Teacher identity and professional development in primary school mathematics

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Abstract

This thesis explores processes of teacher change in primary mathematics, focusing on beliefs and knowledge. Using a case study, a theoretical approach is developed to conceptualise teacher learning. This approach integrates situated theories with notions of identity in order to address weaknesses in existing understandings. Teachers are viewed as active agents authoring their own change in relation to social structures. This provides a more heterogeneous and differentiated picture of teacher development.

The study followed six teachers over four years of professional development. The teachers were involved as teacher-researchers in a project working with academic researchers on lesson development for wider dissemination. The degree of change amongst the six teachers was different and understanding this differential change is a key focus. Qualitative data were collected through interviews and participant observation. Analysis was conducted reflexively through the researcher's role drawing on ethnographic techniques.

Teachers' identities and wider professional networks, or zones of enactment, are seen as key to their engagement and learning. Further factors were the teachers' relationships with school mathematics and, for those whose belief change was significant, their desire to be a different teacher of mathematics. Teachers are conceived of as adapting existing practices in order to make sense of new ideas, to imagine new practices and to act in new settings. Similarity and difference are used as analytic tools to explore the teachers' engagement and to address issues of transfer, motivation and ability to change. Reflection appeared to be an infrequent yet important event in the teachers' learning. Teachers' multiple identities as tutors, curriculum developers and researchers are found to provide opportunities for conscious and explicit reflection.

It is suggested that developing a connected understanding of mathematics is particularly difficult for primary teachers and that sustaining change processes is more problematic and complex than has previously been acknowledged.

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Abbreviations

BERA	British Educational Research Association
BSRLM	British Society for Research into Learning Mathematics
CAME	Cognitive Acceleration in Mathematics Education
CASE	Cognitive Acceleration in Science Education
CPD	Continuing Professional Development
CSMS	Concepts in Secondary Mathematics and Science study
DfEE	Department for Education and Employment
DfES	Department for Education and Skills
FDPRP	Fractions, decimals, percentages, ratio and proportion
GEST	Grants for Education Support and Training
HMI	Her Majesty's Inspectorate
INSET	In-Service Training
ITE	Initial Teacher Education
KS1, KS2, KS3	Key Stage 1, Key Stage 2, Key Stage 3
LEA	Local Education Authority
LNRP	Leverhulme Numeracy Research Programme
NNS	The National Numeracy Strategy
OfSTED	Office for Standards in Education
OISE/UT	Ontario Institute for Studies in Education, University of Toronto
P CAME	Primary Cognitive Acceleration in Mathematics Education
PGCE	Post-Graduate Certificate in Education
PD	Professional Development
QCA	Qualifications and Curriculum Authority
SOLO	Structure of Observed Learning Outcomes
TTA	Teacher Training Agency
TM	Thinking Maths
Y5, Y6 etc	Year 5, Year 6 etc

Chapter 1: Introduction and Background to The Research

1. Introduction

The research reported in this thesis explores the professional change of primary teachers in relation to the teaching and learning of school mathematics. My particular focus is on the development of their beliefs and knowledge about mathematics education. A major concern of the study is on exploring the differential patterns of professional change amongst the teachers.

The research was conducted with a group of six teachers as they underwent professional development in mathematics education between Autumn 1997 and Summer 2001. Three of the six teachers were involved in the research throughout the period, whilst the others were involved for shorter periods of between one and two years.

One feature of my approach in this thesis is that, in place of a standard literature review, I integrate my discussions of the theory and research literature with my analysis of the empirical data. Hence, I discuss the literature throughout the main body of the thesis, in particular in Chapters 4, 5, 6 and 7. In this chapter, therefore, I provide an overview of the literature and theory on which I draw, by outlining mathematics teacher education as a domain in terms of research and policy, and, by introducing the key theoretical ideas on which I draw.

In this chapter I introduce the research. The structure of the chapter is as follows:

- In Section 2, I briefly outline the research study.
- In Section 3, I set out the background to this study placing my work within a broad research and policy arena. I also discuss my own personal motivation for conducting the research.
- In Section 4, I outline the aims of the research.

- In Section 5, I briefly describe the central theoretical themes that I use throughout this thesis: practices and discourses, communities of practice, identity, and beliefs and knowledge.
- In Section 6, I briefly summarise my own conception of good mathematics teaching, an understanding that underlies my analysis.
- In Section 7, I give an outline of the structure of the thesis.

2. The Research Setting

The six teachers, Alexandra, Henrietta, Janice, Lisa, Tony and Ursula, were all involved as teacher-researchers in the Primary Cognitive Acceleration in Mathematics Education (CAME) Project. The professional development they received was directed towards communicating the CAME approach to school mathematics and towards enabling the teachers to become CAME lesson developers and tutors to a further group of teachers. For all six teachers, their involvement in primary CAME, and the professional development (PD) that they received, was both intense and extended. This PD was somewhat unusual in that it was integrated within Primary CAME's research and development work.

The Primary CAME Project was a joint venture between Outertown LEA and King's College London. The teachers were members of the project research team whose other members were a group of King's researchers, the Outertown LEA Mathematics Advisor and myself.

The names of the Local Education Authority, the teachers, schools and other research participants have been changed to preserve their anonymity. However, I use my own name and the real names of the CAME King's researchers: Mundher Adhami, David Johnson and Michael Shayer.

I discuss Primary CAME and outline the research setting in some depth in Chapter 2.

3. Background to The Research

In this section I set this thesis within a research and policy context and, in doing so, outline the need for this study. My intention is to give a brief overview of the field of study and to locate my research questions in terms of this research and policy context. Many of these issues are explored in considerably more depth elsewhere in this thesis.

3.1 Mathematics Teacher Education as a Research Domain

When I began this research in 1997, mathematics teacher education was acknowledged by many commentators to be a neglected field of study, particularly in relation to in-service teacher education or continuing professional development (Askew & Wiliam, 1995; Cooney, 1994a; Grouws & Schultz, 1996). Since then there has been upsurge of interest with the establishment of an international journal in this area, the Journal for Research in Mathematics Teacher Education, and several edited reviews (e.g., Fennema & Nelson, 1997; Jaworski, Wood, & Dawson, 1999). Nevertheless, the field remains a relatively immature one.

One approach that researchers and commentators in mathematics education have taken in order to address and overcome the limitations of the research base is to draw on and review the teacher education literature more generally (e.g., Boero, Dapueto, & Parenti, 1996; Brown & Borko, 1992; Loucks-Horsley, Hewson, Love, & Stiles, 1998). Taking this approach, Clarke (1994), for example, reviews the general teacher education literature to identify ten key and widely accepted principles for the professional development of mathematics teachers. These include allowing opportunities for reflection and of working with groups of teachers rather than isolated individuals. Indeed, reflection, a key component of Primary CAME, is often cited as an essential element of professional change, although the process of enabling teachers to reflect is not well understood (e.g., see Cooney, 1994b).

Clarke's general principles are, however, at a very broad level of good practice and they do relatively little to address issues specific to teacher education in mathematics. Moreover, the focus is on the professional development initiative and it says little about the process of change for individuals. There is, as Clarke notes, substantial evidence that professional change in mathematics education can be extremely difficult. Even in favourable circumstances and with teachers well-disposed to a PD initiative, the extent of professional change may not be significant (e.g., Thompson, Philipp, Thompson, & Boyd, 1994; Wilson & Goldenberg, 1998). Indeed, teachers' self-perceptions of change may be deceptive (Cohen, 1990; Lortie, 1975; Spillane, 1999). In the study reported here, for example, despite the PD programme meeting all of Clarke's (1994) principles, the degree of professional change amongst the group of teachers was very different. One explanation might be that, as Clarke (1997) argues, teachers may have competing needs and wants and, hence, may simply not have the personal space to change. However, whilst this may be an important factor, I feel that it does not fully explain the differential change in this case. Moreover, this approach evokes a model of individual teacher deficit as criticized by Brown (1991), for example.

Although there is considerable evidence that teacher change takes time (e.g., Clarke, 1994), and that extended PD is more effective than short INSET (e.g., Askew, Brown, Rhodes, Johnson, & Wiliam, 1997), there is little research exploring teachers' professional change over a prolonged period of time. One exception is Franke, Carpenter, Levi, & Fennema (2001), although their focus is on the lasting effects of a professional development programme rather than the process of change over time. Another is Ensor (2001), although her focus is on secondary mathematics teachers moving from initial training into their first year of teaching.

Much of the research that does exist in mathematics teacher education focuses on pre-service rather than in-service teacher education (e.g., Brown, McNamara, Hanley, & Jones, 1999; Simon, 1994; Vacc & Bright, 1999). Other research is focused on the continuing professional development of secondary mathematics teachers (e.g., Adler, 1998; Cooney, Shealey, & Arvold, 1998; Hoyles, 1992; Jaworski, 1999; Nolder, 1992). It is important to note, however, that primary teachers as generalists have different professional development needs to secondary subject specialists. Although there are studies focusing on primary teachers' continuing professional change in mathematics (e.g., Clarke, 1997; Millett, 1996; Schifter, 1996; Spillane, 1999), these highlight the need to develop a more substantial research base. Research in the area of how primary teachers change and develop their knowledge of school mathematics is particularly limited.

There is, as Hawley and Valli (1999) argue, a considerable gap between these ideas and the actual practice in much continuing professional development (CPD) for teachers. In a discussion about teacher professional development generally, they highlight that despite an "unprecedented consensus ... among researchers, professional development specialists, and key policymakers on ways to increase the knowledge and skills of educators substantially. ... this vision differs radically from current practice in most schools" (p.127). The vision that Hawley and Valli point to is one in which extended professional development initiatives are set within teachers' school and wider professional networks. Increasingly research in mathematics education is pointing to the role of contextual and social factors within mathematics education, much of it influenced by theories of situated learning (e.g., (Boaler, 2000c). However, little research of this nature has been conducted within mathematics teacher education, a problem which Putnam and Borko (2000) attempt to correct by re-examining the body of existing research from a situated perspective. Jaworski (1999) and Stein, Silver and Smith (1998) are exceptions, but both studies are conducted with subject specialists. Spillane (1999) does begin to explore primary teachers' wider social networks in relation to school mathematics and goes some way towards addressing the issue of differential change by introducing the notion of a teacher's *zone of enactment* to describe and analyse a teachers' wider professional networks and social resources, although this idea needs further explication and development. (See Chapter 4 for a development of this idea.)

In summary, mathematics teacher education is in the process of establishing itself as a research domain and our knowledge in the area of primary teacher CPD in mathematics is particularly fragmented. Moreover, whilst social factors are recognised as important, the exploration of social and situated perspectives is particularly limited in this area. I have highlighted further the need for research exploring the process of teacher change, and for tracking the process of change over an extended period of time. I have also noted that the issue of reflection, although widely promoted, is not well-understood. I used these issues to inform my research questions which are outlined in Section 4 below.

3.2 The Policy Context: A Focus on Primary Mathematics

There has been for some time an intense focus on the teaching of primary mathematics in England. Although evidence from the Third International Mathematics and Science Study (Beaton et al., 1996) is far from conclusive (e.g., see Brown, 1998), government ministers and their advisors amongst others have interpreted the study as providing evidence that UK children's attainment in mathematics is well behind that of other comparable countries. As a result, in 1999, the National Numeracy Strategy (NNS) was introduced with the intention of transforming primary mathematics teaching throughout England both in terms of content and teaching methods in order to significantly raise the mathematical achievement of all 11 year olds.

Recent OfSTED evaluations of the NNS have highlighted poor teacher subject knowledge of mathematics as a continuing difficulty (OfSTED, 2001; see also Earl et al., 2001; OfSTED, 2000). This issue of primary teacher knowledge of mathematics has for some time been a focus of official reports into education (Alexander, Rose, & Woodhead, 1992; OfSTED, 1994) and is reflected in government policy on initial teacher education (TTA, 2002). Indeed, a central element of the NNS is a national 5 day intensive training for selected schools which seeks to "strengthen [teachers'] knowledge of mathematical topics" (NNS, 2000). This focus on the mathematical knowledge of primary teachers is, moreover, not simply a parochial concern, but is reflected internationally in debates on mathematics teaching (e.g., see Shulman, 1999; Wu, 1999).

Since Shulman's (1986) seminal paper introducing the idea of pedagogical content knowledge, there has been considerable debate on the status and uniqueness of teacher subject knowledge (Brown & McIntyre, 1991; Elbaz, 1991; McNamara, 1991; Turner-Bisset, 1999; Wilson, Shulman, & Richert, 1987). There is a substantial amount of research on primary teachers' subject knowledge about mathematics (Askew et al., 1997; Aubrey, 1997; Ball, 1991; Ma, 1999; Prestage & Perks, 2001; Rowland, Martyn, Barber, & Heal, 2000). Broadly this research suggests that the mathematical knowledge which teachers need in order to teach school mathematics well or effectively is different to *and* more profound than the mathematical knowledge necessary to succeed in standard mathematics examinations. Despite this research base, teacher subject knowledge in mathematics is an extremely contested issue both in terms of what constitutes "good" mathematical knowledge and how it can be measured. For example, whilst many studies have found no link between the level of teachers' mathematical qualification and effectiveness of teaching (Askew et al., 1997; Begle, 1968, 1979; Ma, 1999), government policy in England currently places considerable weight on teacher knowledge as measured by success in academic mathematics examinations. There is, moreover, very little research on how primary teachers, as generalists, can develop the knowledge of mathematics necessary for teaching. A further complication is in the area of teachers' beliefs in relation to mathematics. Research suggests strongly that beliefs in this area impact on teachers' classroom practices (e.g., Askew et al., 1997; Thompson, 1992). However, the link between beliefs about mathematics and knowledge of mathematics is far from clear.

The NNS places significant emphasis on the role of primary specialists in its training strategy. Numeracy consultants, whose previous experience is largely as generalist teachers rather than mathematics specialists, for example, lead training sessions addressing teachers' knowledge of mathematics, whilst participants and mathematics coordinators "cascade" this training to the majority of teachers in schools. Yet, the research in this area is meagre. Millett and Johnson (2000b) is one exception, although their focus is on the organizational and management aspects of the mathematics coordinator in this process. There is a pressing need for research into teacher educators' own professional development. Although a number of researchers have begun to address this issue (e.g., Geddis & Wood, 1997; Jaworski, 1999; Prestage & Perks, 2001), this work is limited to teacher educators in the higher education sector.

In summary, a major current policy focus in primary mathematics is on teacher subject knowledge. Yet, despite substantial research in this area, the nature of teacher mathematical knowledge necessary for effective teaching and how this relates to teacher beliefs about school mathematics is not clear. Moreover, the research on how primary teachers develop a better understanding of mathematics is very limited. Finally, I have highlighted the need for research into the professional development of teacher educators, particularly into that of non-subject specialists in mathematics education. I used these issues to inform my research questions which are outlined in Section 4 below.

3.3 Mathematics and Primary Teaching: My Own Research Motivation

My own personal motivation for this study relates to my work as a primary teacher specialising in mathematics. I am unusual within the primary sector in having a mathematics degree. As such I was greeted with open arms by the mathematics specialists on my PGCE course and was rapidly appointed to a mathematics co-ordination role in school. However, my experience was that teaching enabled me to understand mathematics in ways that my mathematics education had not. Moreover, in teaching and working alongside other teachers, I found that almost without exception other primary teachers faced considerable problems teaching mathematics. Their mathematical knowledge was insecure and they experienced considerable anxieties about the subject. For many of the teachers I worked with, teaching mathematics had not been a catalyst for the development of their knowledge of the subject in the way that it had been for me. Yet, the teachers I worked with were on the whole not only willing but keen to develop their mathematics teaching. When I looked for ways of helping these teachers, there was very little available. It was in this context that I became interested in studying teacher education further.

4. The Aims of The Research

The aims of this study strongly reflect the interpretation of the state of the field of primary mathematics teacher education that I have outlined above. The main aim of my research is to explore the ways in which primary teachers change and develop as teachers of mathematics with the overall research question:

> In what ways do primary teachers' beliefs and knowledge change and develop as they undergo professional development in school mathematics?

Within this overall research question, I seek to provide rich descriptions in relation to the following sub-questions:

> In what ways do the wider professional and social contexts in which primary teachers are located influence their professional change?
>
> In what ways do primary teachers' beliefs and knowledge about school mathematics and the teaching and learning of mathematics interrelate and in what ways does this interrelationship change and develop?
>
> What is the nature and role of reflection in teachers' professional change, and how can such reflection be facilitated?
>
> In what ways can the different patterns of change of different teachers be explained?

These research questions reflect the exploratory and interpretative nature of this study. My aim is not to provide a definitive answer to the above questions but rather to contribute to the developing empirical and theoretical knowledge base within the area of mathematics teacher education generally and primary mathematics in particular.

The research questions did develop and evolve over time. In particular, my focus shifted from an emphasis on the professional development intervention, in this case Primary CAME, to focus more on the individual teachers themselves. In part, this was due to the considerable differences in the degree of change amongst the teacher-researchers. As the study progressed, it became apparent that the teachers themselves and their professional networks were crucial factors in their professional change. I discuss the evolution of the research design briefly in Chapter 3.

5. Central Theoretical Themes

Throughout this thesis I will refer to the notions of practices, discourses, communities of practice, identity, knowledge and beliefs. In this section I briefly outline the ways in which I use these central ideas. I take a broadly materialist perspective and, hence, my understanding of these ideas is rooted in social practices.

My intention here is to provide clarification and definition rather than to discuss these ideas in detail. I discuss and develop all these ideas elsewhere in this thesis.

5.1 Practices and Discourses

In referring to practice and discourse, I draw not only on situated learning theories (Lave & Wenger, 1991; Wenger, 1998) but also on more general cultural theories (Gee, 1999; Hall, 1996; Holland, Lachicotte, Skinner, & Cain, 1998). For both terms I use Gee's (1999) broad definition of "Discourses" as:

> Different ways in which we humans integrate language with non-language "stuff," such as different ways of thinking, acting, interacting, valuing, feeling, believing, and using symbols, tools, and objects in the right places and at the right times so as to enact and recognize different identities and activities, give the material world different meanings, distribute social goods in a certain way, make certain sorts of meaningful connections in our experience, and privilege certain symbol systems and ways of knowing over others. (p.13)

Practices and discourses then involve the production and reproduction of meaning and social relations amongst people. Practices and discourses are, moreover, rooted in social spaces or communities. Indeed, practices and discourses are meaningless and empty without the individuals who enact them. Thus, practices and discourses are intertwined with notions of identity.

A further qualification is necessary in relation to the term *practice.* I do not use the term practice to suggest some atheoretical process of acting. Practice includes theoretical understandings (Wenger, 1998). Indeed, theories themselves are practices which are enacted in social settings.

Throughout this thesis, I will use the terms practice and discourse interchangeably.

5.2 Communities of Practice

Community of practice has in recent years been a very influential metaphor within mathematics education, which is used to convey the idea of teachers and pupils sharing a common engagement in a shared mathematics. However, it is often used to promote a vision of how mathematics classrooms should be rather than as a tool for analysing the actual mathematical practices in these classroom.

However, community of practice for all its apparent simplicity is a complex concept. Wenger (1998) describes community as involving three dimensions all of which certainly embody the idea of commonality: mutual engagement; joint enterprise; and, shared repertoire. However, he argues that shared practices do not "imply harmony or collaboration" (p. 85) and that communities are heterogeneous.

Community is most emphatically not homogenous in the sense that I use it. Indeed the community of the research team, the context for my fieldwork, was a setting of argument, disagreement and tension. Lave and Wenger's (1991) conception of community of practice has its roots in the Marxist dialectical notions of class where social groups with opposing interests are bound together within a common enterprise. Hence, mutual engagement does not imply a simple set of common interests nor does a shared repertoire, essentially a common language, imply common interpretation. Indeed, whilst bound within a common enterprise, the fundamental interests of the participants may be very different. (See, e.g., the analysis of claims processing in Wenger, 1998, for an illustration of this.)

Within the research team, participants' interests were very different. Indeed, although Primary CAME was an important aspect of all the participants' professional lives, the teachers and the academics were members of the distinct overarching discourse communities of primary teaching and academia. Hence, however much Primary CAME itself was a shared enterprise, the participants' fundamental professional interests, needs, desires and imagined futures were very different. In particular, each of the teachers except Henrietta saw their career progressing with primary education, whilst the academics' imagined futures were within the world of academia.

Communities, indeed, do not exist in isolation. Participants in the research team were also participants in other communities, including their classrooms, schools, home life, all of which impinged to a greater or lesser extent on the research team itself. Indeed, each of the participants was situated within a unique set of such communities, which Spillane (1999) refers to as an individual's zone of enactment or the "the space" in which teachers "make sense of and operationalize for their own practice the ideas advanced by reformers" (p158).

Communities can be local, as in the research team, or global, as in the community of primary education. A feature of global communities is the absence of direct social intercourse with other members. Drawing on Anderson's (1983) notion of imagined communities, Holland et al. (1998) discuss the powerful sense of community that can be created despite this absence of direct interaction in their notion of *figured worlds*. I will follow Lampert's (1998) and Putnam and Borko's (2000) distinction of using community of practice to refer to local settings, such as a particular school or classroom, and discourse communities to refer to more global communities such as those of primary teaching or mathematics education.

5.3 Identity

Over the past decade, one focus for debate within mathematics education has been the role of social context in teaching and learning (e.g., see Lerman, 2000). Researchers have sought to develop a theory which balances individual and social aspects in education. One approach has been that of social constructivism (e.g., see Cobb, 1994). However, Lave (1993) characterises such theories as *cognition plus* arguing that these approaches merely bolt social interaction on to an what remains an essentially individualist, cognitive approach. As a result, she argues such theories do not fully recognize the interconnection of social and individual or cognitive aspects of learning.

Lerman (2000) neatly encapsulates the issue with his extension of Vygosky's metaphor of mind in society: "mind-in-society-in-mind" (p. 38). Identity is a tool of analysis that is focused on this meeting point of the social and the individual. As such, identity does not refer to an essential and invariant self, but is rather a method for conceptualising change. Wenger (1998) also conceives of identity in terms of change, although he emphasises the co-development of the individual and the social:

> Building an identity consists of negotiating the meanings of our experience of membership in social communities. The concept of

> identity serves as a pivot between the social and the individual, so that each can be talked about in terms of the other. It avoids a simplistic individual-social dichotomy without doing away with the distinction. The resulting perspective is neither individualistic nor abstractly institutional or societal. It does justice to the lived experience of identity while recognizing its social character - it is the social, the cultural, the historical with a human face. (p. 145)

Wenger uses the metaphor of "building" an identity deliberately here to indicate the active nature of identity construction and maintenance. Identity is not "once and for all," rather it is "settled provisionally and continuously, in practice, as part and parcel of shared histories and on-going activities." (Gee, 1999, p. 16).

Using this metaphor, professional change can be seen in terms of changing and developing identities. Hence, fundamental changes in teachers' beliefs and knowledge necessitate fundamental changes to teachers' identities involving far more than "fixing" or "topping up" teachers' "inadequate" knowledge. Professional change, then, involves at least in part becoming a "different" teacher and a "different" person. This touches on an aspect of the difficulty of professional change, for becoming "different" involves letting go of what one has been at the same time as maintaining the more fundamental aspects of one's identity. I will discuss later in this thesis the difficult balancing act some of these teachers' engaged in developing their identities as mathematics teachers whilst at the same time maintaining their identities as primary teachers.

A further feature of identity is that it is fragmented in nature. A teacher has a different identity in different settings, in school and in their personal lives, for example. Indeed, there is a very real sense in which teachers have different identities with different classes. (See Hargreaves, 1994, for an illustration of this.) Griffiths (2000), indeed, sees this fragmentation as a direct result of the non-static nature of identity: "a self cannot be conceived as a unity precisely because a self is always in the process of being constructed in and by particular circumstances" (p.388). Both Holland (1998) and Wenger (1998) argue that this fragmentation is crucial in providing possibilities and the impetus for professional change. This is an issue that I will return to throughout this analysis.

5.4 Beliefs and Knowledge

Underlying the argument of this thesis is an understanding of beliefs and knowledge as situated within social practices. What teachers "know" is not in their heads; "rather, it is spread out (distributed), inscribed in (and *often trapped in*)" classroom routines, habits, resources, text books, activities, and relationships with other people (Gee, 1999, p. 19, emphasis in original). Although it is commonplace to talk about knowledge as a thing, my understanding of knowledge is that it does not have an independent existence outside social practices and discourses. Knowledge is in short a social construction and hence contingent.

Yet, despite this contingency, knowledge is at the same time a useful idea, representing reified practices (Wenger, 1998). This is equally true of mathematical knowledge. Hence, I take a fallibilist epistemological position in relation to mathematical knowledge (Davis & Hersh, 1981; Lakatos, 1976).

Although this thesis is concerned with beliefs and knowledge, it is important to note that I see no absolute distinction between beliefs and knowledge. Askew, for example, argues:

> All propositional statements - beliefs, concepts, knowledge - whatever you call them have the same status of being constructed through Discourse. Where you draw the line between a belief and, say, a fact, cannot be determined by the statement itself, only through the social definition. "Maths is hard" and the "the table is hard" are only separated as a belief and a fact by convention - I can be no more or less certain of the "hardness" of the table than of the maths. (M. Askew. Personal communication, 28 November 2000)

The distinction between beliefs and knowledge is one of social convention and warranting, rather than fundamentally of their epistemological base (See Askew, 1999; see also Edwards & Potter, 1992). Nevertheless, the nature of mathematical knowledge is that this social warranting is exceptionally strong. In this thesis, I use knowledge to refer to specific mathematical concepts, connections and processes together with specific pedagogical practices. Everything else I refer to as beliefs or as orientations to emphasise their more contested and less strongly warranted nature.

6. A Conception of "Good" Mathematics Teaching

Underlying the arguments in this thesis and my analysis of the teachers' change are my own understandings about good or effective primary mathematics teaching. These understandings are informed by the literature outlined in Section 3 above and are developed throughout this thesis. However, since these ideas form the crux of my judgements about teacher change, I briefly summarise these understandings. Since this research is focused on teacher knowledge and beliefs, I confine myself to these areas.

Firstly, in terms of knowledge, I take "good" teacher knowledge to involve an understanding of key mathematical concepts rather than simply a knowledge of procedures, thus enabling the construction rather than simply the use of mathematical procedures. Broadly, this is similar to Lampert's (1986) principled understanding of mathematics and Ma's (1999) profound understanding of elementary mathematics. These ideas are developed further in Chapter 7.

Secondly, I take desirable teacher beliefs to include connectionist beliefs as described by Askew et al. (1997) and beliefs relating to the epistemological status of mathematical knowledge. I draw on two inter-related ideas: authority and authorship. In terms of authority, I refer to beliefs about mathematics as constructed and validated through mathematical argument and discussion (in contrast to be accepted as given by textbooks and experts). In my discussions of authority, I will refer to Cooney's (1994a; see also Cooney & Shealey, 1997) and Povey's (1997; see also Povey, Burton, Angier, & Boylan, 1999) re-working and development of Belenky, Clinchy, Goldberger and Tarule's (1986) work on ways of knowing. Authorship is increasingly used as a metaphor in relation to mathematics education to indicate a conception of mathematics as constructed by a community of learners and of teachers as curriculum-makers rather than simply implementers of the mathematics curriculum (e.g., Burton, 1999; see also Clarke, Clarke, & Sullivan, 1996; Kirshner, 2002). Kirshner, for example, conceives of "teachers as authors of reform … marshaling [sic] accessible, theory-based guidance toward realization of its diverse possibilities" (p. 47). Povey (1997) explicitly links this idea of authorship to notions of authority in mathematics education by developing the notion of author/ity as a way of knowing where meaning is seen as negotiated and co-authored. These ideas are developed further in Chapters 4 and 5.

I also take good teaching to include a motivation both to teach mathematics and to change and develop one's teaching. However, here I touch on one of the key aspects of my research: explicating the different motivations of different teachers in order to understand differential change amongst the teachers. In this thesis I locate motivation in terms of social processes rather than merely individual factors. These ideas are developed further in Chapters 4 and 5.

7. The Structure of The Thesis

In this section I outline the structure of the remaining chapters of this thesis.

In Chapter 2, I discuss the context in which this research was conducted. I outline the national and local setting: the National Numeracy Strategy, Outertown LEA, and, most significant, Primary CAME. I give an outline of the development of the Primary CAME Project and distinguish the research reported in this thesis from that of the Primary CAME Project. I introduce the teachers and other participants, their schools and the LEA, outline their professional development experiences, and discuss the extent to which the teachers changed. I raise a further question in relation to teacher change: the issue of teachers' motivation to change.

In Chapter 3, I discuss the methods that I used. The methodological approach was qualitative and informed by ethnographic and social constructivist theories. I outline the research design and my own role as a participant observer, together with the methods and techniques that I used to collect and analyse data. I also briefly discuss changes to the research design and my approaches to ensuring rigour in the research process.

In Chapter 4, I discuss the research team as a learning environment in the early stages of the project, focusing particularly on the initial group of four teachers. The research questions I address are the role of the teachers' wider professional and social networks in their professional change, the issue of differential change amongst the teachers, and the issue of teacher motivation that I raised in Chapter 2. A central aim of this chapter is to provide a theoretical platform on which to base the discussions in later chapters. To do this, I use and extend the situated theories of Lave and Wenger (1991), Wenger (1998), Boaler (2000a) and others in order to locate the teachers' potential for professional change in social terms. In particular, I use notions of similarity and difference as tools of analysis. I discuss the issue of transfer, recasting this in terms of adaptation. One focus in my discussion is on the teachers' potential to change their beliefs about authority in mathematics. I address the issue of teachers' initial motivation to engage with change using the idea of interest. I raise a further question in relation to teacher motivation to sustain change beyond this initial interest.

In Chapter 5, I shift my focus to teacher identity extending the focus of the previous chapter and using Holland et al.'s (1998) conception of identity as authoring to extend the theoretical approach developed in Chapter 4. My focus is on the ways in which the teachers actively made sense of and constructed new practices in the context of CAME. I discuss how I found that identities provided resources on which the teachers could draw in order to make sense of 'new' practices and act in 'new' situations using and modifying existing practices. Drawing on Lacan's notion of identity as interpreted by Zizek (1992), I investigate a teachers' motivation to sustain change conceiving of this in strong emotive terms. I also discuss the barriers to change using the cases of the teachers for whom change was less significant.

In Chapter 6, I address the question of the role of reflection in teacher change. My concern is with a strong definition of reflection as the reconstruction of experience and knowledge. In particular, I discuss the teachers' motivation to reflect and the process by which reflection can be enabled or facilitated. Here, I focus on the two teachers for whom change was significant: Alexandra and Ursula. Drawing on Schifter's (1996) and Wenger's (1998) approaches to identity, I argue that these teachers' separate identities as teachers, lessons developers and tutors provided different perspectives or stances. These different perspectives provided a way for the teachers to "step outside" one identity in order to reflect on their new or modified practices, enabling reflection and conscious learning and change.

In Chapter 7, I shift the focus on to the teachers' knowledge of specific mathematical concepts. In particular I explore the notion of mathematical knowledge for teaching through two case studies. I examine how these two teachers "held" their mathematical knowledge for teaching. I explore the ways in which this knowledge did and did not change and discuss the implications of this for the professional development of primary teachers.

In Chapter 8, I review the thesis drawing together the arguments in the earlier chapters. I reflect on the limitations of my study. Finally, I discuss the implications of this work and make recommendations both for policy and practice in mathematics teacher education and for further research.

8. Summary

In this chapter, I briefly introduced the research context and the six main participants, although these are both discussed in greater detail in Chapter 2.

I gave an overview of the domain of mathematics teacher education in terms of research and policy. I argued that research in primary mathematics teacher education is fragmented and underdeveloped. Drawing on this discussion, my overall research question is as follows:

> In what ways do primary teachers' beliefs and knowledge change and develop as they undergo professional development in school mathematics?

I identified further sub-questions addressing the following aspects of teachers' professional change:

- the role of teachers' professional and social networks in their change and development
- the interrelationship of teachers beliefs and knowledge about mathematics and mathematics education

- teacher reflection
- differential change amongst teachers

I introduced the central theoretical ideas that I use in this thesis:

- practices and discourses
- communities of practice
- identity
- beliefs and knowledge

I outlined my own conception of good mathematics teaching focusing on teachers' beliefs and knowledge and highlighting a principled rather than a procedural understanding of mathematics together with beliefs about mathematics as a connected discipline and authority and authorship in mathematics.

Finally, I outlined the structure of the thesis.

Chapter 2: The Context for The Research

1. Introduction

In this chapter I describe the context in which the fieldwork took place: the setting, the principal participants and their professional development experiences.

As I have previously outlined in Chapter 1, in this thesis I explore the professional change of a group of six teachers. The focus of this thesis is on these teachers' professional development experiences within the Primary CAME project, the setting for much of the fieldwork. Primary CAME was, however, a separate research project and one aim for this chapter is to distinguish my research from the research programme of the wider project.

The structure of the chapter is as follows:

- In Section 2, I give a brief overview of the context as a whole.
- In Section 3, I describe the national and local scene: the NNS and Outertown LEA.
- In Section 4, I outline the Primary CAME project in general, its background in theory and research, the remaining participants and the research programme. In particular, I discuss the concept of "mathematics without closure" which I developed during the first year of this research to describe the beliefs about school mathematics teaching and learning which Primary CAME sought to promote.
- In Section 5, I focus on my research narrowly: the six teachers, their schools and their professional experiences.

2. The Research Context: An Overview

The six teachers, Alexandra, Henrietta, Janice, Lisa, Tony and Ursula, were involved in the Primary CAME project, participating in the project's research team (and the research reported here.) Alexandra, Lisa and Ursula were involved throughout the four years of fieldwork. Henrietta left after the first year. Janice and Tony joined the research team during the second year.

The teachers each initially worked as a class teacher in one of four schools all situated within Outertown LEA: Beechmount, Brightvale, Meadowside and Parkway. During the course of the project, two of the teachers, Alexandra and Ursula, became Numeracy Consultants working in Outertown LEA's inspection and advisory service.

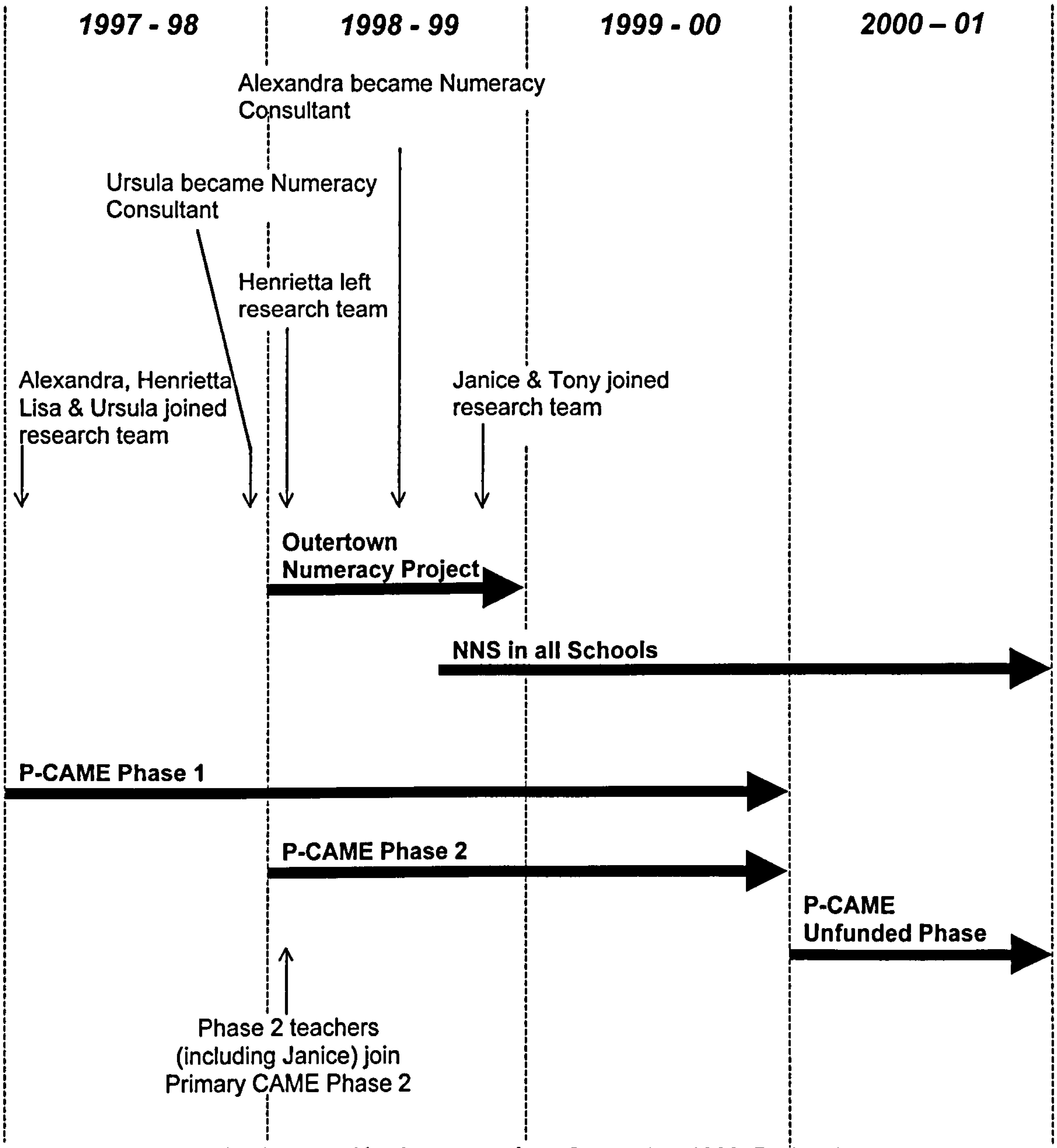
Primary CAME, a joint research project between King's College, London and Outertown LEA, was itself one element of a larger research programme, the Leverhulme Numeracy Research Programme (LNRP). The five-year Leverhulme programme, 1997-2002, focused on attainment in numeracy and on ways of improving numeracy standards. The Programme was built around a Core Project and five Focus Projects: Core: Tracking numeracy; Focus 1: Case studies of pupil progress; Focus 2: Teachers' conceptions and practices and pupils' learning; Focus 3: Whole school action on numeracy; Focus 4: School and Community Numeracies; Focus 5: CAME Primary. The programme was funded by The Leverhulme Trust.

