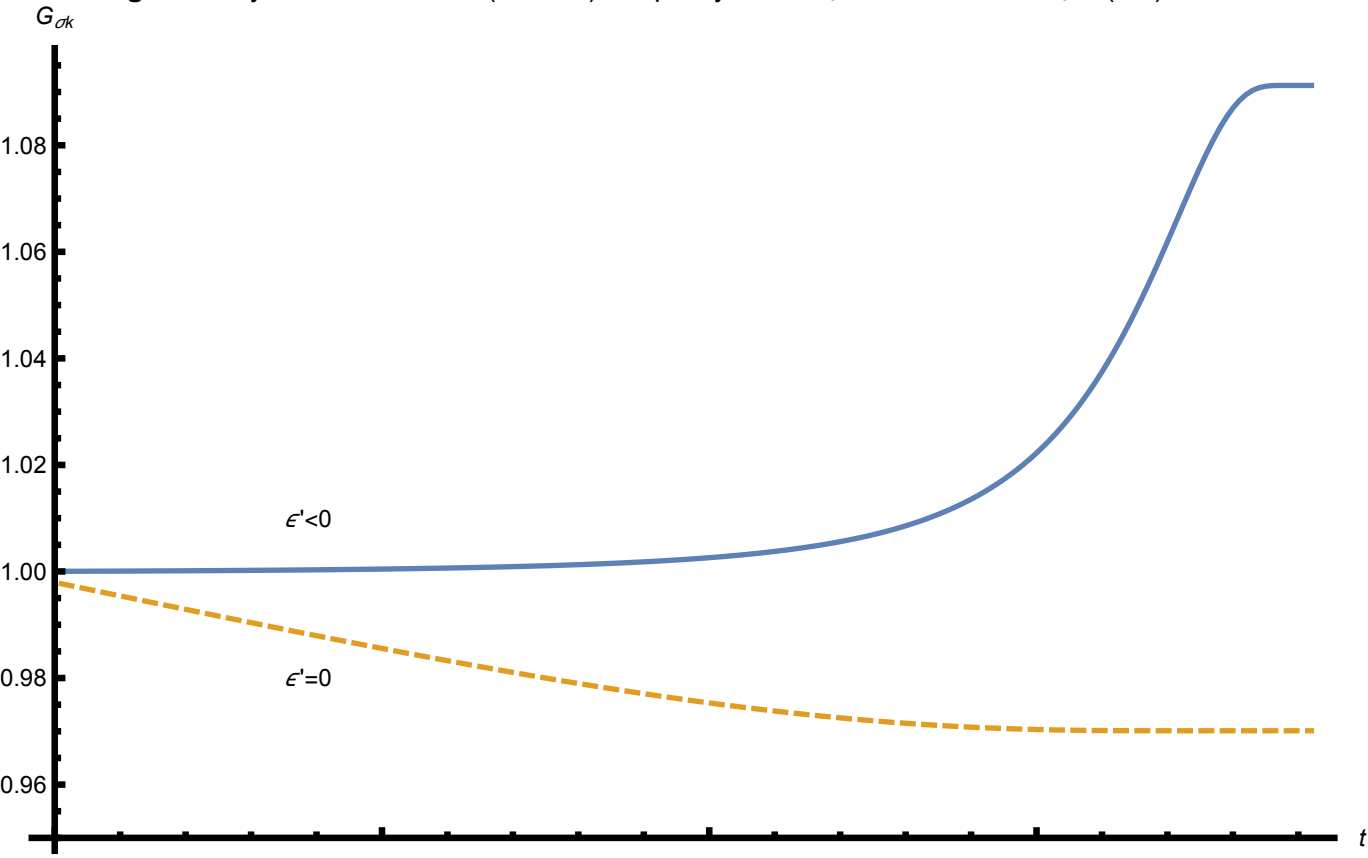
The interest rate over time with (without) endogenous status  
rate of interest



