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### **Photonic spin lattices: symmetry constraints for skyrmion and meron**

# **topologies**

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Symmetry and topology govern many electronic, magnetic and photonic phenomena in condensed matter physics and optics, resulting in counterintuitive skyrmion, meron and other phenomena important for modern technologies. Here we demonstrate photonic spin lattices as a new topological construct governed by the spin-orbit coupling in optical field. The symmetry of the electromagnetic field in the presence of the spin-orbit interaction may result in only two types of photonic spin lattices: either hexagonal spin-skyrmion or square spin-meron lattices. We show that these spin structures correspond to the lowest energy of the electromagnetic field configuration, therefore, energetically stable. We further show that in the absence of spin-orbit coupling these spin topologies are degenerated in dynamic field-skyrmions, unifying description of electromagnetic field topologies. The results provide new understanding of electromagnetic field topology and its transformations as well as new opportunities for applications in quantum technologies, spin-optics and topological photonics.

Topologically nontrivial spin textures appear in various fields from high-energy to condensed matter physics [\[1-](#page-6-0)[5\]](#page-6-1), and skyrmions [\[6-](#page-6-2)[8\]](#page-6-3) and merons [\[9](#page-6-4)[-11\]](#page-6-5) in magnetic materials are their prominent examples. Magnetic skyrmions and merons are topologically protected nanoparticlelike objects with a magnetization swirl in chiral or non-centrosymmetric magnets, important for practical applications in spintronics [\[6-](#page-6-2)[10\]](#page-6-6). Depending on the properties of ferromagnetic materials and/or external magnetic fields, skyrmions, having integer topological number, can exist as isolated entities or condense into a lattice. In contrast to skyrmions, merons are topological constructs with a half-integer topological charge. Because of this, the individual merons cannot be observed and magnetic merons always form a lattice [\[10,](#page-6-6)[11\]](#page-6-5).

Recently, photonic analogies of magnetic skyrmions were observed in different realisations with either optical spin forming a skyrmion-like topology (photonic spin-skyrmions) [\[12\]](#page-6-7), or a lattice of the electric field textures with hedgehog structure (photonic field-skyrmions, which are dynamic field textures with the electric field oscillating in time) [\[13](#page-6-8)[,14\]](#page-6-9). Higher order skyrmoinic structures were also suggested in polarisation textures [\[15](#page-6-10)[,16\]](#page-6-11). Individual photonic spinskyrmions have been recently demonstrated using spin-orbit coupling in the evanescent field [\[17\]](#page-6-12) to achieve topological protection through the generalized photonic quantum spin-Hall effect [\[18\]](#page-6-13), which leads to the locking of the spin to the local momentum of the evanescent wave [\[19,](#page-6-14)[20\]](#page-6-15). The interference of the evanescent electromagnetic fields in the 6-fold symmetry cavity, results in the creation of the field-skyrmions in a hexagonal lattice [\[13](#page-6-8)[,14](#page-6-9)[,21\]](#page-6-16). Until now, there were no reports of photonic analogies of other spin-lattices, such as spin-skyrmion and spin or field merons.

Here we show that it is the symmetry of the field that completely determines the topology of the electromagnetic field of the guided modes (the modes with the evanescent field component) through spin-orbit coupling. We demonstrate both experimentally and theoretically the spinskyrmion and spin-meron lattices formed due to a broken rotational symmetry of the field, with 6-fold symmetry being responsible for a skyrmion lattice and 4-fold symmetry for a meron lattice. In the absence of the spin-orbit coupling (in the case when the field is not carrying topological charge), instead of spin-skyrmions, field skyrmions are formed, thus connecting two types of skyrmion manifestation. We also show that the spin skyrmion and spin meron textures correspond to the lowest energy of the electromagnetic field configuration, therefore, energetically stable. The demonstrated photonic spin-topologies open up new pathways for topological photonics, quantum photonics and metrology and new avenue to understand topological condensed matter systems [\[22\]](#page-6-17).