In addition to the six teachers, the research team consisted of Rhoda, Outertown's Mathematics Advisor, and a group of four researchers from King's College, including myself. Primary CAME was a project in three overlapping phases: Phase 1, the development phase; Phase 2, the main implementation phase; and, a concluding unfunded phase, in which the focus was on writing up lesson materials. In Phase 2, a further group of teachers and schools joined the project to implement lessons developed in Phase 1. The six focus teachers contributed both to the development of lessons and to the dissemination of the approach to the wider group of Phase 2 teachers.

The research took place at a time of considerable change in primary mathematics education in England. During the third year of Primary CAME, the National Numeracy Strategy (NNS) was implemented in schools throughout England with the aim of significantly improving the attainment of all pupils in numeracy at the end of

KS1 and KS2. Although Primary CAME was planned independently of the NNS, the introduction of the NNS had a considerable impact on the project and its research. The NNS was, as Askew, Millett, Brown, Rhodes and Bibby (2001) argue, one of the most significant national intervention in primary mathematics in England and Wales for more than a hundred years. Hence, rather than being, as was initially envisaged, the principal initiative in terms of mathematics teaching for the group of Phase 2 schools and teachers, Primary CAME became secondary to the implementation of the NNS, the NNS PD initiatives which began in May 1999 and the introduction of the daily mathematics lesson from September 1999. In terms of the research reported here, the NNS was influential in the professional change of five of the six principal research participants. All the teachers, except Henrietta who left teaching prior to its introduction, received NNS training and four of the teachers led NNS training sessions for Outertown LEA. Two of the teachers, Alexandra and Ursula, became Numeracy Consultants responsible for the delivery of the NNS PD programme locally during the course of Primary CAME, whilst Tony also became a Numeracy Consultant, although in his case this was at the end of the project in Spring 2001. Ursula was responsible for the delivery of the Outertown Numeracy Project during the second year of the research, in which two of the Phase1 teachers' schools were also involved: Meadowside and Parkway.

In Figure 2.1, I show a timeline of these key contextual events over the four-year of the fieldwork for my research.



[Note: The NNS was implemented in classrooms from September 1999. During the summer term 1998/99, PD was delivered to all primary schools.]

Figure 2.1: Timeline of Primary CAME, Outertown Numeracy Project and National Numeracy Strategy

3. The National and Local Scene

3.1 Outertown LEA

Outertown was a medium-sized LEA located in outer London. Like many London boroughs it was an area with considerable contrasts, in terms of ethnic make-up and indicators of poverty / affluence. In terms of primary mathematics, over the period of this research, the Outertown average results in KS2 national tests were broadly comparable with national averages.

Over the period of the fieldwork, Outertown had a relatively large mathematics support and training service to schools. In 1997, the LEA employed a Mathematics Advisor, Rhoda, and two further part-time mathematics advisory teachers in addition to the Mathematics Inspector. The service's emphasis was on long and medium term projects and extended PD courses in addition to a range of short one-off INSET sessions. Several initiatives focused on transitions between KS1 and KS2 and between KS2 and KS3, as well as mental mathematics and early years numeracy. Of particular importance in terms of the research reported here were three courses, which several of the Phase 1 teachers had participated in. Two of these were 5 day INSET courses run by Rhoda: Tools of Number at KS2, focused on contexts for the teaching of number, and Teaching Mathematically Able Children, focused on Y6 and Y7 teachers and pupils. In addition, until 1998, in common with many other LEAs, the advisory service ran an annual 20 days GEST funded course in mathematics education for primary teachers.

Over the 3 years of the research, although the emphasis of the mathematics advisory service inevitably shifted towards the NNS, the service continued to run a range of projects, courses and other initiatives. Indeed, the Outertown Numeracy Project, a local pilot for the NNS led by Ursula, continued the focus on the transitions by focusing on Years 2, 3, 6 and 7.

Rhoda was funded through LNRP to work 1 day per week on Primary CAME. She played a crucial role within the project's development. In her role as Mathematics Adviser, she promoted the project generally within the LEA. This included, for example, reporting and presenting the work of the project at the Outertown Mathematics Conferences as well as establishing a follow-on CAME PD programme in the LEA. In the role of teacher-researcher, she initiated and trialled several TM lessons. In the role of tutor, she took a leading role in the central PD sessions. Most significantly for the research reported here, however, Rhoda identified the schools and teachers for both phases of the research.

3.2 The National Numeracy Strategy

In the past decade a range of national initiatives has dominated primary education in general, and primary mathematics in particular. These include the introduction of the National Curriculum and its revisions, the OfSTED regime of school inspection, national testing at KS1 and KS2, and the National Literacy and Numeracy Strategies (See Millett and Johnson (2000a) for a discussion of the effects of these initiatives.)

The last of these initiatives, the NNS, aimed to increase primary children's achievement in numeracy. The key features of the strategy were an emphasis on calculation, particularly mental calculation; the introduction of a daily three-part mathematics lesson; a requirement for detailed week-by-week planning with reference to the NNS *Framework for Teaching Mathematics* (DfEE, 1998, 1999). To support the implementation of the strategy, a national training programme was introduced supported by a cohort of newly appointed LEA Numeracy Consultants. Although the NNS was not strictly mandatory, the vast majority of schools in England and Wales, including all those involved in Primary CAME, adopted the strategy. (For further information, see. Askew et al., 2001; Brown, 1999; Brown, Askew, Baker, Denvir, & Millett, 1998; The Numeracy Task Force, 1998a.; 1998b)

The National Numeracy Strategy was particularly significant for the research reported here in two respects. Firstly, all but Henrietta of the research participants were involved to various degrees in its implementation. Hence, the NNS was influential in the teachers' professional development. Secondly, like Primary CAME, the NNS sought to influence pedagogy in mathematics teaching. I will briefly compare the initiatives in section 4.1.3 below. The influence was, however, not all one way and Primary CAME had a considerable impact on the implementation of the NNS locally in Outertown with the primary numeracy team of Rhoda, Ursula and Alexandra all participants in Primary CAME. Moreover, in Summer 2000, Outertown LEA appointed a third Numeracy Consultant, Nicola, who was a participant in Phase 2 of Primary CAME.

4. The Primary CAME Project

In this section, I discuss CAME in general and then the Primary CAME project in detail.

4.1 Cognitive Acceleration in Mathematics Education

4.1.1 Secondary CAME: The Research Background

The primary CAME project drew on a substantial body of applied research and theory. Over the period from 1993 until 1997, the three academic researchers based at King's College, David Johnson, Michael Shayer and Mundher Adhami, developed the CAME approach in the early years of secondary school mathematics (Adhami, Johnson, & Shayer, 1997a, 1998a). At the inception of the research in primary, they had developed a programme of Thinking Maths (TM) lessons for use in years 7 and 8 together with an established PD programme (Adhami, Johnson, & Shayer, 1998b). CAME has since been implemented in over 500 secondary schools in England and has been used as the basis for work overseas (e.g., Mok & Johnson, 2000; Mok, Johnson, Cheung, & Lee, 2000).

This work in secondary mathematics was rooted in a substantial programme of work, Cognitive Acceleration in Science Education (CASE) (Adey & Shayer, 1994). Taking a social constructivist perspective drawn from both Piagetian and Vygotskian psychology, the CASE approach links an analysis of the difficulty of tasks and concepts to a theory of how children's individual learning takes place within a social context. A key idea is the mediation of children's' learning by the teacher *and* by other children.

In CASE, Piagetian levels are used to describe the underlying difficulty of the tasks in the same terms in which they describe the cognitive development levels of students. However, whilst in the CASE programme Piaget's descriptions of formal operational schemata were directly applicable to science education, this was not the case for mathematics education. Hence, CAME has drawn heavily from the literature in the area of children's strategies, conceptions, misconceptions and understandings in school mathematics (Booth, 1984; Brown, 1992b; Hart, 1981;

Johnson, 1989; Kerslake, 1986). Thus, Adhami, Johnson and Shayer (1995) have identified five core strands underlying the school mathematics curriculum: multiplicative relations, number properties, estimation, generalised number (early algebra) and measure (including data handling). (See also Adhami, 2002.) Within these strands, the researchers described a hierarchy of children's achievement in mathematics in explicit Piagetian terms. In the case of the primary work, this also included the neo-Piagetian levels of the SOLO taxonomy (Biggs & Collis, 1982).

In developing the cognitive acceleration approach to mathematics teaching and learning, CAME has supplemented and adapted the general CASE approach with perspectives taken from mathematics education research and theory in the areas of constructivism (Davis, Maher, & Noddings, 1990) and social constructivism (Cobb, 1994; Cobb, Boufi, McClain, & Whitenack, 1997). The role of the teacher is to manage the class in order to maximise the opportunities children have to learn from others. Key elements are small group and whole class discussion, in which children share, discuss and argue about their ideas and strategies.

Both CASE and CAME have been shown to have significant effects on secondary school students' cognitive skills and academic achievement (Adey & Shayer, 1993, 2002; Adhami et al., 1997a). This provides substantial evidence that the Cognitive Acceleration approach to teaching and learning works. Adey and Shayer (1993; 1994; 2002) have further interpreted this as providing strong support for the Cognitive Acceleration theoretical approach. However, other commentators have drawn attention to other potential explanations. Leo and Galloway (1996) suggest that motivation may be a factor. In a commentary on Adey and Shayer (1993), Desforges (1992) suggests that theories of situated cognition might be helpful in understanding the ways in which the approach works. In later chapters of this thesis, I examine the teachers' experiences of professional development from a situated perspective and also consider ways in which teacher motivation may be understood.

4.1.2 CAME, or Thinking Maths, Lessons

TM lessons do not constitute an entire mathematics curriculum. They are intervention lessons intended to be taught every two or three weeks as a complement to students' ordinary "balanced diet of mathematical experience": instructional and investigational lessons (Adhami et al., 1998b, p. vii). The lessons are designed to provide students with an opportunity to engage with challenges just above their level in cognitive terms. Many of the lessons are designed to promote cognitive conflict, in which children are confronted with a contradiction within or between their existing mathematical ideas.

Most TM lessons have two or more episodes, each with a sequence of preparation, small-group construction work and whole-class sharing and reflection. The lesson ends with a concluding whole class reflection discussion, which is conducted whether or not the class has tackled all the episodes in their entirety. To illustrate these ideas, I refer to excerpts from the lesson materials for a typical TM lesson from the primary set of lessons: Share an Apple. This lesson exemplifies many key features of CAME: an emphasis on developing multiple perspectives; using children's informal ideas; and, a cognitively constructed mathematical agenda. (See Appendix A for a full set of lesson notes.)

Share an Apple is intended as a Y5 lesson. The aims of the lesson are to explore representations and comparisons of fractions. The episodes and concluding whole class reflection are described in the following excerpt which is intended to be read alongside the diagrammatic representation of the lesson structure reproduced as Figure 2.2:

> Episode 1: Meaning of unitary fractions and their notations
> Children first rehearse the meanings of 'half' (then a quarter and eighth) through demonstrations on objects, focusing on the equality of parts. They then work independently on meaningfully halving and quartering a book, a glass of water and a coin. Children then rehearse the convention of writing fractions, describing the 'top' and 'bottom' numbers and the partition line, allowing some pupils to reconstruct that convention for themselves. Being confined to halves and quarters (and possibly eighths, for which the 'th' should be highlighted) they should be able to compare fractions and to find which two unitary fractions make 1, or is less or more than 1.
>
> Episode 2: More fractions and their sums
> Children approach the 'one third' practically and as a mental activity. This is intuitively accessible but requires careful handling on paper and in language, especially when combined with 'two thirds' and

comparison of size with earlier simpler fractions. They work on formulating justifications of comparisons of two fractions and deciding whether their sum is more, equal or less than 1. Their reasoning lines are then shared, with the implicit ideas of equivalent fractions made explicit where feasible.

Reflection
Children look back at their work. They verbalise for themselves the meaning of a fraction as a mental image in response to language use and convention of notation on the page. They may talk about how different people would best manipulate and combine fractions on the page.

I note the emphases placed on developing a multiplicity of meanings, on the use and sharing of pupils' informal ideas and on reflection.

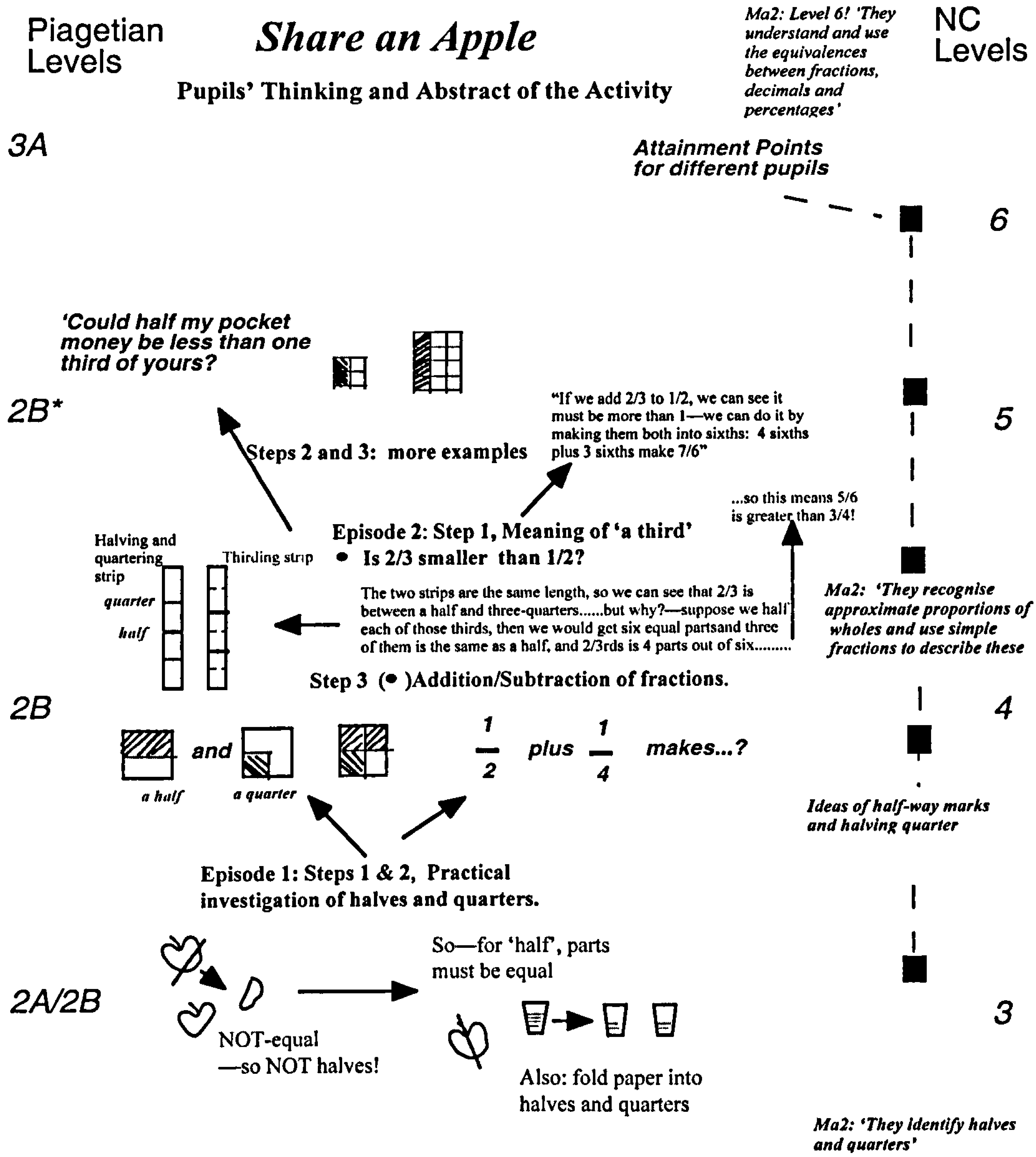


Figure 2.2: The structure of Share an Apple, a typical CAME lesson.

In Figure 2.2, these ideas are illustrated with examples of children's constructions and the cognitive demand of the challenges is linked to both Piagetian levels and to National Curriculum levels. A further note emphasises the importance of discussion both in addressing children's misconceptions and in making connections:

> Remember that the intention is to address existing misconceptions and clarify the connections in this extremely confusing topic for children. Even the issue of equal parts is important since in practice it

> is nearly impossible to achieve in continuous quantities, but it is absolutely exact as a mathematical idea in the head, or on paper! Encourage any expression and clarification of difficulty or insights at any level. For example ideas about 1/4 may include 'one over 4', 'a fourth', ' a quarter', 'one slash four', 'one out of the 4 equal parts', 'the 1 is still inside the 4', or others.

In summary, key features of the CAME approach include the development of multiple perspectives, the use of children's informal ideas, reflection set within a cognitively structured mathematical agenda. Many of these features are shared by other innovative mathematics education projects. In Section 5.2.4 below, I discuss the notion of *mathematics without closure*, an idea that appeared to be somewhat unique to CAME.

4.1.3 Comparing TM Lessons with The NNS Daily Mathematics Lesson

In some ways, the daily mathematics lesson resembles the CAME lesson structure. Both emphasise the whole class teaching and both have a three-part structure. However, there are fundamental differences in the two approaches. In the NNS, the mental / oral starter is intended to "sharpen" children's mental and oral skills (DfEE, 1999, p. 13), whilst in CAME, the preparation phase is intended as specific preparation for the subsequent construction phase in which children develop vocabulary, clarify the task and begin to make conjectures about the task (Mok & Johnson, 2000). CAME lessons normally feature two or more episodes of preparation, construction and sharing / reflection. More fundamentally, however, whilst the focus of the NNS is on step-by-step teaching of separate skills within a logical sequence of instruction, CAME emphasises the development of children's thinking skills. The tendency within the NNS to "fragment the curriculum" is in direct contrast to CAME's emphasis on the "big ideas" in mathematics (Adhami et al., 1998b, p.vii; Brown et al., 1998, p.368). This is not to argue that the two initiatives are necessarily incompatible. However, the surface similarities had at least the potential to obscure quite distinct and subtle differences in approach, thus making the simultaneous introduction of both CAME and the NNS necessarily problematic.

4.1.4 The CAME Approach to Professional Development

The approach to teachers' PD reflects the approach to teaching and learning in the classroom with children. TM lessons are introduced to a group of teachers through a lesson simulation, in which teachers tackle the mathematical activities and tasks that they will present to children. They discuss potential understandings, strategies and misconceptions that children might demonstrate in the light of the teachers' own approaches to the mathematics. Alongside this they consider the teaching and learning in the context of the CAME lesson structure. This initial lesson simulation is followed by each teacher teaching the lesson to a class of children. Having taught the lesson, the group of teachers reflect on the teaching and learning experience. This lesson simulation, teaching and reflection sequence mirrors the preparation, construction and sharing / reflection episodic sequence within TM lessons.

An important element within the CAME materials is the relationship to the standard school mathematics curriculum. The activities in TM lessons are deliberately activities that are in common use in school mathematics. However, CAME aims to "open-up closed activities and to close down open activities" (Mundher, Memo, January 1998). Through using recognisable activities with a difference, the intention is to encourage teachers to re-consider and re-evaluate their teaching of the standard school curriculum both in terms of instructional and investigational approaches.

Within CAME Secondary, there is an established PD programme following this formalised model. The programme is taken a stage further in the CAME tutor development programme which tutors examine their teaching of teachers.

4.2 The Primary CAME Project

As I have already noted, the research reported here was set within the context of the Primary CAME Project, although my research was additional to the work of this project.

The aims of Primary CAME were to develop a series of TM lessons for Y5 and Y6 and to investigate the effects of teaching these lessons on children's intellectual development. A further related aim was to contribute towards teachers' professional development more generally through an in-service teacher education programme centred on the lessons. The research reported here adds to this work by exploring the professional change of six of these teachers in considerable depth.

The research products of Primary CAME were two-fold: firstly, a set of materials, 24 TM lessons, illuminating the approach, and, secondly, an analysis of pupils' attainment using a quasi-experimental pre- and post-test method with control and experimental schools. The main project data included project memos, successive lesson drafts, lesson observations and tests of pupils' attainment in terms of cognitive development and general mathematical reasoning. The lesson materials are in the process of being published (BEAM, Forthcoming), whilst the analysis of pupil attainment has been reported elsewhere (Adhami, 2002). The data that I collected included interviews, fieldnotes and observations of research team meetings, PD sessions, school visits and lessons together with other data on the teachers and schools. These data were additional to that collected by the project generally, although they were shared with the other researchers and I have made use of project data in my analysis.

The primary CAME project was funded over a three-year period covering the academic years 1997/8, 1998/9 and 1999/2000. In fact, the project research team continued to work on lesson materials throughout the academic year 2000/1. The project was in three phases. In Phase 1, the focus was on the development of the approach and a set of draft TM lessons for further trialling in Phase 2. In Phase 2, a second group of Phase 2 teachers and schools joined the project to implement and further trial these TM lessons. Pupil attainment data was collected on pupils in the Phase 1 and 2 classes, together with pupils in a further two control schools. In Figure 2.3, I illustrate this trajectory using a timeline showing the project phases together with the project's principal research products.

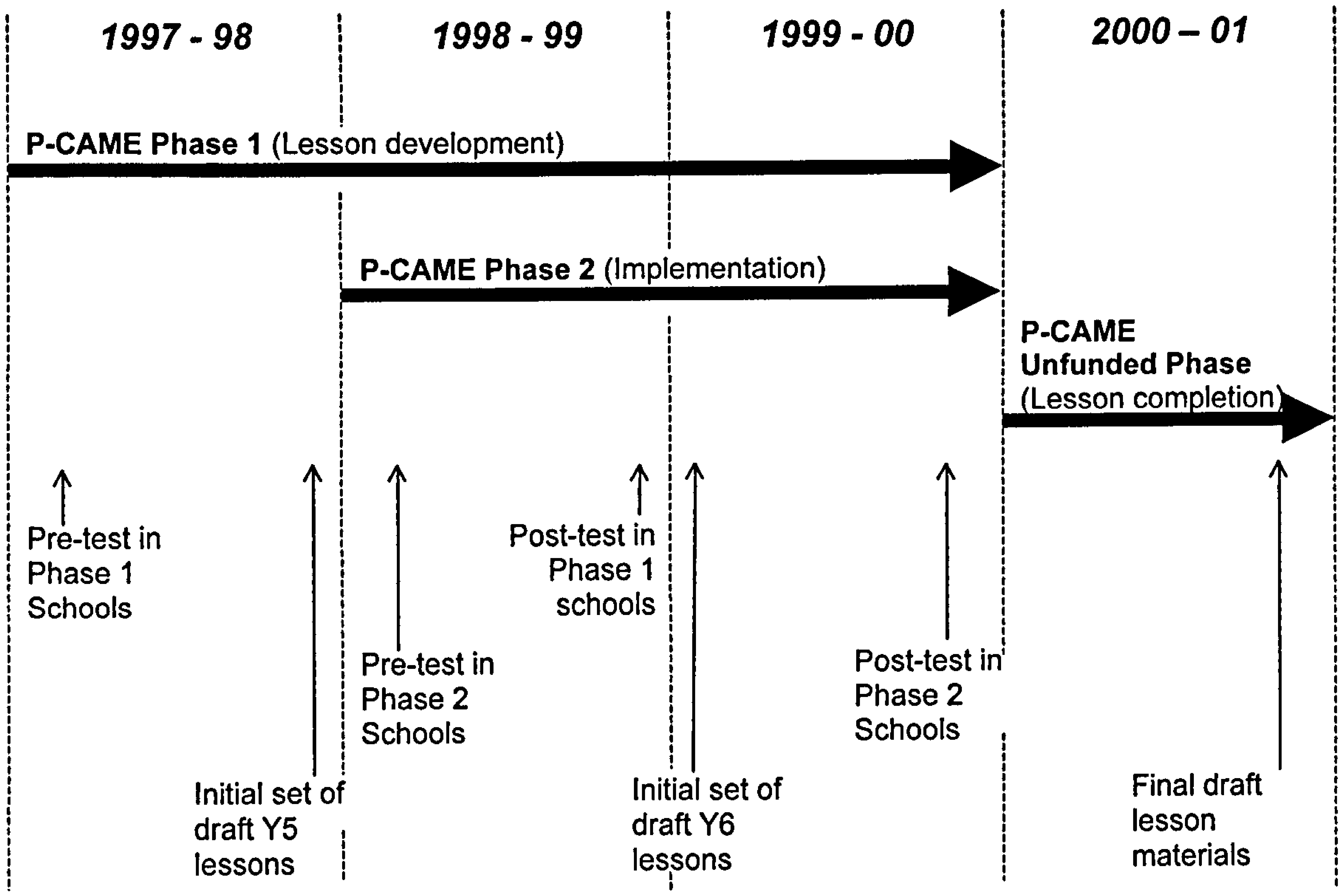


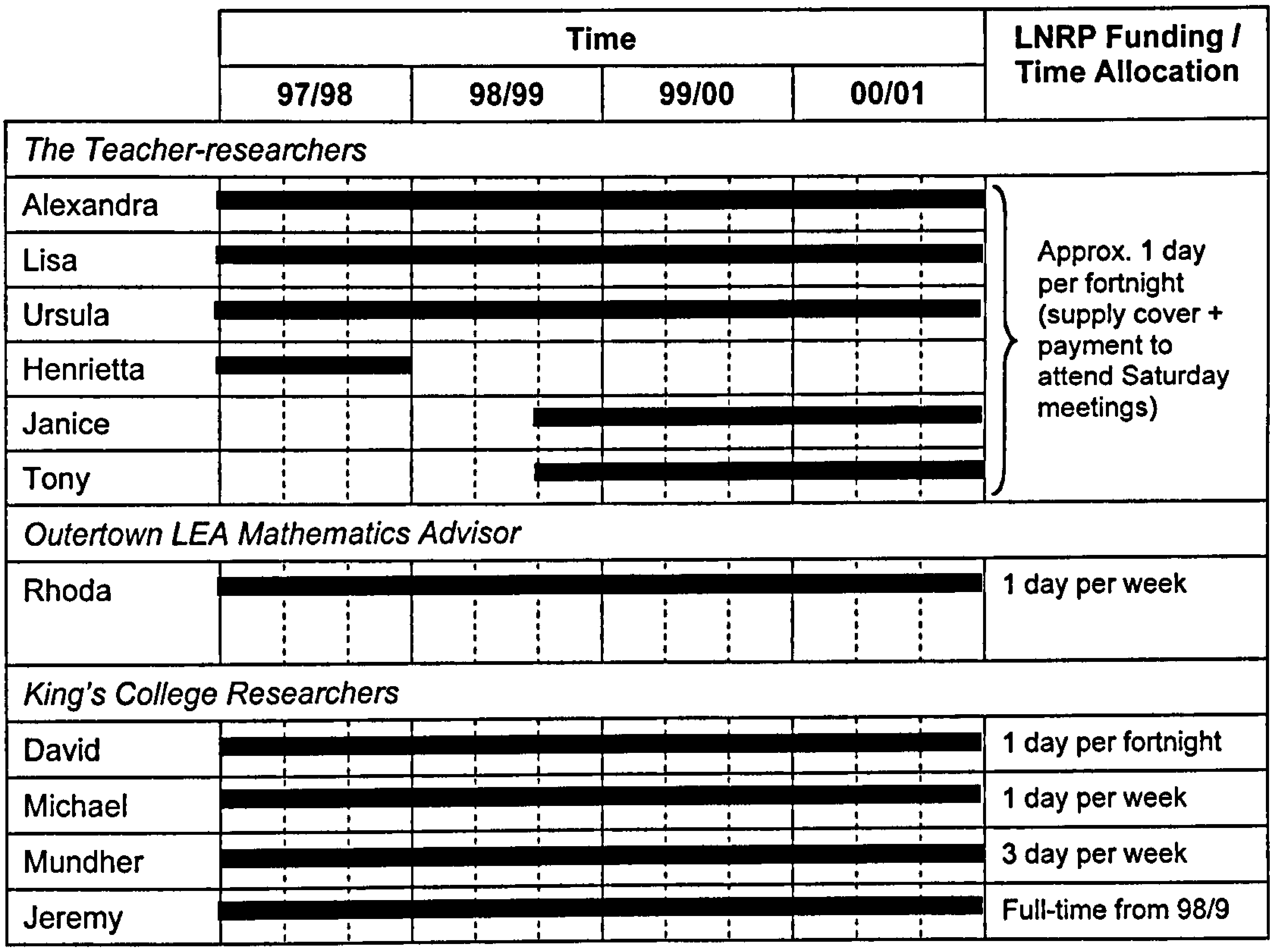
Figure 2.3: Timeline of Primary CAME Phases and data collection

4.2.1 Phase 1: The Research Team

As I have already noted, the project research team initially consisted of four of the teachers (Alexandra, Henrietta, Lisa and Ursula), the Outertown Mathematics Adviser (Rhoda), the three CAME academic researchers (David Johnson, Michael Shayer and Mundher Adhami)[1] and myself, at the time a research student. Henrietta left the research team in July 1998[2] and two further teachers (Janice and Tony) joined the research team in June 1999. In Figure 2.4, I indicate the participation of all team members together with their funded time allocation.

[1] David Johnson was the project director, whilst Mundher Adhami and Michael Shayer were the principal researchers.

[2] Henrietta formally left the project in April 1999. Although she was on maternity leave from July 1998, she had initially been intending to return to teaching and to continue her work in the research team. In the event, however, she decided to leave teaching and was not involved in the project after the first year.



[Notes: Janice was involved as a Phase 2 teacher throughout 1998/9. Rhoda was 50% funded through LNRP and 50% funded by Outertown LEA. I was part-time during 1997/98. From September 1998, I was full-time, funded by a King's Research Studentship.]

Figure 2.4: Timeline of the Primary CAME Research Team members' participation and time allocation

Within the project, these Phase 1 teachers were referred to as teacher-researchers to acknowledge the research element of their role in the project. However, the term teacher-researcher does not fully acknowledge the Phase 1 teachers' role as tutors to the Phase 2 group of teachers. Since my research focus is on the Phase 1 teachers' professional development as teachers, throughout this thesis, I will refer to this group as teachers, except where I need to distinguish them from the Phase 2 teachers or where the discussion focuses specifically on the Phase1 teachers' role as teacher-researchers or tutors within the project.

During the first year, the research team met in fortnightly full-day meetings to assess the feasibility of the approach and to develop primary TM. Initially, two of the academic researchers, Mundher and Michael, slightly adapted four of the Secondary CAME activities, and the teachers taught these themselves to serve as material for discussion at the fortnightly research meetings. Thereafter, the teacher-researchers began to suggest contexts for the generation of new primary TM lessons. The involvement of teachers during this initial development phase of the project was in contrast to the CAME team's work in secondary where Mundher and Michael developed the TM lessons without the direct input of teachers. However, since both Mundher and Michael's teaching background was in secondary education, the King's researchers felt they needed to involve primary teachers more centrally within the lesson development process.

During the second and third years of the project, the research team continued meeting to develop lessons, although the meeting schedule was changed to include Friday / Saturday sessions once per term. The teacher-researchers' roles expanded to encompass delivering PD to the Phase 2 group in addition to lesson development. However, the three remaining Phase 1 teachers found it difficult to fit this increased role within time constraints. Hence, in June 1999, Janice and Tony were recruited to join the research team to contribute to both the lesson development process and to the Phase 2 tutoring. Janice had already been involved as a Phase 2 teacher at Brightvale, and Tony had been identified as a result of Ursula's work in the Outertown Numeracy Pilot.

The final set of Primary TM lessons consisted of lessons that were either adapted from the Secondary materials or involving wholly new ideas. Of the 13 "new" lessons, 7 were initiated by the teacher-researchers, 3 by Rhoda, and 3 by the academics. Briefly, "new" lessons would be presented to the research team either in the format of a lesson simulation or as an activity on which to base a potential lesson. The lesson would subsequently be trialled by one or more members of the team. Further research team discussions would lead to revisions to the original idea and on some occasions to further trialling. In the case of the secondary TM lessons, the process of lesson development was more straightforward. A member of the research team, usually Mundher, would present a lesson simulation. The lesson would then trialled and revisions to the lesson made by members of the research team as a result of this trialling.

4.2.2 Phase 2: Implementing The Approach

In September 1998, a further 19 Phase 2 Y5 teachers and seven schools joined the project[3]. As I have noted above, the aims for this phase were on the implementation of the lessons developed and trialled in Phase 1 and measuring the effects of this intervention on pupils' attainment. The lessons were taught to the cohort of pupils who were in Y5 in 1998 / 99 and in Y6 in 1999 / 00. Although I interviewed and observed several of these teachers as part of the fieldwork, this Phase 2 work was not the focus for my research. In Appendix B, I include a list of the Phase 2 schools together with Phase 2 teachers mentioned in this thesis.

In terms of the research reported here, the PD element of Phase 2 is of particular importance. This was in large part delivered by the Phase 1 teachers. Indeed, a considerable proportion of research team meeting time was devoted to developing the CAME secondary PD approach to the particular needs of primary teachers and schools. A key difference in primary is that teachers are generalists. Mathematics is just one of many curriculum subjects that they teach and their knowledge of mathematics is generally at a considerably lower level than that of secondary mathematics teachers. On the other hand, primary teachers' initial teacher education has a greater bias towards child development and psychological theories including those of Piaget and Vygotsky.

The PD for Phase 2 teachers was in three parts: the communication of the approach through lesson materials; Central PD sessions; and, tutor visits to Phase 2 schools. The communication of the approach was part of the main work of Primary CAME and is reported elsewhere (Johnson, Adhami, & Hodgen, Forthcoming). The Central PD sessions followed the secondary model outlined above and took place on a half-termly basis over the two-year period of Phase 2. Each session consisted of two or more lesson simulations, reflection on teaching and discussions on general aspects of the CAME approach. The Phase 1 teachers led whole group activities and small group discussions. In Appendix C, I attach a summary of a typical meeting together with an excerpt from the transcript for further information.

[3] During the two years of Phase 2, a number of these original teachers left and were replaced by others. Where possible, TM lessons were taught to the classes by the same teacher over the two years.

The secondary CAME approach was less directly transferable to primary tutoring visits. In secondary the focus for these visits is on the mathematics department as a key site for teacher PD. These tutoring visits focus on the departmental meeting as a forum for the wider dissemination of the TM approach within departments through lesson simulation, team-teaching and sharing / reflection discussions. Primary schools do not have an equivalent of the mathematics department and mathematics is managed through a subject co-ordinator. Indeed, in several Phase 2 schools, the mathematics co-ordinator was not involved in CAME. Initially, Phase 1 teachers visited year group teachers in schools. However, in practice, much of this work was on an one-to-one basis with individual Phase 2 teachers, which allowed little opportunity for wider in-school discussion and reflection. Hence, during the second year, a revised model was developed in which the Phase 1 teachers tutored clusters of Phase 2 teachers and schools. Tutoring visits took place in the afternoon following a morning PD session to one of the Phase 2 teachers' classes. The group of teachers planned and team-taught one of the lessons simulated in the PD session, then the Phase 1 teacher led a sharing / reflection discussion.

4.2.3 The Unfunded Phase: Concluding The Work

During the academic year 2000 / 01, the project continued to work on finalising the draft lesson materials. The teacher researchers continued to work with other team members principally on editing the materials. The work during this period was much less intensive than that in the earlier phase of the project and the research team met as a whole on only two occasions. In fact, much of the work during this period was carried out by the King's researchers and Rhoda. A total of 24 lessons was produced and are currently in the process of being published (BEAM, Forthcoming).

4.2.4 Did Primary CAME Work?

In this section, I very briefly summarise the Primary CAME Project's pre and post test research results. These results were a key element of the Primary CAME research and are included here as background. The analysis is summarised in Adhami (2002) and further details are attached in Appendix D.

Two tests were administered as pre and post tests to classes from both Phases of Primary CAME: Piagetian Reasoning Test 1: Spatial Relations, one of the tests used in the Concepts in Secondary Mathematics and Science (CSMS) study (Hart, 1981); and the Mathematics Reasoning Test structured around the SOLO taxonomy (Biggs & Collis, 1982) and based on work by Kevin Collis, Tom Romberg and colleagues at the University of Wisconsin, which was previously used in the UK in the ImpacT research (Watson, 1993).

The pre and post tests for the Phase 1 were conducted with the Y5 classes for each school in Autumn 1997 and again with the same classes in Summer 1999. Both of the Y5 classes in each school experienced TM lessons during the first year, 1997/98, although there were no control classes for these tests. The results on the Piagetian test showed statistically significant gains in comparison with CSMS population norms (Shayer, Küchemann, & Wylam, 1976). The effect sizes for the Parkway and Beechmount classes were 0.83 standard deviations and 0.42 standard deviations, respectively. In the Mathematics Reasoning Test, the results were encouraging in that levels of reasoning had greatly improved in the post test. However, although these results suggest a positive effect, without control classes for statistical comparison these results are only descriptive.

In Parkway, these classes were taught by both Alexandra and Ursula. In Beechmount, the Y5 classes were taught by Henrietta, although for one of these classes the experience was far less extensive. In 1998/99, Alexandra trialled TM lessons with both Parkway classes. In this second year, the Beechmount classes' experiences of CAME was very limited. One class was taught no TM lessons, whilst the other were taught a total of three trial lessons. Although one of these classes was taught as a Y6 class by Lisa, she did not teach any TM lessons after the first year.

For the main study, pre tests for the Piagetian tests were administered to Y5 classes from experimental and control schools in Autumn 1998 with post tests in Summer 2000. The overall comparison between experimental and control classes is statistically significant with a mean effect size of 0.26 standard deviations. Unfortunately, there were problems in the administration of the Mathematics Reasoning post tests and these results could not be used.

5. My Research: The Teachers' Professional Change

In this section I focus on the research reported here: introducing the principal research participants and their professional development experiences. My aim here is to give a succinct overview and much of this is discussed in greater depth in later chapters.

5.1 The Six Teachers, Their Schools and Their Previous Experiences

As I described in Figure 2.4 above, the teachers participated in the Project for differing periods of time with only Alexandra, Lisa and Ursula involved throughout the fieldwork.

