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An LTspice Electrical Circuit Model of the HTS Dynamo

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***Abstract*—The high-temperature superconducting (HTS) dynamo enables injection of large DC currents into a superconducting coil, without the need for bulky and thermally-inefficient current leads. For practical applications of such technology – like energizing superconducting coils in superconducting rotating machines and NMR/MRI magnets – modelling and understanding the behavior while charging a coil is of great interest. From an electrical circuit point of view, the HTS dynamo can be modelled as a current-controlled voltage source with a specified transresistance corresponding to the effective (internal) resistance. The overall charging curves of a coil by an HTS dynamo can be accurately (and quickly, in seconds or less) predicted using such an electrical circuit model by connecting an *RL* load representing a coil and any circuit resistance. In this work, an electrical circuit model of the HTS dynamo built in the circuit simulation software LTspice is described. The model is then used to simulate the charge of an HTS coil under different practical scenarios, and the results validated against both numerical and experimental results published in the literature.**

***Index Terms*—High-temperature superconductivity, HTS dynamo, HTS modelling, electrical circuit model, flux pump, superconducting coil charging**

I. Introduction

THE high-temperature superconducting (HTS) dynamo enables injection of large DC currents into a superconducting coil, without the need for bulky and thermally-inefficient current leads. Because of this important advantage, there is significant interest in using such technology to energise superconducting coils in superconducting rotating machines [1], [2], [3], [4], [5] and NMR/MRI magnets [6], [7].

In recent years, several different numerical models of the HTS dynamo have been developed and have played a key role in elucidating the underlying physics of such devices. Such models also provide useful, cost-effective tools to optimise and improve HTS dynamo designs [8]. For practical applications of such technology, modelling, understanding and predicting the behaviour while charging a coil is of great interest [5], [9], [10], [11]. Coupled numerical models have been developed for this purpose [9], [11]; however, such models can require extensive computational time (several hours or even days) because the full charging process may require 1000s of cycles of rotation.

From an electrical circuit point of view, the HTS dynamo can be modelled as a DC voltage source in series with an effective (internal) resistance [12], [13]. The overall charging curves of a coil by an HTS dynamo can be accurately (and quickly, in seconds or less) predicted using such an electrical circuit model by connecting an *RL* load representing a coil and any circuit resistance, e.g., the resistance of soldered joints.

In this work, an electrical circuit model of the HTS dynamo built in the circuit simulation software LTspice [14] is described. The model is then used to simulate the charge of an HTS coil under different practical scenarios, with the results validated against both numerical and experiment results published in the literature. Section II describes the general electrical circuit model and then the implementation of this model in LTspice. Section III validates the results of the model against the analytical results presented by Ghabeli *et al*. in [9], based on the assumptions of the HTS dynamo benchmark problem [15] applied to charging a load HTS coil. Section IV validates the results of the model against the experimental results reported by Jiang *et al*. in [12].

II. Electrical Circuit Models of HTS Dynamo Coil Charging

A. General Electrical Circuit Model

Fig. 1 shows an equivalent electrical circuit model of an HTS dynamo, represented by a DC voltage source V_{oc} and effective (internal) resistance R_{int}, connected to a load HTS coil, represented by an inductance L_{sc} and equivalent (variable) resistance R_{sc}. R_c represents the circuit resistance, e.g., the resistance of soldered joints connecting the HTS dynamo to the load HTS coil.

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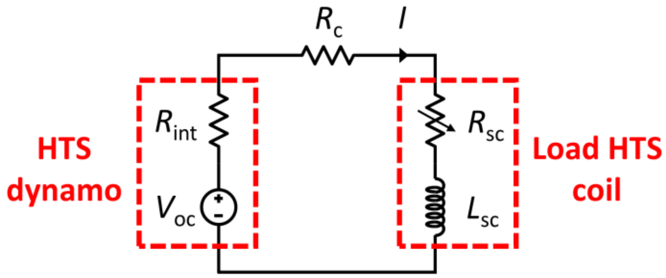

Fig. 1. Equivalent electrical circuit model of an HTS dynamo, represented by a DC voltage source V_{oc} and effective (internal) resistance R_{int}, connected to a load HTS coil, represented by an inductance L_{sc} and equivalent (variable) resistance R_{sc}. R_c represents the circuit resistance, e.g., the resistance of soldered joints.