Status  $\epsilon(k)$  over time with (without) endogenous status



**Figure 5.** Dynamics of wealth (income) inequality when  $k_0$  increases from  $k_0 = (3/4) k^*$  to  $k^*$





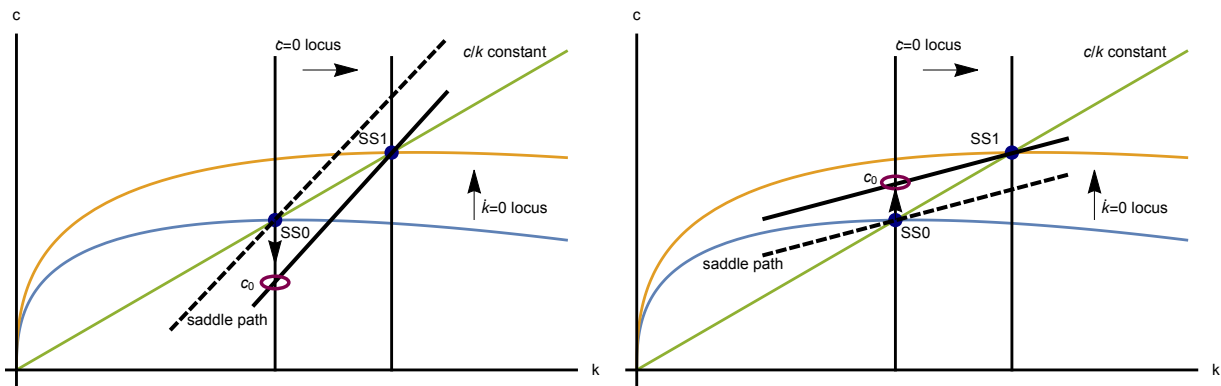
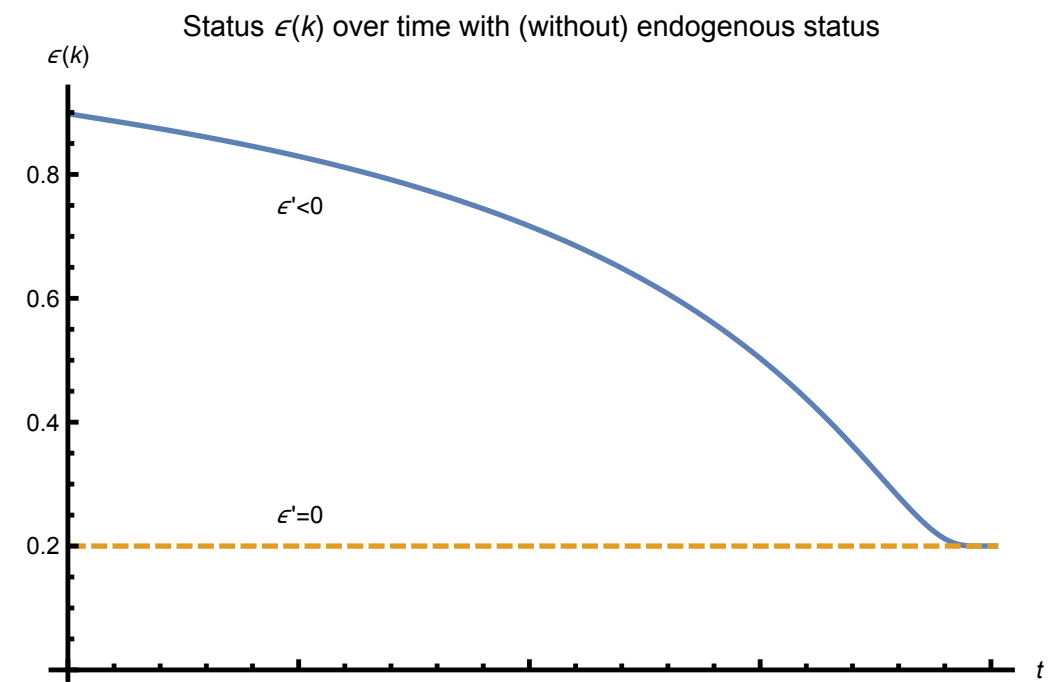
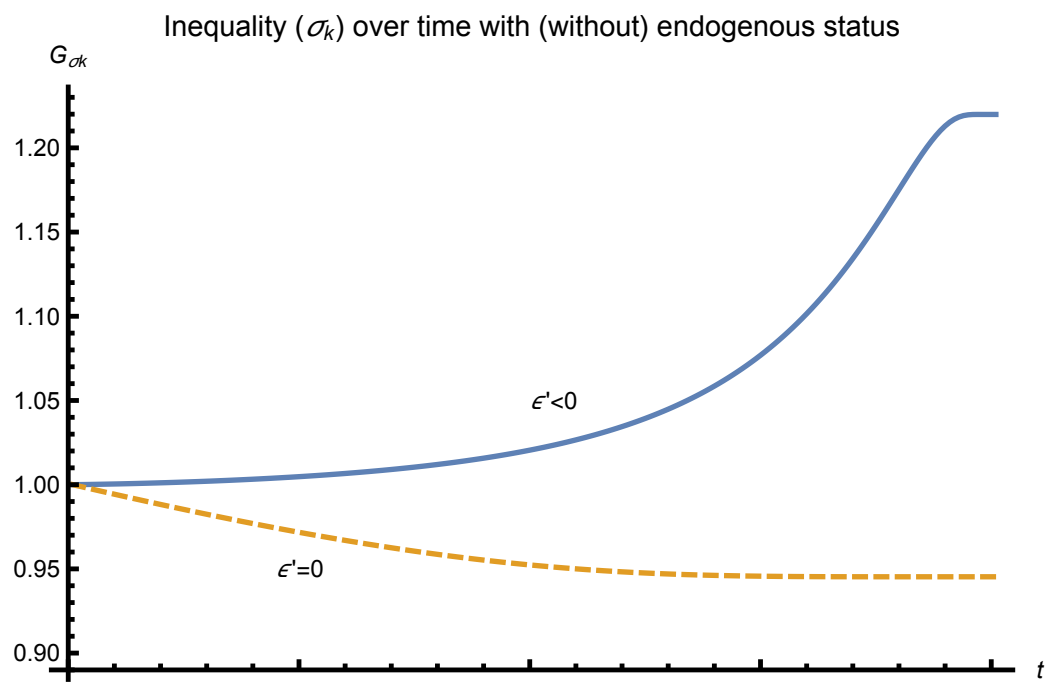
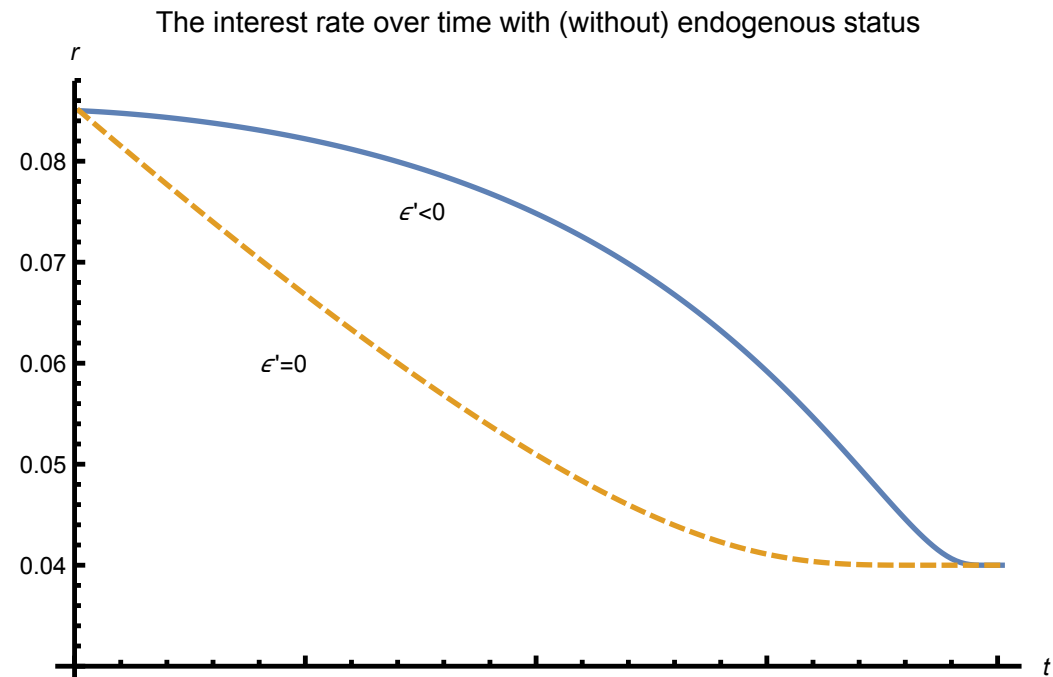
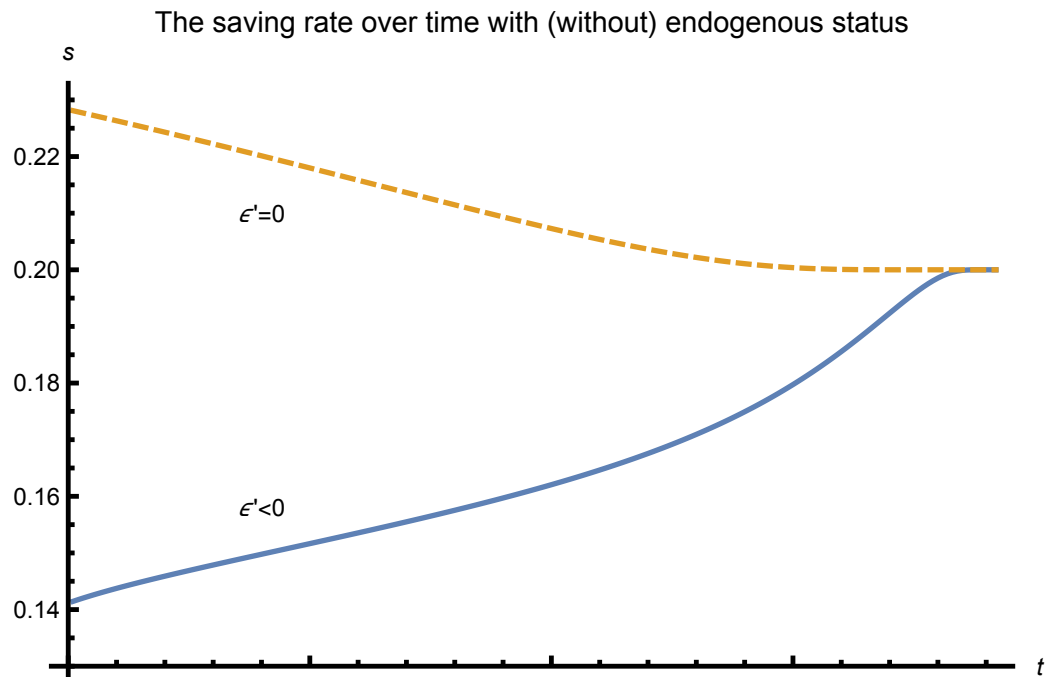