The four original teachers taught in two schools: Alexandra and Ursula at Parkway Junior School; and, Henrietta and Lisa at Beechmount Junior School. Alexandra, Lisa and Ursula were all experienced primary teachers having taught for more than five years, whilst Henrietta was in her third year of teaching, Alexandra and Ursula had worked closely together for a number of years, whilst Henrietta and Lisa had not previously worked together. The two teachers who joined the team later taught in two schools: Janice at Brightvale Girls Junior School; and Tony at Meadowside Primary School. Janice was an experienced primary teacher, whilst Tony was in his third year of teaching. Janice was identified through the Phase 2 programme of work. Tony was identified through Ursula's work in the Outertown Numeracy Project and had not previously been involved in Primary CAME.

In Appendix E, I attach more detailed biographies of the teachers. The six teachers worked in four different schools. In Table 2.1, I show the teachers, the year groups they taught and their schools when they joined the project together with an indication of their schools' involvement in Phase 2.

Teacher	*Year group*	*School*	*School type*	*No. of teachers in Phase 2*
Alexandra	Y5	Parkway	2 form entry KS2	2
Ursula	Y6			
Henrietta	Y5	Beechmount	2/3 form entry KS2	1
Lisa	Y6			
Janice	Y3	Brightvale	3 form entry KS2	7
Tony	Y6	Meadowside	1 form entry KS1 & KS2	0

Table 2.1: The six teachers and their schools

Aside from Alexandra and Ursula, who became Numeracy Consultants, the teachers remained in these schools with these year groups throughout their involvement in the project. Beechmount, Brightvale and Parkway were all involved throughout Phase 2, whilst Meadowside was not. However, in comparison to Beechmount and Parkway, the participation of Brightvale teachers' was very high throughout Phase 2. This was in large part due to Janice's role. As Mathematics Co-ordinator, she used her regular weekly non-contact time to support other teachers' participation in Phase 2.

The teachers had a range of differing experiences prior to Primary CAME. In Table 2.2, I summarise their previous experiences of PD, leading PD sessions for other teachers, posts of responsibility together with their highest school mathematics qualification.

	Previous PD experiences			*Leading PD for teachers*		*Curriculum or management posts*		*GCSE maths*
	Extended PD in Mathematics Education	Extended PD in other areas	Short INSET in Mathematics Education	Within School	Outside school	Mathematics Co-ordinator	Other curriculum or management posts	GCSE Mathematics A - C grade or equivalent
Alexandra	✓	✓	✓	✓	✓	✗	✓	✓
Henrietta	✗	✗	✗	✗	✗	✗	✗	✓
Janice	✓	✓	✓	✓	✗	✓	✓	✓
Lisa	✗	✓	✓	✓	✗	✗	✗	✗
Tony	✓	✗	✓	✗	✗	✓	✗	✓
Ursula	✓	✗	✓	✓	✗	✓	✓	✓

Table 2.2: The teachers' PD experiences and management roles prior to Primary CAME

The teachers' previous experiences are discussed in Chapter 4. However, there are a few key issues that are worth noting at this point. Four of the teachers were experienced teachers, having taught for more than five years; whilst Henrietta and Tony had each taught for three years. All the teachers, except Janice and Ursula, had taught in more than one school, and both Alexandra and Lisa had taught in several schools and LEAs. Four of the teachers, Alexandra, Janice, Tony and Ursula, had all had previous experience of extended PD in mathematics education, a factor that (Askew et al., 1997) found to be associated with effective teaching. I take extended PD to mean five or more days, whereas (Askew et al., 1997) took this to mean twenty days or more. On this stronger test, only Janice and Ursula had previously undertaken extended PD in mathematics education, although Alexandra had participated in several single courses of 5 days or more. When the project began, Lisa had just completed an extended PD course in Thinking Skills (Fisher, 1998). However, as I discuss in Chapter 4, there were distinct and significant differences between this approach and that of Primary CAME. Henrietta, in contrast to the other teachers, had not participated in any extended PD of any kind nor had she undertaken any short INSET in mathematics. Only Alexandra had had previously led PD sessions outside school, although Janice, Lisa and Ursula all had significant prior experience of leading INSET within their schools. Unlike the other three schools, Beechmount's Mathematics Co-ordinator, Jenny, was not involved in the research team. The original intention was that Jenny would have joined the research team and she was involved in Phase 2 of Primary CAME. I discuss this further in Chapter 4. Finally, all the teachers' highest formal qualification was either a GCSE or an O-level, except for Lisa, who had no formal qualification in mathematics having qualified before this became a statutory requirement.

5.2 The CAME PD Experience

A necessary element of the Phase 1 work was the professional development of the six teacher-researchers – as CAME teachers, as curriculum developers and as tutors in the Phase 2 PD programme. This professional development experience forms the backdrop to the research reported here. The discussion here is intended to give a flavour of the teachers' experiences. A more comprehensive account is given in Chapter 4.

The teachers' PD was somewhat unusual in that it was integrated within the projects' work more widely. In other words, the PD was embedded within the lesson development and teacher education work that the research team undertook. This approach was drawn from the King's researchers' work in the secondary CAME tutor programme (Adhami, Johnson, & Shayer, 1997c). A key feature was the notion of reflection, an aspect that I consider in some depth in Chapter 7.

The experiences of the initial group of four teachers, Alexandra, Henrietta, Lisa and Ursula, were different to those of Janice and Tony, who joined later.

5.2.1 Alexandra, Henrietta, Lisa and Ursula

During the first year of the project the research team met fortnightly for day long seminars. A typical meeting included 1 or 2 lesson simulations, a session reflecting on lessons taught and discussion on an aspect of the CAME approach or theory. The seminar agendas were flexible and much of the discussion was open-ended and wide-ranging. Alexandra, for example, described meetings as "usually [having] several conversations going on at once" (Seminar, June 1999). Throughout the first year, Mundher circulated a discussion paper prior to each team meeting. Although the King's researchers made some formal presentations to these Phase 1 teachers, these were relatively infrequent. Indeed, only three such presentations were made during the first year. Team meetings were lively with heated debates about the nature of teaching and learning and the applicability of ideas and activities to the primary classroom. Often the King's researchers would argue amongst themselves about an aspect of mathematics education.

Alongside the research seminars, the teachers taught TM lessons. As I have already noted above, initially these lessons were taken exclusively from the secondary materials, in order to communicate key aspects of the CAME approach. As the project developed, the teachers themselves suggested new contexts for TM lessons. Many of these lesson development trials were either team-taught and / or taught with other research team members observing. Indeed, more than half of the 47 lesson trials in the first year were with other research team members present.

This shift in the research team's focus continued throughout the second and third years of the fieldwork. The team continued to meet for regular seminars five times each term, including a two day extended Friday and Saturday seminar. The extended meeting was focused on lesson development, whilst the remaining meetings concentrated on the Phase 2 teachers' PD and the Phase 1 teachers evolving roles as CAME tutors.

5.2.2 Janice and Tony

Janice and Tony joined the research team in June 1999 after the focus of the activity had shifted towards Phase 1 PD. Janice was at that time already a participant in the Phase 2 programme, whilst Tony worked in a school that was not otherwise involved in Primary CAME. Their PD as teacher-researchers was similar to that of the initial group of teachers in that it was embedded within the Project's work more widely. However, unlike the period of the initial group of teachers' induction, it was not the principal focus of the research team's work. As a result, their PD was significantly less intense. I note several key differences in Janice and Tony's experiences in relation to the research team seminars and their experiences teaching TM lessons.

The focus of the research team seminars was not principally on Janice and Tony's professional development. Indeed, whilst seminars discussions remained open-ended and fluid, the meetings were far more focused on the goals of lesson development and Phase 2 PD. Unlike the first year, for example, Mundher circulated relatively few project memos. Readings were circulated for discussion, although again this was much less frequent and the readings less varied than in the first year.

Like the first four teachers, Janice, as a Phase 2 teacher, had engaged with key aspects of the approach through teaching existing lessons and reflecting on her experiences through the Phase 2 PD, although the Phase 2 was much more formal and structured than the research team seminar discussions. However, Tony had not had this experience. His first experiences of TM lessons were through Ursula using his class in order to trial early TM lessons. Although, like the Parkway teachers, there were opportunities for team-teaching with other research team members, these were much less frequent than in the first year of the project.

5.2.3 Teacher Research Activity

A key feature of the project's development that had a significant impact on my research was the changing pattern of teacher researcher activity and participation over the three years of the project. At the end of the research, Alexandra, Lisa and Ursula, the three teachers involved for the whole period, all looked back at the first year of Phase 1 as an exciting and intense time. All three argued that the project had, in Lisa's words, become "more disparate" during Phase 2 (Fieldnotes, July 2001). Indeed, there was a general perception amongst the academics and Phase 1 teachers that research team meetings were much more infrequent in comparison to the first year. In fact, as I show in Table 2.3, the actual level of activity, appears to have been very similar over the course of the fieldwork. However, as was envisaged, there was a shift from lesson development and a central concern with the Phase 1 teachers' professional development to the professional development of the Phase 2 teachers.

	Phase 1 research team seminars	*Phase 1 TM lesson trials*	*Phase 2 Central PD sessions*	*Phase 2 school tutoring visits*
1997 – 98	**14**	**48**	**–**	**–**
1998 – 99	**14**	**26**	**6**	**29**
1999 – 00	**11**	**9**	**6**	**28**
2000 – 01	**2**	**0**	**–**	**–**

Table 2.3: The shifting balance of research team activity over the period of the fieldwork

Of considerable significance for the research reported here were the differences between the participation of the six teachers. I summarise these differences in Table 2.4 in terms of four aspects of the work: lesson development, Phase 2 PD, Phase 2 teaching and research presentations.

	TM lesson development				*Phase 2 PD*		*Phase 2 lessons*	*Research presentations*	
	TM Lesson trials	**New TM lessons initiated**	**Lesson simulations to Phase 1research team**	**Lesson notes drafted**	**Phase 2 PD activities led**	**Tutor visits to Phase 2 schools**	**Phase 2 lessons taught**	**Outertown / LNRP**	**Academic conferences**
Alexandra	25	5	4	9	5	15	5	2	4
Lisa	11	0	1	3	4	2	0	2	0
Ursula	15	4	4	8	7	7	–	2	4
Henrietta	8	0	0	0	–	–	–	1	0
Janice	4	1	1	2	5	3	22	0	0
Tony	2	0	0	0	3	3	2	0	0

[Note: Only Alexandra, Janice and Lisa taught classes in Phase 2 of the project.]

Table 2.4: The teachers' participation in Primary CAME

It is clear from Table 2.4 that Alexandra and Ursula's participation was significantly greater than that of any of the other teachers, even allowing for their longer involvement. In terms of lesson development, for example, only one of the other teachers, Janice initiated a new TM lesson.

In terms of Phase 2 teaching, I note that only Janice taught a significant number of lessons. Alexandra and Tony did teach some Phase 2 lessons. However, Alexandra was involved only for the first two terms of Phase 2, whilst Tony's school, Meadowside, was not formally in Phase 2 school. Tony had intended to teach the entire Y6 set of 12 lessons to his class, although he actually only taught 4 of these lessons: 2 as trials, and 2 after the Phase 2 PD lesson simulations. Lisa, although involved as Phase 2 teacher, did not teach any TM lessons after the first year. Indeed, whilst Lisa taught a relatively large number of TM lessons, these all took place during the first year of the project. Ursula, as a Numeracy Consultant, was not involved in Phase 2 teaching, whilst Henrietta had left the project at this time.

Although all the teachers, except Henrietta, participated in the Phase 2 central PD sessions, there were significant differences in their tutoring work in schools. In particular, Alexandra participated in a large number of such visits, whilst in contrast Lisa and Tony were involved in very few visits. I discuss and contrast Alexandra and Lisa's activity in the Phase 2 PD in Chapter 5.

Only Alexandra and Ursula made presentations to academic audiences beyond Outertown and the LNRP. These involved presentations at the British Educational Research Association (BERA) Annual Conference, a British Society for Research into Learning Mathematics (BSRLM) day conference and research days at King's College London. Lisa did participate in two presentations during the first year of the project: one to the Outertown Annual Mathematics Conference; and, one to the wider LNRP research team at King's College.

From early on in the project's development, there were quite distinct differences between the teachers from Beechmount and Parkway in terms of lesson development. At Parkway, Alexandra and Ursula regularly team-taught TM lessons with each other or with Mundher. Both were very vocal within the research team. At Beechmount, in contrast, Henrietta and Lisa generally taught lessons individually either on their own or with Michael as an observer. Although Lisa was initially vocal in research team meetings, she gradually became quieter, contributing less frequently over the course of the year. Henrietta rarely made any unprompted contributions. Neither Henrietta nor Lisa presented any of their own ideas for potential TM lessons to the team. Lisa did work on lesson development, although she focused on amending lessons from the Secondary TM Teacher's Guide. Moreover, Mundher largely worked with the Parkway teachers, whilst Michael largely worked with the Beechmount teachers. The differing approaches of these two King's academics appeared to further emphasise the differences between the pairs of teachers. These issues are discussed and analysed in some depth in Chapter 5.

During the summer term of the first year, the focus of the team's work shifted towards the preparation for the project's work in Phase 2. This had two elements: writing lesson materials, and planning the Phase 2 PD programme. In May 1998, the team agreed on a format for the lesson materials, which had a set of teaching notes together with a set of more theoretical background notes on the mathematics and the children's learning. The teachers took responsibility for producing the teaching notes, whilst the King's researchers took responsibility for producing the background materials, a division of labour which was to continue throughout the lesson development process. However, as a result of an OfSTED inspection at Beechmount at the beginning of June, Henrietta and Lisa had limited time during the summer term and much of the teaching notes were at this stage largely written by Alexandra, Ursula and Rhoda. During this period, Lisa prepared the first draft of one set of teaching notes. Henrietta did no writing, although I should note that she was in the latter stages of her first pregnancy during June and July.

Although Janice and Tony were recruited in large part to give a renewed impetus to the lesson development process, Tony did not actually play a large part in this aspect of the project.

5.2.4 Mathematics Without Closure

During the first year of my research, a key focus for my work was to describe the beliefs about mathematics teaching and learning which Primary CAME sought to promote through an analysis of the academics' presentation of CAME to the teachers. In doing this through the development of the construct *mathematics without closure*, which I used to encompass a set of beliefs about children's learning, about TM lessons and about the teacher's role (See (Hodgen, 1999) for a more detailed discussion.) It is important to emphasise that this construct was developed out of my research and was not a term used previously by CAME.

Many aspects of the CAME approach are shared by other initiatives in mathematics teaching. A focus on methods and understanding over "right" answers, the importance of questioning and discussion, and on the class as a community of mathematicians are all familiar themes within mathematics education, reflecting the valuing of principled over procedural knowledge (Lampert, 1986). However, the academic researchers were emphatic in their characterisation of CAME as distinctive. A key feature was in their references to the approach as one which did not "seek closure" (David, Research team, January 1998). At the same time, the CAME approach was described as being in marked contrast to open-ended investigational work, although much of the language - an absence of closure, open-ended conceptually - used to describe the CAME approach was very similar. Indeed, this tension was highlighted by several of the teachers during the first years. Ursula, for example, commented : "It's quite hard, because having spent a year trying to open up Mathematics ... I find it really quite hard to try and close it in again which is what I feel like I'm doing [with TM lessons]" (Interview, March 1998). Similarly, Alexandra commented in relation to lesson development:

> What I find quite hard is whether we're satisfying CAME aims. We're still not quite clear about that. So I guess most of all what I would feel is we need to go back now to the experts and say, you know, is this fulfilling CAME aims. ... Is this sufficiently open? And closing it down? Have we done that sufficiently? I suspect we might have led a little bit too much for it, so that might be an issue, I don't know. (Interview, March 1998)

In the main, these ideas were communicated in the course of discussions about particular lessons or about general aspects of the CAME approach. However, presentation also included formal presentations and through regular project memos and circulated reading which were then discussed at research team meetings. Readings included Secondary TM materials (Adhami, Johnson, & Shayer, 1997b), other curriculum materials (e.g., Brown, 1992b), articles aimed at teachers (e.g., Middleton, van den Heuvel-Panhhuizen, & Shew, 1998), research reports (e.g., Askew et al., 1997; Johnson, 1989) and more theoretical articles aimed at the research community (e.g., Cobb et al., 1997).

Children's learning was presented as *without closure.* Mundher, for example, commented in a research memo:

> There will always be occasions where the child's thinking structures are disturbed by an episode. ... We should judge effects only at a distance. ... [Let] the repercussions of the lesson settle down in the mind first, since they are not simple ideas to be added but new ways of looking at things that need adjusting mental structures to (Memo, December 1997, p. 3).

Similarly, TM lessons were characterised by the university researchers in terms of an absence of *closure* in that there was not generally a common end point or learning objective to be achieved by all. The lessons were argued to be "open-ended in the points children reach conceptually" and may be "sowing seeds" for later work. TM lessons are distinctly different to "ordinary open-ended investigations," although the vocabulary used to describe the lessons appears very similar. "The conceptual challenges for children are very specific" and promote "very specific mathematical connections and generalisations" (David, Research team, November 1997). Where lessons had a clear end point, this would often be beyond the reach of the majority of the children who would be "struggling on the way towards the concepts ... [and] gaining insights at different levels of complexity" (Adhami et al., 1998b, p. vii.). In CAME "we are go[ing] deeper but in this direction rather than wider doing different sorts of applications of the same. ... That sounds like narrowing, but not in the same sense as closure" (Michael, Research team, September, 1998). In fact, the tightly focused agenda of a TM lesson appeared to be at odds with at least some accepted thinking on investigational work. Ahmed (1987), in an influential study aimed at describing and disseminating good practice in the UK, argues that "a rich mathematical activity ... should not restrict pupils from searching in other directions" (p. 20). In fact, the contrast that the academics made between CAME and open-ended investigations was at least in part due to a focus on investigations within secondary mathematics education in England and Wales during the late 1980s and early 1990s (See Johnson & Millett, 1996, for a discussion.)

In common with other projects in mathematics education the teachers' role was described as that of a mediator and facilitator of learning rather than an instructor (e.g., Brown, Collins, & Duguid, 1989). A key focus for CAME was teacher subject knowledge. The belief in mathematics as a connected subject was emphasised (Askew et al., 1997). However, it was in relation to a teachers' knowledge of the school mathematics curriculum as a whole that CAME appeared to be distinctive. The academic researchers stressed the importance of teacher knowledge of key mathematical concepts in order that she can "frame ... the specifics of each task so that 'the road ahead' does lead in the right direction" (Adhami et al., 1998c, p. x). Here, CAME goes beyond Ma's (1999) longitudinal coherence within the primary curriculum to include aspects of "big ideas" in secondary mathematics. For example, in Gardens, one of the Secondary lessons adapted for primary, an activity exploring simple two step number patterns was linked to later work on linear functions and the concepts of intercept and gradient. (This lesson is discussed in Chapter 4 and the materials for this lesson are attached in Appendix A.)

Whilst teachers' mathematical knowledge was presented as crucial, however, the teachers' role "is not to tell or instruct. It is developmental." (Mundher, Research team, January 1998). Mundher referred to this tension between knowing and not telling as "paradox[ical]. We do need the strands but we don't teach them." (Mundher, Research team, March 1998).

Hence, by mathematics without closure, I identify a set of beliefs relating to mathematics learning, mathematics pedagogy and the role of teacher mathematical knowledge. These centre around CAME contributing to the wider picture of children's developmental learning. Whilst not unique to CAME, the project's emphasis on this aspect of mathematics education does distinguish the CAME approach from other mathematics education initiatives. I stress, however, that as I have noted above mathematics without closure does not encompass all the beliefs about school mathematics encompassed within CAME. I discuss the teachers' engagement with the CAME approach and in particular the notion of mathematics without closure in Chapter 5.

5.3 The NNS and Other Experiences During the Research

As I have already noted above, the impact of the NNS locally on the CAME programme of research was significant. Two members of the research team, Alexandra and Ursula, became Numeracy Consultants. Whilst they remained involved in the research team, their numeracy work inevitably had a very considerable influence on their role within CAME. At a purely practical level, without their own classes, it became more difficult to organise the teaching of TM lesson trials. More significantly, teacher education in the form of NNS training sessions and in-school demonstration lessons, became a central part of their wider professional role.

Whilst Henrietta had left the project by the time the NNS was implemented, the strategy had a significant impact on the three who remained as class teachers. All three implemented the daily mathematics lesson in their classrooms and received in-school INSET on key aspects of the strategy. Janice and Tony's involvement was more considerable. As Mathematics Co-ordinators, they received initial 3-day training along with their Headteachers and Special Needs Co-ordinators and delivered INSET to their school's staff groups. Both of their schools were identified for intensive training, although Janice used this opportunity to send a group of other teachers on the training. In Tony's case this was as part of the Outertown Numeracy Project, on the basis that his school, Meadowside, had a strength and interest in numeracy. Moreover, both Janice and Tony led NNS Booster training for groups of teachers from other Outertown schools. In Table 2.5, I summarise Janice's, Lisa's and Tony's involvement in NNS training initiatives.

	Participation in NNS PD			Leading NNS PD for others	
	Within School	3 day training	5 day intensive training	Within School	Outside school
Janice	✓	✓	✗	✓	✓
Lisa	✓	✗	✗	✗	✗
Tony	✓	✓	✓	✓	✓

Table 2.5: Janice's, Lisa's and Tony's involvement in NNS PD

5.4 Did The Teachers Change?

The teachers' professional change is the focus of later chapters in this thesis, in particular Chapters 6 and 7 in which I focus on the teachers' beliefs about and knowledge of school mathematics. However, in this section, I give a very brief indication of these findings.

Given the intensity of all the teachers' profession development experiences, I had expected some degree of change for all of the teachers. However, although all the teachers reported change and there was evidence of some change for all the teachers, change was only extensive in the cases of Alexandra and Ursula. This was particularly evident in terms of their beliefs about mathematics education. The contrast with Lisa, the only other teacher to be involved throughout the fieldwork, is particularly significant. In Chapters 4 and 5 I address the question of how such differential change may be explained.

However, despite these changes, all the teachers' mathematical knowledge appeared to develop very slowly. Indeed, when I explored Alexandra's knowledge of multiplicative relations at the end of the project, I found it to be largely procedural.

I make three other brief points:

Firstly, all of the teachers, except Henrietta, who left teaching after the first year, were promoted during the life-span of the project. As I have already noted, Alexandra and Ursula became Numeracy Consultants. Towards the end of the fourth year, Ursula became Deputy Headteacher in an Outertown school. Lisa was also promoted to Headteacher of another Outertown school. Janice became Deputy Headteacher at Brightvale, her existing school. Tony became Acting Deputy at Meadowside and later moved to become a Numeracy Consultant for another LEA. It is noteworthy that in terms of promotion to a management position, Lisa was the most successful of the teachers despite her limited professional change in terms of mathematics education. This is particularly significant since both Alexandra and Ursula were unsuccessful in several interviews for similar positions.

Secondly, the teaching of the lessons appeared to make a very significant difference to the Phase 1 children's cognitive abilities, as I highlighted in Section 4.2.4 above. I note, however, that this testing did not involve Lisa's or Ursula's classes. This is not a direct measure of professional change. However, it would appear to suggest that the teachers were enabled to teach TM lessons successfully.

Finally, a feature of this research is the very extensive period that the teachers were involved in this professional development initiative. Yet, as I discuss in Chapters 4 and 5, the teachers experienced the process of professional change as a difficult one. Indeed, a focus for the discussion in these and the later chapters is to understand and explain the difficulty of professional change.

6. Summary

In this chapter I have described the local and national context to this research:

- the National Numeracy Strategy, which was introduced during the course of the research and had a significant impact on the professional development of five of the teachers in this study
- Outertown LEA, the authority in which the research was set
- the Primary CAME Project

Of these, the Primary CAME Project was the most significant to this research in that my six principal research participants were teacher-researchers in the project's research team and the majority of the professional development experiences that I studied were in the context of this project. Primary CAME was a project in three phases:

- Phase 1 focused on lesson development and assessing the feasibility of the approach to primary
- Phase 2 focused on implementing the approach with a further group of Phase 2 teachers and schools
- an unfunded phase focused on finalising the lessons

Given the fact that I was attached to Primary CAME, itself an ongoing research project, one function of this chapter was to distinguish my research from that of Primary CAME more generally. Whilst my research is focused on the teachers' professional change, the research aims of Primary CAME were concerned with the development and extension of the CAME approach into the primary years of schooling. Specifically, Primary CAME's objectives were:

- to develop a series of Thinking Maths (TM) lessons for Y5 and Y6
- to investigate the effects of teaching these lessons on children's intellectual development through a set of pre and post tests
- to contribute towards teachers' professional development more generally through a in-service teacher education programme centred around the lessons

Although teacher professional development was an aim of the project, the data that the project collected in this area was limited. In the next chapter, I further distinguish my research by discussing and differentiating the data that I collected from that of Primary CAME more generally.

Since my research is focused on changes to the teachers' belief and knowledge, a key aim for my research was to describe the beliefs underlying the CAME approach to school mathematics. In doing so I introduced the construct of mathematics without closure in order to describe CAME's focus on the wider picture of children's mathematical development. I have also raised the issues of teacher motivation and of situated learning, which I discuss in Chapters 4 and 5.

I have introduced the principal research participants, Alexandra, Henrietta, Janice, Lisa, Tony and Ursula, and described their professional development experiences within the Primary CAME Project. These experiences related to teachers' participation as lesson developers and as tutors to Phase 2 group of teachers. I have noted that that change appeared to be extensive for just two of the teachers: Alexandra and Ursula. Hence, I have highlighted the question of explaining differential change, one of the research questions described in Chapter 1, Section 4. I address this question in Chapters 4, 5 and 7.

Chapter 3: Methodology

1. Introduction

In this chapter, I discuss and outline my research methodology.

This was an exploratory study and both the aims and methods evolved over the course of the research. I summarise this development in section 3 below.

The chapter is structured as follows:

- In Section 2, I summarise the aims of the research.
- In Section 3, I briefly outline the major changes to the research design over the period of the research.
- In Section 4, I discuss the methodological approach and the overall research design.
- In Section 5, I describe the different methods of data collection together with the way these different methods contributed to the study and the ways in which the different data sources were used and initially analysed.
- In Section 6, I describe the analysis that I undertook.

2. Aims

The aims of this research were to explore the nature of teacher professional change in primary mathematics with a particular focus on teachers' beliefs and knowledge about school mathematics. Within this, I wanted to look at the ways in which differences between teachers' professional change might be explained and the influence of teachers' wider professional networks on their professional change.

The research questions for this study are outlined in Chapter 1, Section 4. The principal research question is as follows:

> In what ways do primary teachers' beliefs and knowledge change and develop as they undergo professional development in school mathematics?

As I discussed in Chapter 1, I approached this question from an epistemological perspective of knowledge as a social practice and, hence, both contingent and socially negotiated.

As I indicated earlier, this is an exploratory research project and the open-endedness of the research question reflects this. My aim is not to provide a definitive answer to the question but rather to contribute to the developing empirical and theoretical knowledge base within the area of mathematics teacher education generally and primary mathematics in particular.

3. Changes to The Research

Given the open-ended nature of the research, I expected the research aims and questions to evolve over the period of the research. In fact these changes were quite considerable, although the general focus on continuing teacher education in primary mathematics remained constant. These changes in focus were guided by empirical findings, theoretical insights from the literature and my ongoing analysis together with more pragmatic and practical concerns. The principal changes were as follows:

- My initial research aim was to focus on the CAME professional development initiative and explore the ways in which this facilitated teacher change. However, the empirical work in the first year emphasised that the teachers themselves seemed to be catalysts in their own learning and that Primary CAME was just one element in an ongoing process of professional change, an understanding that resonated with the mathematics education literature. (See, for example, (Brown & Borko, 1992).) Hence, I shifted the focus of my research from the PD initiative to the teachers themselves.

- I had initially intended to increase the sample of key informants to include a number of the Phase 2 teachers during the second year of the project, with the first year's research in part acting as a pilot for this later phase. This proved practically difficult, because four of the six Phase 2 teachers with whom I had started working left at the end of the second year. From a more theoretical perspective, my early work confirmed the findings in the teacher change literature that professional change is a difficult and lengthy process. (See, e.g., Clarke, 1994.) A focus on the teacher-researchers, whose professional development experience was both extended and intense, would enable me to focus on the possibilities for professional change in extreme cases. By extreme here, I indicate that these teachers' experiences as teacher-researchers were, for primary teachers, unusually intense and extended. Although these experiences would not be directly generalizable to the majority of primary teachers, this atypical case could, through the intensity of the teachers' experiences, enable a focus on the process of professional change.

- My original aim was to explore the interrelationship of teachers' beliefs, knowledge, understandings *and* practices. However, Alexandra's and Ursula's promotion to Numeracy Consultants meant that they were no longer class teaching on a regular basis. Although I focused for some time on their practices as teacher educators, I decided that a focus on their knowledge, beliefs and understandings would make the research more manageable.

4. Methods

Given the nature of this study and my aim of developing understanding of the process of professional change, the choice of qualitative methods seemed most appropriate. I have made use of ethnographic methodologies and in particular techniques of data collection and analysis drawn from grounded theory (Charmaz, 1995; Strauss & Corbin, 1998) It has not been my intention to conduct a full blown ethnography and the approach is best described in Bloome and Green's (1997) taxonomy as *using ethnographic tools* rather than *doing ethnography*. In a sympathetic critique of grounded theory, Charmaz (2000) argues that grounded theory has a tendency to ignore or underplay influences on the research process from theory. Concepts do not simply arise from data, but rather researchers bring their knowledge of the research area, literature and theory to aid understanding of the data. However, by underplaying this source of theoretical understanding of the research domain, grounded theory can render the development of theory somewhat opaque. Influenced by this critique, in addition to using techniques from Strauss and Corbin's (1998) grounded theory approach, I have drawn on constructs from the research literature in order to make sense of and supplement those arising from the data. I have also drawn on Kvale's (1996) social constructivist approach both in informing the study as a whole and in designing data collection methods.

4.1 Participant Observation: The Researcher as Research Instrument

My principal strategy for data collection was that of participant observation, by which I mean that I engaged in the activities of the research participants but *in a limited way*. For example, I attended and participated in the research team seminars. However, in comparison to both the teachers and the King's researchers, my participation was both different – in every setting I took copious notes – and more limited – I contributed to discussions much less frequently and then largely only in response to direct questions. A particular feature of participant observation that makes it appropriate to this study is its "orientation to discovery" (Ball, 1990, p.157). By getting close to and to an extent participating in the practices of the group, the participant observer has the possibility of gaining understandings not easily accessible to the more distant observer.

My aim was to be a largely "peripheral member" in order to "observe and interact closely enough with members to establish an insider's identity without participating in those activities constituting the core of group membership" (Adler & Adler, 1998, p.85). However, as Nolder (1992, p.20) notes "the quality of data obtained [is] dependent on the quality of the research relationship" and the researcher's "acceptance" by the other participants. Therefore on occasion, I became what Adler and Adler (1998) refer to as a more "active member ... assuming responsibilities that advance the group, but without fully committing [myself] to members' values and goals" (p.85). So, for example, on occasion I taught lessons and actively participated in discussions with Phase 2 teachers. Throughout the fieldwork I agonised over the extent of my participation in the wider Primary CAME project. (See, e.g., the research memos attached in Appendix F.)

A key issue for participant observation is to "to balance involvement with detachment, familiarity with strangeness, closeness with distance" (Adler & Adler, 1998, p.84). This balance is somewhat difficult to achieve, a dilemma which is aptly captured in Bourdieu's notion of "disinterested interest" (Bourdieu, 1993, quoted in Grenfell, James, Hodkinson, Reay, & Robbins, 1998). Brown and Dowling (1998) urge researchers to leave their "motivational baggage" as they enter the research process to be "collected at the other end of the research process", to allow a "cooling out of the anxieties and political interests that shaped" the original research motivation (p. 155). In contrast Lampert (1998) argues that such a strategy can lead to limited understandings: "when one strives ... to make definitions, hypotheses, and arguments 'precise enough to avoid any misunderstanding' the conversation moves away from knowledge of and for practice" (p. 61). Lampert argues that it is her position as a practising teacher *and* a researcher *and* a teacher educator "being a university professor who also teaches fifth grade mathematics" that allows her "to view teaching as a thinking practice" (p. 54, original emphasis). I have considerable sympathy with Lampert's arguments. My own motivation to do this research comes from my experiences as a primary teacher with a mathematics specialism and many of the insights I gained were due in part to my own teaching and learning experiences. Nevertheless, there is a danger in this position of losing one's research perspective and the research lacking rigour (Delamont, 1992). It is crucial to be conscious of these dilemmas and, thus, to ensure rigour in the research process. Ball (1990) refers to this technical rigour as self-conscious engagement or *reflexivity* and recommends the use of a research biography:

> The problems of conceptualising qualitative research increase when data, and the analysis and interpretation of data, are separated from the social process which generated them. In one respect, the solution is a simple one. It is a requirement for methodological rigor [sic] that every ethnography be accompanied by a research biography, that is a reflexive account of the conduct of the research which, by drawing on fieldnotes and reflections, recounts the processes, problems,

> choices, and errors which describe the fieldwork upon which the substantive account is based. (p.170)

My research biography took a number of forms. Firstly, I regularly recorded my difficulties, dilemmas and strategies in the form of a series of research memos. Not only did these form a record of the issues themselves, but I found the process of writing them invaluable in overcoming the difficulties that I encountered during the research process. Secondly, I used these memos as the basis for longer and more considered analytic memos on my own participation in the research. Thirdly, from time to time, I wrote and updated my research biography recording the research process based on these research memos. I attach several examples of research memos in Appendix F.

4.2 The Research Design

4.2.1 The Fieldwork: November 1997 – July 2001

The fieldwork for this research was conducted in three phases over a four year period covering the school years 1997/98 – 2000/01. In Figure 3.1, I show a timeline for these phases together with the involvement of the teachers. These phases mirror the three phases of the Primary CAME Project. Although Primary CAME was actually only funded for the three years 1997/98 – 1999/00, a significant amount of work was done the fourth year in finalising and writing up lesson materials. In the exploratory phase, my aim was to identify key issues in relation to the teachers' professional change and to develop research techniques and methods to use in the later phases of the project. My focus was on the teacher-researchers' initial engagement and on exploring the PD element of their early experiences. This reflects my initial focus on the PD as outlined in section 3 above. In the main phase my focus shifted towards the teachers themselves and their professional change. In the concluding phase, I focused on tying up loose ends, completing any interviews and observations not carried out during the main phase, and on checking and clarifying issues arising from my analysis.

	97/98	*98/99*	*99/00*	*00/01*	***Teacher participation***	***Research focus***
Exploratory phase [P-CAME Phase 1]					Initial: CAME teachers & lesson developers	Teachers' initial engagement
Main phase [P-CAME Phase 2]					Full: CAME teachers, lesson developers, tutors	Teachers' professional change.
Concluding phase [P-CAME unfunded phase]					Partial: Lesson developers	Tying up loose ends

Figure 3.1: A timeline of the fieldwork

4.2.2 The Sample of Teachers

As I have already noted, the research setting and hence sample of teachers was an opportunistic one (Silverman, 2000). My research interest was in the professional development of primary teachers in mathematics. The opportunities presented by Primary CAME seemed to be an ideal setting in which to explore this research interest. The sample of teachers that I focus on was made up of the six teacher-researchers involved in the project research team. Four of these teachers, Alexandra, Henrietta, Lisa and Ursula, joined the project at its inception in November 1997, although Henrietta left teaching for personal reasons at the end of the first year. The two further teachers, Janice and Tony, joined the project in June 1999. In Figure 3.2, I indicate the different teachers' participation over the period of the fieldwork.

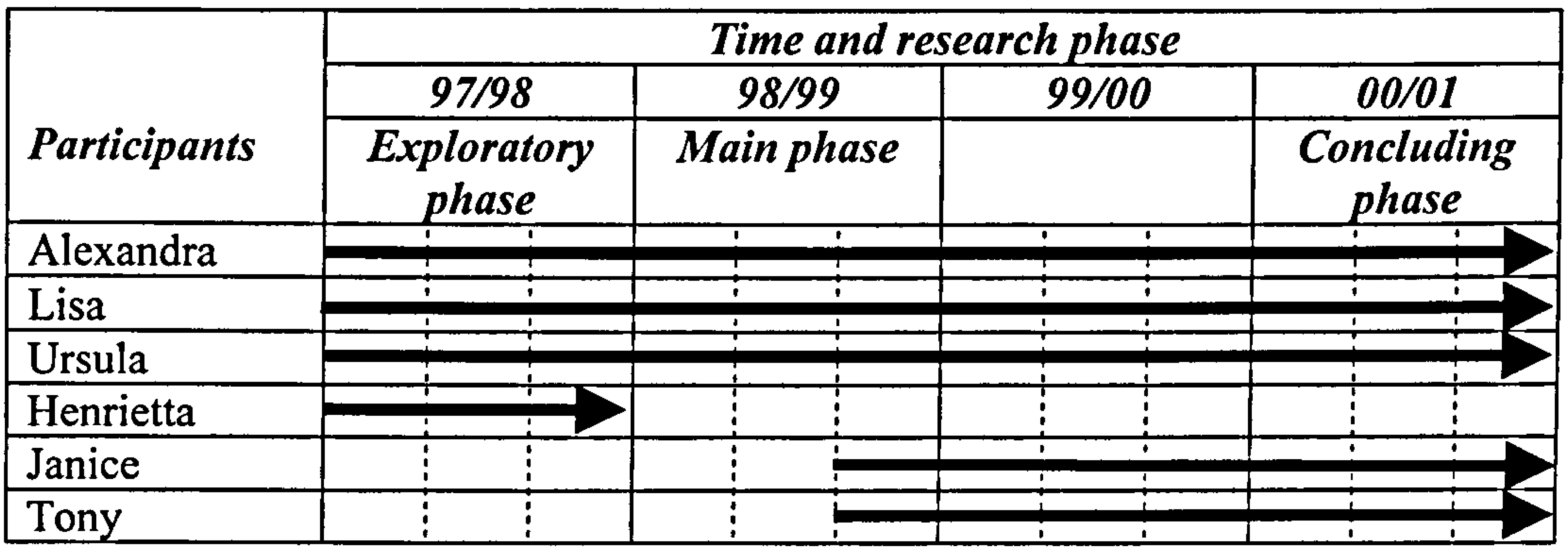


Figure 3.2: The participation of the 6 teachers

The group of six teacher-researchers was an unusual one. In particular, their experiences within mathematics education within the project (and beyond) were extremely intense. As recipients of what Nolder (1992) describes as "accelerated professional development" over an extended period, one might expect this to provide favourable conditions for professional change to take place. (Although as I have already noted in Chapter 2, change appeared to be significant for only two of the teachers, Alexandra and Ursula, and, at least for Alexandra, change in terms of their understanding of specific mathematical concepts appeared to be very limited.) The intensity of these teachers' experiences provided an ideal opportunity in which to study the possibilities for professional development for primary teachers in mathematics education. In terms of the generalizability of this case study, my interest is in identifying processes of professional change rather than in suggesting that these teachers' intense experiences should be replicated. This case represents what Mitchell (1984) describes as a "telling" rather than a "typical" case. I address these issues of generalizability more directly in Chapter 8.