For an ideal coil and/or assuming the circuit current $I(t) << I_{c,coil}$, the critical current of the load HTS coil, R_{sc} can be neglected (see Section III). In this case, $I(t)$ is limited by the maximum value of current that the HTS dynamo can deliver [9]. The current flowing in the circuit can then be calculated by

$$I(t) = I_{sat}[1 - e^{-t/\tau}] \quad (1)$$

where $I_{sat} = V_{oc} / (R_c + R_{int})$ is the saturation current and $\tau = L_{sc} / (R_c + R_{int})$ is the time constant of the circuit, which determines its charging rate.

To consider saturation of the current when limited by $I_{c,coil}$ (see Section IV), the equivalent (variable) resistance R_{sc} can be included, which can be determined from the measured current-voltage relationship, $V_{coil}(I)$, for the coil, as described in [12].

B. LTspice Model

LTspice is a free and widely-used circuit simulation software based on SPICE (Simulation Program with Integrated Circuit Elements). It has been previously used successfully to model the HTS transformer-rectifier flux pump in [16], [17]. Fig. 2 shows the LTspice implementation used here in this work of the equivalent electrical circuit model in Fig. 1.

The HTS dynamo is represented as a current-controlled voltage source using three voltages sources. Firstly, **Voc** is a DC voltage source with a voltage value set to the open-circuit voltage, V_{oc}, from the dynamo's *V-I* curve (note this circuit element's series resistance is set to 0 Ω). Next, **H1** is a current-controlled voltage source. This circuit element applies a voltage between its nodes equal to the value of its transresistance (input as $-R_{int}$) times the current flowing through the voltage source **Vref** which here acts as a reference (or dummy) voltage source for this purpose. **Rc** is simply a resistor (representing the circuit resistance). Finally, **Rsc** and **Lsc** are a resistor and inductor representing the equivalent (variable) resistance and the inductance of the load HTS coil, respectively.

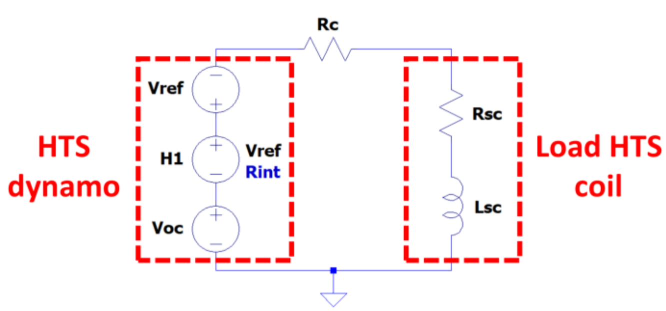

Fig. 2. LTspice implementation of the equivalent electrical circuit model shown in Fig. 1. The HTS dynamo is represented as a current-controlled voltage source with a specified transresistance corresponding to the effective (internal) resistance.

III. Validation Against Benchmark Problem

To validate the LTspice model, we first compare the results of the model with the results presented in [9]. In [9], the *V-I* curves of the modelled HTS dynamo – based on the assumptions of the HTS dynamo benchmark problem [15], which sees a single permanent magnet rotating past a single HTS wire – are calculated using two numerical modelling frameworks (MEMEP and the segregated **H**-formulation). These curves then provide the parameters V_{oc} and R_{int} for the equivalent electrical circuit model.

Table I summarises these parameters (V_{oc}, R_{int}) as calculated in [9] for rotation frequencies of 4.25, 25 and 50 Hz and an HTS dynamo airgap of 3.7 mm. The airgap corresponds to the distance between the permanent magnet face and the inner surface of the HTS wire. An ideal HTS coil is assumed with L = 0.24 mH (and R_{sc} = 0 Ω), and the circuit (joint) resistance R_c = 0.88 μΩ. Also included in Table I are the calculated values for I_{sat} and τ for each frequency.