Figure 6. The impact of a positive technology shock,  $\Delta A > 0$ , on the initial response of consumption,  $c(0)$ .

Figure 7. Dynamics of saving, rate of return, inequality, and degree of positionality



## APPENDIX

This Appendix provides some of the technical details and proofs. Unless needed for clarity, time indexes are omitted.

### A.1. Degree of Positionality

Recalling Assumption 1,

$$(A.1) \quad U(C_i, S_i, \varepsilon(k)) = U(C_i, C_i/\bar{c}, \varepsilon(k)) = C_i^R V(\bar{c}, \varepsilon(k)),$$

so that differentiating both sides of (A.1) with respect to  $C_i$  yields the denominator of the definition of the DOP in (2)

$$(A.2) \quad \frac{\partial U}{\partial C_i} + \left( \frac{\partial U}{\partial S_i} \right) \left( \frac{\partial S_i}{\partial C_i} \right) = \frac{\partial [C_i^R V(\bar{c}, \varepsilon(k))]}{\partial C_i} = R C_i^{R-1} V(\bar{c}, \varepsilon(k)).$$

Next, differentiating both sides of (A.1) with respect to  $\bar{c}$  yields

$$\left( \frac{\partial U}{\partial S_i} \right) \left( \frac{\partial S_i}{\partial \bar{c}} \right) = C_i^R V_{\bar{c}}(\bar{c}, \varepsilon(k)),$$

and using the fact that  $\partial S_i / \partial \bar{c} = -(C_i / \bar{c}) \partial S_i / \partial C_i$  we obtain

$$(A.3) \quad \left( \frac{\partial U}{\partial S_i} \right) \left( \frac{\partial S_i}{\partial C_i} \right) = C_i^{R-1} V_{\bar{c}}(\bar{c}, \varepsilon(k)) \bar{c}.$$

which represents the numerator of (2). Dividing (A.3) by (A.2) yields (3) of the text.