However, it is important to note that, whilst the choice of teachers participating in Primary CAME was outside my control, the decision on the research sample of teachers was a deliberate one, albeit a constrained choice. Indeed, one aim of the exploratory phase was to develop key issues and constructs in order to bound the case study by theoretical means (Silverman, 2000). I have already noted above, however, that due to Phase 2 teachers leaving the project I had to re-think my initial decision to focus on a group of Phase 2 teachers in addition to the Phase 1 group.

Other decisions on bounding the case study were more practical and pragmatic. I chose to focus on the teachers' involvement in Primary CAME and their mathematics teaching and learning more generally. I did not set out to collect data on the teachers' personal lives and their teaching and learning of subjects other than mathematics or indeed their participation in school life more generally except as these issues were raised by the teachers themselves. As a sole researcher, I judged that, whilst these issues were of undoubted importance, collecting data on these aspects of the teachers' lives would be spreading myself too thinly.

4.2.3 Methods of Data Collection

In Table 3.1, I summarise the data collection and methods over the course of the fieldwork. At the heart of this thesis is my analysis of the observations and interviews with the case study sample of teachers. I show these main sources of data in bold in Table 3.1. In addition, I collected a large amount of secondary data that has been only partially analysed, which is shown in Table 3.1 in ordinary type. I conducted interviews with other research team members and Phase 2 teachers, lesson observations and observations of tutoring visits. Largely these were used for purposes of triangulation and confirmation. Finally, I have used some data from other sources within Primary CAME and LNRP more widely together with internal project papers, draft lesson materials and memoranda. These data collection methods are discussed in more detail in Section 5 below. A more detailed schedule of this data indicating timelines and respondents in more depth is attached in Appendix G.

Time	*Method*	*Number*
1997/98	**Observations of research team seminars**	**14**
	Interviews	**4**
	Lesson observations	5
	Interviews	2
	Mathematics interview (pilot)	1
1998/99	**Observations of research team seminars**	**13**
	Observations of PD sessions	**6**
	Interviews	**2**
	Observations of Phase 2 tutor visits	8
	Teachers' taped reflections of Phase 2 visits	2
	Lesson observations	20
	Interviews	1
1999/00	**Observations of research team seminars**	**11**
	Observations of PD sessions	**6**
	Observations of NNS PD sessions	**2**
	Interviews	**6**
	Group interviews	**3**
	Mathematics interview	**1**
	Lesson observations	16
	Interviews	5
2000/01	**Observations of research team seminars**	**1**
	Observations of NNS PD sessions	**2**
	Mathematics interview	**1**
	Questionnaire and informal discussion	**1**

[Note: The main data set is indicated in **bold**.]

Table 3.1: Overview of the research methods used.

Due to the opportunism of this sample and the fact that Primary CAME began work shortly after I entered the doctoral programme at King's, I had little time to develop research tools and instruments prior to the initial period of fieldwork. Indeed, I began data collection in the form of observations and fieldnotes almost immediately I embarked on the research. Research methods were therefore developed during the course of the fieldwork. As I noted above, one aim of the initial exploratory phase was to develop techniques of observation and interview to use in the main phase of the study.

In deciding on and developing forms of data collection, I was aware of the difficulties in researching teachers' beliefs and knowledge and that these difficulties would be particularly acute in relation to primary mathematics (Bibby, 1999; Thompson, 1992). There are, however, many well-developed and validated structured techniques used to elicit teacher's beliefs. In a review, Fang (1996) argues that, whilst such techniques are useful in determining teachers' beliefs, they do not address how the experiences of teachers influence and shape these views. Since my research interest was on the process of teachers' changing beliefs and knowledge and given that I had considerable access to the teachers, I felt that observations and semi-structured interviews were more appropriate to my research aims. Thompson (1992) recommends the use of multiple methods in researching teachers' beliefs. (See also, Aubrey, 1997; Richardson, 1996.) I therefore adopted a variety of methods; I observed the teachers in a variety of settings; I spent time in their schools, classrooms and work environments; I looked at their involvement in mathematics education beyond the immediate confines of Primary CAME. I also interviewed and observed other key participants in Primary CAME and used data collected by others.

Miles and Huberman (1984, pp.42-3) argue that having few prior formal research tools and a reliance on open-ended fieldnotes can be more appropriate to an exploratory study with a small sample. However, they also note that there are disadvantages to this approach. The lack of prior instrumentation can make the project susceptible to problems such as the collection of too much superfluous or unfocused data or to limited comparability with other similar studies. In order to minimise such problems, I used and developed techniques for observation and interview from previous studies at King's (e.g., Millett, 1996; Nolder, 1992) and from the wider Leverhulme Numeracy Research Programme (e.g., Askew & Millett, 2001).

The purpose of the multiple sources of data was to gain multiple perspectives on the teachers' professional change. Hence, the intention was for the different types of data to address the research questions as a whole rather than for each source to address a specific set of research questions. Nevertheless, some sub-sets of the data set were used in particular ways. For example, a major focus for the group interviews was on collaboration. In the other hand, the individual interviews had different foci partly reflecting both changes in emphasis in the overall research aims and the teachers' professional change over time and partly reflecting that the interviews built on my evolving understandings of teacher change.

5. Data Collection and Initial Analysis

In this section I discuss and outline the different methods of data collection that I used. I focus on the ways in which I recorded the data, using transcription, fieldnotes and on how I recorded my ongoing observations and reflections. All methods of data collection entail selection and the transcripts and fieldnotes produced are "interpretative constructions" (Kvale, 1996, p.165). My aim here is to be explicit about the ways in which I engaged with, initially analysed, sampled and thus constructed the data during the process of data collection and initial recording. I stress that I examine the collection of data and the *initial* analysis involved in this. In the next section, I focus on the process of data analysis as a whole.

Over the first three years of the fieldwork, I generally saw each of the teacher participants at least every two or three weeks during term-time, much of this in day-long research seminars. One possible danger with such an extended research period was the potential for data overload. In this regard, a key part of the data collection process was the reduction of the data to manageable proportions whilst maintaining the richness and depth of the data.

5.1 Observations of Research Team Seminars and PD Sessions

I first describe in some detail the methods I used for recording the research team seminars. Other meetings were recorded in similar ways and these are then discussed in less detail.

5.1.1 The Research Team Seminars

Central to the participation of the teacher-researchers were the research seminars in which the research team met to discuss issues, reflect on work outside the seminar and to work on lesson materials. Over the four years of the research, there were 41 seminars (33 whole day and 8 half-day meetings) of which I attended all but one. These seminars were characterised by free-flowing and open-ended discussion with frequent interruptions and concurrent conversations. Often participants disagreed or argued. Most discussions involved many aspects of the project's work. In addition, much of the more informal chat over lunch or coffee was interesting and relevant. This open-endedness made the recording of fieldnotes all the more complex.

Given the length of these meetings, I was very conscious of the need to be selective in relation to data collection. Prior to the seminar, I made a list of issues that I would focus on during the seminars together with any specific questions or clarifications to raise with the teachers. During these seminar, I took hand-written notes of conversations and activities, recording the on-going focus of discussions, the gist of participants contributions, any annotations made on the board or on paper, and my impressions of participants' body language and engagement with the discussions. Whilst these fieldnotes were detailed, I found it difficult to record interchanges and prolonged conversations in depth and with sufficient detail. As a result, I audio taped large portions of the seminars from midway through the first year. However, these audiotapes presented a further problem – I have approximately one hundred 90 minute audiotapes of 41 day / half-day research seminars. Moreover, the quality of the tapes was at times poor. Transcribing them in entirety would not only have been extremely time consuming but would have resulted in a huge amount of data, much of it superfluous. Hence, a key objective during the meeting was to make an initial decision to highlight items of interest. In my fieldnotes, I recorded such items of significance and portions of the audiotape that I might want to listen to and transcribe (with an indication of the time and the tape counter). In the evening I would write up a summary of the meeting together with one or two pages of on-going reflections. These summaries provided both a simple and brief précis of the meeting together with a degree of what Kvale (1996) refers to as "clarification of the material, making it meaningful to analysis ... eliminating superfluous material, distinguishing between the essential and the non-essential" (p.190). The next day I would listen to the selected portions of transcript, again making a decision on whether to transcribe the tape, simply add to the fieldnotes or to ignore it. I then read through the notes, cross-referenced the computer file to my hand-written fieldnotes and added further reflections. Depending on time constraints, the actual transcription was often carried out at a later date. There were occasions when I found that I had decided not to record in my computer fieldnotes something that later turned out to be significant. In such cases, this cross-referencing was invaluable in quickly accessing the relevant sections of my hand-written fieldnotes and if necessary of audiotape. In addition, this enabled me to easily and quickly check the reliability of transcriptions.

Although my intention in this process of data reduction was to make the data manageable, I still found this resulted in a still considerable amount of notes. A further strategy that I used was to record the topics of discussion on a spreadsheet summarising all the seminars.

The research seminar data contributed to the development of the research as a whole. They were particularly useful in highlighting contradictions either between the participants or within what one participant had said on different occasions.

5.1.2 Phase 2 Professional Development Sessions

During the main phase of the research, there were 12 Phase 2 PD half day sessions. During these sessions, I took hand-written notes of conversations and activities. In addition I made audio tape recordings of any lesson simulations led by the teachers and any reflection sessions on lessons taught. During the first year, I tracked key informants from the Phase 2 group of teachers, although much of this material was not used after I decided to focus more centrally on the group of teacher-researchers. Since the meetings were far more structured and discussion less open-ended, the fieldnotes were considerably easier to write up and transcribe than the research team seminars. I used a form of fieldnotes and summaries developed from Millett's work (1996). Again although these PD sessions contributed to the study as a whole, when conducting this initial analysis I was particularly focusing on the teachers' mathematical knowledge, their questioning approaches, and any disagreements as highlighted above.

5.1.3 National Numeracy Strategy Professional Development Sessions

In the latter stages of the project, I observed several NNS training sessions: one in-school session at Brightvale led by Janice together with three sessions led by Alexandra and Ursula. These sessions were written up in detail as for the Phase 2 PD sessions above. Summaries of these session notes were then shared and discussed during an interview with the teacher leading the session. This discussion focused on the changes the teachers had made to the NNS training guidance and on their judgements about the course participants' mathematical knowledge. These sessions were particularly useful as a comparison to Primary CAME.

5.2 Interviews

In this section, I first describe the main set of individual semi-structured interviews and the approaches that I used to record and summarise them. Then I look at the development of the more focused and structured interviews – the mathematics interviews and the group interviews.

5.2.1 The Individual Interviews

Over the course of the project I conducted 17 semi-structured interviews with the main group of teachers. My intention was to track the teachers' experiences and understandings of professional change over the course of the project by interviewing each teacher once each year during the three funded years of the project. The first interview focused on the teachers' initial engagement with the project, whilst the later two explored the teachers' reflections on their professional change. A detailed list of the interview schedules together with a summary of the key issues for discussion is attached in Appendix H.

In order to structure the interviews, I prepared a schedule of interview questions and prompts. I used Kvale's (1996) typology of interview questions as a model. In particular, I asked teachers to comment on my interpretations of things they had said or done in previous interviews and other settings. As a result, the schedules differed slightly for each teacher.

Each interview was between an hour and two hours long. The interviews were taped and transcribed. I used a fairly simple transcription format, which is attached in Appendix I. Initially I transcribed these tapes myself, although in later interviews I used a transcriber. Once transcribed, whether by a transcriber or myself, I compared the transcript to the tape to ensure reliability.

Immediately after conducting the interviews I wrote short reflection notes. The aims of these notes were twofold: to remind myself of the interview setting and any factors that might not be clear from the transcript, for example, whether the interviewee seemed nervous; and, secondly, to record any discussions of significance that were not taped. These reflections were also useful in checking and interpreting the transcripts. Once the tape was transcribed, I produced interview summaries in a form developed from Millett's work (1996). I attach an excerpt from a transcribed interview in Appendix J.

Once transcribed the interviews were given to the teachers for respondent validation with the aim of prompting them not only to correct errors, but also to comment further on issues that were discussed, and to clarify or amend what they had said (Goodson, 1992, p.245). Despite my encouragement, only Janice actually made any direct comments on her interviews when she corrected a couple of factual transcribing errors. Given this lack of a response, I experimented with alternative approaches to respondent validation. In later interviews I invited the teachers to comment on excerpts from earlier interviews. I also asked them from time to time to comment on my on-going analysis of their interviews and I recorded these discussions in the form of fieldnotes as detailed above. As Silverman (2000) argues, respondent validation does not provide direct refutation or validation and I therefore used these fieldnotes as further sources of data.

As a further reliability and validity procedure, for each teacher, I checked at least one tape. I first listened to the tape highlighting issues of importance and compared this to my interview summary. Then I compared the tape to the transcript.

Although my intention was to conduct these interviews each year, in the event, only Alexandra and Ursula were interviewed three times. Henrietta left after the first year and was, therefore, only interviewed once. Lisa was also interviewed only once. It proved very difficult to arrange an interview, an issue that I discuss below. Janice and Tony, having joined at the end of the second year, were each interviewed twice: at the beginning, and at the end of the third year.

5.2.2 The Mathematics Interviews

In the third year of the project I conducted mathematics interviews with Alexandra and Janice. The purpose of these interviews was to explore these particular teachers' understanding of mathematics concepts in the area of multiplicative reasoning. The interview schedule was similar that used in LNRP Focus 2 Project, with some amendments and additions (Askew & Millett, 2001). This interview schedule was itself developed from a previous doctoral study at King's (Bibby, 2001) and I had trialled an early version of this interview during the first year of this study. I discuss the reasons for the focus on multiplicative reasoning in Chapter 7. The interview schedule and the mathematical problems discussed are attached in Appendix K.

My analysis of other data indicated that Alexandra's beliefs about school mathematics had changed in various ways, whilst Janice's beliefs had changed much less significantly. My aim was to explore the extent to which the changes to Alexandra's beliefs were mirrored by developing understandings of mathematical concepts – or indeed whether Janice's understanding of mathematical concepts had developed despite the less dramatic changes to her beliefs. In particular, I wanted to assess the extent to which these teachers' subject knowledge could be described as profound in Ma's (1999) terms or as teacher-knowledge in Prestage and Perk's (2001) terms. I discuss these issues further in Chapter 7. I should note that, since I had not conducted similar interviews when the teachers joined the project, this was not in a pre and post test format. However, I could compare their performance with other data on the teachers' subject knowledge as well as their perceptions of change.

Throughout the study I gather data on the teachers' subject knowledge. As I noted above, during the first year of fieldwork, in April 1998, I trialled an early version of the mathematics interview with Kate, a teacher unconnected to Primary CAME. This pilot interview combined a concept mapping exercise as used in Askew et al. (1997) together with a series of mathematics questions. However, Kate, a primary mathematics specialist with a Masters in Mathematics Education, found the interview uncomfortable and at times threatening, an experience which resonates with other work in this area (e.g., Bibby, 2001; Brown et al., 1999; Buxton, 1981). Given that the teachers had already expressed anxieties and feelings of discomfort about research team discussions about mathematics, I judged that to conduct these interviews so early on in the fieldwork would be counter-productive in terms of the wider Primary CAME project's aims. In place of the interview, I intended to observe the teachers doing mathematics more informally in the course of lesson development, preparation and reflection rather than conducting a formal interview. However, although I did gather some data of this type, it proved very difficult to observe the teachers doing mathematics on a regular basis. It was partly as a result of these difficulties that I decided during the third year of the project to conduct mathematical interviews.

I had intended to conduct the mathematics interview with Ursula as well as Alexandra and Janice, although this proved difficult. Moreover, both Alexandra and Janice expressed considerable anxieties before, during and after these interviews. I discuss these issues as well as the reasons for the choice of the teachers to conduct this interview with in Chapter 7.

The interviews were written up in a similar format to that described the interviews above. In addition I annotated the teachers' notes, recording and jottings from the interview.

5.2.3 The Group Interviews

In the third year of the fieldwork, I conducted three group interviews with the teachers: a joint interview with Alexandra and Ursula and two interviews with the teachers as a group. However, Tony was unable to be present at the first of these, so the group interviews were with the following groups of teachers: Alexandra, Janice, Lisa and Ursula, and Alexandra, Janice, Lisa, Tony and Ursula. The purpose of these interviews was to explore the issue of collaboration.

A secondary aim was to find a way to interview Lisa, with whom, as I discuss below, I had difficulty arranging an interview.

As I noted above, these interviews made use of focus group techniques (Morgan, 1997). Since I wanted to explore the issue of collaboration, I aimed to ask fewer questions than in the other interviews, making particular use of open-ended introductory questions. Aide-memoires for the group interviews are attached in Appendix H. The interviews were transcribed with the aid of fieldnotes and summarised as described above.

5.2.4 Informal and Impromptu Interviews

In addition to the aboveI had many informal discussions with the teachers. Often these were impromptu discussions after visiting a Phase 2 school together or over lunch during a research team seminar. At other times, I had asked a teacher specifically to comment on my on-going analysis or to confirm or clarify a point. On these occasions, I took hand-written notes during or shortly after the conversation and recorded my immediate recollections and reflections on tape immediately afterwards. These fieldnotes were written up as part of a general fieldnote or as a brief standalone memo.

5.3 Lesson Observations, Phase 2 Visits, Interviews and Fieldnotes

During the course of the fieldwork, I collected data in a range of settings. I observed lessons with both the main sample of teachers and the Phase 2 group of teachers. I observed the teacher-researchers' tutoring visits to Phase 2 schools. I conducted interviews with other research team members and Phase 2 teachers. This data was largely used for purposes of triangulation or as prompts for interview discussions.

In addition I made use of data collected by other research team participants in the form of lesson observations and interviews in addition to project memos and drafts of lesson materials. The lesson observation data are detailed in Appendix G.

5.4 Collecting Data on Lisa and Henrietta

I had much more limited data in relation to Lisa and Henrietta than any of the other teachers. The two cases are quite different. Henrietta left the project (and teaching) after the first year. Lisa, on the other hand, was involved in the project throughout and led several Phase 2 PD activities, but became very much less involved in the research team seminars. Moreover, I was only able to interview her individually once, whereas, in contrast, I individually interviewed Alexandra on four occasions, Janice and Ursula on three occasions and Tony twice. Although Lisa said that she was willing to be interviewed, I found it difficult to arrange an interview with her during the second and third years of the project. I discuss this issue in Chapter 5.

In order to address the issue of limited data, I used the following strategies:

I collected all the data on Henrietta's and Lisa's participation in research team seminars on separate files and coded these in detail. In Lisa's case I included her contributions to the Phase 2 PD sessions.

I coded and analysed in some depth the lesson observations of Henrietta and Lisa from the first year. These amounted to three lesson observations for each and in each case one of these observations was my own, whilst the remaining two were by Michael Shayer.

I tried whenever possible to conduct more informal interviews with Lisa at breaks during research team seminars or PD sessions, although I was only able to do this on a three occasions.

Although the focus of the group interviews was on collaboration, a secondary motive was to involve Lisa in interview. Indeed, she was very keen to participate in these interviews in contrast to her apparent reluctance to be interviewed individually. These group interviews were not equivalent in depth to individual interviews. However, they covered similar topics to the individual interviews.

Finally, towards the end of the fourth year, I asked Lisa to complete a questionnaire covering her activity in the project. I then discussed her responses informally with her.

I discuss the strategies that I used to analyse Lisa's participation in Section 6.2 below.

5.5 Summary of Data Collection Methods and Purposes

In Table 3.2, I summarise the data collections methods and purposes as discussed in this section.

Method	*Purposes*
Observations:	
Research team seminars	Tracking the teachers' professional change
Phase 2 PD sessions	Mathematical knowledge Questioning approaches Teachers' development as tutors
NNS sessions	Contrast and comparison with Primary CAME Questioning approaches Generating prompts for interviews
Interviews:	
Interviews with focus teachers	Tracking the teachers' professional change
Group interviews	Collaboration Tracking Lisa's professional change
Mathematics interviews	Mathematical knowledge
Informal interviews	Tracking the teachers' professional change
Other data:	
Lesson observations, Phase 2 school visits, teachers' taped reflections of Phase 2 visits; interviews with other participants, fieldnotes, project memos, TM lesson materials	Triangulation Generating prompts for interviews
Data collected by others	Triangulation Tracking Henrietta's and Lisa's professional change

Table 3.2: Summary of data collected and purposes

6. Methods of Data Analysis

My analysis used methods drawn from ethnographic research. My initial analysis used open coding (Strauss & Corbin, 1998). This was initially done by hand using scissors and envelopes. Thereafter, I collated excerpts of data using computer cut and. The analysis was focused on the main data set. Interviews were coded line by line, whilst observations and fieldnotes were more selectively coded using summaries to select excerpts that I felt to be important. This initial open coding produced a large set of descriptive codes (Strauss & Corbin, 1998). They included categories such as team teaching, reflection and questioning, key elements of TM lessons. Many of the codes related to issues raised by the participants themselves such as closure, confidence and struggle and challenge.

I used a number of different techniques in order to develop this open coding into a more coherent analysis. I wrote memos describing these codes and my early analysis of the data. For example, the notion of mathematics without closure, which I use to describe the CAME approach to school mathematics, began life as a memo. Other memos covered issues drawn from the wider literature, such as participation, that cut across codes.

Charmaz (2000) argues that a potential danger of line by line coding is that the original text becomes atomised and much of the meaning lost. Mindful of this, I referred to summaries and original transcripts throughout the analysis. I wrote vignettes that drew upon the data more holistically. These vignettes took a variety of forms. For example, on several occasions Alexandra talked about her experiences with a particular teacher. Drawing upon Kvale's (1996) technique of narrative structuring, I used these different accounts to produce a coherent story. Some of these vignettes described and interpreted individual teachers' experiences, whilst others compared the different teachers in relation to a particular aspect of Primary CAME, for example, tutoring.

The process of data collection and analysis was both concurrent and interactive. Ideas from analysis were used to frame and structure ongoing observations and interviews. In addition, I asked teachers to comment on aspects of this analysis.

Through this process of progressive focusing, I began to develop more analytic codes and constructs (Strauss & Corbin, 1998) These drew together the early descriptive codes and related them to theoretical ideas and concepts from the existing research literature, thus interpreting common issues from across the data set. The final set of these codes is attached in Appendix L together with an example of their use. As the analysis progressed, I drew on Miles and Huberman's (1984) diagrammatic techniques in order to interpret the ways in which these key analytic constructs inter-relate and interact. I attach examples of these formative diagrams together with explanatory memos in Appendix M.

6.1 Rigour in The Research Process

It is important to stress that the process of data analysis is complex and difficult, involving "risk, uncertainty and discomfort" (Ball, 1990, p.157). The interpretative nature of qualitative research makes the need for rigour important, an issue that Ball discusses in depth. My initial idea was to provide rigour by triangulation comparing multiple sources of data (Ball, 1990). However, this proved to be more problematic than I anticipated. Comparison between and within individual sources of data produced ambiguous and at times contradictory results. However, as the analysis developed I began to see this ambiguity as a strength rather than a weakness. As Denzin and Lincoln argue,

> The combination of multiple methodological practices, empirical materials, perspectives and observers in a single study is best understood, then, as a strategy that adds rigor [sic], breadth, complexity, richness and depth to any inquiry. (Denzin & Lincoln, 2000, p. 5)

Indeed, it was through considering disagreements, dissonances and contradictions that I generated more useful and powerful ideas and constructs.

Nevertheless the need for rigour in qualitative data is of paramount importance. I have already noted several strategies that I used: the writing of a reflexive research biography; the checking and re-coding of transcripts and fieldnotes; and, the use of respondent validation by feeding back ideas from analysis into the data collection process. In addition, through student support networks at King's, I asked other doctoral students, to code excerpts of my data and to comment on my own interpretations.

6.2 Lisa: Conflicting Evidence

A further strategy was the identification and consideration of what Silverman (2000) calls deviant cases, which appear to negate or contradict the researcher's interpretations or analysis. In this regard, Lisa was of particular interest. In contrast to Alexandra and Ursula, the two other teachers involved throughout the research, Lisa's engagement in Primary CAME and her professional change appeared to be very much less. At times she appeared less interested in the project than any of the other teachers. There were two further specific issues. Firstly, as I discussed above, I could not interview her after the first year. Hence, I took steps to counter this as outlined in Section 5.4 above. As a result, I had very much less individual interview data in relation to Lisa, although taken as a whole my data on her change was still considerable.

Secondly, and much more significantly, was that in several instances, Lisa's account of events appeared to completely contradict evidence from other sources. This was in the context of what I expected to be relatively uncontested 'factual' data. For example, Lisa claimed on several occasions to have visited the Phase 2 teacher at Cheston in the afternoon following a visit to Roseway, which I had observed. She had recorded on a reflection tape: "Tutor visit to Cheston school. I am observing in Tom's classroom" (Taped reflection, November 1998). However, no other comments were recorded on the tape. At the subsequent research team meeting, I asked her about the Cheston visit and she responded: "It was good. Yes, it went well" (Research team, December 1998). However, Rhoda later confirmed that Lisa had actually only claimed supply cover for the Roseway visit, and it was, therefore, extremely unlikely that she had visited Cheston. My judgement is that Lisa did not visit Cheston, although, in terms of my analysis as a whole, this specific judgement is relatively unimportant. What is important, however, is that Lisa's activity was a very contested issue within the research team. In dealing with this conflict over Lisa's activity, I used triangulation techniques drawn from Miles and Huberman's (1984) approach and compared a number of data sources in order to highlight the areas of conflict.

7. Summary

In this chapter I have outlined the methods of data collection and analysis that I used in this research. The overall research question is as follows:

> In what ways do primary teachers' beliefs and knowledge change and develop as they undergo professional development in school mathematics?

As I discussed in Chapter 1, I approached this question from an epistemological perspective of knowledge as a social practice and, hence, both contingent and socially negotiated.

I used a qualitative methodology drawing on both ethnographic approaches (e.g., Charmaz, 2000) and social constructivist approaches (e.g., Kvale, 1996). Drawing on the approach of Ball (1990), I took the role of a participant observer. I collected data in a variety of ways. The principal sources of data were through observation and semi-structured interviews as follows:

- observations of research team meetings
- observations of Phase 2 professional development sessions
- observations of NNS INSET sessions
- individual interviews
- group interviews
- interviews probing the teachers' mathematical knowledge

In addition to generally tracking and exploring the teachers' professional change, these multiple data sources were used for a variety of purposes including triangulation, generating prompts and investigating specific issues. These purposes are summarised in Table 3.2 above. I used the different perspectives provided by these multiple sources of data as one way of ensuring rigour in the research process (Denzin & Lincoln, 2000).

Initially, the data was analysed through open coding methods (Strauss & Corbin, 1998). As the research progressed, I developed a more coherent analysis through writing memos and vignettes, using narrative methods, drawing on both the data and the wider research literature (Kvale, 1996).

Chapter 4: The Learning Environment: Understanding Teacher Change

1. Introduction

In this chapter I explore how the teachers experienced Primary CAME as a learning environment and use this context to develop a theoretical approach to understanding teacher change.

I address two of the research questions identified in Chapter 1: the role of the teachers' wider professional and social networks in their professional change; and the issue of differential change amongst the teachers. Specifically, I discuss the extent to which the common setting of Primary CAME was different for the teachers and, thus, explore how a teacher's potential for change can be understood in social terms. I begin to examine the role of motivation, a question that I raised in Chapter 2. All these questions are also addressed in later chapters, in particular Chapter 5.

I largely focus on the four original teachers, Alexandra, Henrietta, Lisa and Ursula. I emphasise my concern here is with the early stages of the project and the teachers' initial experiences. In later chapters, I take a more holistic view on the teachers' change. A central purpose of this chapter is to describe and analyse the teachers' social setting as a platform on which to base these later discussions. To do this I use and develop theories of situated learning in order to understand the teachers' experiences.

The chapter is structured as follows:

- In Section 2, I discuss the theoretical approaches on which I draw, and, in doing so highlight a number of further questions.
- In Section 3, I focus on the formation of the research team, exploring it as a *potential* learning environment for the teachers. In this section, I particularly highlight the contrast between the teachers from the two initial schools: Beechmount and Parkway.

- In Section 4, I consider the teachers' professional development, particularly the teaching and tutoring by the academics.
- In Section 5, I explore the teachers' learning through an exploration of their early engagement with and interpretations of the lesson development process and more briefly the Phase 2 tutoring. In this section, I draw more of a contrast between the individual teachers, in particular between the two Beechmount teachers: Henrietta and Lisa.
- In Section 6, I very briefly extend the analysis to Janice and Tony.
- In Section 7, I discuss and reflect upon the approach developed in this chapter.

2. Communities, Practices and Teachers' Resources

Although many commentators and researchers acknowledge differential professional change amongst teachers, there is, as I noted in Chapter 1, little research that seeks to explain such differential change in the context of mathematics education. Cooney's work (1994a) is a notable exception. However, Cooney's approach is focused on secondary pre-service teachers and his explanation is focused almost exclusively on teachers' individual relationships to authority in mathematics. Notions of authority are certainly important in teacher change. Indeed, I have already noted this in Chapter 1, Section 6 in explicating my own beliefs about mathematics teaching. However, Cooney's approach, although recognising the importance of context, is somewhat limited in the range of social contexts considered (principally the classroom and pre-service education). I consider Cooney's position further in Section 2.2 below.

Other researchers, whilst their analyses are limited, do posit explanations for differential change amongst mathematics teachers. Largely these explanations point to individual and personal factors, including, for example, motivation and the difficulty of sustaining the process of change (e.g., Stocks & Schofield, 1996), individual capacity (e.g., Earl et al., 2000) and teachers' discomfort with the uncertainties of change (e.g., Goldsmith & Schifter, 1997). These explanations certainly have some merit. However, they are, like Cooney's approach, focused on individual factors. In this chapter and Chapter 5, I develop an analysis that locates these individual factors, or personal resources, within a social context. In the current chapter, my focus is largely on motivation, individual capacity and beliefs about authority in mathematics. In Chapter 5, I extend my analysis of these issues and additionally consider the issues of coping with uncertainty and sustaining change.

2.1 Teachers' Social and Professional Networks: A Situated Perspective

A number of approaches point to the importance of teachers' social and professional networks (e.g., Nelson, 1996). Thompson et al. (1994), for example, highlight the difficulty of implementing innovative practice within school environments that are not themselves innovative. Spillane (1999) focuses on teachers' professional networks, which he terms *zones of enactment*, in analysing differential change amongst elementary teachers implementing an innovative approach to mathematics teaching. He highlights the importance of the breadth of teachers' zones of enactment together with teachers' involvement in "rich deliberations about the substance ... a practising of reform ideas with other teachers and reform experts includ[ing] material resources or artefacts that support [these] deliberations" (p.171). However, Spillane's analysis, whilst useful in highlighting how teachers' professional networks mediate policy initiatives, is almost wholly concerned with the teachers who changed. Moreover, his concern is at a policy level and his approach, as it stands, appears to lack the fine tuning necessary to explain differences between the individual teachers in my fieldwork. For example, as I have already noted, despite an unusually favourable and extended PD setting in which they had access to both rich deliberations *and* teaching resources supporting these deliberations, only two of the six teachers in my study appeared to change significantly.

In this chapter I develop an approach which, like Spillane's, relates teachers' professional change to their wider professional networks, but which overcomes the difficulties I have identified. To do this I draw on the situated approach of Lave and Wenger (1991). (See also Boaler, 1997, 2000a, 2000b; Greeno, 1998; Lave, 1996; Wenger, 1998.) The strength of the situated approach is that it recasts teachers' personal resources, motivation and individual capacity, for example, in social terms. However, there are few studies that explore either the applicability of this approach to teacher education in general or to mathematics education in particular. In order to address this problem, Putnam and Borko (2000), in a review article on teacher education in general that draws heavily on examples from mathematics teacher education, re-examine existing approaches to teacher education from a situated perspective. However, their review highlights a need for further empirical research informed by situated theories in order to further develop situated understandings of teacher change. One study which does consider mathematics teacher education from a situated perspective is that of Stein et al. (1998). They demonstrate how Lave and Wenger's approach can usefully be used to describe the process of teacher change in a middle school mathematics department, although their account, directed towards demonstrating the potential benefits of Lave and Wenger's ideas, is somewhat uncritical in its application of the theory. This is particularly so in relation to their treatment of the differential change of individuals in that they analyse different teachers' learning in terms of what appear to be largely homogenous and uniform learning trajectories. In this and subsequent chapters, I take a more critical approach to situated learning drawing on a number of other approaches in addition to that of Lave and Wenger in order to develop a more comprehensive theoretical approach.

The notions of participation and enculturation are fundamental to the situated perspective. In many senses the central ideas of situated learning stand in direct contrast to the metaphor of acquisition in teaching and learning (Boaler, 2000b; Kirshner, 2002; Lave, 1997; Sfard, 1998). Learning is seen as a process of enculturation and refining practice rather than one of acquiring knowledge. However, the notion of situated learning implies more than that learning takes place in contexts:

> In our view, learning is not merely situated in practice – as if it were some independently reifiable process that just happened to be located somewhere: learning is an integral part of generative social practice in the lived-in world. (Lave & Wenger, 1991, p.35)

Central to situated theory is the notion of a community of practice, which I have already discussed briefly in Chapter 1. Social practices are set within communities of practice (Lave & Wenger, 1991; Wenger, 1998). Communities are centred around shared practices, shared understandings, shared discourse, joint enterprise and mutual engagement (Wenger, 1998). As I noted in Chapter 1, despite the focus on shared practices, a community of practice is not a homogeneous group. Participants may not only have diverse roles and differing engagement with the practices of the community but are likely also to have different imagined futures and may have different fundamental interests. Lampert (1998), for example, argues that a significant problem in educational research is that teachers and education academics belong to different communities with very limited shared discourses. Hence, any community involving teachers and academics, like the CAME research team in my study, is likely to face challenges in terms of communication and developing shared understandings.

In their early work, Lave and Wenger (1991) focus on the ways in which individuals through participation 'learn' to become full participants in a community of practice. They conceive of learning as a process of developing an identity within a community of practice and draw on notions of learning as apprenticeship. They refer to *old-timers* and *newcomers* in preference to teachers and learners. Newcomers learn, for example, by telling and re-telling significant stories, by working alongside more experienced old-timers, by tackling modified but nevertheless *authentic* tasks, and through old timers (often indirectly) drawing their attention to significant aspects of their practice. A central theme with Lave and Wenger's work is the rejection of the notion of transfer in education. (See, in particular, Lave, 1988.) Boaler (1997), in a study of school mathematics, extends this analysis by focusing on the *reformation* of practices and the extent to which pupils can *adapt* school mathematics practices in new situations.

Lave and Wenger use the term *legitimate peripheral participation* to describe the ways in which old-timers modify practices to enable newcomers to participate in the activities of the community of practice. Hence, the focus is not simply on old-timers' didactic explanations of practices (although this is certainly one way in which practices are modified). Rather, their analysis highlights the multiplicity of ways in which old-timers help and guide the participation of newcomers in order to give "an approximation of full participation" (Wenger, 1998, p.100). Wenger defines learning for individuals more widely as "engaging in and contributing to the practices of their communities" (1998, p.7). He introduces the notion of *trajectories of participation* to describe and analyse different individual histories of learning and practice. Trajectories, he argues, include both the inbound trajectories of apprenticeship leading to full participation and peripheral trajectories which "by choice or necessity ... never lead to full participation" (1998, p.154). Despite Wenger's recognition of different participation patterns, his work is theoretical and lacking in empirical foundation. Moreover, his analysis is largely focused on positive paradigmatic trajectories of participation, whether or not these lead to full participation. Indeed, although he is critical of formal education and schooling, his discussion in this area is largely aspirational and speculative. As I indicated above in my discussion of Stein et al.'s (1998) work, there is a need to develop a more in-depth and empirically founded understanding of differential change amongst teachers.

In their early work, Lave and Wenger (1991) focus their analysis on participation within single communities. However, a community of practice does not stand in isolation. Indeed, individuals participate and develop identities in many different communities which make up "a complex social landscape of shared practices, boundaries, peripheries, overlaps, connections and encounters" (Wenger, 1998, p.118). The teachers in my study, for example, participated in both professional and personal communities. As generalist teachers, they were involved in far more than mathematics teaching. They had personal lives with friends and families as well as lives in school and beyond. Whilst individuals may behave and think differently in these different communities, Wenger (1998) argues that an individual's identities within different communities, whilst distinct, are not wholly separate:

> An identity is thus more than just a single trajectory; instead, it should be viewed as a nexus of multimembership. As such a nexus, identity is not a unity but neither is it simply fragmented. ... Considering a person as having multiple identities would miss all the subtle ways in which our various forms of participation, no matter how distinct, can interact, influence each other and require co-ordination. (p.159)

Confronted by tensions between the different aspects of their identities, individuals are compelled to negotiate and reconcile these different forms of participation and meaning in order to construct an identity that encompasses the membership of different communities. This process of identity reconciliation is central to an individual's ability to make connections and transfer meaning and knowledge between practices. Hence, continuity and discontinuity are central to learning across differing practices. Learning new practices is likely to be "easier" if the new practices are similar to an individual's existing practices, whilst conversely it is likely to be more difficult to adapt very different practices. Boaler (2000a), for example, uses the idea of difference to examine why some pupils appear not to be able to transfer school mathematics into real world situations. She argues that the differences between the practices of some mathematics classrooms and the mathematics encountered outside school are in some cases so significant as to make the mathematics learned in class of no use outside school. I find these notions of similarity and difference to be useful. However, other researchers suggest that these issues of similarity and difference are complex. Evans (2000) argues that difference forms a crucial role in enabling change in that for change to take place teachers need to recognise *how* new practices are different. Moreover, Spillane's (1999) analysis in focusing on the importance of rich deliberations appears to suggest that similarities in the *form* of practices may be more important than similarities in the content. Commenting on Boaler's (1997) discussion of transfer as adaptation or reformation, Evans (2000) also highlights this issue of form and argues that transfer is better conceived as *translation* or *transformation.* His metaphor of translation is useful in highlighting that the translation, or transfer, of practices between different settings may be either strict or free and the "transferred" practices themselves changed in the process of learning. I discuss this issue of the form versus content in Chapter 5.