Individual photonic spin-skyrmions formed due to the spin-orbit coupling in the guided waves with the evanescent field component [Fig. 1(a)] can be described by a Hertz vector potential with a helical phase term in the cylindrical coordinate (*r, φ, z*) as [\[23\]](#page-6-18)

$$
\Psi_0 = AJ_t(k_r r)e^{il\varphi}e^{-k_z z},\tag{1}
$$

where *A* is a constant,  $k_r$  and  $ik_z$  are the transverse and longitudinal wave-vector components satisfying  $k_r^2 - k_z^2 = k^2$  $k_r^2 - k_z^2 = k^2$  with *k* being the wave-vector, and *J*<sub>*l*</sub> is the Bessel function of the first kind of order *l*. The Neel-type photonic spin-skyrmions are formed for  $l \neq 0$ , with the spin up or down in the center of the skyrmion, determined by the sign of *l*. Equation (1) describes an evanescent optical vortex (eOV) in a source free, homogeneous and isotropic medium. If the rotational symmetry of the field is broken, individual eOVs will interact and may condense in a lattice. In order to reveal the topological features under different types of symmetry, we consider superpositions of the individual Hertz vector potentials with each lattice point fed by an eOV:  $\Psi = \Psi_0 * \sum \delta(\mathbf{r} - \mathbf{r}_{mn})$ , where  $\mathbf{r}_{mn}$  is the position of each lattice point,  $\delta$  is the Dirac delta function

and \* represents the operation of convolution. In two dimensions, only two Bravais lattices exist with equal lattice constants corresponding to point groups  $D_6$  and  $D_4$  having hexagonal and square lattices, respectively [Figs. 1(a) and 2(a)]. In view of translational and rotational symmetry imposed by a lattice, the total Hertz potential can be obtained as [\[24\]](#page-6-19)

$$
\Psi = A \sum_{n=1}^{2N} e^{il\theta_n} e^{ik_r \mathbf{r} \cdot \mathbf{e}_n} e^{-k_z z} , \qquad (2)
$$

where  $\theta_n = n\pi/N$ ,  $\mathbf{e}_n = (\cos\theta_n, \sin\theta_n)$  with  $N=3$  for hexagonal and  $N=2$  for square lattice. The Hertz potential in Equation (2) exhibits rotational periodicity with total angular momentum *l* (we consider  $l = 1$  in the following). The amplitude distributions of the Hertz potential manifest the lattice symmetry features [Figs. 1(a) and 2(a)] with zero points corresponding the singularities of the phase distribution [Figs.  $1(b)$  and  $2(b)$ ].

The Poynting vector  $P=Re(E^* \times H)/2$ , which represents a directional [energy flux](https://en.wikipedia.org/wiki/Energy_flux) of an [electromagnetic field](https://en.wikipedia.org/wiki/Electromagnetic_field) in the lattice, can be calculated through the Hertz potential as [\[24\]](#page-6-19)

$$
\mathbf{P} = \frac{\omega \varepsilon k_r^2}{2} \operatorname{Im} \left( \Psi^* \nabla \Psi \right) \sim \left\langle \Psi \left| i \nabla \right| \Psi \right\rangle, \tag{3}
$$

where *ω* is the angular frequency of the wave and *ε* is the absolute permittivity of the medium. The spin texture of the electromagnetic field can be obtained considering the Poynting vector distribution in the lattice since for TM/TE polarized evanescent waves, the intrinsic spinmomentum coupling yields a relationship between the SAM **S** and the Poynting vector **P** as [\[19\]](#page-6-14)

$$
\mathbf{S} = \frac{1}{2\omega^2} \nabla \times \mathbf{P} \,. \tag{4}
$$

The generalized spin momentum relationship in Equation (4) is a manifestation of the conservation law of SAM, which indicates that the SAM of an evanescent field originates from the vortices of the electromagnetic energy flow. Equation (4) can be expressed in terms of the

Hertz potential as 
$$
\mathbf{S} = \frac{\varepsilon k_r^2}{4\omega} \text{Im}(\nabla \Psi^* \times \nabla \Psi) \sim \langle \nabla \Psi | \times i | \nabla \Psi \rangle
$$
, revealing that the SAM of an

evanescent field is in correspondence to the Berry curvature of the Hertz potential [\[25\]](#page-6-20). Consequently, a nontrivial spin texture is a consequence of winding of the Hertz potential in the presence spin-orbit coupling (e.g. formation of individual spin-skyrmion as *l*≠0).