### A.2. Derivation of Equation (6)

Optimizing (4) subject to (5) with respect to  $C_i$  yields the first order optimality condition  $R C_i^{R-1} V(\bar{c}, \varepsilon(k)) = \mu_i$ , where  $\mu_i$  is the individual's shadow value of wealth. Taking the time derivative of this condition yields  $(R-1) \frac{\dot{C}_i}{C_i} + \left( \frac{V_{\bar{c}} \bar{c}}{V} \right) \frac{\dot{\bar{c}}}{\bar{c}} + \left( \frac{V_{\varepsilon} \varepsilon'(k)}{V} \right) \dot{k} = \frac{\dot{\mu}_i}{\mu_i}$ . As all agents face the same rate of return,  $\dot{\mu}_i / \mu_i = -(r - \beta)$ , individual consumption growth rates are independent of household characteristics, i.e., they are identical across households. Consequently, individual and average consumption growth rates coincide:  $\dot{C}_i / C_i = \dot{c} / c$ . Considering  $\bar{c} = c$  in equilibrium, and recalling (3), leads to  $-(1-R) \frac{\dot{C}_i}{C_i} - R DOP \frac{\dot{C}_i}{C_i} + \left( \frac{V_{\varepsilon} \varepsilon'(k)}{V} \right) \dot{k} = -(r - \beta)$ . Rearranging terms yields (6).

### A.3. Proof of Proposition 1

We note the policy function  $c(k)$  and simply denote  $IES(c(k),k)$ ,  $E(c(k),k)$ , and  $\xi(c(k),k)$  by  $IES$ ,  $E$ , and  $\xi$ , respectively. Recall that the steady-state saving rate is  $s^* = \alpha\delta / (\beta + \delta)$ . To examine the behavior of  $s$  as capital is being accumulated in the presence of the dynamic status effect it is convenient to focus on the consumption to output ratio,  $z = c/f(k)$  where  $z \equiv 1 - s$ . In the case of the Cobb-Douglas production technology,  $f(k) = Ak^\alpha$ , we obtain  $z = cA^{-1}k^{-\alpha}$ , and taking the time derivative yields  $\dot{z}/z = \dot{c}/c - \alpha\dot{k}/k$ . Substituting for the equilibrium  $\dot{c}/c$  from (10b') into this relationship and using the definition of  $\xi$  given in (13) we obtain

$$(A.4) \quad \frac{\dot{z}}{z} = IES \left[ f'(k) - (\beta + \delta) \right] - \xi \alpha \frac{\dot{k}}{k}.$$

Next, rewriting the capital accumulation in (10a) as  $\dot{k}/k = (1 - z)f(k)/k - \delta$ , and recalling (i) that for the Cobb-Douglas production function  $f'(k) = \alpha k^{\alpha-1} = \alpha f(k)/k$ , and (ii) definition of  $s^*$ , we obtain

$$\alpha \frac{\dot{k}}{k} = sf'(k) - (\beta + \delta)s^*.$$

Substituting this expression into (A.4) and using  $z = 1 - s$ ,  $\dot{z} = -\dot{s}$ , we obtain the following equation describing savings dynamics:

$$(A.5) \quad \frac{\dot{s}}{1-s} = (\beta + \delta) \left[ IES - \xi s^* \right] - f'(k) \left[ IES - \xi s \right].$$

In the absence of static and dynamic status effects ( $\varepsilon = 0$ ,  $\varepsilon' = 0$ ,  $\xi(c, k) = 1$ ), (A.5) reduces to

$$(A.5') \quad \frac{\dot{s}}{1-s} = (\beta + \delta) \left[ \frac{1}{1-R} - s^* \right] - f'(k) \left[ \frac{1}{1-R} - s \right],$$

which corresponds to the standard case, considered by Barro and Sala-i-Martin (p.136, 2004). From (A.5'), they conclude that if  $s^* < (1-R)^{-1}$  then  $\dot{s} < 0$  during the transition, while if  $s^* > (1-R)^{-1}$  then  $\dot{s} > 0$  during the transition. They find that for typical estimated parameter values the former is likely to hold, implying that the increasing capital stock is associated with a declining saving rate.

The presence of status effects modifies (A.5') in two ways. First, the intertemporal elasticity of substitution depends on the degree of positionality:  $IES = (1 - R(1 - \varepsilon))^{-1}$ . Second, expression  $\xi \geq 1$  is included in (A.5). The saving rate dynamics may be monotonic or non-monotonic.

(i) *Monotonic dynamics.* Here, we consider the case  $s^* > IES(k) / \xi(k)$  for all  $k$ , which implies

$\dot{s} > 0$  for some  $k$  during transition. The case with reversed inequalities follows parallel reasoning.

Suppose the proposition does not hold. Then  $\dot{s} < 0$  for all  $k$  during transition. Consequently, for  $s(t)$  approaching the steady state saving rate  $s^*$ ,  $s(0) > s(t) > s^*$ . That is,  $[s(t) - IES(k)/\xi(k)] > [s^* - IES(k)/\xi(k)] > 0$ . Moreover, during transition,  $f'(k) > (\beta + \delta)$ . Thus, (A.5) implies  $\dot{s} > 0$ . This however, contradicts  $\dot{s} < 0$  for all  $k$  during transition.