TABLE I. Assumed Parameters for Validation Against the Benchmark Problem in [9]

HTS dynamo	4.25 Hz	V_{oc}	10.177 μV
		R_{int}	0.324 μΩ
		I_{sat}	8.45 A
		τ	199.34 s
	25 Hz	V_{oc}	59.219 μV
		R_{int}	1.7235 μΩ
		I_{sat}	22.75 A
		τ	92.18 s
	50 Hz	V_{oc}	117.911 μV
		R_{int}	3.3415 μΩ
		I_{sat}	27.93 A
		τ	56.85 s
Load HTS coil	Inductance, L_{sc}		0.24 mH
	Resistance, R_{sc}		0 Ω
Circuit (joint) resistance, R_c			0.88 μΩ

Fig. 3 shows a comparison of the calculated charging current curves of the HTS coil in LTspice and the results of the analytical equation (1), for three HTS dynamo frequencies of 4.25, 25 and 50 Hz, and an airgap of 3.7 mm. There is excellent agreement between these two methods.

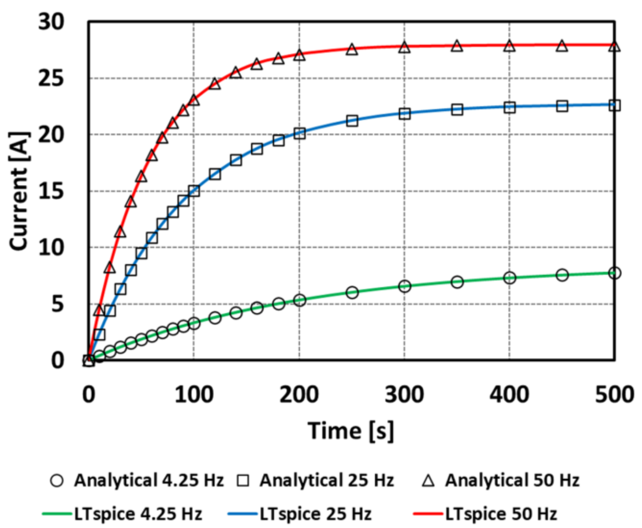

Fig. 3. The calculated charging current curves of the HTS coil in LTspice, compared with the results from the analytical equation (1), for three HTS dynamo frequencies of 4.25, 25 and 50 Hz, and an airgap of 3.7 mm.

IV. Validation Against Experimental Results

To validate the LTspice model further, we now modify the assumed model parameters to enable a comparison with the experimental results reported in [12]. Firstly, we calculate the parameters V_{oc} and R_{int} for the experimental HTS dynamo – which consists of two co-aligned circular arrays of 12 N38 NdFeB permanent magnets that rotate past a single HTS wire – for rotation frequencies of 48, 192 and 600 Hz, from the *V-I* curves measured in [12]. These parameters are listed in Table II. In addition, the experimental HTS coil has an inductance L = 2.4 mH, and the limitation of the coil due to its critical current $I_{c,coil}$ is taken into account via its equivalent resistance R_{sc}. R_{sc} is calculated using the measured *V-I* curve of the load HTS coil in [12], shown in Fig. 4(a), which is fitted in Fig. 4(b) using the function:

$$R_{sc}(I) = R_0(I/I_{c,coil})^n \qquad (2)$$

where R_0 is the equivalent resistance at the critical current, $I_{c,coil}$ is the measured critical current, and n represents the steepness of the transition between the superconducting and normal state. Since the coil length is 4000 cm, for a characteristic electric field of 1 μV/cm, the corresponding voltage is 4000 μV, which in turn corresponds to $I_{c,coil}$ = 55.315 A. Thus, R_0 = 72.3 μΩ, and trial and error found n = 35 to fit the curve best. R_{sc} is programmed into the circuit element **Rsc** in the LTspice circuit as R = 7.23e-5*(I(Vref)/55.315)**35. Finally, the circuit resistance R_c is the same as Section III.