The phase singularity of the Hertz potential pins the center of each Poynting vector vortex (Eq. 3) and, depending on the symmetry of the lattice, the in-plane Poynting vector forms different types of vortex distributions. For a hexagonal lattice, the Poynting vector possesses a rotational self-similarity with characteristic angle of  $\pi/3$  as  $\hat{R}(\frac{\pi}{3}){\bf P}({\bf r}) = {\bf P}[\hat{R}(\frac{\pi}{3}){\bf r}]$  $\hat{R}(\frac{\pi}{2})\mathbf{P}(\mathbf{r}) = \mathbf{P}[\hat{R}(\frac{\pi}{2})\mathbf{r}]$ , where  $\hat{R}(\theta)$  is the rotation matrix along *z*-axis. A hierarchical structure of the Poynting vector vortices is obtained with two sublattices, due to the multiple vortex centers in each unit cell of the lattice [Fig. 1(b)], which can be considered as an optical analogue of the Abrikosov vortex lattice [\[26\]](#page-6-21). The appearance of two sub-sets of the Poynting vector vortex lattice can be attributed to the different compositions of the wave numbers [\[24\]](#page-6-19). While for a square lattice, the Poynting vector possesses a *π*/2 angled rotational self-similarity as  $\hat{R}(\frac{\pi}{2})\mathbf{P}(\mathbf{r}) = \mathbf{P}[\hat{R}(\frac{\pi}{2})\mathbf{r}]$  $\hat{R}(\frac{\pi}{2})\mathbf{P}(\mathbf{r}) = \mathbf{P}[\hat{R}(\frac{\pi}{2})\mathbf{r}]$ . The energy flow in each unit cell exhibits a vortex with winding number  $\pm 1$  [Fig. 2(b)], which is an optical analogue of the so-called staggered-flux in condensed matter systems [\[27](#page-6-22)[,28\]](#page-6-23). The junction of the vortices in Figs. 1(b) and 2(b) can be found where the Poynting vector is zero but the Hertz potential is non-zero.

These two types of vortices in the Poynting vector distributions determine the topology of the spin textures. The topological invariant *Q*, which is known as the skyrmion number, can be obtained as  $Q = (1/4\pi) \iint \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) dxdy$ , where  $\mathbf{n} = S/|S|$  represents the unit vector in the direction of a three-component spin. Since each vortex center of the Poynting vector is the phase

singularity of the Herz vector, where the Poynting vector is zero, only *z*-component of the SAM is present in the center of the vortex [Eq. (4)], with sign of  $S<sub>z</sub>$  determined by the rotation direction of the Poynting vector vortex. For a hexagonal lattice, two skyrmion sub-lattices are observed, according to the structure of the Poynting vector distribution, with skyrmion number  $Q = 1$  for each unit cell of the sublattice where the spin vectors vary progressively from the central "up" state to the edge "down" state, manifesting a Neel-type photonic spin-skyrmion [Figs. 1(c) and (d)] without increased spin flux. For other angular momenta with non-zero SAM (*l*=2, 4, 5), the two skyrmion sub-lattices will emerge with different signs of skyrmion numbers, demonstrating four realisations of spin-skyrmion topology [\[24\]](#page-6-19).

On the other hand, the staggered-flux configuration of the Poynting vector observed in a square lattice gives rise to the formation of a photonics spin-meron lattice. From Eq. (4), the SAM distribution can be obtained as  $S \propto (k_z \sin(k_z x) \cos(k_z y))$ ,  $k_z \cos(k_z x) \sin(k_z y)$ ,  $k_z \cos(k_z x) \cos(k_z y)$ ), which possesses distinct domains where *z*-component of the local spin orientation goes to zero and merons are confined in a unit cell  $[Fig. 2(c)]$ . In each unit cell, the spin vector tilts progressively from the central 'up' or 'down' state to the edge where *z* component of SAM  $S_z = 0$ . The skyrmion number calculated for each unit cell in the spin texture is  $Q = \pm \frac{1}{2}$ , corresponding to the Neel-type photonic 'core-up' or 'core-down' spin-meron topology with a nonzero spin flux [\[24\]](#page-6-19). These spin configurations in each unit cell can be smoothly mapped on a paraboloid *z* = −  $r^2/2$  by converting the in-plane spin vector to an infinite circle with substitution

$$
\frac{k_z}{k_r} \tan(k_r x) = r \cos \varphi
$$
 and 
$$
\frac{k_z}{k_r} \tan(k_r y) = r \sin \varphi
$$
 [Fig. 2(d)]. Similar spin configurations can be

obtained for *l*=3 with opposite signs of skyrmion numbers in each unit cell [\[24\]](#page-6-19). The obtained square spin-meron lattice with alternating Poynting vector vorticities is in analogy with the lowestenergy multi-meron configuration in frustrated magnets [\[29](#page-6-24)[,30\]](#page-7-0).