The dynamic status effect implies  $\xi > 1$ , with no upper limit to  $\xi$  on the basis of theory. For this reason,  $s^* > IES(k)/\xi(k)$  can be satisfied for arbitrary parameter values. In particular, for every value of the elasticity of intertemporal substitution, there exists a  $\xi > 1$  for which the inequality is satisfied, and consequently  $\dot{s} > 0$ .

(ii) *Non-monotonic dynamics.* Existence and types of non-monotone behaviors of the saving rate are discussed in Corollary 1.

#### A.4. Proof of Corollary 1

We note that the ratio  $IES(k)/\xi(k)$  may vary (non-monotonically) as  $k$  increases. We first show the existence of an inverted U-shaped path of the saving rate.

(i)  $IES(k)$  and  $\xi(k)$  are continuous functions.

(ii) By the intermediate value theorem, there exists a  $\hat{k}$  satisfying

$$\frac{IES(\hat{k})}{\xi(\hat{k})} = s^*, \quad k(0) < \hat{k} < k^* .$$

Recalling (A.5),  $\text{sgn } \dot{s}(\hat{k}) = \text{sgn } f'(\hat{k}) [\xi(\hat{k})s(\hat{k}) - IES(\hat{k})]$ . Depending on  $s(\hat{k})$ , which in turn depends on the development of  $IES(k)/\xi(k)$  during transition,  $\dot{s}(\hat{k}) \leq 0$ .

(iii) There are four cases: Case I with  $s(\hat{k}) > s^*$ ; Case II with  $s(\hat{k}) < s^*$ ; Case III with  $s(\hat{k}) = s^*$ ; Case IV with more complex transitional patterns of  $IES(k)/\xi(k)$ .

*Case I:* According to (A.5), as along the transitional path,  $s(k) \rightarrow s^*$ , for  $k > \hat{k}$ , there exists a range of  $k$ , for which  $\dot{s}(k) < 0$ . Specifically, as  $s(\hat{k}) > s^*$ , there exists  $\tilde{k} > \hat{k}$  such that for all  $k$  with  $\tilde{k} < k < k^*$ ,  $\dot{s}(k) < 0$ . According to (A.5),  $\tilde{k}$  is implicitly given by

$$(A.6) \quad f'(\tilde{k}) [\xi(\tilde{k})s(\tilde{k}) - IES(\tilde{k})] = (\beta + \delta) [\xi(\tilde{k})s^* - IES(\tilde{k})] .$$

This case gives rise to an inverse U-shaped transitional path of the saving rate.

*Case II:* According to (A.5) as along the transitional path,  $s(k) \rightarrow s^*$ , for  $k > \hat{k}$ , there exists a range of  $k$ , for which  $\dot{s}(k) > 0$ , generating a U-shaped path of the saving rate.

*Case III:* If  $s(\hat{k}) = s^*$ ,  $\dot{s}(\hat{k}) = 0$ . However, as  $k$  increases over time,  $IES(k)$  and  $\xi(k)$  change. Consequently,  $\left[ IES(k) - \xi(k)s(\hat{k}) \right] \neq 0$ , and  $\dot{s}(k) \neq 0$ . As a consequence, the dynamics of the saving rate are non-monotonic.

*Case IV:* Depending on the transitional behavior of  $IES(c(t), k(t)) / \xi(c(t), k(t))$  other forms of non-monotonicities are possible. If  $IES(c(t), k(t)) / \xi(c(t), k(t))$  behaves sufficiently non-monotonic, for example, there might be two (several) values  $\tilde{k}$  for which (A.6) is satisfied. In such a case, the saving rate follows a “repeatedly non-monotonic” transition path.