Situated learning theories have been the subject of many forceful critiques and equally forceful defences. (See, e.g., the recent debate in the journal Educational Researcher: Anderson, Greeno, Reder, & Simon, 2000; Anderson, Reder, & Simon, 1996, 1997; Cobb & Bowers, 1999; Greeno, 1997; Kirshner & Whitson, 1998). Anderson, Reder and Simon (1996), for example, criticise the rejection of the notion of transfer and cite examples of situations where transfer has been shown to take place. However, Greeno (1997) argues this is based on a misunderstanding of the situated position. Situated theorists, he argues, do not reject the idea that transfer as such takes place; rather, they reject transfer as a tool for analysing the process of learning. Boaler (2000b) argues that focusing on transfer is necessarily to consider knowledge as separate to the social settings in which it is "learned":

> Situated theorists do not imply that knowledge is not transferable, nor that all teaching should take place in "complex, social environments." What is fundamental to the situated perspective is an idea that knowledge is co-produced in settings, and is not the preserve of individual minds. Situated perspectives suggest that when people develop and use knowledge, they do so through their interactions with broader social systems. ... The different activities in which learners engage co-produce their knowledge, so that when students learn algorithms through the manipulation of abstract procedures, they do not only learn the algorithms, they learn a particular set of practices and associated beliefs. ... It is inadequate to focus on knowledge alone, outside of the practices of its production and use." (p. 3)

Thus, an understanding of transfer is integrated within the broader social analysis in the situated perspective. Nevertheless, this process of transfer is complex and not well understood (Lerman, 2000). This is particularly so in relation to teacher education (Putnam & Borko, 2000). In this and the next chapter I use notions of similarity and difference to explore whether and in what ways the teachers were able to adapt and modify their existing practices, and thus the extent to which they were able to "transfer" aspects of their knowledge.

Lerman (2000), in a sympathetic critique of situated learning, argues that Lave's theory, in particular, does not sufficiently account for practice as a form of social regulation. Wenger (1998), in his more recent work, goes some way towards addressing these issues emphasising that membership of a community "is not necessarily a positive, elevating, or empowering process" (p.297). Nevertheless, Lerman's critique has merit and in Chapter 5 I draw on theories of discourse, culture and identity that more adequately account for discourse and practice as forms of social regulation (e.g., Gee, 1999; Gee, Hull, & Lankshear, 1996; Hall, 1996; Holland et al., 1998).

In summary, in this section I have given a brief outline of the situated approach to learning. I have noted that there are few studies that examine mathematics teacher education from a situated perspective. I have also highlighted several areas where this approach requires extension or further explication:

- understanding of differential change amongst participants
- understanding the issue of transfer
- a need to more adequately locate learning within a wider social and cultural setting (and to account for issues of power and authority in mathematics)

I have further suggested that exploring similarity and difference and a focus on the form of practices are potentially useful analytic approaches.

2.2 Authority in Mathematics Education

As I noted above, Cooney (1994a) focuses on authority as a tool for conceptualising teacher change. He draws on the work of Perry (1970) and Belenky et al. (1986) (itself a reaction to Perry's work) in order to conceptualise the ways in which mathematics teachers position themselves in relation to the validation and construction of mathematical knowledge. He focuses on the extent to which teachers see authority in mathematics as dependent on context and self. Belenky et al. (1986) conceive of five different ways of knowing. However, Povey (1997) argues that these are not all applicable to mathematics teachers. Drawing on this earlier work, Povey, Burton, Angier, and Boylan (1999) simplify Belenky et al.'s approach to three positions: *silence*, *external authority* and *author/ity*.

Silence is a position in which teachers perceive themselves as powerless and cut off from mathematics similar to the feelings of extreme anxiety described in Buxton's (1981) work. In this position teachers "do not see themselves as developing, acting, learning, planning or choosing (Povey et al., 1999, p. 233). Although Povey et al. regard this way of knowing as less likely for secondary mathematics teachers, Bibby

(1999) has shown this alienation from mathematics to be found amongst primary teachers.

External authority is a way of knowing where teachers perceive knowledge to be validated by experts and as fixed and absolute. In this position teachers are "deeply dependent on others, especially authoritative others" (Povey et al., 1999, p. 234). Cooney and Shealey (1997) refer to this as received knowing and relate this to Ernest's (1991) absolutist beliefs about mathematics.

Author/ity is a position where teachers understand mathematical knowledge as negotiated and constructed with others. External sources of authority, textbooks or expert mathematicians, for example, are critically evaluated. Povey (1997) relates this position explicitly to authorship in mathematics as follows:

> Author/ity links back together two words that have a common root, but which have come be read very differently from each other. An author is one who brings things into being, who is the originator of any action or state of things. Authority is linked with power and the validity of knowledge. Linked together they lead to the construction of an epistemology which recognises each of us as the originator of knowledge. (p. 332)

Cooney and Shealy (1997) refer to this as connected knowing and relate this to Ernest's (1991) fallibilist beliefs about mathematics.

In this chapter, I locate the teachers' beliefs about authority in terms of their social practices and explore the potential for developing author/ity at the outset of the project.

2.3 Motivation

As I have already noted above, one possible explanation for the teachers' differential change is motivation. Although many commentators point to the importance of teachers themselves recognising the need to change (e.g., Clarke, 1994; Stocks & Schofield, 1996), Goldsmith and Schifter (1997) argue that motivation is a particularly neglected issue within the fields of research and practice in mathematics teacher education. There is, moreover, little research within mathematics education more generally that seeks to understand and theorise motivation (McLeod, 1992; Middleton & Spanias, 1999). Often, where motivation is considered, it is treated somewhat simplistically in terms of individual factors or external rewards (e.g., Earl et al., 2000). Drawing on the work of Ames (1992) and Dweck (1986), Middleton and Spanias (1999) describe this performance and reward orientation as *extrinsic* motivation and contrast this to *intrinsic* motivation. Intrinsic motivation is focused on enjoyment of learning and conceptual understanding. Middleton and Spanias (1999) argue that theoretical studies of motivation in mathematics education tend to treat motivation as a given and unchanging individual factor and do not explore why or how individuals are motivated, how motivation changes over time, or how motivation can be integrated within social theories of learning. Stein et al. (1998) suggest that situated theories offer the possibility for locating motivation in social terms, although the detail of their analysis of motivation is undeveloped. In this chapter I draw on situated theories to develop a theoretical understanding of teacher motivation in terms of practices. In Chapter 5, I extend the understanding developed here by drawing on related theories of identity. Throughout, my focus is on intrinsic motivation.

3. The Formation of The Research Team

In this section I focus on the four original teachers. I analyse the formation of the research team as a learning environment. I use the notion of communities of practice to explore the social networks within which the teachers were located and, thus, analyse the implications for professional change of differences in the social networks of the four teachers at the outset of the project. In particular, I examine the overlaps and interconnections between the teachers' existing practices and the "new" practices of Primary CAME and the ways in which the teachers perceived these practices using ideas of difference and similarity identified in Section 2 above.

The research team was embedded within a rich and diverse set of practices and the participants themselves were members of wider discourse communities, including primary teaching, mathematics education and academia. The research team was itself a far from homogenous group. Although the research team had just formed, it already had several 'old-timers' or full participants. The academics, David, Michael and Mundher, had developed the CAME secondary materials and took primary responsibility for the teachers' professional development. Rhoda was less familiar with the CAME approach, although she had attended several sessions on CAME as part of her MA at King's. Indeed, her enthusiasm for the approach was a factor in the choice of Outertown as the LEA partner for Primary CAME. More importantly, however, Rhoda was the LEA link for the project and had the central role of identifying the initial four teachers. Indeed, although Rhoda wrote materials, trialled lessons and undertook tutoring, thus taking a teacher-research role, her participation (like David's as project director) was more administrative and financial. As a result, she can be seen as a full participant from the project's inception, although, looked at strictly in terms of the CAME approach, she was a newcomer. Moreover, two of the academics and CAME old-timers, Michael and Mundher, were newcomers to primary education.

Throughout the project's development, participation was very different for the teachers and the academics, a difference that in part reflects their membership of the wider discourse communities of primary teaching and academia and the divergent interests of these groups which I have already noted that Lampert (1998) highlights. This was reflected in the division of labour in the production of lesson materials, with the teachers writing the teaching notes and the academics focusing on the more theoretical background notes.

3.1 The Research Team as an Envisaged Community of Practice

As I have noted above, Rhoda, as the LEA contact, identified the initial group of four teacher-researchers. These negotiations took place during the Summer term 1997. Rather than directly choosing interested teachers, she first identified the two Phase 1 schools. This process was informed both by pragmatic concerns, such as the proximity of the schools to the Outertown Teachers' Centre, by her own ideas of good practice in teacher education and by the CAME approach to professional development. Rhoda's primary aim at this point was to identify schools that would provide a supportive environment for the research and for the four teachers' professional development. As I noted in Chapter 2, in the CAME secondary model mathematics departments are seen as key sites for teacher PD through discussion, sharing and reflection. In transferring this approach to primary, the academics together with Rhoda identified a supportive Headteacher, a strong Mathematics Co-ordinator, and an otherwise supportive environment, which would facilitate the sharing of the approach beyond those directly involved.

Beechmount and Parkway, the two schools chosen had both previously been involved in local mathematics initiatives; both had Headteachers that Rhoda judged would be strongly supportive of the research; each had a strong Mathematics Co-ordinator teaching in Y5 or Y6, the year groups Primary CAME was to focus on; and both were schools with which she had an established professional relationship that went beyond contact with the mathematics co-ordinator. In discussions with the respective Headteachers, four teachers were identified: Jenny and Lisa from Beechmount, and Alexandra and Ursula from Parkway. All were experienced teachers and all had participated in relevant extended INSET courses. Jenny and Ursula were Mathematics Co-ordinators for the respective schools and were regular participants in Rhoda's termly mathematics meetings. Although Lisa had not attended any extended INSET in mathematics education, she had participated in Fisher's (1998) Thinking Skills training, an issue which I discuss in Section 3.2.1 below. Alexandra had attended extended INSET in mathematics education, although her particular expertise was in language education. She was, at the time of the project's inception, undertaking a Diploma in Reading in which she engaged with theories of cognitive development, in particular those of Vygotsky.

Rhoda's intention was to choose teachers with wide professional networks. However, much of the discussion in this chapter relates to how, in comparison to Alexandra and Ursula, Henrietta's and particularly Lisa's professional networks were limited *in relation to* CAME. I emphasise, therefore, that Rhoda considered Lisa's experiences as both broad and relevant, as she commented later:

> I [hadn't] worked with [Lisa] previously, but [Paul, her Headteacher] was very impressed with Lisa's work in terms of developing thinking. I mean, she's ... done all this other stuff, hasn't she. (Interview, July 1999)

Lisa had "done all this other stuff." She had not only taken part in the Thinking Skills, but was, in fact, involved in a wide range of activities. She was attending management and headteacher training and was Acting Deputy when she joined the project. As the schools' art co-ordinator she attended the borough-wide co-ordinator meetings. She had taught in several schools and LEAs and, moreover, had taught across the primary, secondary and tertiary education sectors. Hence, viewed in a general sense out of the specific context of CAME, Lisa's professional network was in many ways broad.

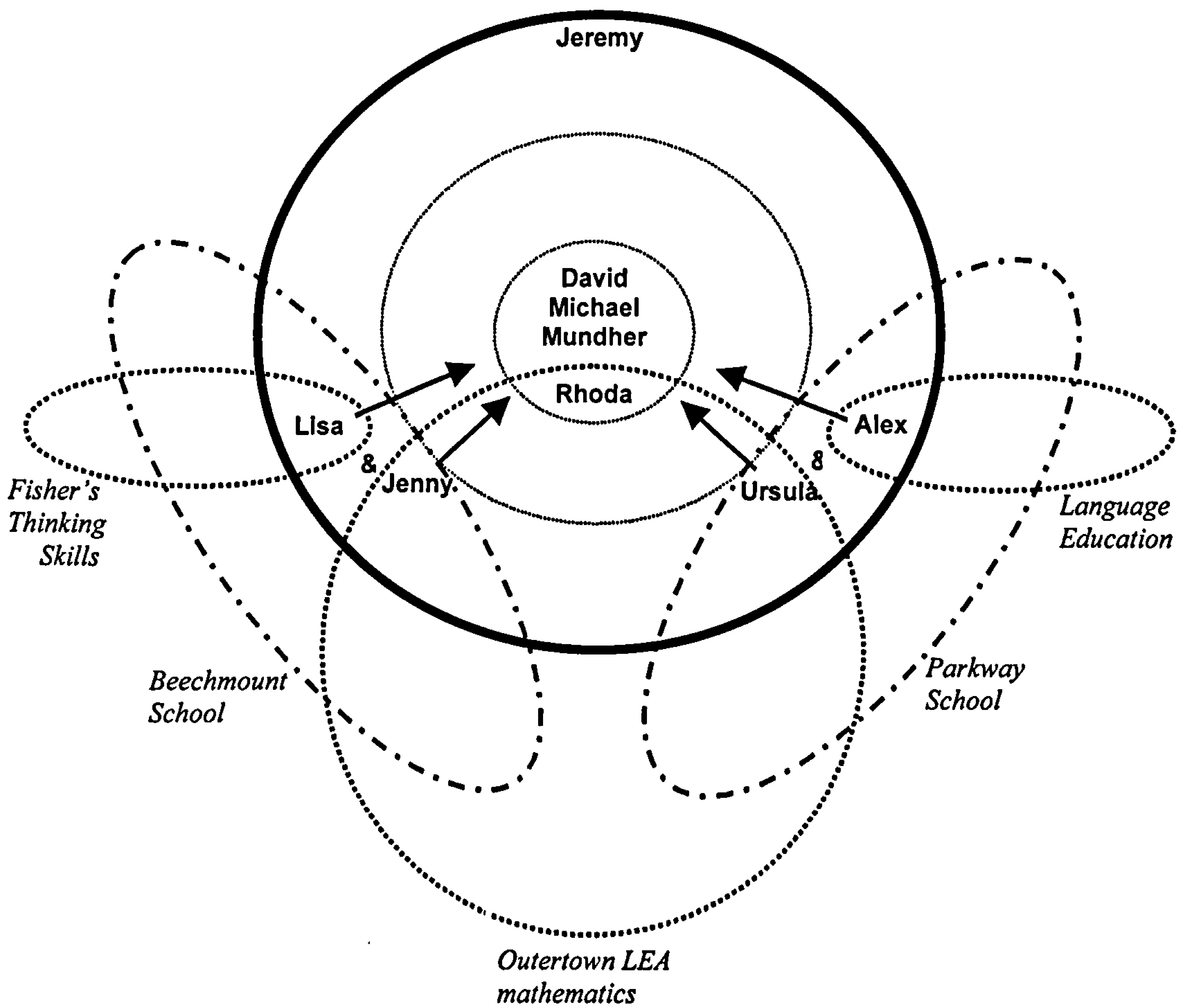


Figure 4.1: The research team as an anticipated community of practice

In Figure 4.1, I illustrate my interpretation of how the research team was anticipated by Rhoda and the academics prior to its actual formation in October 1997. David, Michael, Mundher and Rhoda are shown as full participants. I show myself at the periphery to reflect my position as participant observer. The teachers from each school are shown in collaborative school groups at the periphery of the community. Jenny and Ursula, both mathematics co-ordinators and participants within the Outertown LEA mathematics education community (and, hence, key players within their respective schools) are shown as closer to full participation. I note the rich set of overlapping practices which all of the teachers might draw on in making sense of, and engaging with, CAME. There is, moreover, a symmetry between the schools and the pairs of teachers within the school with the mathematics specialists balanced by the different but relevant specialisms of thinking skills and language education respectively. As a result, all four of the teachers were anticipated to be on inbound learning trajectories towards full participation as CAME primary teacher-researchers as indicated by the arrows (Wenger, 1998).

3.2 The Research Team as an Actual Community of Practice

The actual research team was, however, somewhat different to this plan. Despite Rhoda's primary aim of choosing schools that would facilitate and support the teachers' involvement in the project, there were significant differences between the two schools. These were emphasised by differences between the teachers' own practices within the schools. These were further compounded by events at Beechmount. Jenny was not able to become involved in the project, partly because the Beechmount Headteacher left and she became Acting Headteacher and partly because she was due to be on maternity leave for a large part of the first year. Jenny was replaced by Henrietta, the remaining Y5 teacher at Beechmount, although she too later went on maternity leave and left both the project and teaching.

In order to analyse the actual research team, I first consider differences between the teachers and differences between the mathematics education at the two schools.

3.2.1 Differences Between The Beechmount and Parkway Teachers

There were considerable differences between the four teachers, and in particular between the two school groups: Alexandra and Ursula at Parkway, and Henrietta and Lisa at Beechmount.

Ursula had not only attended a 20 days GEST-funded mathematics course, she had participated in Rhoda's local BEAM Education writing group and had written mathematics materials with other Parkway teachers. Indeed, in her role as Parkway's mathematics co-ordinator, she had been praised in the school's 1997 OfSTED report for the support that she gave other teachers in terms of investigative teaching. Whilst Alexandra had not originally been identified as having a mathematics specialism, she had nevertheless attended extended PD in mathematics education run by Rhoda. In addition, she had jointly planned a mathematics week at Parkway with Ursula. In contrast, neither Lisa nor Henrietta had attended any mathematics INSET outside their school. Indeed, Lisa had no academic qualification in mathematics having qualified before this became necessary. Henrietta, moreover, a relatively inexperienced teacher in her third year of teaching, had not undertaken any extended INSET subsequent to her initial training.

In contrast to the other teachers, Henrietta appeared to be in what Huberman (1989) terms the survival phase of her professional career in that she exhibited an intense concern with issues of classroom management. Indeed, as I discuss in Chapter 5, she perceived a conflict between the CAME approach to discussion and her own desire for classroom control.

Alexandra and Ursula were both teachers that Rhoda knew well from their contacts with the Outertown Advisory Service. Alexandra had previously led INSET for the LEA. In addition, the three had a personal relationship from their joint visits to a local gym. In contrast, Rhoda had not worked with either Henrietta or Lisa. Although Lisa had undertaken extended PD organised through the advisory service and had taught in several Outertown schools, she was not well known within the advisory service.

As I noted above, Lisa had already been involved in a thinking skills programme, which Rhoda had anticipated would have similarities with the CAME approach. However, despite their surface similarities, the two approaches were quite distinct and different. The thinking skills programme had been organised Outertown-wide by Robert Fisher from Brunel University and was directed at teaching philosophy to children through discussion (Fisher, 1998). Fisher's Thinking Skills shares much of its key terminology and jargon with CAME. Both approaches stress classroom culture, the sharing of ideas and reflection. However, as Grimmett (1988) notes in relation to reflection, a common terminology does not imply shared meaning. Indeed, Thinking Skills bases its approach on a model of Socratic questioning and is more akin to Bruner's notion of the scaffolding of children's learning by knowledgeable *adults* (Bliss, Askew, & Macrae, 1996) than to CAME's interpretation of Vygotsky and its focus on the mediation of learning by *peers*. (See Chapter 2, Section 4.1, for a discussion of the CAME approach to teaching and learning.) A further difference between the two approaches relates to the stress both place on multiple methods, which in part reflect differences between the disciplines of mathematics and philosophy. Set within the terrain of philosophy broadly, Thinking Skills places considerable emphasis on children recognising the different perspectives, values and views of others and on developing "divergent thinking". Whilst different perspectives are important within CAME, these form the basis for the identification of mathematical connections and commonalities rather than simply the recognition of different perspectives. Indeed, as I discuss in Chapter 6, Lisa appeared to interpret CAME through the lens of her previous thinking skills work and, thus, failed to perceive fundamental differences between the two approaches. I note that the two approaches are certainly not completely incompatible. However, similar terminology is used to describe quite distinct teaching approaches.

In contrast to the Thinking Skills programme, Alexandra's Diploma in Language Education was broader and more generic. She was able to draw on this experience to make sense of the classroom discussions within TM lessons and in particular to engage with aspects of what Yackel and Cobb (1996) term socio-mathematical norms: for example, mathematical difference, mathematical definition and mathematical explanation.

A further contrast between the teachers lay in the differing relationships of the pairs from the two schools. Alexandra and Ursula's relationship pre-dated CAME, having begun when Alexandra had been Ursula's mentor in her first year of teaching. They had then worked closely together for six years at Parkway and had planned and team taught parallel year group classes. They presented their relationship as an intense collaboration that was "more than a professional relationship" and often talked of "finishing each other's sentences." For example, Ursula described their first year of working together as follows:

> We began doing quite a lot of wandering in and out of each other's classrooms as well, that year. And I think that year we used to just leave our class and wander through and have a chat and have a joke with the other class and wander back again. (Joint Interview, May 2000)

This was not only a relaxed relationship, but also one in which interruptions were welcomed. They argued and discussed ideas:

> I mean we don't agree. I mean, Alexandra and I don't agree on everything by a long chalk. We tend to argue things out for the good of the idea, if you know what I mean. (Interview, March 1999)

The relationship was so strong that one of Alexandra's formative experiences in mathematics teaching was the 20 days mathematics course which Ursula, rather than Alexandra herself, had attended:

> Key [in my development] as a maths teacher I think ... in a way I think before Leverhulme, not that I've done a GEST [20 days mathematics] course, but the fact that Ursula had and, you know, we had this close relationship and we, we talked things through and practice was changing here generally, I think that was quite key and I do think that Leverhulme has actually built on that. (Interview, March 1999)

As Alexandra describes, such 20 days extended mathematics courses have been found to be significant experiences for teachers who have attended them (e.g., Askew et al., 1997). However, there is little evidence to suggest that such courses have any significant effects on course participants' colleagues (Harling & Kinder, 1992). For this to be a formative experience for another teacher is very unusual and is, therefore, an indication of the closeness of Alexandra and Ursula's relationship.

Their planning was extended and lengthy and, as they describe in the following, "entwined" within other activities:

Alexandra:	this makes us sound like really sad people, but because we planned outside, it almost became a social, I don't mean a social event, 'Oh come on. Let's go for our planning.' But
Ursula:	Well it did, because we used to have lunch and we used to have a bottle of wine and
Alexandra:	Yeah, that's what I mean ... and our planning session used to take a long time, because they would be entwined with lunch and eating

(Joint Interview, May 2000)

The open-ended approach had similarities with the open-ended, extended and extensive discussion that typified the research team meetings, as I described in Chapter 2.

In contrast, although Henrietta and Lisa taught in the same school, they had not worked directly together previously, since Beechmount's organisational and planning focus was on year group teams. And, given the school's tight organisation by year groups, they had no reason to work together on anything but primary CAME. Moreover, their relationship was not of two equal peers. At the project's inception, Lisa was acting deputy head, whilst Henrietta was still a relatively new and inexperienced teacher.

3.2.2 Mathematics Education at Beechmount and Parkway Schools

There were many differences between the two schools, which I summarise in Appendix N. However, in terms of Primary CAME, the most significant of these related to the teaching and learning of mathematics.

Firstly, in terms of external measures of attainment in mathematics, Parkway was more successful than Beechmount. Over the period of the fieldwork for the years 1997-2000, the percentages of pupils at Beechmount achieving level 4 and above in

KS2 national tests were consistently significantly below average, whilst in contrast the performance of Parkway pupils was consistently well above average. For example, in 1998, the first year of the project, the figures were 43% at Beechmount compared with 77% at Parkway and a national average of 58%. The OfSTED inspection reports from 1998 for Beechmount and from 1997 for Parkway emphasise this difference further. OfSTED found that, whilst Beechmount's intake was below average in terms of mathematical achievement, by the end of KS2 children's achievement was well below average. Parkway's OfSTED inspection judged that children's achievement at intake was above average. However, by the end of KS2, their achievement was well above average. Although far from conclusive, the large difference in the mathematics attainment of pupils at the two schools does suggest that mathematics teaching at Beechmount was aimed at a lower level than that at Parkway. Indeed, on at least one occasion Lisa commented to the Phase 2 teachers that Halving and Thirding, a lesson focusing on the multiplication of fractions, was not accessible to the wide range of children (PD session, 3 Feb 00).

Secondly, at Beechmount mathematics teaching was taught in sets. Hence, although the two teachers taught their own classes for CAME, these were different groups to their regular mathematics lessons. Moreover, both teachers were accustomed to teaching mathematics to groups which they perceived to have a more limited range of attainment than their CAME classes.

Thirdly, and very significantly, mathematics teaching at the two schools was very different. Parkway had for some time made extensive use of mathematical investigations and was not heavily influenced by published schemes. Ursula, as Mathematics Co-ordinator, had recently written the school's scheme of work in mathematics, which promoted the use and adaptation of a range of published and unpublished resources. In contrast, Beechmount's mathematics teaching as a whole appeared to be very heavily dependent on the Heinemann published scheme. These differences were evident in the different teachers' responses in March 1998 to an interview question asking them to give an example of a good non-CAME mathematics lesson. Alexandra and Ursula both gave examples of investigations, each citing a lesson she had taught and *adapted* from an open-ended starting point taken from Straker (1993) and BEAM (1988), respectively. Lisa's example of a "good" mathematics lesson was an exercise she had taken *unadapted* from the Heinemann workbook. Henrietta gave an example of a cross-curricular data-handling lesson. However, the lesson appeared to be largely skill-based and focused on teaching the procedure for constructing bar charts. Indeed, in a further lesson that I observed in March 1998, the children appeared to be measuring without any purpose, a laissez-faire discovery approach to investigative work as described by Askew (1996). It seems likely that these examples of investigative work were largely for my benefit, since an examination of her children's exercise books suggested that her mathematics lessons largely consisted of the children working through the school's Heinemann scheme. Jenny, Rhoda's first choice of teacher at Beechmount, had been developing an alternative approach with some support from Rhoda. However, at the time of the project's inception, this work had yet to have a wider impact on mathematics teaching at the school. This contrast was reinforced by the schools' OfSTED inspection reports. Whereas Parkway was judged to have strong and consistent mathematics teaching with particular strengths in investigational work, Beechmount was judged to have had low expectations in mathematics and teaching that was over-reliant on a commercial scheme. Using Millett and Johnson's (1996) categories, Beechmount's mathematics teaching was largely scheme-driven, whereas Parkway's was largely low-scheme use. In particular, whilst Henrietta and Lisa's existing approaches to school mathematics appeared to be very reliant on the school's Heinemann mathematics scheme and to following through the ideas of others, Alexandra and Ursula worked in an environment where they adapted and interpreted mathematical activities.

Bowe, Ball and Gold (1992), drawing on the work of Barthes, provide a useful way of contrasting the approaches of the two schools in their description of "readerly" and "writerly" approaches to the National Curriculum. Beechmount's mathematics teaching was readerly in that the teachers' engagement with and interpretation of mathematics curriculum resources was largely limited to the acceptance or rejection of the resource. Indeed, where Henrietta attempted investigative work, the resources she could draw on in order to interpret and provide meaning for this work were very limited. In contrast, the Parkway teachers took a writerly approach to the mathematics curriculum by "join[ing] in, co-operat[ing] and co-author[ing]" the resources and materials of others (Bowe et al., 1992, p.10-11). Hence, prior to their involvement in CAME, the two Parkway teachers already had some understanding of school mathematics as constructed, at least in terms of activities, and, thus, some potential to develop Povey's (1997) sense of author/ity.

3.2.3 The Actual Research Team as a Community of Practice

Using the differences between the teachers and schools discussed above, I now turn to analyse the research team as a potential community of practices contrasting this with the anticipated community outlined in Section 3.1 above. Essentially, as I illustrate in Figure 4.2, the potential for the early participation of the four teachers was very different.

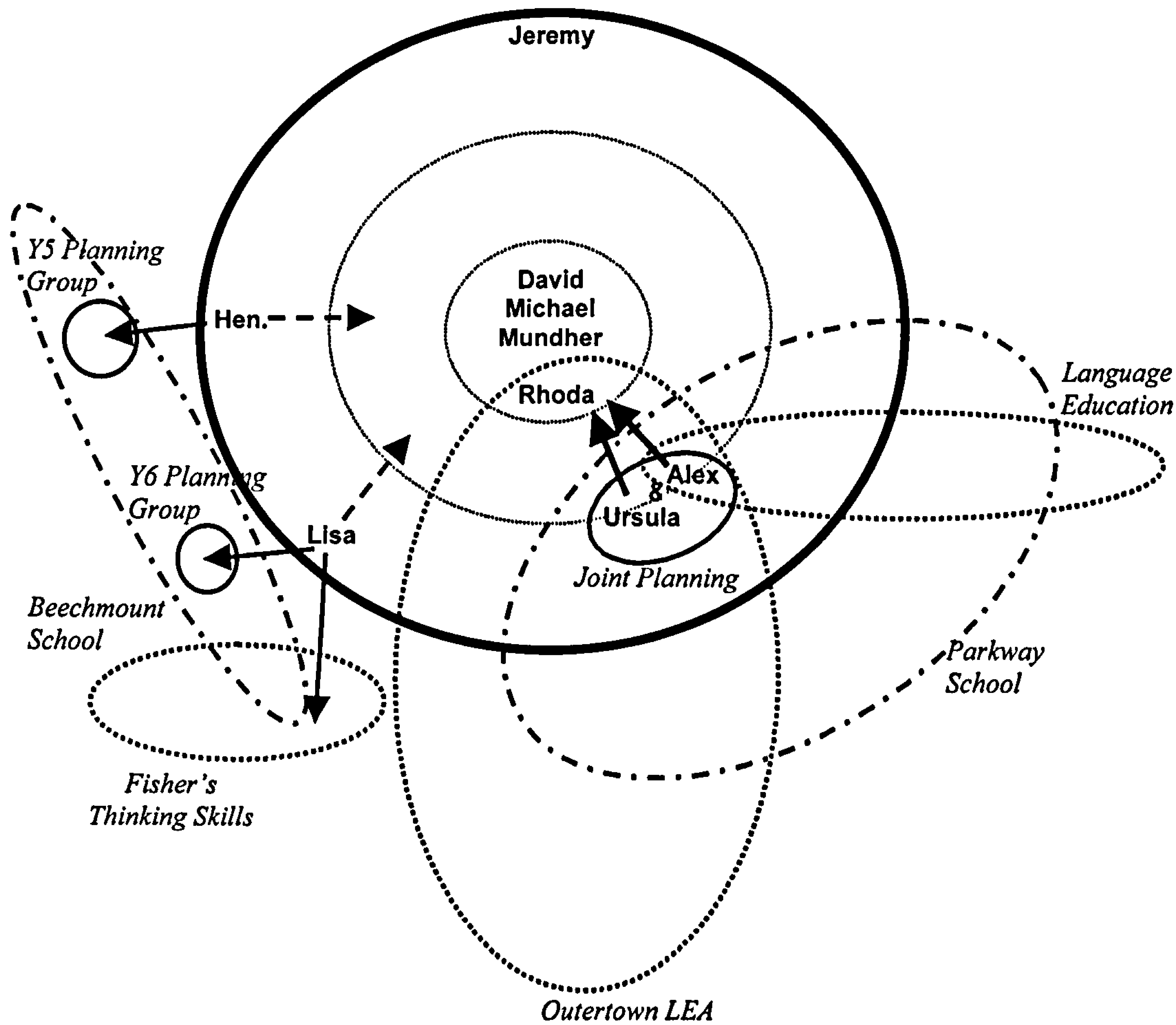


Figure 4.2: The actual practices of the research team

For Alexandra and Ursula, there were many overlaps and similarities between their existing practices within Parkway school, their own relationship, their history of joint planning and the practices of Primary CAME. Moreover, these practices extended beyond Parkway through their participation in Outertown LEA initiatives. The teachers' own relationship was close, but, more significantly, their joint experiences of open-ended lengthy planning sessions had similarities with many of the open-ended and unstructured research team planning discussions. Their existing practices in relation to mathematics teaching and learning were investigative, writerly and not scheme dependent. Alexandra and Ursula's existing practices had considerable potential to enable them not only to make sense of the new practices within CAME, but also to collaborate and interact with the other research team members. For, as Griffiths (2000) argues, collaboration is rooted in links and connections between the social practices of collaborators:

> processes of collaboration can best be understood as being based on a dense set of connections between interlocking but nonetheless discrete spaces. Setting up a collaborative project creates a new space ... but this space depends on and contributes to others. A project is not more public than any of its constituent spaces, though it draws on them. It neither subsumes them, nor is it subsumed by them. Rather it creates a complex web of interconnections. Individuals from some spaces join together with individuals from others. (p. 393)

I discuss the issue of collaboration further in Chapter 5. For the moment, however, I note simply that, for Alexandra and Ursula, their own collaborative relationship and the mathematical practices at Parkway created the potential for this dense set of connections with the new practices of CAME. Hence, although they are still peripheral participants, both the Parkway teachers were, at the time of the project's inception, each on a very definite inbound trajectory towards full participation as shown by the bold arrows on Figure 4.2.

In contrast, Henrietta and Lisa's existing practices both as teachers generally and in terms of mathematics education were very different to those of Primary CAME. Beechmount's scheme-driven and more 'readerly' and individualistic approach to the mathematics curriculum did not prepare them for lesson development. Thus, like

Boaler's (2000a) students, the mathematics Henrietta and Lisa taught at school, and the ways in which they taught it, were of limited use to them in engaging with and making sense of CAME. The two teachers had not worked together previously and worked in separate year group planning teams. Moreover, Lisa's "relevant" specialism, her previous Thinking Skills work, despite surface similarities, was based on a very different approach to that of CAME. Thus, the interconnections between the Beechmount teachers' existing practices and those of CAME were at best sparse, providing limited potential for collaboration. Henrietta and Lisa's trajectories of participation were even at this early stage far from certain. Indeed, as I indicate on Figure 4.2, the practices were so distinct and different that they were being pulled in different directions.

The contrast with the anticipated community of practice, as illustrated in Figure 4.1, is a stark one. In contrast to the symmetry of the planned project, the teachers from the two schools were in asymmetric positions, with the Parkway teachers much closer to the centre, reflecting their greater potential for full participation. It is important to emphasise, however, that this analysis, particularly in relation to Lisa, was far from self-evident to the participants at the time (including myself). As I noted above, the teachers were chosen with the deliberate aim of having wide professional networks that would support their participation in the research team. In other words, using Spillane's (1999) analysis, the intention was that all four teachers' or zones of enactment would be broad. Indeed, given her involvement in Thinking Skills, LEA-wide art initiatives, management training initiatives and her wide teaching experience, Lisa had what appeared to be such a broad network. The crucial difference between her network and those of Alexandra and Ursula was not the breadth of professional activities but rather the *depth* and *range* of interconnections between their existing practices and those of CAME. Viewed in terms of her development as a manager, Lisa's zone of enactment was in many ways broad and deep. Viewed in terms of Primary CAME, however, her zone of enactment was much less rich, thus providing much less potential for her to change and develop in this area.

It is interesting to speculate whether the balance between the two schools might have been very different if Jenny, Rhoda's original choice, had been able to join the project. She was developing a more investigative and less scheme dependent approach to mathematics teaching at the school. She had worked with both Henrietta and Lisa at Beechmount and with Rhoda in the LEA. For Lisa, in particular, Jenny was a peer with whom she had worked for a number of years. It is possible that Jenny's involvement might have created a network of overlapping and collaborative practices at Beechmount similar to, if perhaps not as intense as, those at Parkway. There is an important point to be made here in that the space in which a teacher can change or develop is social and dynamic rather than individual and static. A key feature for both Alexandra and Ursula was not simply their individual zones of enactment, but the way in which these combined and interacted through their joint participation in the project. In a similar way, Jenny's participation might have transformed Lisa's participation, through providing depth as well as breadth to her zone of enactment *in relation to* Primary CAME.

Whilst I have highlighted the similarities between practices at Parkway and those of CAME, these communities were nevertheless still very different. I must emphasise that both Alexandra and Ursula experienced considerable difficulties in coming to terms with the CAME approach despite being more attuned to the practices of CAME. An interesting feature of their engagement with CAME was the way in which they seemed to perceive CAME as both different and similar to their exiting practices. For example, in an interview in March 1998, Alexandra expressed the difficulties she had in understanding CAME as follows:

> What I find quite hard is whether we're satisfying CAME aims. We're still not quite clear about that. So I guess most of all what I would feel is we need to go back now to the experts and say, you know, is this fulfilling CAME aims. ... Is this sufficiently open? And closing it down? Have we done that sufficiently? I suspect we might have led a little bit too much for it, so that might be an issue, I don't know. (Interview, March 1998)

Yet, despite this, later in the same interview she said: "I'm not sure what is special about CAME." This neatly encapsulates the tension in Alexandra's (and Ursula's) position. The interconnections (and similarities) between their practices at Parkway and those of CAME enabled her to "see" the differences between the two. At the same time, she found aspects of these differences hard to grasp, again because of these interconnections and similarities.

In contrast, Lisa often commented on the similarities between CAME and both her existing teaching and learning practices and Fisher's Thinking Skills approach. Despite this, there was, as I discuss in Chapter 5, considerable evidence that she did perceive CAME as something of a threat to her professional competence (Nolder, 1992). In fact, there was considerable evidence that she perceived the approach as different to her existing mathematics teaching, but that she had difficulty identifying what those differences were. Her position was a mirror image of that of Alexandra. Lisa was aware of some differences with CAME, but, because of the lack of interconnections and similarities with her existing teaching, the resources she could draw on to make sense of CAME were limited. I discuss this together with how Lisa resolved this tension in some depth in Chapter 5.