It is worth noting that in the case of  $l = 0$ , the SAM vanishes and, in the absence of spin-orbit coupling, the spin skyrmion and meron are not present. In this case, the considered 6-fold and 4 fold symmetry lattices of the Herz potential result in the oscillating electric field patterns which exhibit field-topologies corresponding to a field-skyrmion lattice for  $D_6$  and a field-meron lattice for D<sup>4</sup> symmetry [\[24\]](#page-6-19)(Fig. S1). This provides connection between the concepts of spin-skyrmions formed due to the spin-orbit coupling [\[12\]](#page-6-7) and dynamic, time-dependent field-skyrmions formed due to the interference effects [\[13](#page-6-8)[,14\]](#page-6-9).

The configuration of a square meron lattice can be understood from the thermodynamic perspective considering the lowest-energy theorem. For magnetic skyrmions, the energy Hamiltonian is given by the nonlinear sigma model together with the Dzyaloshinskii-Moriya (DM) interactions for the antisymmetric exchange between two neighboring magnetic spins [\[8](#page-6-3)[,31\]](#page-7-1). A meron lattice can be stabilized in chiral magnets with anisotropy [\[32](#page-7-2)[,33\]](#page-7-3). Considering the generalized photonic quantum spin-Hall effect for transverse spin which reveals an analogy between photonic and electron spin [\[18\]](#page-6-13), the energy for a photonic skyrmion texture is described by the Hamiltonian in the classical nonlinear sigma model [\[34\]](#page-7-4)

$$
H = \frac{1}{2} \int (\nabla \mathbf{n})^2 dxdy, \qquad (5)
$$

where **n** represents the unit spin vector. The individual spin-skyrmion solution can be obtained in

the system with the rotational symmetry as  $\theta(r)$ =2arctan(*r*) by using Euler–Lagrange (EL) equation for the lowest energy [\[24\]](#page-6-19) and applying boundary conditions  $n_z(0)=1$  at the core and *n<sub>z</sub>*(∞)=−1 at infinity [Fig. 3(a)]. If we chose the boundary conditions corresponding to a square spin lattice:  $n_z(0)=\pm 1$  at the core and  $n_z(\infty)=0$  at the edge of the unit cell [Fig. 3(b)], the exact numerical solution for a meron state can be obtained behaving as  $\theta(r) \approx \arctan(r)$  or  $\theta(r) \approx \pi$ –arctan(*r*) [Fig. 3(c)]. These solutions correspond to the SAM configuration of a square lattice in Fig. 2(c) (Ref. [\[24\]](#page-6-19)). It was not possible to find an analytical solution to Equation (5) because of the difficulties with defining boundary conditions due to the composite sub-lattice structure.

We experimentally demonstrated the spin lattices of the evanescent vortex beams on the example of surface plasmon polaritons (SPPs) sustained at a dielectric-metal interface [\[35\]](#page-7-5). A radially polarized laser beam with helical wavefront (topological charge *l*=1, wavelength *λ*=532 nm) is tightly focused onto a thin silver film surface, which provides an excitation of SPPs with OAM at air/silver interface. Under such conditions, individual photonic spin-skyrmions are formed on the metal interface in the case if rotational symmetry is not broken [\[12\]](#page-6-7). To break the rotational symmetry, the incident beam was modulated with intensity masks comprised of either six- or four-fold symmetry apertures. The spin-skyrmion or spin-meron textures are formed in the case of six-fold or four-fold symmetry, respectively, due to interference of the SPPs in the presence of the spin-orbit interaction [Fig. 4(a)]. A spin-resolved near-field scanning optical microscope was used with a dielectric nanosphere as a near-field probe to characterize the spin-texture of the vortex lattice [\[36\]](#page-7-6) (see Ref. [\[24\]](#page-6-19) for the details of the experiment). The measured  $S_z$  distributions and the retrieved local spin vector orientation exhibit a skyrmion-type lattice for the field of sixfold symmetry and reveal spin-meron lattice for four-fold symmetry [Figs. 4(b-e)]. These distributions correspond well to the simulation results for respective symmetries of the plasmonic field [cf. Figs. 4(c,e) and 2(c), 3(c)], revealing to interleaved hexagonal skyrmion lattices for a hexagonal lattice and square lattice of positive/negative merons.