#### **A.4.1. Some Numerical Examples of Non-monotonic Dynamics**

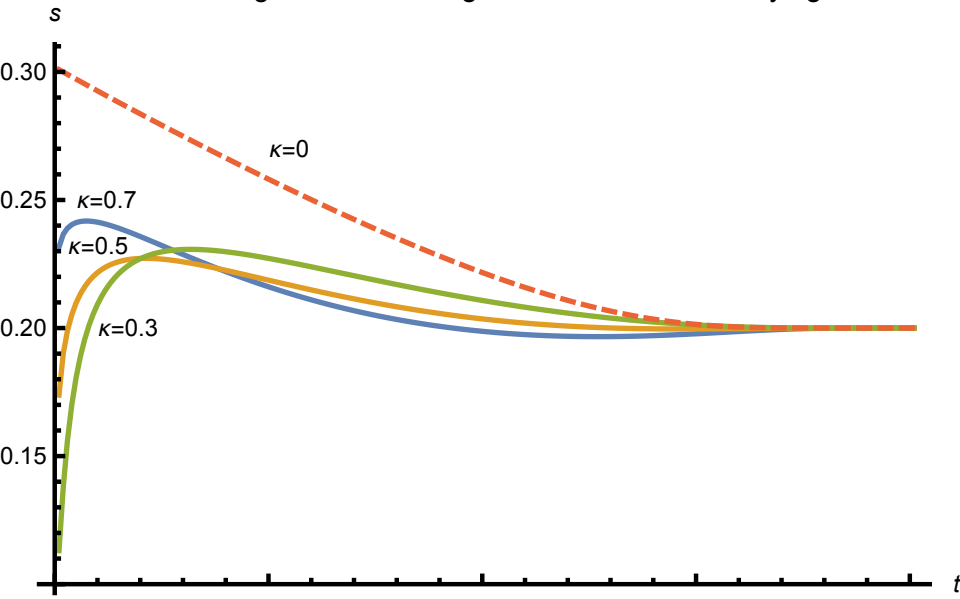
We now provide numerical examples for the non-monotonic dynamics. In doing so we parameterize the explicit status function as in the text. In addition, we set  $R = -1.5$  (pure intertemporal elasticity of substitution 0.4) and we vary only the status strength parameter,  $\kappa$ , for values 0, 0.3, 0.5, and 0.7. Figure A.1 illustrates the cases of our theoretical results of this proof. It illustrates the dynamics of the saving rate for varying values of the strength of dynamic status,  $\kappa$ , when  $k(t)$  increases from  $k_0 = 1/4k^*$  to  $k^*$ . The time unit is one year. Each tick on the horizontal axis corresponds to 10 years. In all simulations,  $\Delta = 0.01$ .

Consider  $\kappa = 0.3$  (the reasoning for other values of  $\kappa$  follows parallel arguments). In this case,  $0.014 = IES(k(0)) / \xi(k(0)) < s^* = 0.2 < IES(k^*) / \xi(k^*) = 0.243$ . According to Corollary 1 the transitional path of the saving rate is non-monotonic. Specifically,  $\hat{k} = 7.32 < \tilde{k} = 8.78$ .<sup>28</sup> At  $\hat{k}$ , the saving rate is given by  $s(\hat{k}) = 0.22 > s^* = 0.2$ . As  $s(\hat{k}) > s^*$ ,  $\dot{s}(\hat{k}) > 0$ . As, along the transitional path,  $s(k) \rightarrow s^*$ , there exists a range of  $k$ , for which  $\dot{s}(k) < 0$ . Specifically, there exists  $\tilde{k} > \hat{k}$  such that for all  $k$  with  $\tilde{k} < k < k^*$ ,  $\dot{s}(k) < 0$  numerically justifying Corollary 1.

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<sup>28</sup> Notice that  $\hat{k}$  denotes the value of  $k$  for which  $s = IES / \xi$ , and  $\tilde{k}$  denotes the  $k$  at which  $\dot{s}(k) = 0$ , and the sign of  $\dot{s}(k)$  switches (see proof of Corollary 1).

Fig. A.1 The saving rate over time with varying  $\kappa$



### A.5. Proof of Proposition 2

Following García-Peñalosa and Turnovsky (2008), equation (19) shows that inequality rises over time ( $\sigma_k(t) > \sigma_{k,0}$ ) if  $h(k^*) > 0$  and declines over time if  $h(k^*) < 0$ . For the Cobb-Douglas technology,  $f''(k)k = -(1-\alpha)f'(k)$ . Moreover, in steady state,  $c^* = f(k^*) - \delta k^*$ . Thus,

$$\begin{aligned} h(k^*) &= -f''(k^*)k^* - v_1 w^* / c^* = (1-\alpha)f'(k^*) - (\beta - \mu^*) [f(k^*) - k^* f'(k^*)] / [f(k^*) - \delta k^*] \\ &= (1-\alpha)f'(k^*) \left[ 1 - (\beta - \mu^*) / (f'(k^*) - \alpha\delta) \right]. \end{aligned}$$

Using the steady-state condition,  $f'(k^*) = \beta + \delta$ , we obtain

$$h(k^*) = \frac{(1-\alpha)(\beta + \delta)}{\beta + \delta(1-\alpha)} [(1-\alpha)\delta + \mu^*].$$

Therefore  $\text{sgn}[h(k^*)] = \text{sgn}[(1-\alpha)\delta + \mu^*]$ .