I now turn from this examination of the potential for the teachers' participation (and, hence, change), to an analysis to their early engagement with CAME. Firstly, I discuss the ways in which the academics modified CAME practices through their teaching and tutoring and how this was experienced by the different teachers. Then, I explore the teachers' initial engagement *as learners* and legitimate peripheral participants with the CAME practices of lesson development and, more briefly, tutoring Phase 2.

4. The Academics' Teaching and Tutoring

The key focus of the research team's initial work was the professional development of the four teachers and, thus, inducting the four into the CAME approach. This was led by the academics and informed by their previous work in secondary CAME. In this section, I focus on this initial professional development work with the teachers. In particular, I focus on the academics' teaching and examine this in the context of two aspects of the professional development: the discussion of the teachers' first teaching experiences and the academics' approach to tutoring. Here, I use the term tutoring to refer to the academics' work in school observing and supporting the teachers to teach TM lessons. In particular, I discuss the differences in this professional development for the four teachers. In the light of the discussion in Section 3.2 above, I examine the extent to which this compensated for or exacerbated the differences between the pairs of teachers from the two schools.

I draw on Lave and Wenger's (1991) notion of legitimate peripheral participation to interpret the ways in which the academics, as old-timers, interacted with the teachers, as newcomers and legitimate peripheral participants. Through this I explore the ways in which the academics modified the CAME practices in order to open up the practices of CAME and, thus, enabled the teachers to participate as CAME teachers. Wenger (1998) describes this notion as follows:

> Peripherality provides an approximation of full participation that gives exposure to actual practice. It can be achieved in various ways, including lessened intensity, lessened risk, special assistance, lessened cost of error, close supervision, or lessened production pressures. It can involve explanations and stories, but there is a big difference between a lesson that is *about* the practice but takes place outside of it, and explanations and stories that are *part of* the practice and take place within it. (p. 100, original emphasis)

Thus, the peripherality focuses on ways in which newcomers' participation can be seen as within the actual and authentic practices of the community rather than contrived or artificial activities. A key feature here is the notion of the teaching being conceived of as inside the practices rather than simply an external commentary on those practices.

There are two qualifications to this analysis. Firstly, the teachers' PD was a necessary but secondary aim of the project. A greater and more immediate concern for the academics at this early stage was whether the CAME approach would be applicable to primary teaching. Secondly, this PD was, as I have already noted in Chapter 2, integrated within the general work and discussions of the research team. There were very few formal presentations and very little explicit teaching. Largely, this was conducted through discussions on general aspects of the approach and of primary teaching in general. Hence, my analysis is directed at drawing out the teaching.

4.1 The Discussion of the Teachers' First TM Lesson

As I have already noted in Section 3.2.3 above, the ways in which Alexandra and Ursula collaborated at Parkway had many similarities to the ways in which the research team discussions were organised. In this section, I explore this in the context of the research team discussion of the teachers' first experiences of teaching a TM lesson.

The first lesson taught by all the teachers was Roofs, a lesson "used for 'seed-sowing' some ideas on counter-examples and generalised number" (Adhami et al., 1997b, p.11). Mundher had presented a lesson simulation of Roofs at the first team meeting a week earlier. These first lessons took place on the morning of the second research team meeting in November 1997. Alexandra and Ursula team-taught Roofs to Ursula's class. Lisa taught the lesson individually to her own class, whilst Henrietta, who had missed Mundher's lesson simulation, observed Rhoda teaching the lesson to her own class.

Alexandra and Ursula's lesson was recorded on video and later watched by the research team. So, in contrast to Lisa's individual experience and Henrietta's more passive one, Alexandra's and Ursula's first experience was a collaborative and active experience, which was then shared with the research team as a whole. The video recording itself reflects differences between the teachers. When the idea of the video was raised at the first meeting, Alexandra and Ursula volunteered, whereas Lisa appeared not to be keen on the idea. Henrietta was not present at this first meeting. Although all the teachers reported back on their experiences the video was inevitably more immediate and striking and, hence, produced a richer discussion. Moreover, the video was greeted with enthusiasm by the two academics present, David and Michael. The video was stopped, started, rewound and fast-forwarded and the accompanying discussion took the form of an annotated commentary, largely from the academics. Michael took the video as evidence that the CAME approach would be applicable to primary: "This is very encouraging. This looks like a CAME lesson." David commented on the team-teaching, an aspect of the approach that he had previously highlighted in his first presentation to the teachers: "I like the way you're both sharing the teaching." Both David and Michael were particularly excited by the activity of two children who were, David commented, "getting very close to proof", thus making further links between the mathematical practices at Parkway and the CAME approach to big mathematical ideas. (All quotes from research team November 1997.) Whilst the academics' comments were intended to draw *all* the teachers' attention to key aspects of the CAME approach, they were necessarily more meaningful to the Parkway teachers who had actually taught the lesson. In addition, whilst this was certainly not the academics' intention, their excitement and enthusiasm had the effect of valuing the Parkway lesson over the Beechmount experiences.

For Alexandra and Ursula, their first teaching experience was extremely positive and the academics' reactions emphasised the overlaps and similarities between their existing mathematical practices at Parkway and those of CAME and the research team. Henrietta's and Lisa's first experiences of Roofs were certainly not negative ones. In fact, both had been in lessons that each judged had been a success. However, an unintended consequence of the discussion of the lesson at Parkway was to emphasise the differences between the teachers at the two schools. Indeed, the modifications to the teachers' practice, which were largely verbal comments on the video of Alexandra and Ursula's lesson, served to facilitate the Parkway teachers' participation in the project. The academics' excitement about the children's learning and the links they made with CAME theory served to make this an "authentic" experience for Alexandra and Ursula (Brown et al., 1989). Whilst there was no criticism of the Beechmount teachers' lessons, there was no equivalent excitement or interest, and, thus, there were relatively few explicit, or implicit, links made with the teaching experiences at Beechmount. Yet, it was the Beechmount teachers, rather than the Parkway teachers, who were in greater need of connections being made. In addition, it seems highly likely that, in the light of the animated discussion about the Parkway lesson, Henrietta and Lisa compared themselves at least to an extent unfavourably with Alexandra and Ursula.

4.2 The Phase 1 Tutoring by the Academics

A second aspect of the teachers' initial professional development was the in-school tutoring by the academics. During these visits, the academics observed the teachers teaching a TM lesson, made notes and subsequently discussed the lesson with the teacher. It is important to note that these visits were exploratory in several senses. As I have already noted in Chapter 2, whilst this was informed by the professional development work in secondary CAME, one key aspect of this model did not readily transfer into primary since it was centred on secondary mathematics departments for which there is not a direct equivalent in primary. Hence, the academics had no worked out model for this primary tutoring and they deliberately explored different strategies. Moreover, an over-riding concern for the academics at this point was the lesson development: whether or not the lessons were suitable and what changes might be needed.

Michael and Mundher shared the tutoring. However, in practice, most visits to Parkway were made by Mundher, whilst most visits to Beechmount were made by Michael. There were significant differences between the two academics' approaches to tutoring.

Michael observed lessons without taking part in the teaching. He, thus, produced very detailed notes in the form of transcripts together with a commentary on the appropriateness of the activity as a TM lesson. These notes were "directed to expressing problems and potentialities in the Activity itself, rather than on assessing the teacher" (Michael, Personal communication, 9 August 2001) and were shared at a later date with the teacher. As a result, they were not so much directed at modifying and supporting the teachers' participation *as CAME teachers*, but at modifying the lesson in general. His comments on the teaching were deliberately general and abstract and, thus, almost tangential to the specifics of the teachers' practices. For example, his summary comments on one of Lisa's lessons concluded as follows:

> What can we say about the class management of this exercise which will enable pupils from NC level 2 to NC level 5 each to benefit from working on the task and then, by hearing and seeing each others' struggles and strategies, each to go one more step from wherever they are toward a better appreciation of number relationships? (Lesson observation, February 1998)

This was intended as a starting point for a research team discussion on whether the particular activity was appropriate to a mixed ability primary class. However, in this more general focus, Michael said very little about Lisa's specific orchestration of whole class discussion. Moreover, Michael often discussed the lesson in depth with the teacher once the notes were produced, rather than at the time of teaching. As Henrietta commented: "It was a madcap day and ... there wasn't a lot of time to talk. I mean, he just waved at me. I only saw his notes when we were up at King's later in the week" (Henrietta, Interview, March 1998). As a result, when the teaching was discussed, it seems likely that the teacher, whether Henrietta or Lisa, had herself forgotten many of the specific details of the lesson.

Mundher, in contrast, took much less detailed notes and took a more active teaching role in the lessons. His lesson notes focused on key features of the lessons and took the form of commentaries rather than complete transcripts. These were circulated as part of his more extended project memos to the whole team. Typically, Mundher would interject a question during a whole class discussion and would often then take the discussion over from the teacher. Immediately after the lesson the discussion would focus on these incidents and the types of questions Mundher had asked. Both Alexandra and Ursula identified this as a formative and valuable learning experience. Alexandra, for example, commented as follows:

> He always does [get involved], doesn't he. Yeah and I mean that was good because he could draw ... out some interesting points and he's so good at questioning the children as well. And I mean I find that a valuable learning experience myself just to listen to him, to hear him asking the questions and ... sort of delving a little bit deeper and he is very, very skilled at that. (Interview, March 1998)

By getting involved in the teaching, Mundher was able to comment on the teaching from within *as a participant*, and through this to exemplify and comment upon key aspects of the approach. Alexandra went on to comment on the way this support enabled her to take risks:

> You're making yourself almost vulnerable in a sense, because you're not asking closed questions and ... you're not necessarily knowing what you're going to get from the children and ... it's not the case of guess the teacher or anything like that, and I think that's where if

someone was less confident, they'd need prompts and support to deal with that because I suppose at times I felt that a bit and you know I've spent a lot of time with Mundher and I've been glad because he's known the next question to ask really to move where we've got on a bit further. (Interview, March 1998)

Thus, through prompts and discussion as part of the teaching process, Mundher's modifications lessened the risks of teaching new and potentially difficult lessons and, hence, enabled Alexandra to cope with her associated feelings of vulnerability. Indeed, Mundher's tutoring approach was closer to the professional development model of "formative peer evaluation" developed in Phase 2 of the project (Adhami, 2000).

It is important to note that these differences were as a result of the development of an approach applicable to the primary setting. Both academics' intentions were to enable all the teachers to participate fully in the practices of the project. The differences in their approaches were deliberate, but the intention here was to explore the value of different approaches to tutoring. In addition, my analysis here is not directed at evaluating whether Michael or Mundher's "teaching" was "good" or "bad" and I stress again the exploratory and developmental nature of this professional development work. Moreover, Michael's tutoring work was in a sense aimed at the teachers as teacher-researchers. The notes and the more general and abstract commentaries that accompanied them were directed at including the teachers within the CAME lesson development process. Indeed, on two occasions during this first year, Alexandra followed Michael's practice at Parkway of producing detailed notes with a transcript and general commentary of trial lessons taught by Ursula.

The differences between the tutoring practices of Michael and Mundher at the two schools were highly significant in terms of the teachers' professional development. Whilst Mundher's approach facilitated Alexandra and Ursula's peripheral participation as CAME teachers, Michael's approach did not support Henrietta or Lisa in the same way. Yet, the teachers at Beechmount needed the greater support, given the distinct differences between their existing mathematics teaching and the CAME approach that I highlighted in Section 3.2 above. However, the two Parkway teachers' existing relationship was one in which they not only team-taught each other's classes, but they interrupted, argued and discussed their teaching. Thus, Mundher's tutoring practices were in many respects very similar to the way the two Parkway teachers already worked together. This way of working had few overlaps with either Henrietta's or Lisa's individual teaching at Beechmount. Indeed, on the occasions that Mundher did work with Lisa and her class at Beechmount, Lisa chose to observe him teach a lesson.

4.3 Summary: The Teachers' Experiences of The Academics' Teaching

As I have demonstrated above, the academics modified CAME practices with the aim of facilitating the teachers' participation. As I have discussed in Section 3.2 above, the potential for the Parkway teachers' participation at the project's inception was much greater than that for the Beechmount teachers. However, the academics' teaching, both in meetings and in schools, served to further emphasise these differences.

The tutoring approaches of the academics were very different. In Mundher's case, this was largely directed at the teachers' participation as CAME teachers; in Michael's case, their participation as CAME teacher-researchers. The way in which Mundher approached the tutoring appeared to fit with the existing practices of the Parkway teachers, in particular their collaborative relationship. However, this was not the case for Michael's tutoring of the Beechmount teachers. I have suggested, however, that Mundher's approach to tutoring would not have been as successful with either Henrietta or Lisa.

The contrasts between the teachers were rooted in the their different zones of enactment that I discussed in Section 3 above: the Parkway teachers' dense set of interconnections with CAME and the Beechmount teachers' very different and separate practices. However, again as I noted above, these contrasts were not evident at the time of the projects' inception. And, thus, although this was not the academics' intention, these contrasts were compounded rather than compensated for by the academics' tutoring practices. Whilst the Parkway teachers quickly began to establish further connections between their existing practices and those of CAME, the Beechmount teachers did not. In comparison to Alexandra and Ursula, Henrietta and Lisa had fewer opportunities to derive meaning within CAME and more opportunities to experience contrasts, thus emphasising rather than ameliorating their perceptions of difference.

5. The Teachers' Learning: Engaging with The Practices of Primary CAME

I now turn to examine the teachers' initial engagement as learners with two key aspects of Primary CAME: lesson development and the Phase 2 tutoring. Here I focus on the teachers as learners and participants using the notion of legitimate peripheral participation. In particular, I explore how the teachers drew on their existing practices in order to act as lesson developers and tutors. I emphasise here that I am concerned with the initial stages of their engagement and exploring the impact of their early participation on the potential for their change and development as CAME teacher-researchers. In later chapters, I explore and analyse the change process as a whole.

5.1 Becoming Lesson Developers

The lesson development process had two aspects: adapting pre-existing secondary TM lessons to the context of primary mathematics, and developing new primary TM lessons. However, the process appeared to be understood in somewhat different ways by the teachers from the two schools.

Alexandra and Ursula were very involved in the process of developing activities into new primary CAME lessons and both expressed considerable interest in this area. In fact, both saw the development of new lessons as very much more interesting than the reworking the existing secondary TM lessons. Ursula, for example, commented:

> And I like particularly the ones where we're being asked for ideas. I'm quite looking forward to developing stuff and thinking my way round stuff, 'cos that's more me than to keep going over old ground if you know what I mean. (Interview, March 1998)

For Ursula, new lessons were forward-looking in contrast to the "old ground" of the secondary lessons. New lessons required her to "think" and I suggest to become more of an author. But this interest was not a simple matter of personal preference or indeed free choice. Rather, Alexandra and Ursula were *interested* because they could already draw on a range of social resources, practices and discourses with which to make sense of this lesson development process. Alexandra, for example, commented on her interest in lesson development in the context of Half-time Scores, originally a GAIM activity entitled Final Score (Brown, 1992a), and an idea which she had encountered during extended PD run by Rhoda:

> [The development of new lessons has] been quite - quite interesting. The one we did the other day, the Final Score, I felt that was really quite successful, more so perhaps than the fractions or the networks, and maybe that's because I'd already taught it at the teachers' centre. Again team teaching it with Ursula, she was picking up on bits perhaps that I hadn't developed enough. I think that was really valuable and possibly I think, if we are doing the new lessons, maybe that is a good approach, to team teach them so that somehow you're not missing out so many bits and you've also got that other person there observing you and picking up that there might be a shortfall in certain cases. (Interview, March 1998)

In explaining her interest in this new lesson, Alexandra drew on two aspects of her existing social network: her experiences of extended PD in mathematics through teaching the activity "at the teachers' centre", an, her relationship with Ursula. Moreover, she relates these to a process of adapting, crafting and re-teaching the lesson through team-teaching and collaboration. Her motivation and interest was created and supported by the breadth and depth of her professional network.

In contrast, Henrietta and Lisa had a limited role in the development of "new" activities, each teaching only 3 trials of wholly new lessons in comparison to Alexandra's 15 and Ursula's 10. Indeed, the reactions of both Beechmount teachers to this element of the lesson development work were negative and, unlike the Parkway teachers, expressed a strong preference for the re-working of the existing TM lessons. Henrietta, for example, commented:

> I personally think we've spent too long on [new lessons] than necessarily we need to, whether we're getting a little bogged down. I

> mean they're very interesting but I don't know whether it detracts from what we're actually supposed to be doing. (Interview, March 1998)

Although Henrietta referred to these new activities as "interesting", she also distanced herself from this interest. Indeed, she went further to identify this work as a distraction to the main purpose of the project, the adaptation of the secondary lessons.

Lisa felt that the new ideas were "old ideas" lacking in originality in comparison to the secondary TM lessons.

> [New lessons are] my least favourite, because I personally feel there's some originality to the activities in this booklet [The secondary TM draft materials (Adhami et al., 1997b)], but I'm not convinced that the new lessons are that original. I think that they are old ideas being pulled out and we're trying to turn those into CAME lessons. (Interview, March 1998)

Lisa's characterisation of the new activities as "old ideas" is correct. All these "new" activities were based on existing ideas and activities commonly used in both primary and secondary mathematics and drawn from the participants' teaching and professional development experiences. It is, however, worth noting at this point the adage that there are no new ideas in teaching. Indeed, none of the activities involved here were wholly new or original. The secondary materials were largely adapted from GAIM activities, which were themselves adaptations of activities commonly in use in schools. As I discussed in Chapter 2, many of the TM lessons were set within deliberately unoriginal contexts in order to encourage teachers to re-think and re-evaluate their mathematics teaching beyond the CAME initiative. Lisa's reference to the originality of these materials certainly highlights her restricted and scheme-dependent experiences of teaching mathematics. More significantly, her characterisation of the secondary lessons as "original" suggests that she experienced CAME as very different to her existing mathematics teaching.

The four teachers' understandings of, and reactions to, this process of lesson development reflected the schools' contrasting writerly and readerly approaches to the mathematics curriculum that I identified in Section 3.2.2 above. The

Beechmount teachers' *readerly* approach led them to prefer the more limited activity of adapting the secondary materials, whereas having a *writerly* approach, the Parkway teachers preferred the development of new ideas. Moreover, I suggest that the development of "new" activities was *not interesting* to Henrietta or Lisa, because the practices they could draw on to make sense of this were very limited. In contrast, new lesson development was *interesting* to Alexandra and Ursula, not only because of their overlapping practices, but also because they could nevertheless perceive contrasts and tensions with their existing mathematics teaching. However, Alexandra and Ursula's perception of difference was very distinct to that of Henrietta and Lisa, because the Parkway teachers had social tools with which to begin to make sense of this difference. Thus, new lessons were interesting not simply because they required these teachers to think, but also because they had resources with which to carry out this thinking. A further implication of these overlaps was that, for both Alexandra and Ursula, their participation in CAME shed new light on their existing practices.

I now explore the teachers' engagement with the lesson development process in more depth in the context of three lessons involving Alexandra and Ursula working together, and Lisa and Henrietta working individually.

5.1.1 Fractions at Parkway

In this section I consider the development of two fractions lessons by the Parkway teachers.

This lesson was typical of lessons developed at Parkway in that Alexandra, Ursula and Mundher worked collaboratively on the lessons, sharing working ideas and drafts at research team seminars. There was a brief period of intense collaborative activity when the lesson was team-taught four times. During this period, the teaching experiences were formally discussed as part of the agenda at three research team meetings. These discussions involved considerable argument and disagreement. This initial period was followed by a longer and less intense period of re-drafting and further trials. (See Table 4.1, for a timeline of the lesson development.)

Date	Activity
January '98	First trials of the initial fractions lesson. First research team reflection discussions. Alexandra's diagrammatic solution to Whisky & Water. Reflection discussion about children's errors, strategies and misconceptions in the area of ratio and proportion
April '98	Trials of revised Share an Apple and Halving & Thirding lessons.
May '98	Second reflection discussion at research team meeting about children's errors, strategies and misconceptions in the area of ratio and proportion.
October '98	Lesson simulation of Share an Apple to Phase 2 teachers. Phase 2 Share an Apple lessons taught.
January '99	Informal reflection discussion with Alexandra about the Whisky & Water problem following tutor visit to Phase 2 school.
August '99	Alexandra and Ursula's joint academic paper written. Preparation for Alexandra and Ursula's presentation at an academic conference.
February '00	Lesson simulation of Halving & Thirding to Phase 2 teachers. Phase 2 Halving & Thirding lessons taught.

Table 4.1: A timeline of the development of the Parkway fractions lessons

This initial fractions lesson focused first on different diagrammatic representations of fractions, then moved on to explore the multiplication of fractions. The lesson concluded with the children tackling and discussing the following Whisky and Water problem:

> I have two glasses. One glass contains whisky, whilst the other contains water. If you pour half of the whisky into the water, mix it up, then pour half of that quantity back into the original whisky glass, which glass now has more whisky?[4]

Following a research team discussion, a second trialling of the lesson placed a greater emphasis on the construction of a variety forms of representing fractions. A further reflection session followed at which the team decided to split the initial lesson into two more focused lessons: a Y5 lesson entitled "Share an Apple", and a Y6 lesson entitled "Halving & Thirding".

In Share an Apple the focus is on representations and comparisons of fractions. So, for example, children are asked to consider various ways of representing and comparing the magnitude of simple fractions of everyday objects. In Halving & Thirding the focus is on developing and connecting different representations for the multiplication of fractions, in particular $^1/_2 \times {}^1/_2$, and repeated multiplication by $^1/_2$ and $^1/_3$. Halving & Thirding concludes with the children tackling and discussing the original Whisky and Water problem in the context of mixing different coloured paints. The lesson notes for both lessons are attached in Appendix A.

In the following quote, Ursula described the origins of the lesson:

> It was ... the one with ... the pocket money. The sort of question which I thought would actually only take my class twenty minutes. ... Somebody who has half of somebody else's pocket money. ... Is it possible for one person to have half, [and that] half [be] more than a third? ... I think it was when we were talking CAME and looking for examples to use ... that the Whiskey and Water came up. And I thought the Whiskey and Water actually linked directly to that lesson that I had done [and] having already done that with my class. (Interview, March 1998)

[4] Note this is different to the well-known symmetrical problem in which a quantity of whisky is poured into the water and the same quantity of the resulting mixture poured back into the whisky. This revised problem is a considerable simplification accessible to a greater range of abilities than the

This pocket money activity [Can a third of one person's pocket money be more than a half of someone else's?] was precisely the kind of mathematics investigation at which Parkway, and Ursula in particular, was strong. Indeed, for Ursula it was the fact that the children "were using a lot of Maths that I hadn't even thought about in the first place" that persuaded her of the possibilities for a CAME lesson exploring fractions (Interview, March 1998). In addition, Ursula had first met both activities, Pocket Money and Whisky and Water, on the 20 days extended PD she had attended. Thus, in coming up with lesson ideas, Ursula was drawing from a range of her professional practices both within Parkway and beyond.

In January 1998, the initial lesson was greeted with some enthusiasm by the academics. Their reaction was certainly of some motivational importance for the two teachers. Of more significance, however, was the way in which they used the lesson as a vehicle for communicating and illustrating key ideas about the CAME approach. Mundher, for example, in a memo circulated to the research team, commented:

> The richness of Ursula's new 'halving' lesson is in the different ideas pupils expressed through the teacher's insistence on them to find *more than one way*. ... The context is rich in that it allows informal visual attempts at a solution. ... The thinking demand is potentially very high since it contains the notion of the Cartesian product, in fractional terms. (Memo, January 1998, p. 5-6, original emphasis).

Thus, he highlighted three key aspects of CAME: multiple perspectives through "the teacher's insistence on them to find *more than one way*"; the use of children's informal ideas through "informal visual attempts at a solution"; and, the longitudinal coherence of extended mathematical agenda through the high "thinking demand" and the "Cartesian product." I note that the term "Cartesian product" here functioned largely as a metaphor for high level mathematics, since none of the teachers knew what the term meant.

Both David and Michael also used this lesson to make connections with the notion of *mathematics without closure*, a key idea within the CAME approach, as I

original due to its potential for visualisation. It is much more open to a wholly diagrammatic solution and produces a more concrete set of end results.

discussed in Chapter 2. David, for example, commented favourably on the lack of closure within the lesson and the focus *away* from the end point: "I was worried about closure in this activity. I'm impressed that [the children] weren't so interested in getting the right answer" (Fieldnotes, January 1998). Michael commented further on the absence of closure within Alexandra and Ursula's lesson development process: "This is like the CAME approach. We haven't closed everything off. We have left the discussion open so that you can develop the ideas that you think are potentially fruitful" (Fieldnotes, January 1998).

The significance of the academics' comments is not simply that they were highlighting key aspects of CAME. It is rather that, as with the first teaching experiences in Section 4.1 above, they were situating these ideas within the Parkway teachers' practices (Lave & Wenger, 1991). In doing so, they were drawing attention to similarities with the Parkway teachers' more established professional practices. An unintended consequence of this was that they were at the same time drawing attention to the contrasts with the Beechmount teachers and thus adding to their perceptions of difference.

I have noted above the teachers' joint practices of extended planning and as a pair they brought these practices into the lesson development process. Indeed, they reported having been up "until after midnight" planning the first lesson (Fieldnotes, Research team, January 1998). Between January and September 1998, the lessons were trialled six times. Each time the lesson was trialled, it was team-taught. Ursula commented on this joint experience:

> I'm doing the full one [Halving & Thirding] as we've developed it with the other parallel Year 6 class and Mundher's coming in with some notes and some ideas. And then Alexandra will be doing the abridged version that ... she introduced to Year 5, with far more cutting and slicing and looking at fractions. So we've split it into two really and she'll be doing that in Year 5 next week with Mundher. (Interview, March 1998)

It seems clear from this comment that Ursula regards the lesson development as a joint process involving Alexandra, Mundher and herself. Indeed, by this stage both she and Alexandra saw their relationship with Mundher as an extension of their own close relationship. I stress, however, that this perception was supported and facilitated by the interconnections between their practices.

A further comment from Mundher highlights the complexity of relationships:

> My next task is to write up the fraction lesson, with the help of the remarkably detailed notes Alexandra has provided, again restructuring the planned lesson. The notes, by focusing on the questioning and responses, highlighted indirectly what both of us felt missing: what should the gist or the main challenge be. Pupils were active and involved and doing and thinking, but there was something missing still, but it is actually there. ... The section when the fraction 'One Third' was introduced into the picture showed the potential. The focus may well be the comparison of size of fractions. and how to explain or to find out which fraction is bigger than another. (Memo, May 1998, p.2)

Mundher's identification of the mathematical potential was dependent on and mediated by Alexandra's observations from her perspective as a primary practitioner. The challenge became apparent not through an analysis of the task, but through Alexandra's lesson notes. This illustrates the way in which the practices within the research team were becoming symbiotically interlinked for the Parkway teachers. The academics were not simply "teaching" these teachers about CAME. Rather, drawing upon the academics' practices within CAME *and* the teachers' primary practices, the participants were constructing primary CAME as a "joint enterprise" (Wenger, 1998). This in turn increased both Alexandra's and Ursula's potential to develop Povey's (1997) sense of author/ity in mathematics. Thus, by co-authoring the materials and, through "mutual engagement" (Wenger, 1998) with the academics on this process, Alexandra and Ursula had the potential to develop a different and more critical relationship with mathematics as socially constructed and negotiated.

However, this process was not one that fully included Henrietta and Lisa. Indeed, as I have already noted, the Beechmount teachers had developed a somewhat different understanding of the lesson development process. I now shift my focus onto, first, Lisa's and, second, Henrietta's engagement with lesson development.

5.1.2 Lisa's Development of Gardens

As I discussed above, Lisa from an early stage expressed a strong preference adapting the secondary TM lessons over the development of new lessons. In this section, therefore, I discuss her adaptation of one such lesson, Gardens. See Table 4.2 for a timeline of the lesson development.

Date	Activity
May '98	Lisa's first trial of Gardens.
June '98	Lisa's second Gardens lesson observed by OfSTED. Discussion of Beechmount OfSTED visits at research team seminar.
June '99	Lisa's lesson simulation of Gardens to the research team.
October '99	Teaching notes for Gardens written by Lisa.
February '00	Lesson simulation of Gardens to Phase 2 teachers. Phase 2 Gardens lessons taught.

Table 4.2: A timeline of the development of Gardens

Gardens is a lesson in which children explore the linear relationships $y = 3x + 2$ and $y = 2x + 1$ in simple concrete and pictorial contexts. They are then asked to compare and contrast algebraic, tabular and graphical forms or representing these relationships. (See Appendix A for a full set of the lesson notes.)

In terms of the lesson development process, Gardens was very much Lisa's lesson. She chose to teach the lesson individually and without reference to the team, taught the lesson without a lesson simulation and was never observed teaching the lesson by any of the research team members. At the end of the second year, Lisa simulated the lesson to both the research team and to the Phase 2 teachers. It was, moreover, the only TM lesson that she taught twice. On both occasions she was observed teaching, first by her headteacher and subsequently by an OfSTED inspector. On both occasions she received positive praise. Indeed, the lesson was mentioned positively in Beechmount's OfSTED report. However, in contrast to the Parkway teachers' experience discussed above, Lisa worked wholly individually on the lesson. She described during a PD session for Phase 2 teachers how she had chosen to work on Gardens:

> I'll tell you a little story about the lesson first. I'm actually going to share my experiences of teaching it with you. What happened was, when the original Phase 1 group were actually given the booklet and some of us tried some lessons, I chose to trial this one without having simulated it as a group, I just went off and did it with my class and I really liked it, because I felt it was accessible to a wide range of children. (PD Session, February 2000)

The contrast with the Parkway fractions experience is striking. Whilst Ursula's initial idea was rooted in her classroom experiences, Lisa identified Gardens purely from the TM teachers' guide. Whilst Ursula had highlighted the mathematical potential in the fractions activity, Lisa here highlighted the Gardens lesson's accessibility. Moreover, Lisa stressed the individual nature of the process. I note, however, that Lisa presented this individuality very positively. This is hardly surprising since her existing mathematics teaching was individual. However, Lisa further emphasises the process as *separate* to the research team as whole. Indeed, these comments appear to place value on her individuality and independence. The contrast with the collaborative practices of the Parkway teachers *and* the academics is very striking.

For Lisa, nonetheless, the lesson was a success. She received positive praise from both her headteacher and the OfSTED inspector. Indeed, the lesson was given external approval with a positive mention in Beechmount's OfSTED report. Lisa reported back on OfSTED's reaction to the lesson at the subsequent research team meeting:

> The OfSTED inspectors were really very impressed. I saw the lesson in the secondary booklet and I really liked it. It looked very clear and seemed accessible to all children. I have to admit that I practised it

first in the other Y6 class. Yes, the inspector really liked it. (Research team, June 1998)

However, although Lisa was keen to share her general experiences of the OfSTED inspection, she did not discuss her specific experiences of teaching this lesson. This was in contrast to her previous practice. This may in part be due to the collective and unreflective sigh of relief felt by teachers after an OfSTED inspection (Jeffrey & Woods, 1998). It may also have been partly Lisa's choice. It may also partly reflect an understanding by Lisa of teaching as performance: there was no need to discuss this lesson since it had received the authoritative approval from OfSTED. Certainly other lesson trials, by Mundher, Alexandra and Ursula, were discussed at that meeting.

The individuality of Lisa's experiences of Gardens provided very limited opportunities for the academics to draw connections between Lisa's experiences of teaching Gardens and the CAME approach more widely – as they had done with the Parkway teachers' fractions lesson. The original secondary TM lesson had been developed by the academics and certainly, therefore, epitomised many key aspects of the approach. However, in relation to the Parkway experiences, the academics were able to highlight aspects of the CAME approach by situating these within the teachers' own practices. In the absence of any mutual engagement on Gardens, together with Lisa's uncritical readerly approach to implementing the lesson, there was almost no possibility for modifying the CAME approach in relation to this lesson, or for suggesting ways in which primary practices might be adapted.

Lisa's teaching notes provide a further insight. Her lesson notes are very heavily reliant on the lesson notesheets, which were unchanged from the secondary teaching materials. Indeed, whilst the format of the lesson notes was different to the secondary materials, the lesson itself was unchanged. This further emphasises the readerly nature of Lisa's approach to lesson development.

Lisa was, like both Alexandra and Ursula, drawing on her established professional practices in the lesson development process. However, since Lisa's existing mathematical practices were individual, scheme-driven and readerly, this was how she approached the lesson development process. Moreover, the individual nature of this process provided few opportunities for either Lisa or the academics to make connections between her practices and those of CAME. Rather, this process appears to have further emphasised the contrasts and dis-connections between the two. That Lisa seemed to perceive the process as a success attests to the similarities between the individuality of these particular experiences and her everyday mathematics teaching. Hence, the adaptation of existing lessons enabled Lisa to overcome her perceptions of difference by providing an alternative, more readerly approach to lesson development, which was nevertheless acceptable to the other research team participants. However, Lisa overcame these perceptions of difference not by confronting them, but by passively ignoring them. Moreover, this success, validated as it was by the academics, her headteacher and OfSTED, appeared to further confirm Lisa's notions of authority as external in relation to mathematics. Thus, the potential for Lisa to engage critically with the nature of mathematical authority and to develop Povey's (1997) author/ity was limited.

5.1.3 Henrietta and Lesson Development

Henrietta, the other Beechmount teacher, took a very limited role in lesson development. Like Lisa, she too expressed discomfort with wholly new lessons and the lessons she taught were principally adaptations of existing secondary lessons. However, unlike Lisa, Henrietta did not write any lesson materials and her contributions to research team discussions of lesson development were limited. Hence, in this section I briefly discuss her experience of teaching one new lesson. The lesson, Triangles, was only trialled once, when Henrietta taught the lesson with Michael observing. The activity was suggested by Rhoda, who had found the starting point in The Mathematics Association publication, Mathematical Pie, which is aimed at pupils aged 10 to 14.

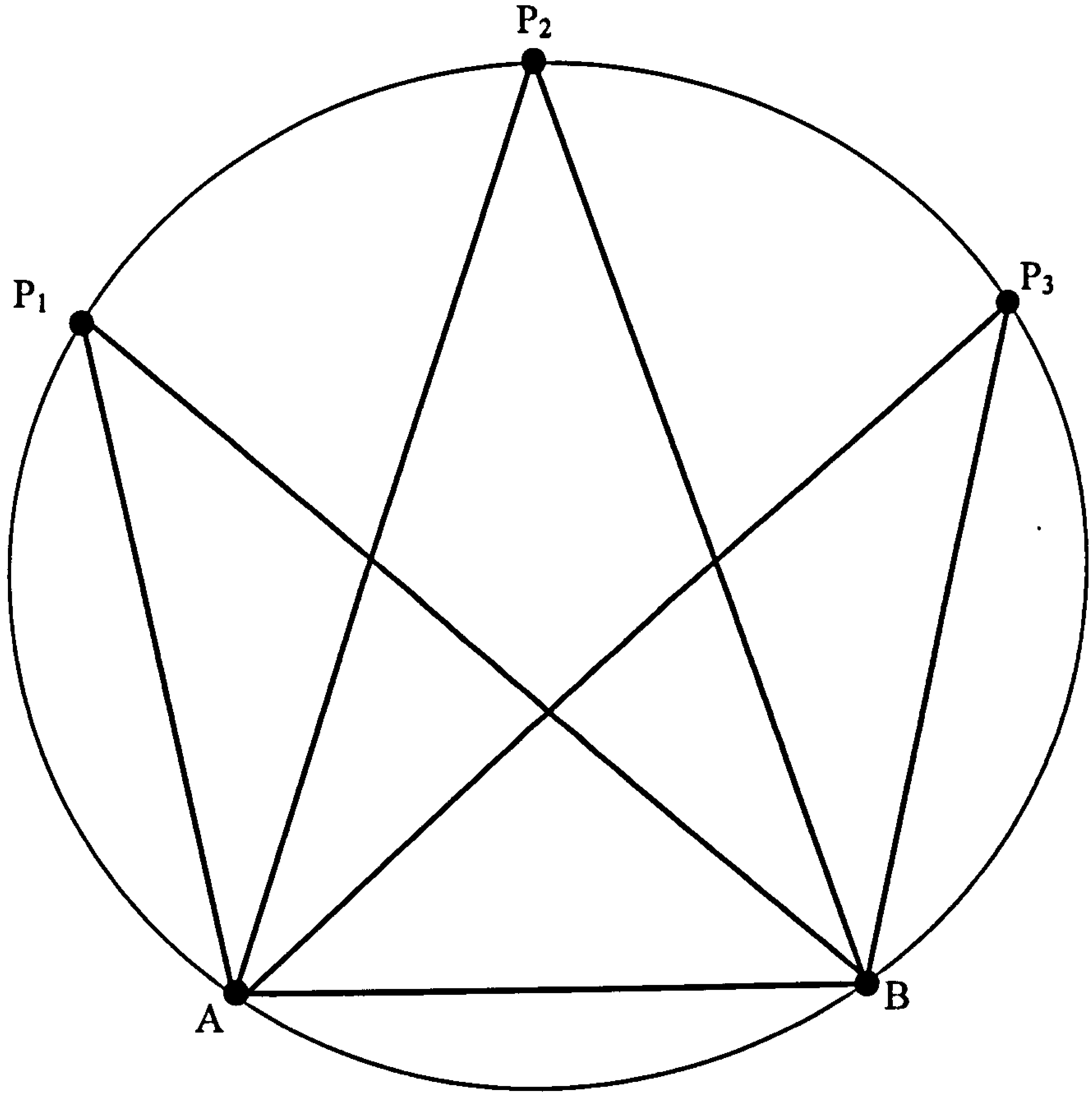
Date	Activity
December '97	Henrietta's lesson trial of Tournaments, a triangular numbers investigation.
January '98	Henrietta's lesson trial of Rectangles, an area / perimeter investigation.
March '98	Henrietta's lesson trial of Triangles. Reflection discussion at research team meeting.