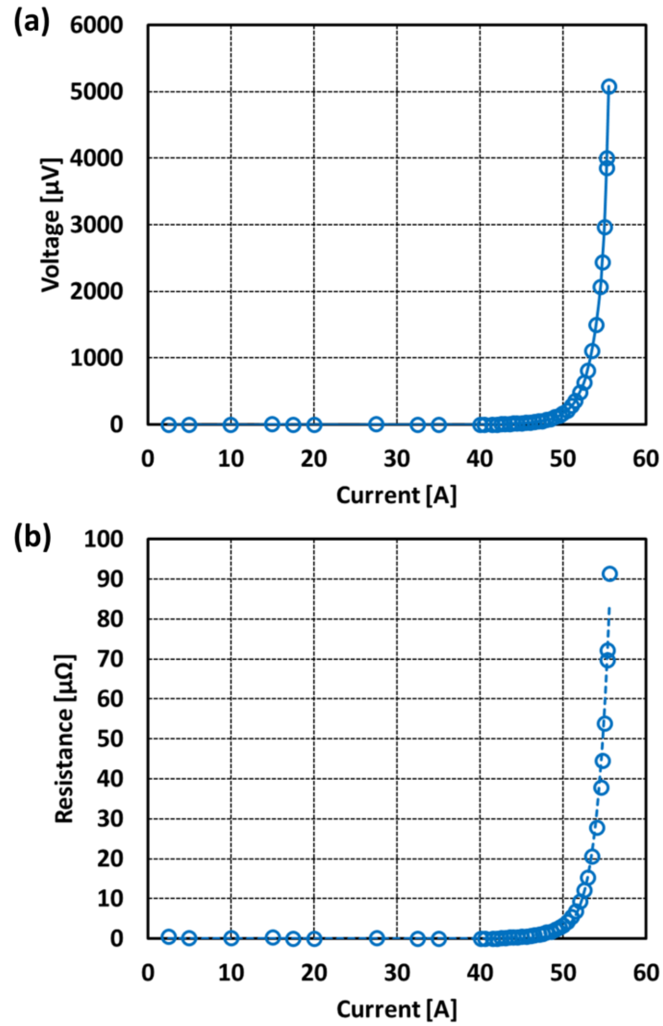

Fig. 4. (a) Measured *V-I* curve of the load HTS coil in [12], (b) Calculated (open symbols) and fitted (dashed line) equivalent resistance for the coil, used as R_{sc} in the equivalent circuit (see Fig. 1) and LTspice model.

TABLE II.
Assumed Parameters for Validation Against the Experimental Results in [12]

HTS dynamo	48 Hz	V_{oc}	0.75 mV
		R_{int}	13.2 μΩ
		I_{sat}	56.9 A
		τ	170.5 s
	192 Hz	V_{oc}	2.84 mV
		R_{int}	45.6 μΩ
		I_{sat}	62.3 A
		τ	51.6 s
	600 Hz	V_{oc}	8.45 mV
		R_{int}	130.8 μΩ
		I_{sat}	64.8 A
		τ	18.2 s
Load HTS coil	Inductance, L_{sc}		2.4 mH
	Resistance, R_{sc}		See Fig. 4(b)
Circuit (joint) resistance, R_c			0.88 μΩ