In conclusion, we demonstrated the formation of photonic spin-skyrmion and meron lattices in real-space based on spin-orbit coupling in the evanescent waves in the environment with broken rotational symmetry. We showed that the field symmetry is the key for determining the photonic spin-topology, resulting either 6-fold symmetry spin-skyrmion lattices or 4-fold symmetry of spinmeron lattices. These two kinds of spin-lattice topologies were verified on the example of SPP waves in the presence of spin-orbit coupling in the broken rotational symmetry conditions. In the absence of spin-orbit coupling, the spin lattices are degenerated to time-dependent field-skyrmion or field meron lattices. Thermodynamic considerations confirm stability of spin-textures of the evanescence waves. These new topological features of electromagnetic waves may provide new insights on the properties of skyrmion and meron topological structures and their transformations in condensed matter physics, where they may be difficult to realize, as well as new applications in metrology (e.g. for magnetic domain observations [\[37\]](#page-7-7)), topological and quantum photonics and spin optics.

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All the data supporting the findings of this study are available in the main text and Supplemental Material.

#### **References:**

- <span id="page-6-0"></span>1. D. Hsieh et al., Science **323**, 919-922 (2009).
- 2. M. R. Matthews et al., Physical Review Letters **83**, 2498 (1999).
- 3. L. Lu, J.D. Joannopoulos, M. Soljačić, Nature Photonics **8**, 821-829 (2014).
- 4. D. Hsieh et al., Nature **452**, 970-974 (2008).
- <span id="page-6-1"></span>5. J. E. Moore, Nature **464**, 194-198 (2010).
- <span id="page-6-2"></span>6. N. Nagaosa, Y. Tokura, Nat Nanotech. **8**, 899-911 (2013).
- 7. A. Fert, N. Reyren, V. Cros, Nature Reviews Materials **2**, 1-15 (2017).
- <span id="page-6-3"></span>8. S. Mühlbauer, Science **323**, 915-919 (2009).
- <span id="page-6-4"></span>9. N. Gao et al., Nat Commun **10**, 1-9 (2019).
- <span id="page-6-6"></span>10. X. Z. Yu et al. Nature **564**, 95-98 (2018).
- <span id="page-6-5"></span>11. K. Moon et al., Phys Rev B Condens Matter **51**, 5138-5170 (1995).
- <span id="page-6-7"></span>12. L. Du, A. Yang, A.V. Zayats, X. Yuan, Nature Physics **15**, 650-654 (2019).
- <span id="page-6-8"></span>13. S. Tsesses et al., Science **361**, 993-996 (2018).
- <span id="page-6-9"></span>14. T. J. Davis et al., Science **368** (2020).
- <span id="page-6-10"></span>15. M. Król et al., Optica **8**, 255 (2021).
- <span id="page-6-11"></span>16. W. Lin, Y. Ota, Y. Arakawa, and S. Iwamoto, Physical Review Research **3** (2021).
- <span id="page-6-12"></span>17. K. Y. Bliokh, F. J. Rodríguez-Fortuño, F. Nori, A. V. Zayats, Nature Photonics **9**, 796-808 (2015).
- <span id="page-6-13"></span>18. K. Y. Bliokh, D. Smirnova, F. Nori, Science **348**, 1448-1451 (2015).
- <span id="page-6-14"></span>19. P. Shi et al. Proc Natl Acad Sci U S A **118** (2021).
- <span id="page-6-15"></span>20. T. Van Mechelen, Z. Jacob, Optical Materials Express **9**, 95 (2018).
- <span id="page-6-16"></span>21. S. Tsesses et. al., Nano Lett **19**, 4010-4016 (2019).
- <span id="page-6-17"></span>22. T. Van Mechelen, W. Sun, and Z. Jacob, Nature Communications **12** (2021).
- <span id="page-6-18"></span>23. A. Ishimaru, Electromagnetic wave propagation, radiation, and scattering from fundamentals to applications. (Wiley, 2017).
- <span id="page-6-19"></span>24. See Supplemental Material at http://link.aps.org/supplemental/..... for eigen-mode analysis of the Helmholtz equation with 6-fold and 4-fold symmetry; analysis of the Poynting vector vortices with different symmetry; skyrmion feature with different symmetry; coupling and transition between SAM and Poynting vector; energy considerations for a spin-meron lattice configuration; experimental setup for exciting and measuring the photonic spin lattices; Figs. S1-S5.
- <span id="page-6-20"></span>25. D. Xiao, M.-C. Chang, and Q. Niu, Reviews of modern physics **82**, 1959 (2010).
- <span id="page-6-21"></span>26. B. Rosenstein, D. Li, Reviews of Modern Physics **82**, 109-168 (2010).
- <span id="page-6-22"></span>27. C. C. Chang, R. T. Scalettar, Phys Rev Lett **109**, 026404 (2012).
- <span id="page-6-23"></span>28. L. K. Lim, C. M. Smith, A. Hemmerich, Phys Rev Lett **100**, 130402 (2008).
- <span id="page-6-24"></span>29. Y. A. Kharkov, ). O. P. Sushkov, M. Mostovoy, Phys Rev Lett **119**, 207201 (2017).
- <span id="page-7-0"></span>30. U. K. Rossler, A. N. Bogdanov, C. Pfleiderer, Nature **442**, 797-801 (2006).
- <span id="page-7-1"></span>31. X. Z. Yu et al., Nature **465**, 901-904 (2010).
- <span id="page-7-2"></span>32. S.-Z. Lin, A. Saxena, C. D. Batista, Physical Review B **91**, 224407 (2015).
- <span id="page-7-3"></span>33. M. Ezawa, Physical Review B **83**, 100408 (2011).
- <span id="page-7-4"></span>34. R. Rajaraman, Solitons and Instantons (North-Holland, Amsterdam, 1982).
- <span id="page-7-5"></span>35. A. V. Zayats, I. I. Smolyaninov, A. A. Maradudin, Physics Reports **408**, 131-314 (2005).
- <span id="page-7-6"></span>36. C.C. Li, P. Shi, P., L. Du, X. Yuan, Nanoscale **12**, 13674-13679 (2020).
- <span id="page-7-7"></span>37. X. Lei, L. Du, X. Yuan, and A. V. Zayats, Nanophotonics (2021) doi.org/10.1515/nanoph-2021- 0201.