Table 4.3: Key events leading to Henrietta's Triangles lesson

The problem is as follows:

> Starting with a chord formed by joining 2 points on the circumference of a circle, how many triangles are formed by joining the initial points to 1,2, 3 ... n further points on the circumference of the circle.

Although the activity is set in the context of triangles created by the chords joining n points on the circumference of a circle, it is actually an investigation into number sequences. There are two somewhat different possible interpretations of this problem. If the triangles have the original chord as the base, the sequence formed is the triangular numbers. This is a relatively simple sequence, which both Henrietta and her class had met in a different context in an earlier TM lesson, Tournaments. The solution can be justified by enumerating the total points formed by the lines crossing inside the circle. In Figure 4.3, I illustrate the problem with 3 additional points. In this case, there are 6 triangles formed with the original chord as their base.



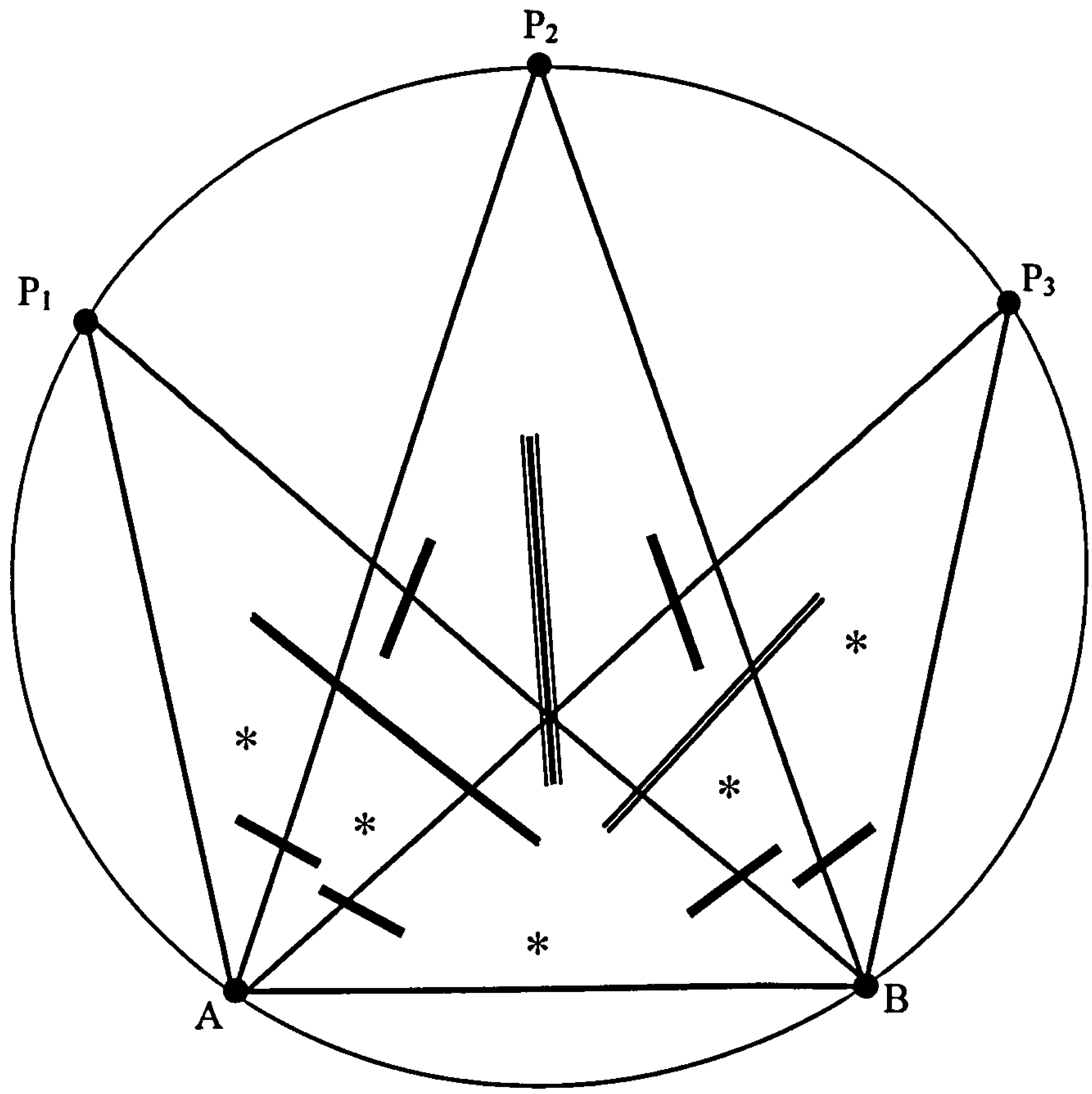
[The original chord is AB and the additional points are P_1, P_2 and P_3.

Figure 4.3: The triangles investigation.

A second interpretation is to identify all the triangles formed within the circle. This produces a very much more complex sequence, the pyramid numbers, or the sum of the first n square numbers. The solution to this is a cubic function[5], a level of mathematics that was certainly at a fairly high level for most children in a Y5 mixed ability class, although some children could certainly have spotted the successive differences as square numbers. Moreover, the process of counting the triangles in this second interpretation is complex and a justification of the sequence difficult. As

[5] The successive totals create the sequence: 1, 5, 14, 30, 55, 91 ... The nth term of this sequence is the sum of the first n square numbers, or $P_n = n(2n + 1)(n + 1) / 6$.
Solution: Calling the first two points A and B, say, there are then n further points on the circle, P_1, P_2, ... P_n. These produce n large triangular regions each with A, B and one of the other points as vertices: AP_1B, AP_2B, ... AP_nB. Within this diagram there are many sections, some triangular and some not. Triangles are formed by one or more sections. Adding one extra point, P_{n+1}, close to B, produces one extra triangular region, with n + 1 sections all of which are triangular. Hence in this region there are $(n + 1)(n + 2) / 2$ new triangles: n + 1 one section triangles, n two section triangles etc. Then, looking at the other triangular regions, starting with AP_1B, the line AP_{n+1} produces n + 1 new triangles, one of which is in the region $AP_{n+1}B$ and already counted, hence there are n extra triangles in AP_1B. Repeating this for the regions AP_2B, ... AP_nB, produces a total of $n(n + 1) / 2$ extra triangles. Thus, adding point P_{n+1} creates a total $(n + 1)^2$ new triangles and proof by induction follows.

can be seen from Figure 4.4, the process of enumerating the 14 triangles formed by just 3 additional points in this interpretation is fairly complex.



[Note: In the diagram, the triangles are enumerated by identifying traingles formed by one, two, three and four sections. The one section traingles are marked with an asterix (*); the two section with a single line (-); the three section with a double line (=); and the four section with a trib=plie line (≡). There are 5 one section triangles, 6 two section triangles, 2 three section triangles, and 1 four section triangles. All the children in Henrietta's class who correctly counted 14 triangles used this strategy.]

Figure 4.4: Enumerating the triangles for 3 additional points by marking.

Henrietta took the second, more challenging interpretation, although it seems likely that this was by accident rather than design. The lesson notes taken by Michael suggest that Henrietta had some difficulty in planning and teaching the activity. However, the evidence suggests that Henrietta actually planned the lesson carefully. The problem was not a lack of planning as such, but rather that she had very limited resources on which to draw to make sense of the activity. As I have noted in Section 3.2.2 above, Henrietta had a discovery approach to investigative work (Askew, 1996). Moreover, Beechmount's teaching generally was weak in the area of investigative work. Hence, in planning her teaching of this lesson, Henrietta drew not from her experiences at Beechmount but from her experiences of teaching CAME lessons. Unfortunately, the strategies she used were inappropriate to the Triangles lesson.

Henrietta began the lesson with an activity similar to one used in the introduction to Rectangles, a TM lesson, which she had taught three months previously. She asked the pupils to discuss the characteristics of triangles, which led to the production of the following definition of a triangle: "3 corners, 3 sides, 2D shape, no gaps" (Lesson observation, Michael, March 1998). In Rectangles, the equivalent activity is one in which pupils produce a sufficient definition of a rectangle and notice that this definition includes a square. This is directly related to the subsequent activity in which children explore the relationship between perimeter and area. However, in the Triangles investigation, whilst children need to be able to recognise triangles, the subsequent activity does not draw on the definition of a triangle. Hence, in Henrietta's activity, the children were engaging in mathematics they would not use later in the lesson. The preparation episode would have been better focused on a more central feature of the mathematical challenge: number sequences, square numbers, or systematic recording.

Having produced the definition of a triangle, Henrietta set the main problem to the class. Although the children found this problem difficult, particularly in relation to the enumeration of the triangles, the class did in fact generate the first three terms of the sequence: 1, 5 and 14. At this point, Henrietta ended the lesson by asking:

> So with the numbers of triangles, one, five and fourteen, think about continuing the pattern to ten points and let me know after assembly. (Lesson observation, Michael, March 1998)

It would have been challenging for any Y5 children to mentally work out even simply the next term in the sequence simply on the basis of the first three terms, since the square number differences had not been highlighted. The tenth term, 385, would have been extraordinarily difficult. This strongly suggests that Henrietta had not worked through the sequence herself beyond these first three terms. However, in asking this question Henrietta appeared to be drawing on her experiences of another

TM lesson, Tournaments, which she had taught four months previously. In Tournaments, a triangular numbers investigation, the children are asked to predict subsequent terms. However, the context in which they do this is considerably more structured than in this Triangles lesson. Having explored the 2^{nd}, 3^{rd} and 4^{th} triangular numbers (3, 6 and 10, respectively) with a focus on systematic recording, they are asked to predict the 5^{th} triangular number (15) *using* their systematic method of recording. This did not transfer easily into the more complex Triangles activity, because, in contrast to Tournaments, the children had no structure to enable them to engage with this predictive question.

In part, this lesson points to differences between Mundher's and Michael's tutoring approaches. Several of the primary lessons were developed from mathematical investigations at a level beyond that of most primary children. In similar situations at Parkway, Mundher got involved in the lesson and looked for different challenges at the children's level. Indeed, in the case of the fractions lesson discussed above, the initial Whisky and Water challenge was at a very high level and, in part due to an intervention from Mundher, the lesson was extended and split into two.

More significantly, this highlights the limited resources on which Henrietta could draw in order to interpret and make sense of CAME. As I noted above, the issue here is not so much that Henrietta did not plan the lesson carefully. Her lack of success in this lesson was not that she did not plan the lesson, but rather that she planned the lesson inappropriately. In the absence of applicable mathematical resources on which she could draw, she adapted techniques from previous CAME lessons. However, whilst those strategies were appropriate in their original context, they did not transfer directly into the Triangles activity. As a result, they were of little use to either Henrietta or her class in engaging with the lesson.

Perhaps unsurprisingly, Henrietta's perception of this lesson was one of failure:

> I did one. It wasn't one that I'd sort of thought up. It was a lesson that, you know Rhoda gave us some sheets of a lesson, of a little task to do, a while back. Actually, I don't think you were there. I'm not sure. And I chose to do that as my lesson last time and it was terrible. I haven't developed one of my own. I mean I've looked at negative numbers, but I couldn't really say it was a CAME lesson. It was just

> a sort of dabble into negative numbers. It wasn't really a particular lesson. (Interview, March 1998)

This comment also highlights Henrietta's lack of author/ity in mathematics education. For Henrietta, Rhoda was, as the LEA Mathematics Advisor and as a teacher who had successfully taught CAME lessons with Henrietta's class, a key external authority in mathematics education. She expected, therefore, Triangles, Rhoda's "little task", to work. When it did not, Henrietta was left with a feeling of not only being let down by Rhoda, but of her own personal failure. Thus, her beliefs about external authority in mathematics, combined with her experience of this lesson, created for Henrietta a powerful sense of silence (Povey et al., 1999). Indeed, I suggest she perceived CAME not simply as different but alien to her existing teaching practices. Her potential for developing a sense of author/ity was further restricted rather than enabled through her experience of lesson development.

It is worth noting that all the other teachers taught some lessons for which they appeared not to have clearly worked through the mathematics themselves. Ursula, for example, had found this in relation to the original fractions problem:

> [The children] were trying to find some sort of percentage or fraction or ratio terminology to sort of express how much the difference of money would always have to be for it to work. I mean they couldn't, and they didn't and they lost me and they were, they knew, they kind of knew what they were doing in an odd sort of way. They just carried it along and they felt like they were getting somewhere. And that to me was quite a good Maths lesson. They were using a lot of Maths that I hadn't even thought about in the first place. (Interview, March 1998).

For Ursula, unlike Henrietta, such experiences appeared to be a catalyst for further work and, indeed, were evidence of successful mathematics lessons. It is important to note that Ursula's interest was underpinned by a wealth of investigative practices. Henrietta, on the other hand, had very limited resources with which to interpret the children's mathematics.

I emphasise here that Henrietta's experience was very different to that of Lisa. Like Lisa in relation to Gardens, Henrietta attempted to make sense of the Triangles lesson using her existing experiences. However, unlike Lisa, who was able to use her existing school mathematics experiences to interpret Gardens, Henrietta, as an inexperienced teacher, had limited resources with which to interpret, adapt and *transfer* these teaching resources to the new setting of the CAME lesson. Hence, she attempted to make sense of the new lesson in terms of her very limited CAME experiences. As a result, and unlike Lisa, who experienced success at lesson development through adapting Gardens, Henrietta's experience of lesson development, and I suggest of CAME, was one of failure, alienation and silence.

5.1.4 Contrasting the Teachers as CAME Lesson Developers

In summary, the lesson development process compounded the differences between the teachers. For Alexandra and Ursula, the interconnections between their existing practices and those of CAME provided possibilities for them to develop further links and interconnections, creating the potential for learning and change. There were many similarities between their existing practices and those of CAME. Crucially, however, these practices were sufficiently different for the development of new lesson to be of considerable *interest* to Alexandra and Ursula. Hence, this combination of similarity and difference provided the initial motivation for the Parkway teachers' participation and change. In contrast, for the Beechmount teachers, the difference between their existing practices and those of CAME were too great to create such interest. Indeed, where there were apparent similarities for Lisa and her Thinking Skills work, these surface similarities in terminology obscured differences to the point where Lisa may have regarded CAME as largely identical to the Thinking Skills approach. I discuss this issue further in Chapter 5.

I have described how the process of lesson development became a "joint enterprise" (Wenger, 1998) for the Parkway teachers and the academics. However, although lesson development was important for both Henrietta and Lisa, their interpretation of this process was somewhat more limited. For the Beechmount teachers, the most important element was the adaptation of the existing secondary TM lessons for primary. Thus, the joint enterprise of lesson development was interpreted in quite different ways by the Parkway and Beechmount teachers. Crucially, the Parkway teachers writerly interpretation was closer to that of the CAME academics.

The potential for change amongst the teachers was very different. I have described how the experience of lesson development provided the potential for Alexandra and Ursula to develop an understanding of author/ity in relation to mathematics, whereas Henrietta's and Ursula's experiences did not appear to provide the potential for them to challenge their existing perceptions of authority in mathematics education as external.

Nevertheless, by drawing on her Beechmount practices, Lisa approached the development in an individual and readerly way, making very limited changes to the secondary lesson. In doing this, she was able to experience success in the development of the Gardens lesson. Henrietta, however, experienced the lesson development process in terms of failure, thus not only emphasising her beliefs about authority as external but also creating alienation and silence.

All of these teachers were, however, active learners. All drew on their existing practices in their attempts, successful or otherwise, to make sense of the lesson development process. Indeed, given this activity, it would be difficult to conceive of the teachers themselves as in any way "in deficit" (Brown & McIntyre, 1991). It is rather their social networks that constrain or enable their interest and their ability to interpret and make sense of the "new" practices of CAME.

5.2 Becoming Phase 2 Tutors

In this section, I briefly extend the above analysis to the teachers' tutoring role by comparing Alexandra's and Lisa's early experiences of tutoring.

5.2.1 Tutoring: Comparing Alexandra and Lisa

The Phase 2 tutoring visits that I consider took place in October and November 1998, with Alexandra visiting Greenbank School and Lisa visiting Roseway School. For both teachers, this initial tutor visit was their first experience of doing PD in another school, although Alexandra had led INSET for Outertown LEA.

My fieldnotes record that both Alexandra and Lisa appeared to find the first tutoring visit uncomfortable and awkward. As newcomers to the role of tutoring, they seemed uncertain about what to do. There had been some discussion of team-teaching as part of the tutor visits to schools and at the first PD session Rhoda had said that the tutors "will come in and work with you on the lessons. This might involve teaching a lesson with you observing or team-teaching the lesson together" (Rhoda, PD session, October 1998). Indeed, during their interviews in March 1998, both Alexandra and Lisa had said that team-teaching would be the most important strategy in introducing new teachers to Primary CAME.

Despite this, neither Alexandra nor Lisa team-taught the lesson. They both observed the Phase 2 teachers taking copious lesson observation notes, but not intervening or talking to children. Following the lesson, they had a discussion with each of the Phase 2 teachers for 10 to 15 minutes, during which they appeared to be following a fixed and predetermined agenda. However, during each of the visits, the Phase 2 teachers expressed dissatisfaction with this approach. For example, Gudhreer, at Greenbank, said that she had asked for help during the lesson and Alexandra had not responded: "I needed help. ... It's difficult being observed. ... I'd like to work on this together" (Fieldnotes, October 1998). Similarly, John, at Roseway, said to Lisa: "I'd like you to join in and teach the lesson with me not just observe. I'd find that more useful." (Fieldnotes, November 1998).

Alexandra's and Lisa's reactions to the experience of these first visits were quite different. At her next school visit, Alexandra "just team-taught it [the lesson] and it was much better" (Alexandra, Fieldnotes, Research team, December 1998). Lisa, however, cancelled her next tutor visit and did not visit another school until February 2000, over a year later.

For Alexandra, the first tutor visit triggered a connection between tutoring and team-teaching. She was able to draw on her existing practices at Parkway and within the research team in order to develop her practices as a tutor. Of course, tutoring practices were distinct from her practices in other communities. Her tutoring role was focused on the professional development of other teachers, rather than on lesson development or class teaching. Indeed, her apparent awkwardness during the first visit suggests that she perceived the role as extremely different to her established practices. Gudhreer's request for help and to "work on this together" enabled Alexandra to identify similarities between the practices. Lisa had no team-teaching experiences to draw on and John's request for her to "teach the lesson with me" intensified her feelings of difference to the point at which she felt unable to undertake any further tutoring visits. Indeed, I suggest that, like Henrietta in relation to lesson development, Lisa here experienced alienation and silence.

6. Extending the Analysis to Janice and Tony

In this section, I briefly consider the learning environment as experienced by Janice and Tony.

6.1 Janice

Initially, it appeared that the interconnections and overlaps between Janice's existing practices and those of CAME were rich and dense, similar in many ways to those of Alexandra and Ursula. Janice was an experienced teacher who had attended several extended PD courses in mathematics education. She was the mathematics co-ordinator in the Phase 2 school, which had the greatest number of teachers involved in Phase 2. Moreover, Janice was given non-contact time as part of her curriculum management role to implement the approach. Janice characterised the school as an intensely collaborative environment. For example, in a presentation to the Phase 2 teachers, Janice described her school's approach as follows:

> This is my team at Brightvale Girls. ... We do have to kind of co-operate to organise time for the lessons. ... We try to organise things so that there are at least 2 adults in the room for a CAME lesson. So sometimes this means that a teacher can actually go and watch the lesson happening in one class and then go and teach it again in her own room, which has been really helpful. ... It has happened sometimes that we have actually had as many as 4 or 5 adults in the room when a CAME lesson is taking place and that is really, really good. ... It's also been really good to have had the opportunity to watch each other teach and also team teach. ... CAME has created a lot of discussion about maths in our staff. Other teachers are intrigued when they hear us talking, They keep saying, "What's CAME? What's CAME?" So it does crop up quite a few times that we're explaining what we're doing. (Fieldnotes, June 1999)

Her emphasis is on co-operation, team-teaching and discussion amongst the team. Indeed, she went on to describe a system of CAME buddies, in which more experienced CAME teachers tutored new and less experienced teachers. However, although on my many visits to the school I found it very friendly and welcoming, I found no evidence of collaboration on CAME or mathematics in general, except where prompted by an outside event or requirement. Indeed, the CAME lessons that I observed at the school were largely taught by Janice and observed by other teachers, who took no, or an extremely limited, part in the lesson. Moreover, whilst not a wholly scheme-dependent school like Beechmount, Brightvale was still very reliant on schemes. Indeed, during the second year of the project, Janice purchased a new mathematics scheme for the school that she described as follows:

> We've just bought a scheme that is excellent. Somebody has spent a lot of time really, like CAME really, they've made the lessons for us, and ... they're lessons that you can give something of yourself too. ... It's excellent. ... You have three lessons, each week it's a unit and there are three lessons for the week. But because it's three lessons, you look at those lessons and you don't necessarily follow them, but they are, they give you the ideas for the week. It is all tied in with the numeracy strategy. Somebody has, like with CAME, people have spent hours doing these wonderful lessons. For me it's an excellent scheme, but ... I'm allowed to give something of myself to it. (Interview, November 1999)

Janice stressed the scheme's facility to "give something of yourself to it", thus suggesting a scheme-assisted approach, implying less of a wholescale dependence on the scheme. Elsewhere she described this process of "giving something of herself" to a lesson as follows:

> [I] make it mine ... by putting it in context. I don't know if you've noticed I always like to have a story - I always like to look at it. I mean a lot of them are in context, but you always have those little things that somehow make it yours. (Interview, June 2000)

My lesson observations provide further confirmation that what Janice actually meant by giving something of herself was not authorship but the contextualisation of lessons through stories. Hence, her approach to mathematics materials was closer to the readerly approach of Beechmount than to the writerly approach of Parkway.

Nevertheless, Janice did express considerable interest in lesson development. However, whilst Janice did develop one new lesson, much of her focus appeared not to be on the mathematical challenge but on the preparation of a detailed and intricate story as a context for a mathematical puzzle involving the recognition of prime numbers.

Janice's emphasis that the authorship of lessons lay elsewhere with others and, indeed, that she was *allowed* to give something of herself to it, suggests a readerly approach to the curriculum, coupled with a strong suggestion that for her authority in mathematics education very much lay elsewhere in the hands of experts. Thus, Janice's relationship to the curriculum, like that of Lisa, was one of external authority (Povey, 1997) and, moreover, her experience of CAME, written by experts, appeared to confirm these beliefs.

Like Alexandra, Ursula and Lisa, Janice was able to draw on a range of her existing practices in interpreting and making sense of CAME. However, her interpretations were constrained by the limitations of her existing mathematical practices. Hence, like Lisa, although her professional networks were relatively wide, the interconnections with CAME were limited and her zone of enactment, viewed in relation to CAME, was shallow and limited. Hence, her potential for professional change was limited.

6.2 Tony

Tony was identified as a teacher-researcher from Ursula's work with schools as part of the Outertown Numeracy Pilot during Spring 1999. Like Henrietta, he was an inexperienced teacher in the early years of his teaching career. He was also teaching in a one-form entry school where none of the other teachers were involved in CAME. His perceptions of difference are very striking. For example, he described his own teaching as "old-fashioned" and "rambling" in contrast to the "modern" and "focused" approaches of CAME and the NNS (Interview, November 1999). Moreover, he seemed to perceive himself as separate and distinct to the project. For example, in the following quote he described his own professional development:

> it's useful to see the actual process - of lesson development, in phase one, that is really useful, because I think that gives you such an insight into, and that's what actually occurred to me at the – at the phase two meeting, and at the peer tutoring, that - you do realise how - you know you are getting quite a good understanding of how it's all working, through being involved in the phase one, which is, which is tremendous. (Interview, November 1999)

There is an overwhelming sense of an outsider looking in on the processes of lesson development and tutoring, getting "insight" into "how it's all working." Indeed, although Tony always expressed interest in the project, his involvement beyond research team meetings was very limited. He developed no new lessons and wrote no lesson materials. He taught very few lessons and took part in only three tutoring visits, in none of which did he discuss the lesson with the teachers. However, he did not experience silence and alienation as Henrietta, the other inexperienced teacher, did. Indeed, he was welcomed into the research team by Ursula and this appeared to enable him to develop a strong identity with research team meetings, contributing frequently and forcefully despite his limited role outside these meetings. His beliefs about mathematics and mathematics education, however, remained one of external authority. I discuss this further in Chapter 5.

7. Discussion

At the beginning of this chapter, I raised a number of questions relating to differential change amongst the teachers and to the role of the teachers' wider professional networks in their professional change. In order to address these issues I have analysed the teachers' early experiences of Primary CAME from a situated perspective. In doing so, I have begun to describe their personal resources, in particular the teachers' individual capacity to change, in social rather than purely individual terms.

I have largely focused on the four original teachers. Of these four teachers, the professional networks of Alexandra, Lisa and Ursula were all broad. In contrast, as a relatively inexperienced teacher, Henrietta's professional network was narrow and limited. As a result, she had few resources to draw upon with which to make sense of CAME. Despite its breadth, Lisa's professional network was very different to that of Alexandra and Ursula. In particular, in contrast to the positions of Alexandra and Ursula, there were few overlaps between Lisa's existing practices and professional resources and those of Primary CAME. Hence, although Lisa's professional network was in many ways broad, the interconnections with CAME were not deep. As a result, whilst Alexandra and Ursula had the potential to change considerably, the possibilities for Lisa's professional change at the start of the project appeared to be significantly more limited. Considering Spillane's (1999) approach, this analysis would suggest that, in addition to being broad, teachers' zones of enactment need to have deep interconnections with new practices in order that teachers can engage with and make sense of reform initiatives. Without such depth, a teacher can, like Lisa, be party to rich deliberations with academics, for example, yet be unable to relate these discussions to her everyday mathematical practices.

My analysis has been largely focused on the teachers' early engagement with the project. Thus, I have explored the teachers' *potential* for change rather than the actual process of change as a whole. In Chapter 5, I extend this analysis to consider the process of professional change.

A central purpose of this chapter was to develop this theoretical approach on which to base the discussions on teacher change in the next three chapters. Nevertheless, in doing so, I have begun to address several of the issues which I identified in Section 2 above: transfer, teacher motivation, and teachers' ways of knowing in mathematics and mathematics education.

In highlighting the problematic issue of transfer in Section 2.1, I suggested that transfer could be understood in terms of the adaptation or reformation of existing practices. In this chapter, I have used notions of similarity and difference to explore the potential for this adaptation of existing practices. In my analysis of the research team as a potential learning environment, I described how Alexandra's and Ursula's existing practices had many similarities to those of Primary CAME, particularly in relation to their own collaboration and to the *writerly* approach to mathematics teaching at Parkway. Thus, they had access to a range of existing practices, which they could draw on and adapt in order to make sense of CAME in ways that matched the understandings of the academics. In contrast, both Henrietta's and

Lisa's practices at Beechmount, in particular their *readerly* approaches to the mathematics curriculum, were very different to those of CAME. In consequence, at the outset of the project, both Beechmount teachers had limited resources on which they could draw to make sense of CAME. This was less surprising in Henrietta's case since her professional network was limited. However, in Lisa's case the overlaps between her practices and those of CAME were limited. Hence, despite her broad professional network, she had limited resources that were potentially useful in making sense of the particular practices of CAME.

In retrospect, the differences between the Beechmount and Parkway teachers indicated a need for the academics to differentiate their tutoring of the four teachers. In reality, however, the academics' exploration of teaching and tutoring approaches reinforced and intensified differences between the teachers. One factor in this is that, whilst in the early stage some contrasts between the teachers and schools were apparent, these did not appear at the time to the participants (including myself) to be as widely different as my analysis here suggests. My analysis suggests that one important strategy in teacher education is to identify and analyse the differences in teachers' professional networks in order to differentiate their PD in appropriate ways. However, the fact that differences between the teachers were difficult to perceive in such an intense environment as Primary CAME, suggests that identifying such differences and hence differentiating teachers' experiences are neither easy nor straightforward. Indeed, I have tentatively suggested that Lisa might have had considerably more resources had Jenny, Beechmount's mathematics co-ordinator, been involved as a teacher-researcher rather than Henrietta.

In my discussion of the teachers' early engagement as learners, I again found the notions of similarity and difference to be analytically useful. Similarities between their existing practices and those of CAME enabled Alexandra and Ursula not only able to adapt and modify their existing practices, but also to perceive ways in which their existing teaching approaches were different to those of CAME. Thus, they were able to recognise how their existing approach to mathematics teaching needed to change. In contrast, Lisa, drawing on her readerly and scheme-driven approach to teaching mathematics, interpreted the different approach of CAME as fundamentally similar to these existing practices. Hence, despite the intensive experiences of

Primary CAME, she taught TM lessons in a largely similar way to her existing teaching. Moreover, I have suggested that surface similarities in language and terminology may have further obscured these differences. Henrietta, as an inexperienced teacher with limited resources to draw on, experienced CAME as largely alien. Similarly, in relation to developing as a teacher educator, Lisa, who had very limited experience of teaching other teachers, appeared to perceive CAME tutoring as alien to her existing practices. I explore all these issues further in Chapter 5.

In terms of the teachers' developing ways of knowing and beliefs about authority in mathematics, I have related this in particular to the teachers' working relationships and their existing approaches to mathematics teaching and learning. For Alexandra and Ursula, their intense personal collaboration together with their investigative and collaborative approach to mathematics enabled them to begin to develop an understanding of mathematics as constructed through argument and discussion and, thus, the potential to develop what Povey et al. (1999) describe as author/ity in relation to mathematics. In contrast, Henrietta's and Lisa's working relationships were individual and their mathematics teaching was predominantly scheme-driven. My analysis, thus far, suggests this provided little potential for them to change their absolutist notions of authority in mathematics as external. Moreover, their experiences of the academics' tutoring appeared to reinforce rather than challenge these beliefs. Henrietta's perception of CAME as extremely different and alien to her existing teaching appeared to cause her to experience a sense of alienation and powerlessness, or what Povey et al. (1999) describe as silence. This discussion leaves further questions as to the process of the teachers' changing beliefs about authority. I address these issues in Chapters 5 and 6.

In terms of motivation, I have described how, for Alexandra and Ursula, the combination of similarity and difference between their existing practices and those of CAME created interest and, thus, provided what Goldsmith and Schifter (1997) describe as "a compelling reason to undertake the task of transforming their practice" (p. 46). In doing so, they made a commitment to CAME that was more than simply "being there"; thus recognising themselves as legitimate peripheral participants in the research team. In contrast, Lisa, who perceived CAME as fundamentally similar to her existing practices, did not appear to experience any significant degree of interest. Indeed, she appeared to view the CAME approach as largely a non-issue for herself. On the other hand, for Henrietta the differences between her existing teaching and CAME were too extreme to generate interest. However, I have in this chapter been concerned with the teachers' initial motivations to change. Yet, as important as this initial motivation is, perhaps more important is understanding what motivates teachers to persevere in experiences that can at times be painful or frustrating (Goldsmith & Schifter, 1997). Despite their interest, both Alexandra and Ursula did find aspects of CAME difficult and at times frustrating or boring. In Chapter 5, I use these teachers' cases to investigate teacher motivation to sustain and continue change.

8. Summary

At the beginning of this chapter I highlighted two broad questions: understanding the role of teachers' professional networks in change and exploring differential change amongst the teachers. In order to address these questions, I have used and extended the situated approach of Lave and Wenger (1991), Wenger (1998), Boaler (2000b) and others to explore the research team as it was experienced as a learning environment by the teachers in the early stages of the project. Specifically, I have explored the following aspects of their experiences:

- the formation of the research team
- the teaching and tutoring provided by the academics
- the teachers' initial engagement with lesson development and Phase 2 tutoring

One particular feature of my analysis has been a consideration of the teachers' beliefs about authority in mathematics using Povey's (1997) development of Belenky et al.'s (1986) work.

Although my focus has been on the original group of four teachers, I have briefly extended this analysis to the remaining two teachers: Janice and Tony.

I have shown that the common setting was actually very different for the different teachers. Hence, the potential for professional change amongst the group of teachers was very different. Whilst the breadth of the teachers' wider professional networks, or zones of enactment (Spillane, 1999), was certainly important, a further crucial factor was the depth of the interconnections between their existing professional practices and those of CAME. In short, the teachers' zones of enactment in relation to Primary CAME needed to be both broad *and* deep in order to provide the potential for significant professional change.

Drawing on Boaler (2000a), I have used similarity and difference as ways of analysing issues of transfer and motivation. Similarity, if not too close, and difference, if not too great, were important characteristics in creating the potential for the adaptation of existing practices, and in creating interest and thus the initial motivation to change.

My discussion has been focused on the teachers' early experiences and, hence, their initial potential for change. The question of differential change remains: in what ways, if any, did the differences in the way the teachers experienced Primary CAME as a learning environment entail differences in the teachers' learning and professional change? I address this question in the next three chapters.

In this chapter I have raised several further issues detailed below.

Firstly, whilst I have explored the teachers' initial motivation, I have not explored the question of the teachers' motivation to continue with the change process in the face of difficulties. I address this in Chapter 5.

Secondly, I have suggested that a useful approach to understanding teacher change and the issue of transfer may be through a consideration of similarities and differences between the form of practices as opposed to simply their content. I address this further in Chapter 5.

Finally, I have raised issues of differentiation in mathematics teacher education, which I discuss further in Chapter 5.

Chapter 5: Changing Teacher Identities: Enculturation and Meaning-making

1. Introduction

In this chapter, I focus on the process and patterns of change amongst the teachers. Specifically, I examine the teachers' learning and change with a particular focus on their beliefs and orientations towards school mathematics and its pedagogy.

I address the questions of the process of change and the differential change of the teachers. I address the question of motivation to sustain change that I raised in Chapter 4. I also discuss the barriers to professional change using the cases of the teachers for whom change was less significant. In order to address these questions I extend the theoretical approach developed in Chapter 4, using notions of identity and emphasising the teachers as authors of new meaning.

The structure of the chapter is as follows:

- In Section 2, I give a very brief outline of my use of the idea of identity, a notion that I develop further during the course of the chapter.
- In Section 3, I focus on Alexandra and Ursula, the two teachers for whom change was significant, focusing on the process of their change.
- In Section 4, I discuss the other teachers. Here, my focus is on why change was less significant and analysing the barriers to change.
- In Section 5, I draw on this analysis to discuss more general issues of teacher change in primary mathematics.

2. Identity, Enculturation and Authorship

In this analysis, I extend the situated approach to understanding professional change that I developed in Chapter 4. One criticism of the work of Lave and Wenger (1991) is that, because their focus is on the community of practice as a learning environment, their approach does not account for differential change amongst learners. Individuals' learning trajectories are presented in a somewhat undifferentiated way and their success in learning, or move towards full participation, is presented as dependent on the extent to which the community as a whole allows them to participate in authentic, but modified activity.

Wenger (1998) goes some towards addressing these problems in his earlier work with Lave by analysing how communities themselves change and develop and by highlighting individual identity as being located in a variety of communities. He sees individual development and change as a process of reconciling conflicts and discontinuities between these different identities. He sees a role for human agency in learning and change, conceiving of this in terms of imagination. However, Wenger's analysis lacks both specificity and empirical foundation.

Holland et al. (1998) take the idea of identity and agency further by using the notion of co-development to emphasise the ways in which the change of individuals, practices and communities are inter-linked and interdependent. They point to the space for human agency in inventing and authoring new practices and ideas. Thus, they conceive of teachers not only actively making sense of new situations, but also constructing new meaning in the process. They refer to two aspects of identity, which they describe as follows:

> We make an analytic distinction between aspects of identities that have to do with figured worlds - story lines, narrativity, generic characters, and desire - and aspects that have to do with one's position relative to socially identified others, one's sense of social place, and entitlement. These figurative and positional aspects of identity interrelate in myriad ways. Sometimes they are completely coincident; sometimes one dominates over the other. (p. 125)

Here, I use identity to refer to both these aspects: the positional identity which Boaler and Greeno (2000, p. 173) describe as the ways in which people "comprehend and enact their positions in the worlds in which they live"; and, the figured aspects, involving Wenger's (1998) use of imagination and which enable individuals to adapt aspects of their positional identities in order to make sense of, interpret and invent new practices. Moreover, in conceiving of change, I add the metaphor of authorship to the metaphors of participation and enculturation highlighted in Chapter 4.

3. Alexandra's and Ursula's Professional Change

In this section, I discuss professional change in the context of the two teachers, Alexandra and Ursula, for whom change was significant. I begin with an overview of their change, focusing on two aspects of meaning-making: trying out new practices, and story-telling. Here, I flesh out the notions of authorship and co-development described above. Then, I discuss how Alexandra and Ursula adapted and extended their collaborative relationship into CAME. I focus on how similarities in the form of their working practices were valuable in this transformation. Finally in this section, I discuss motivation to sustain change and introduce the notion of desire. I argue that the maintenance of separate and different identities was an important factor.

There is, of course, ample evidence that effective professional development initiatives engage with teachers' individual needs and enable teachers to draw on their existing experiences (e.g., see reviews by Clarke, 1994; Hawley & Valli, 1999; Loucks-Horsley et al., 1998). In this analysis, however, I discuss *how* the teachers drew on these experiences.

3.1 Meaning-making, Exploration and Authoring

Through analysing and understanding the process of professional change for Alexandra and Ursula, I use and develop the metaphor of meaning-making, something that Ursula highlighted in the following comment:

> I think somehow you have to make meaning of it whatever you get in a professional development session, you've got to able to be given a chance to make meaning of it – and to practise it for yourself, to try it out. (Interview, March 1999)

It is commonplace within teacher education to refer to the importance of teachers developing ownership of ideas. However, the metaphor of ownership is problematic in that it suggests an external relationship with beliefs and ideas, in which ideas, as things in themselves, are adopted unchanged. The notion of meaning-making suggests, in contrast, that ideas and beliefs are adapted rather than adopted. Indeed, the fundamental feature of both Alexandra's and Ursula's engagement with CAME was that they did not simply adopt, or enact, the new practices of CAME: rather they interpreted and made sense of them by trying them out. I use the term "trying out" to emphasise two aspects of this process: firstly, its provisional and exploratory nature; and, secondly, that it involved real and authentic activities. Alexandra and Ursula tried out CAME teaching as teacher-researchers, albeit novice ones. Lave and Wenger (1991) argue that this process of legitimate peripheral participation is one in which the practices of the community are modified by the old-timers, the academics in this case. However, I found that the process to be equally one in which the practices were modified and interpreted by the teachers themselves. Hence, the change was a two-way process. These teachers did not simply become CAME teachers: what counted as a CAME teacher was changed in the process. This was particularly true in this case since the notion of a primary CAME teacher, as distinct from the more developed notion of a secondary CAME teacher, was itself very much in the process of formation.