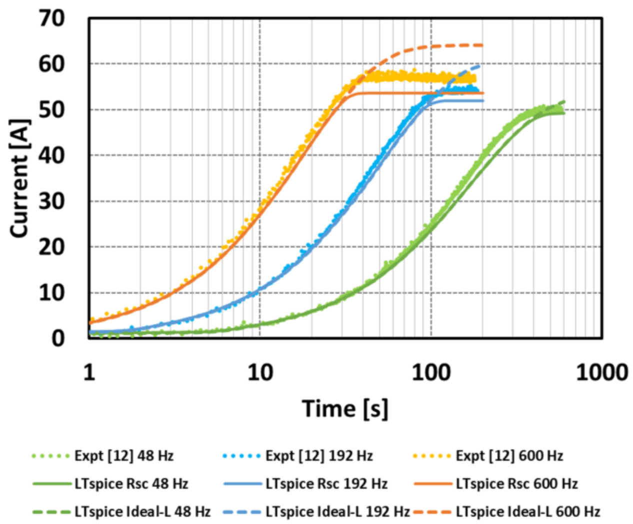

Fig. 5. The calculated charging current curves of the HTS coil in LTspice, compared with the experimental results reported in [12], for three HTS dynamo frequencies of 48, 192 and 600 Hz.

Fig. 5 shows a comparison of the calculated charging current curves of the HTS coil in LTspice and the experimental results reported in [12], for three HTS dynamo frequencies of 48, 192 and 600 Hz. For all frequencies, there is excellent agreement between the LTspice model and experimental results for much of the charging curves, until the current approaches the saturation current of the circuit. For the lowest frequency (48 Hz), there is also excellent agreement as the current saturates; however, for the higher frequencies (192, 600 Hz), the LTspice model tends to under-estimate the saturation current by a few amperes.

One possible explanation for this is that simply treating the resistive voltage of the load HTS coil as a resistor based on its measured *V-I* curve may not be wholly appropriate, especially at higher frequencies. Since the coil's *V-I* curve is obtained from a DC measurement, some frequency scaling may be required to take into account any frequency dependence of the critical current [18] and hence the coil's equivalent resistance. However, in this case – where the HTS dynamo is charging a load HTS coil – the current waveform is not a simple sinusoid like that analysed in [18]. The charging current curves shown in Figs. 3 and 5 actually contain ripples within each cycle, as described in detail in [9], which cannot be easily captured via analytical methods or electrical circuit models. This is beyond the scope of this paper, but should be investigated in future work.

For reference, the current charging curves when treating the load HTS coil as ideal (as per Section III, where $R_{sc} = 0$) are also included in Fig. 5 as dashed-line curves. In this case, the circuit is limited by the maximum value of current that the HTS dynamo can deliver, and it is clear that the LTspice model will significantly over-estimate the saturation current. This indicates the need for the inclusion of the equivalent (variable) resistance R_{sc} when saturation of the current is limited by $I_{c,coil}$.

It should also be noted that the HTS dynamo can also be modelled directly, as shown in Fig. 1, as a DC voltage source with a value of V_{oc} and a series resistance with a value of R_{int}. For the linear *V-I* curves presented in this work – from [9] and [12] – both approaches are equivalent in terms of the circuit behaviour. However, complications arise when the dynamo has a nonlinear *V-I* curve, like in [19]. In the case of the current-controlled voltage source implementation, nonlinear curves may be implemented easily using a polynomial fitting function for the *V-I* curve in the **H1** element (other methods exist too).

V. Conclusion

The overall charging curves of a load HTS coil by an HTS dynamo can be accurately (and quickly, in seconds or less) predicted using an electrical circuit model. In this work, an electrical circuit model of the HTS dynamo – which can be modelled as a current-controlled voltage source with a specified transresistance corresponding to the effective (internal) resistance – is built in the circuit simulation software LTspice.

The model is firstly validated against the analytical results presented by Ghabeli *et al.* in [9], where an ideal HTS coil is assumed, and excellent agreement is obtained. Next, the model is validated against the experimental results by Jiang *et al.* in [12], where an equivalent (variable) resistance R_{sc} of the load HTS coil is included in the electrical circuit to take into account the limitation of the coil due to its critical current $I_{c,coil}$. The model shows excellent agreement for much of the charging curves, but tends to under-estimate the saturation current by a few amperes for higher rotation frequencies. To improve the model's accuracy, future work should investigate frequency scaling of R_{sc} to consider the ripples contained in the charging current curves contained within each cycle.

VI. Data Statement

The data supporting this article is openly available from the King's College London research data repository, KORDS, at https://doi.org/10.18742/24018519.

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