Next, we consider the Jacobian to the dynamic system (10a, 10b'), evaluated at steady state:

$$J \equiv \begin{bmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial c} \\ \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial c} \end{bmatrix} = \begin{bmatrix} \beta & -1 \\ j_{21} & j_{22} \end{bmatrix},$$

where  $j_{21} \equiv IES(c^*, k^*) c^* f''(k^*) - \beta j_{22} < 0$ ,  $j_{22} \equiv -IES(c^*, k^*) c^* E(c^*, k^*) / k^* \geq 0$ , and  $j_{22} > 0$  if and only if  $\varepsilon'(k^*) < 0$ . As  $\mu^* = 2^{-1} \left\{ \beta + j_{22} - \sqrt{(\beta + j_{22})^2 - 4(\beta j_{22} + j_{21})} \right\}$  is the smaller eigenvalue, we know that  $h(k^*) > 0$  is equivalent to  $2\delta(1-\alpha) + (\beta + j_{22}) > \sqrt{(\beta + j_{22})^2 - 4(\beta j_{22} + j_{21})}$ . Squaring both sides of the inequality, rearranging terms, and using the definition of  $j_{21}$  yields:

$$(A.7) \quad (1-\alpha)\delta [(1-\alpha)\delta + \beta + j_{22}] > -IES(c^*, k^*) c^* f''(k^*).$$

Recalling the definition of the equilibrium consumption-output and capital-output ratios given in (11b), we have  $c^*/k^* = [(\beta + \delta)/\alpha - \delta]$ . Using this relationship, we may write

$$c^* f''(k^*) = (c^*/k^*) k^* f''(k^*) = -(c^*/k^*) (1-\alpha) f'(k^*) = -[(\beta + \delta)/\alpha - \delta] (1-\alpha) (\beta + \delta).$$

Introducing this result into inequality (A.7) and solving for  $j_{22}$  implies

$$(A.8) \quad j_{22} > \left[ IES(c^*, k^*) (\beta + \delta) / (\alpha\delta) - 1 \right] [\beta + (1-\alpha)\delta].$$



Finally, recalling the above definition of  $j_{22}$ , and the steady-state  $c^*/k^*$  ratio, and saving rate  $s^* = \alpha\delta / (\beta + \delta)$ , (A.8) after simplifying can be written as

$$(A.9) \quad h(k^*) > 0 \Rightarrow -\frac{E(c^*, k^*)}{\alpha} > \left[ \frac{1}{s^*} - \frac{1}{IES} \right].$$

All the above steps can likewise be done for the reversed inequality

$$(A.10) \quad h(k^*) < 0 \Rightarrow -\frac{E(c^*, k^*)}{\alpha} < \left[ \frac{1}{s^*} - \frac{1}{IES} \right],$$

establishing that (A.9) and (A.10) imply (21).

### A.6. Proof of Proposition 3

From Propositions 1 and 2, it follows that  $h(k^*) > 0$  ( $h(k^*) < 0$ ) implies  $\dot{s} > 0$  ( $\dot{s} < 0$ ) approaching its steady-state level from below (above). We need to show that

$$(A.11) \quad \left( \beta - \frac{\mu^*}{s^*} \right) \geq \frac{c^*}{k^*} \Leftrightarrow h(k^*) \leq 0.$$

Substituting the steady state solutions for  $c^*/k^*$  and  $s^*$  from (11b) yields

$$\beta - \frac{\mu^*}{s^*} - \frac{c^*}{k^*} = -\frac{(\beta + \delta)}{\alpha\delta} [(1 - \alpha)\delta + \mu^*],$$

and hence

$$\text{sgn} \left[ \beta - \frac{\mu^*}{s^*} - \frac{c^*}{k^*} \right] = -\text{sgn} [(1 - \alpha)\delta + \mu^*].$$

Since  $\text{sgn}[(1 - \alpha)\delta + \mu^*] = \text{sgn}[h(k^*)]$  (A.11) immediately follows.

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