# **FIGURES.**



Fig. 1. (a) Schematic of the eOV lattice of hexagonal symmetry with a lattice constant  $2\lambda_r$  ( $\lambda_r$  is in-plane wavelength) and the distribution of the amplitudes of the resultant Hertz potential. Each lattice point (red dots) is fed with an eOV described by a helical Hertz potential  $\Psi_0$  corresponding to isolated spin-skyrmion (inset). (b) Poynting vector direction (arrows) and magnitude (arrow colors) in the generated optical vortex lattice in (a). Background color represents the phase distribution of total Hertz potential. Red and yellow dots denote the center of the anticlockwise Poynting vector vortices (due to two sub-lattices), where the phase of the Hertz potential is singular; blue dots denote the center of clockwise ones. Green dots show the junction of different Poynting vector vortices where  $P=0$  with nonzero Hertz potential. The scale bar is  $\lambda_r$ . Note that thin vertical line in each vortex is an artifact of the colour representation where phase is the same on both sides. (c) Optical spin orientation distribution corresponding to the lattice in (a,b) showing a spin-skyrmion lattice. The arrows indicate the direction of the unit spin vector. (d) Two sub-lattices of the skyrmion lattice forming the spin distribution in (c). Central "up" state in each sub-set can be referred to red and yellow dots in (b), respectively.



Fig. 2. (a) Schematic and the distribution of the Hertz potential amplitude for the evanescent optical vortex lattice of square symmetry with a lattice constant  $\lambda_r$ . (b) Poynting vector direction (arrows) and magnitude (arrow colors) for the square lattice in (a). The scale bar is  $\lambda r/2$ . (c) Optical spin orientation distribution corresponding to the lattice in (a) and (b). The dashed red and blue squares indicate unit cells of the spin-meron lattice with skyrmion number  $Q = \pm \frac{1}{2}$ , which can be smoothly mapped on a paraboloid in (d).



Fig. 3. (a) Isolated skyrmion configuration in a cylindrical system derived from lowest energy theorem in the nonlinear sigma model. (b) Coordinate transformation from an infinite circle to a confined square when a square spin lattice is formed. (c) Two meron configurations confined in a square corresponding to two solutions of Eq. (4) with  $Q = \pm \frac{1}{2}$ , forming the spin-lattice in Fig. 2(c).



Fig. 4. (a) Schematic diagram of the experiment. SPPs are excited on a surface of a thin silver film by a tightly focused, radially polarized beam modulated by the intensity masks comprised of six- or four-fold symmetry apertures. A dielectric nanosphere is employed as a near-field probe to characterize the spin-texture of the spin lattice, formed at the center area induced by the spin-orbit interaction in SPPs. (b-e) Measured  $S_z$  component (b,d) and local spin orientation (c,e) in the lattice obtained with the field of six-fold (b,c) and four-fold (d,e) symmetry. The scale bar in (b) and (d) is the SPP wavelength. In c and e, the arrows indicate the orientation of the normalized spin vectors, with the *z*-direction colour-coded to the same colour scale as (b) and (d).