In developing lessons, both Alexandra and Ursula recorded their teaching experiences using methods previously employed and demonstrated by the academics. Indeed, they experimented with both Michael's and Mundher's approaches: at times using Michael's broad and holistic style in which all contributions were detailed in long and extended notes; and, at other times, adopting Mundher's more partial approach in which he explored one or two key ideas in depth. In a sense, they adopted these almost wholesale using the original style as a template. In March 1998, for example, Alexandra and Ursula jointly produced a set of notes for Half-time Scores, a lesson referred to in Chapter 4. These were set out in a very similar format to Michael's style of notes. Like Michael, they used one column for contributions by the teacher, and another for those of the children, together with a commentary intended to illuminate the lesson development process, although the two teachers' notes contained very little of the cognitive analysis that typified Michael's notes. However, they added their own twist to the notes. At one level, this was simply the use of primary teaching discourse and jargon, for example, referring to "tops, middles and bottoms" in categorising children's ability. More fundamentally, the notes were taken and written up as a joint exercise by both teachers, reflecting the fact they team-taught the lesson and took turn to observe and teach. A further feature was that the notes contained many references to their own more intimate relationship with and knowledge of the children. They used children's names and referred to the children's previous mathematical, and other, learning experiences. More than this, the notes were peppered with references to aspects of their classroom as a specific community of practice. For example, the conclusion to the lesson notes included the following comment:

> Whole class (teachers n'all) spontaneous applause. All the children know that they have succeeded – right on the dot of lunchtime. Loads of praise given. We told them we were going to stop after they had found a suitable method of grouping, thinking they would be unable to cope with the rest. (Lesson notes, Alexandra & Ursula, March 1998)

Of significance here was the way in which they included their own and the children's pleasure and sense of achievement within the notes, in terms of the excitement and enthusiasm encapsulated by the "spontaneous applause" by "teachers n'all", and in terms of the sharing with the children of their surprise at the children reaching a conclusion. I note that this surprise was at several children reaching the end-point of an algebraic generalisation rather than at their engagement with the mathematical ideas. Alexandra and Ursula were, at this early stage, still struggling with the ideas of mathematics without closure. Nevertheless, this is still significant and was, I contend, not simply pleasure at the lesson's success, but at the children's and their own success in doing the lesson.

> That's when I think the kids have enjoyed a lesson - but I don't know whether that goes back to, you know, having the class that I used to have, and teaching Alexandra's class as well that used to break into spontaneous applause when they actually managed to sort something out (Ursula, Interview, March 1999)

Thus, the notes highlighted aspects of Alexandra's and Ursula's practices with these particular children and their engagement in what Wenger (1998) refers to as "joint enterprise" of this community and their practices of school mathematics as distinct to the community and practices of CAME. The notes were written not simply from the perspective of primary teachers but as primary teachers working with these children within their ordinary practices of school mathematics.

This insider perspective, an intense celebration of their achievement *as* a community, was not present in Michael's or indeed Mundher's lesson notes. Michael did comment on a few occasions about children's enthusiasm. However, the difference was that he commented as an outsider – he was not a participant in the enthusiasm. Mundher, in particular, was dismissive of such enthusiasm commenting as follows in relation to a child thanking him for teaching a lesson:

> There have been a few occasions of this kind. Normally I pay little attention to them, perhaps unjustifiably. It either feels "I don't want to encourage a kind of hypocrisy, or endearing oneself to teacher" or it is felt prompted by the class teacher. (Adhami, 2001)

Mundher went on to argue that such enthusiasm was of relevance *if* it related to a specific mathematical concept. So, to the academics, children's enthusiasm was important but only as it related to the more general issues of the lesson's development as a TM lesson. Hence, the notes were different to Michael's style in that they were interpretations from the perspective of two collaborative insiders rather than one neutral outside observer. At the same time, they were similar in that they were detailed records of what took place, together with a commentary directed at TM lesson development.

The two teachers were in a very real sense "authoring" and "inventing" new practices (Holland et al., 1998). Faced with the new situation of lesson observation for the purposes of TM lesson development, they tried out Michael's style of lesson observation. In doing so they created a different style of lesson notes by drawing on a range of their existing practices: their own collaborative relationship; their practices as teachers with these specific teachers; and, the discourse of primary teaching. At the same time as drawing on these practices to interpret CAME, they asserted their identity within these distinct communities. Thus, the notes reflected the two teachers' identity as CAME teacher-researchers alongside, and separate to, their existing identities as primary teachers within school. The maintenance of this separation was, I suggest, a crucial factor in their professional change, an issue which I explore further in Sections 3.2 and 3.3 below.

A second important feature of Alexandra's and Ursula's meaning-making was story-telling, an issue which Lave and Wenger (1991) stress as important. (See also Stein et al., 1998, for an analysis set within mathematics teacher education.) Throughout their involvement in the project, both Alexandra and Ursula told and re-told various stories about their experiences before and during CAME. They did this in a range of settings: individually, in groups, with other research team members, and with Phase 2 teachers. A feature of this story-telling was the way in which it appeared to enable the teachers not only to interpret and re-interpret the events being told, but to do this collaboratively with others.

For example, Alexandra repeatedly related an incident from her classroom in which Mundher had pretended to be a ballerina. Mundher had done this during a discussion about a point having no space. Alexandra told the following story about this incident as part of a group discussion contrasting CAME teaching with instruction, involving Janice, Lisa, Tony and Ursula:

> I had quite a long discussion in my class. It was when Mundher was in. There's my vision of Mundher as a ballerina. Typical Mundher. [LAUGHTER] – We were actually sort of *doing* it, but *through* a discussion really, not through saying you know this is what you need to do, but it came through, through the questions that were asked. And that, I mean that's different to instruction, isn't it. (Group interview, June 2000, original emphasis)

An important feature of this story was that it was amusing. Indeed, Alexandra's and Ursula's stories frequently used a humorous image to engage their audience. However, more significant was her use of the anecdote to highlight and illuminate several key aspects of CAME: the interlinking of doing mathematics with mathematical dialogue as in "actually sort of *doing* it, but *through* a discussion"; the contrast of this approach to instructional teaching as in "not through saying you know this is what you need to do"; and, the way in which she herself had engaged with this by doing it with Mundher.

This story had several purposes. For Alexandra herself, it was an opportunity for her to construct her new identity as a CAME teacher in contrast to her previous identity as an instructional teacher, at least in terms of mathematics. For the others, it was an opportunity to share vicariously in Alexandra's understanding and experiences. Here the anecdote was intended to enable the engagement of the others. Although they had not experienced Mundher as a ballerina, they had certainly experienced Mundher in their own classrooms. As Alexandra emphasised, this anecdote typified Mundher's behaviour in class: he did unusual things and behaved in odd ways. His behaviour as a teacher, whilst normal in terms of CAME, was different to the norms of ordinary primary school mathematics. This abnormal behaviour related both to unusual ways of communicating mathematical ideas through images like the ballerina *and* to ways in which he conducted mathematical discussions. By aligning herself with Mundher's odd and unusual behaviour, Alexandra was making a strong public statement of her identity as a CAME teacher-researcher.

I should note that the *intention* was to engage the other teachers. However, whilst the other teachers had certainly all experienced teaching with Mundher, as discussed in Chapter 4, this anecdote was unlikely to resonate with Janice, Lisa and Tony as powerfully as it did with Alexandra and Ursula. Yet, I suggest that the anecdote served a further purpose here in providing an enticing and desirable image of CAME as different, yet attractive. It thus functioned to facilitate the construction of what Lave and Wenger (1991) refer to as "a communal form of memory and reflection", an essential feature of a common shared identity as a teacher-researcher (p. 109).

In their analysis, Lave and Wenger place considerable emphasis on the crafting of extended stories, paying particular attention to the notion of "war stories" in which individuals give a "personal account of an arduous, but illuminating, work-related experience" (Stein et al., 1998, p. 39). However, in contrast I found anecdotes to be retold frequently in different ways, in different settings and for different purposes. For example, on another occasion, Alexandra referred to the same incident as part of a seminar discussion about pace in TM lessons:

> Do you remember the time Mundher pretended to be a ballerina? I've talked about it over and over, because I love the idea of Mundher

> doing a pirouette. But what was important there was we took the time with the class to talk about a point having no space or whatever and actually when we're doing maths what we're doing is imagining together. You know that thing Mundher is always talking about. So in a sense there, yes, we were taking our time, but we were taking our time to go deeper into the maths. (Research team, June 1999)

Here, the issue of mathematical talk is certainly still a central focus. Alexandra used the anecdote to illuminate the issue of pace as "taking our time to go deeper into the maths," an issue which she considered to be a key difference in her teaching. In doing so she juxtaposes the more commonplace limited understanding of pace as speed with the slower image of "taking our time." The image of Mundher as a ballerina here performed the additional function of emphasising this issue in that he had the time to do a "pirouette" and to play the fool with the children in order to illuminate this mathematical concept. Crucially, however, she linked this slower pace to a going deeper into the mathematics. Indeed, she specifically highlighted the issue of a collective mathematical imagination. So, as in the first example, the issue of mathematical talk is certainly still the focus. However, in this latter case, Alexandra used the anecdote to illuminate a quite different issue. In both cases, the ballerina incident itself was used as a point of reference from which to interpret different aspects of CAME. Moreover, in both cases the story was used as part of a collaborative "diagnosis" of a complex and contested issue (Lave & Wenger, 1991, p. 109).

Again, Holland et al.'s (1998) metaphor of authoring is useful. The metaphor of authorship implies an audience. Alexandra was, indeed, authoring these stories not simply for her own benefit and to make meaning for herself, but also for the benefit of the other participants. Through these stories, she was constructing an identity as a CAME teacher, an identity that was for herself *and* the other teachers, in particular Ursula, itself in the process of development.

Thus, for Alexandra and Ursula, the process of professional development was one of meaning-making. However, I have stressed here that this meaning-making was a social rather than an individual process. It was one of involving negotiations with others.

3.2 Extending The Form of Alexandra and Ursula's Collaborative Relationship

Collaboration amongst teachers is widely argued to be a key factor in enabling professional change both within mathematics education (e.g., Ball & Rudquist, 1993; Clarke, 1994; Grouws & Schultz, 1996; Jaworski & Wood, 1999) and beyond (e.g., Hargreaves, 1992; Hawley & Valli, 1999; Putnam & Borko, 2000). Indeed, images of collaboration underlie many descriptions of reformed mathematics classroom (e.g., Brown et al., 1989; Lampert, 1990; Stein et al., 1998). However, collaboration is an idea that is fraught with problems. Collaboration, if it involves fundamental belief change, is difficult and painful. It is, as Welch (1998) argues, "messy, unpredictable and uncomfortable" and collaborators "constantly argue and bicker" (p.116). Despite this messiness, collaboration is widely presented as a preferred method of working in contrast to the individualistic and isolated norms of teaching. Hargreaves (1992), for example, describes collaboration within school staff teams as follows:

> When they are in full flow, collaborative cultures exude an apparently "natural warmth" in human relationships. But they do not arise by a kind of emotional spontaneous combustion; they have to be created and sustained. Like good marriages, they have to be worked at. (p. 226)

Certainly, Hargreaves is attempting here to convey a sense that collaboration, whilst desirable, is both hard to define and hard to achieve. However, one problem with his description is that the association with vague images like "natural warmth" or culturally specific notions like "good marriage" is unhelpful either in recognising collaboration or in enabling it to take place. Moreover, Griffiths (2000) argues that the term collaboration is often used loosely to describe a vast array of very different practices around which there has developed a mystique, something which is evident in Hargreaves' description.

A further example of the mystique attached to collaboration is the way in which it appeared to be an important element in Janice's identity as a Brightvale teacher, locating her practices as different to and "better" than the neighbouring boys' school. She portrayed the Brightvale teachers' Phase 2 CAME work as an extension to their existing collaboration: they worked as a team on CAME, team-taught TM lessons, and "constantly" discussed their CAME mathematics teaching (e.g., PD session, June 1999). However, whilst Brightvale was certainly a friendly and welcoming environment, I actually found no evidence of such collaborative practices. Indeed, on my limited experience of one visit to the boys' school during a joint mathematics day, my judgement was that mathematics teaching in the two schools seemed to be broadly similar. The main difference between the two schools was that, whilst Brightvale was in one building with large open corridors, the boys school was spread out amongst several buildings themselves shared with an infant school. This environment was, as a result, more isolating and less welcoming and friendly. Indeed, Janice's presentation of Brightvale as collaborative and, hence, "better" than the boys school appeared to me to be at least in part a defence to counter attempts by the LEA to merge the two schools.

In Chapter 4, I suggested that there were strong parallels between Alexandra and Ursula's relationship and the working practices of the research team. In particular, their teaching and planning approaches took the form of open-ended, extended and extensive discussions that had many similarities with the format of research seminars. There were several key images that the two teachers each used repeatedly to describe their relationship: "more than a professional relationship"; "bouncing ideas off each other"; classroom talk as "conversations between us and the children"; and, discussions "going off at a tangent." They used the phrase "finishing each other's sentences" particularly frequently as a metaphor for their relationship. The repeated use of these images conveyed a sense of their relationship as deep, intense and collaborative, which was at least in part a larger than life exaggeration. Indeed, in using these images, Alexandra and Ursula were making a very strong public statement of their identities as primary teachers. Moreover, they were locating themselves as different to the norm of isolation and individualism in primary practice (Lortie, 1975). Their identity as different was further emphasised by their presentation of themselves as distinct and different to other teachers at Parkway where they were know as "the girls [who] always worked together" (Joint interview, May 2000). In the following discussion, they discussed the difficulties this caused:

Alexandra:	I think one of the problems in a sense was, for us, I don't know, while we were in agreement on so much of educational ideology, we were friends, those two things, I can't explain myself very well, weren't in conflict, I mean it was just the way it was, but the educational views that we might express in a staff meeting or whatever were about what we believed about education. It wasn't, because we were friends, but for some people they perceived it as
Ursula:	they perceived as if we were sticking up for one another, I think ... but actually I mean our views aren't exactly the same ... but they are
Alexandra:	similar
Ursula:	similar. And we've both got, we're both very child orientated, I think, and we're both very led by what we think would be good for the people as opposed to necessarily the school
	(Joint interview, May 2000)

Here, they presented themselves not only as different to other members of the staff group, because they collaborated as "friends," but, more importantly, their aims were different. Hence, whilst they did not necessarily agree: "our views are not exactly the same," their fundamental beliefs about education as "child orientated" and for the "good of the people" were shared and distinct to those of other members of staff. The educational project they were engaged in was at odds with that of other Parkway teachers. So, they were different to other teachers at Parkway not simply because they collaborated, but also because what they collaborated about was different. Indeed, I suggest their identification with the unusual practices of CAME, as discussed in Section 3.1 above, was itself facilitated by their existing identity with Parkway as unusual and different.

Alexandra's and Ursula's positioning here is very significant. Although they placed their own disagreement within shared fundamental aims, I suggest that they were themselves bound to the school community. They defined their aims as loftier and almost altruistic in contrast to the more pragmatic concerns of others. Indeed, their collaboration was in large part structured and maintained by their definition of themselves as in contrast to the rest of the school.

Both teachers referred to the importance of this relationship in enabling their engagement with CAME. In particular, they both frequently characterised Primary

CAME as an extension of their own relationship. For example, Alexandra described working with Mundher as follows:

> I think this thing about linking what children say with their mathematical thinking and the development they make through what they are saying. And I think round about that time there were a couple of sessions where we were having really in depth conversations my class, me, Mundher. We tend to go off at a bit of a tangent sometimes especially when we were trying to develop some of the lessons and just ... thinking things through and what things actually mean and to see children do that is quite fascinating, I think. And I think ... that ability to articulate your thinking just consolidates it so much for children and I think as well as that just the sheer enthusiasm the children had for some of the activities. (Interview, March 1999)

The similarities to the ways in which Alexandra described her relationship with Ursula are very striking. These similarities go beyond what is being described to the way in which she describes it. The phrases she used, for example, are ones both teachers used to describe their relationship: "in depth conversations," and "going off at a bit of a tangent." Indeed, if Mundher's name were replaced with Ursula's, this passage would not be out of place as a description of her relationship with Ursula.

However, the transfer of their collaborative practices into CAME was not a simple or straightforward process. Here, I draw on Greeno's (1998) notion of constraints and affordances. He argues that different situations, or different communities in this case, present different constraints and affordances that both structure and provide resources that enable different forms of participation. A key feature here is that individuals can predict the outcomes of their own and other's actions. An individual's attunements to the constraints and affordances, or regular patterns of participation, enable that individual to respond and act in appropriate ways, and thus to interpret and make sense of the situation.

For example, one important aspect of Primary CAME was the exploratory nature of TM lesson development. In exploring new ways of teaching and new activities, the teachers were often faced with situations and problems for which they had no immediate solution, yet the situation of teaching in the classroom demanded an immediate response. This was, in fact, something that both Alexandra and Ursula expected. Ursula described how she and Alexandra dealt with this problem as follows:

> You know sometimes you hit those barriers in teaching, where you look round for help. And just someone else stepping in, not necessarily with a clear idea, just moves things along a bit, nudges it and gives you a bit of space to think. (Fieldnotes, September 1998)

Hence, when they were faced with such problems, they had a pattern of behaviour that enabled them to begin to overcome the problem. Indeed, the significant factor here was more than that the two teachers expected interruptions and more than that they expected to be at times confused and uncertain of what to do: it was that such interruptions were a way of approaching problems. Moreover, these patterns of behaviour were very similar to the ways in which Mundher, in particular, worked in class. Thus, on the one hand, their existing practices of "stepping in" enabled them to engage together as novice CAME teachers, and, on the other hand, it enabled them to begin to work with other research team members.

Exploration, and trying things out, was an important aspect to Alexandra and Ursula's existing joint collaborative practices. For example, in Chapter 4, I noted how Alexandra perceived Ursula's 20 days mathematics course as a formative experience in her own development as a mathematics teacher. Ursula too described the impact of the course in terms of her relationship with Alexandra:

> I think that 20 day course was so significant because it gave me so many things to try in the classroom. And what I actually did, it comes back to the working with Alexandra, because I would go along to Alexandra and say this was good. I tried this and she'd say why? And that's the key to it. Is that why you did that? Why was it good? And then she'd try it and we would have something to talk about. (Interview, March 1999)

Ursula here placed the course's significance for her in terms of a dialogue with Alexandra: Alexandra would ask why; they would both try things out in their classrooms; and, they would than have "something to talk about." Ursula emphasised this exploratory work as one of shared meaning-making. That Alexandra and Ursula expected to engage with new ideas in a sense-making and exploratory way, which was similar to their work in the research team, is important in itself. Of further importance is that their meaning-making was not necessarily centred around actual shared experiences. The 20 days course was experienced by Ursula individually and not Alexandra. The classroom trials of the 20 days activities suggested by Ursula were conducted by the two teachers working individually with their classes. However, from their discussion about these individual activities, the two teachers constructed a shared understanding. Indeed, Alexandra's role as an outsider, asking *why* questions, appears here to have been crucial. This was important in that the work in research team seminars largely involved sharing, interpreting and constructing a shared meaning out of the actual experiences of others.

Thus in transferring their collaborative practices, it was the form of these practices, rather than their specific content, that enabled the teachers to draw on these practices. Indeed, an important feature of their practices of collaboration was that they extended across the primary curriculum and were not exclusive to mathematics education. In order to examine this issue, I quote at length from a discussion taken from an interview in May 2000, in which they described their working practices as a planning team at Parkway. I use this to illustrate the form of their collaboration alongside their own perception of it. In the first excerpt, they contrast their different individual contributions and discuss how they balance out:

Alexandra:	We are quite different temperaments. I mean Ursula's creative and she'd be the one that would come up with all the hair-brained schemes.
Ursula:	Which you used to run with. [LAUGHTER]
Alexandra:	Yeah, I used to run with them thinking, "Oh my God! Let's totally cover a child with plaster, that's a good idea - so we can make a sarcophagus." – Em - And I think, I don't know I think that I
Ursula:	It was quite good that, because I did have hair-brained schemes and Alexandra used to calm them down a little bit. And she'd put the important bit of sort of like, the basics, of, "Ursula, we still need to do this, and this, and this." Whereas I'd quite happily sort of run riot in the classroom with a load of masking tape, sort of thing.

I note how their contributions build off each other and how their own metaphor of "finishing off each other's sentences" aptly describes their conversation. This pattern of building off and expanding on each other's contributions typified many of their conversations. In the next excerpt, which follows on from the last, they discussed their approach to planning together:

Alexandra:	But I think what happened sort of quite quickly really, I guess, was that we would have this sort of over-embracing sort of like curriculum map, where we were really trying to do quite a lot of cross-curricular things, weren't we?
Ursula:	We did a huge amount of cross-curricular things. The basis of our planning, actually, was that things had to link, and be worthwhile otherwise we wouldn't do it. -
Jeremy:	So you didn't start from subjects, you started from topics, or?
Ursula:	The curricular map in the school gave you headings, if you like, so it just gave you headings. And things like maths and English were very much, at that point, I think, you could almost choose which bits you wanted to do when. You had a kind of rough guide as to what you would do. And that was all.
Alexandra:	I mean, it was very much, when I went there, everything had to be linked. It was an integrated day, you wouldn't be expected to have children doing the same subject
Ursula:	You had to have five particular areas in the classroom and five groups working in those subjects.

Their approach to planning was one which shared many features with CAME: the emphasis on the big picture, "the over-embracing curriculum map"; the making of connections, "everything had to be linked"; the risk-taking, with Alexandra, in particular, running with Ursula's schemes despite her instincts.

In the next section, they point to differences in relation to mathematics:

Alexandra:	And the, I mean, it was very much, everything had to be linked. ... I think sometimes the difficulty was, you know, well maybe you need to teach, say maths topics, independent of, you know, teach the skills sometimes outside. I mean, people had difficulties with that.
Ursula:	Although we still linked where we could, things like shape and space we linked wherever we could, measures we linked where we could.

Significantly, mathematics appeared to be different to other curriculum areas. In contrast to the connectedness of the majority of the curriculum, mathematics was an "exception" and often had to be taught "independent to" and "outside" their more general integrated approach.

Whilst Alexandra and Ursula had been developing open-ended, investigative practices at Parkway, these practices were at CAME's inception considerably less-sophisticated than those of CAME and did not, for example, constitute Lampert's (1986) notion of a principled approach towards school mathematics. Indeed, the excerpt above strongly suggests that their shared investigative practices were considerably more developed in relation to aspects of the primary curriculum, which were *not* mathematics.

These practices were powerful factors in their development as CAME teachers despite the fact that they were not exclusively, or indeed primarily, mathematical. Indeed, because these practices were more general, their collaboration was attuned to a greater range of constraints and affordances in a greater range of different contexts. Hence, I suggest the non-specialist and less narrowly situated nature of these practices facilitated transfer. Investigative mathematics, on the other hand, provided a vision of what school mathematics could be like, enabling them to imagine different practices. In part, this vision itself was powerful through its association with the 20 days course that Ursula described as "inspirational" (Interview, March 1999). The point that I am making here is not that Alexandra and Ursula easily transferred their existing practices into CAME. They most emphatically did not. However, their existing practices did provide ways to engage with CAME. Thus, they were not only able to get started by acting like CAME teacher-researchers, but they were able to envisage at least partially what this new way of teaching was like.

Lave and Wenger (1991) make a similar distinction between form and content in distinguishing between learning *to* talk and learning *from* talk, that is useful here. Alexandra and Ursula's practices, their ways of talking, across the curriculum were similar to ways of talking within CAME. Because of these similarities in *form*, they could at an early stage begin to talk like CAME teachers, although the mathematical *content* of what they were talking about was unclear to them at the time.

I stress that I am not arguing that mathematics pedagogy is the same as pedagogy in other subjects. CAME was still a very different way of teaching mathematics to these teachers' existing practices. Good mathematics teaching involves, I believe, an attention to the specifics of mathematics as a discipline and, in particular, to issues of the validity of knowledge particular to the discipline. However, I am suggesting that the similarities between the working practices inside and outside CAME were less about the pedagogy per se and more about their ways of talking, thinking and engaging in these pedagogic practices *together* and *collaboratively.* So, the ways of talking, doing *and* problem-solving were similar in terms of interruption, experimentation, disagreement etc., although the content in terms of both specific mathematical pedagogic practices and knowledge "being learnt" was very different.

3.3 Motivation and Desire

In this section I discuss the issue of motivation to sustain change and introduce the idea of desire. I explore how motivation and desire were maintained through Ursula's construction of a new and different identity within CAME. The discussion centres around two accounts of Ursula's first experiences of teaching the fractions lessons that I introduced earlier in Chapter 4. (See this chapter for a statement of the Whisky and Water problem discussed here.)

My consideration of desire arose from two directions. Firstly, Alexandra and Ursula, and to a lesser extent, Janice, all referred to mathematics in strong emotive terms. They talked about their "love" for doing or teaching mathematics and used strong emotive stories from their past to illustrate this. Yet, the professional change experience was at times painful. Secondly, in the literature, Middleton and Spanias (1999) associate intrinsic motivation with a love for the subject and the "sheer joy of learning" (p. 66). (See also, Ames, 1992; Dweck, 1986.) Stocks and Schofield (1996) argue that teachers need to have "a deep desire to change" (p. 291) in order to continue with a process that is at times difficult and painful.

In order to conceptualise desire, I draw somewhat loosely on the Lacanian psychoanalytic approach to identity and desire, as interpreted by Zizek (1992), and discussed in Brown, Hardy and Wilson (1993), Brown (2001) and Evans (2000). For Lacan, imagination, fantasy and desire are fundamental to understanding human action. He conceives of identity in terms of an unattainable completeness:

> For Lacan the human subject is always seen as incomplete, where identifications of oneself are captured in an image: as an individual I am forever trying to complete the picture I have of myself in relation to the world around me and the others who also inhabit it. (Brown & Jones, 2001, p. 10)

Lacanian theory is particularly useful here, because of the way in which pleasure is seen as dialectically linked to pain. Thus, it provides a way of locating the motivation to sustain change in relation to the very real difficulty of this for teachers. In conceptualising desire, I build on Evans's (2000) suggestion that transfer, the adaptation and transformation of existing practices, may be "facilitated by fantasy" (p. 224). I use desire to represent a strong and emotive motivating force for an imagined, but only partially grasped, potential future. For Lacan it is precisely this tantalising gap between the actual and the imagined self that defines desire. In this case study, this imagined future was as a different teacher of mathematics.

During the first year of the project, Ursula presented her mathematics classroom as a motivating and exciting place in which to learn. For example, she described the first fractions lesson as follows:

> They were really noisy. I had stand up arguments between children about the maths, shouting at each other. If anyone had come in, they'd have thought it was chaos, but I loved it. (Research team, January 1998)

The image presented here was certainly an exciting one in which children were engaged in mathematical talk. However, the way in which she expressed this message is very significant. Schools and classrooms are generally characterised by order, control and turn-taking. "Chaos" and children "shouting at each other" are the very antithesis of what classrooms are expected to be like. Certainly, Ursula used these descriptions in order to emphasise that mathematics in this incident was different to ordinary primary mathematics lessons. But, this, I suggest, went beyond mere emphasis. Her description of the children's mathematical talk was framed in language that implicitly challenged her own authority as the teacher. Indeed, she presented the children as arguing about mathematics without apparent teacher intervention. This is in marked contrast to the culture of many mathematics classrooms where authority for what is right or wrong, together with what counts as mathematics, rests with the teacher and the textbook. Thus, in this brief description Ursula pointed to three inter-related issues in relation to school mathematics: the children's control of the mathematics; the contrast with other people's mathematics lessons; and, her own strongly expressed belief in this way of working. This combination was typical of the descriptions she gave of TM lessons during the early stages of the project.

However, equally important to these messages was the form in which she presented them. In particular, she presented the fractions lesson not simply as an alternative to the norm, but in terms that emphasised this event's status as a deviant case. These points about authority, and the marked contrast with ordinary mathematics lessons, were emphasised by her statement that "anyone," implying, I suggest, anyone who taught in the ordinary way, would have judged the episode as chaotic. This highlights the intuitive and undeveloped nature of Ursula's beliefs in relation to mathematical authority at this stage. Ursula believed that, contrary to her own experiences, authority should be dependent not on the teacher but on mathematical discussion. However, whilst she believed this to be the case, she did not *know* it to be the case and would have had difficulty justifying this belief to others. So, despite her presentation of these beliefs as strongly held, they were not strongly warranted. Indeed, Edwards and Potter (1997) argue that strong and emotive claims like Ursula's "I loved it" are typical of the way such contested and weakly warranted beliefs are presented. Contested and uncertain beliefs need such strong support. The more consensual and certain beliefs are, the less they need such support.

I tentatively suggest here that Ursula's emergent image of a different form of mathematics teaching and, hence, this mathematical desire pre-dated her participation in Primary CAME. There was a strong suggestion in her interview responses that both were fostered by her experiences on the 20 days course that I discussed in Section 3.2 above. On this course she was able to reflect on her own negative and painful experiences of school mathematics. In particular, she began to challenge external authority figures in the form of the course tutor and through him to construct a more positive image of mathematics teachers. Bibby and I discuss this issue further elsewhere (Bibby & Hodgen, 2002).

Eighteen months later, writing a paper for an academic conference with Alexandra, Ursula described the same incident very differently:

> There followed a brief silence and then uproar. Many of the children made intuitive guesses but the result was that of an equal split between the whisky glass, the water glass and them both holding the same amount of liquid. Very quickly the pupils attempted to explain their answer and the majority of them instinctively began to draw their various glasses. Some children used colour, others used fractions, a few used ratio. (Joint paper, August 1999)

The contrast with Ursula's earlier description of the lesson is striking. In this later writing, she was certainly celebrating the children's excitement and linking this to positive changes in children's attitudes towards mathematics. However, in this extract she explicitly emphasised the mathematical activities of explanation and representation. The earlier "chaos" and "stand up arguments" have been replaced with the "uproar" of "intuitive guesses" and "pupils attempt[ing] to explain their answer" using a range of different approaches to tackling the problem. This description was preceded with a general explanation of the CAME approach of which the following is an excerpt:

> One of the outstanding features of Thinking Maths lessons ... is the enthusiasm with which classes tend to greet them. ... For several children these lessons have changed the way in which they view maths, engendering a far more positive attitude to the subject than previously held. ... In many ways this excitement reflects not just the content and structure of the lessons but, equally importantly, the fact that one important feature is the dialogue that is central to this way of teaching and learning, not just between teacher and pupil but also between pupil and pupil. The lessons are about all members of the class, including the teacher, exploring mathematical ideas and challenges together in a climate in which everyone's views are valued. (Joint paper, August 1999)

Here, the children's excitement was related explicitly to the central importance of mathematical dialogue and to the climate of the mathematics classroom. Here, unlike the earlier quote, she explicitly included the teacher in the dialogue.

Of course, these descriptions had both different audiences and different purposes. The writing of an academic paper is of a different genre and requires greater crafting than a contribution to a seminar discussion. However, a key difference here is that Ursula, working with Alexandra, was *able* to write in this academic genre, to craft the description and to provide the necessary justifications to her position. Moreover, this fluency and confidence featured widely in her later more developed discourse. The following quote, in which Ursula looked back on her earlier confusion, exemplifies this:

> To begin with the confusion with me was the lessons seem relatively closed. You seem to be closing down on children when, you know, I understand from courses, the 20 day course and looking at investigative ways of doing things where you're trying to open things up all the time. The big confusion for me with CAME is you're closing things down for children and narrowing it down all the time. Whereas now I think in any maths lesson, I think I see the questions in two ways and I can choose now which one I want to do. I can quite happily close down the children and focus in a little more. Or I can open it up wider. So I've got more the repertoire of both things, I think. (Research team, September 1998)

Here, in contrast to her earlier comment in which she invoked her existing open-ended and investigative teaching and the CAME approach as binary opposites, she saw them rather as complementary parts of her mathematics teaching repertoire: the investigative approach seeking *breadth*; and, CAME seeking *depth*. Her earlier confusion was that in seeking depth, CAME appeared to be "closing down on children." Despite the presentation by the academics of CAME as progressive and open, this apparently more closed approach seemed to her to be at times closer to the normal practice in school mathematics than to the investigative approach she had been developing.

A crucial difference in these accounts was the way in which Ursula contrasted CAME with ordinary primary mathematics teaching. In both the later accounts, rather than as deviant to the norm, she presented CAME not only as a viable alternative but also as one that encompassed and developed ordinary primary practice. The descriptions were presented less emotively and more neutrally. Moreover, in integrating the importance of mathematical dialogue with explicit mathematical behaviours, the later comments demonstrate the confidence and fluency explanations that Berliner argues are characteristic of experienced, expert teachers' discourse (cited in Brown & McIntyre, 1991). Ursula had resolved her earlier confusion about the CAME approach and her earlier intuitive ideas about authority in school mathematics were more strongly held, and, more significantly, were more strongly warranted.

The use of the strong emotive term of "love" in these extracts is of further significance. It points beyond Ursula's uncertainty and suggests that she herself held competing beliefs in relation to mathematical authority. In the first description, I suggest that she was making a strong statement about her identity as a mathematics teacher. Here, I draw on Hall's (1996) notion of identity as representing possible futures:

> Actually identities are about using the resources of history, language and culture in the process of becoming rather than being; not "who we are" or "where we came from" so much as what we might become, how we have been represented and how that bears on how we might represent ourselves." (p.4)

Within the constraints and affordances of the past and present, an individual can "explore, take risks and create unlikely connections." (Wenger, 1998, p. 185) Indeed, an individual's identity, and ultimately legitimacy, within a community depends not simply on their acceptance by the old-timers as in Lave and Wenger's (1991) early work, but on the individual's identification with the community. In expressing "love" for her image of the chaotic and different practice of CAME, a practice which as a newcomer she could only imagine, Ursula was articulating a *desire* not only for this different way of teaching but also to be a different teacher herself. Yet, because of the differences between CAME and her ordinary practices, CAME could only be imagined and partially realised.

As I argued in Section 3.2, Ursula's ability and motivation to imagine this new way of working was itself rooted in overlaps and similarities between her existing practices and those of CAME. However, Ursula's use of such a strong term to identify herself with the deviant practices of CAME indicates both the difficulty of her position and the strength of her motivation to become this different, imagined teacher.

I use the term *desire* deliberately to emphasise not just the personal and emotional investment in professional change but also the compulsion to change that Ursula experienced. There is considerable evidence in the literature that the process of professional change is confusing and painful. (See, e.g., the teachers' stories in Schifter & Fosnot, 1993.) Indeed, Ursula herself expressed extreme pain and confusion at various times during the first year of the project. Yet, at the same time, Ursula found the possibility of change deeply attractive in terms of her teaching, through excitement and interest, and beyond the classroom in terms of her professional status and future career. Despite the pain of this process, Ursula was driven to engage with CAME, a drive she expressed as love. Thus, she experienced what Lacan calls "jouissance ... which simultaneously attracts and repels." (Zizek quoted in Brown, Hardy, & Wilson, 1993, p. 14) Ursula's motivation to change was not simply that she perceived the need nor even an active choice because the new practices were interesting: it was rather that she was compelled to change through this powerful emotive and motivating force of desire.

The mixed feelings that Ursula experienced in relation to CAME are evident in the following comments about leaving a discussion "in the air." Commenting on the same Whisky and Water activity after her second experience of teaching it, she had referred to the lack of closure in the discussion as a strength: "That's what's nice about Whisky and Water. You can leave it in the air." (Research team, January 1998) However, despite her characterisation of this as nice, she later commented on her earlier discomfort with this approach, referring to Roofs, the first TM lesson, which I discuss in Chapter 4:

> I've had a big shift actually in the fact that I used to like things like Roofs 'cos you had a really exciting answer at the end of it and the

> kids were pleased, but that was it. And I actually like the lessons now where you ask them a question and they go away still talking about it much more. But I used to feel very uncomfortable with those, they used to feel that there was no conclusion to my lesson and there was nothing going for it. ... I used to love to get to the end. ... I'm much more comfortable now about just leaving up in the air. (Group interview, June 2000)

As an aside, it is interesting that here Ursula emphasised her changed beliefs by placing her desire firmly in the past, using the emotive "love" to describe her previous practice of looking for a clear end result. At the time of this interview, towards the end of her involvement in the project and as she was beginning to apply for primary management posts, she appears to have achieved some degree of closure on her mathematical desire.

However, of more significance for this argument is the way in which Ursula described herself as having working with two competing and contradictory approaches within TM lessons: a comfortable one, as in Roofs, where she was looking for closure with a "really exciting answer at the end of it"; and, a very uncomfortable one, as in these fractions lessons, where there was no conclusion and she left the mathematics "up in the air." The first approach, that of closure, was comfortable, because it was closer to the norm in school mathematics, and to her existing practices of teaching, which, although investigative, nevertheless sought closure. The latter approach, CAME's mathematics without closure, was *very* uncomfortable in part because it was *different* to her existing practices. Indeed, the "up in the air-ness", the very thing that was attractive, was also painful. This discomfort was increased by the way in which Ursula constructed this new identity as deviant to her own ordinary practices in school mathematics. Despite this pain, it is evident from the earlier comment that she found this new approach attractive. Indeed, I suggest this attraction stemmed in part from the way in which she could only glimpse these new ideas. Again this glimpsing is itself both painful, because of the uncertainty and unpredictability, and attractive, because the unpredictability is interesting. A key feature here is that the desire is for reconciliation in order to understand and overcome the unpredictability.

Yet, this desire was not simply an external force acting on Ursula. Indeed, she actively created and maintained this compulsion to change. For example, in the following quote, she commented on how she needed to convince herself of CAME's difference:

> There are bits sort of similar, but I have to make myself convinced that it is different - somehow. But I don't know why I have to do that, but I do. I have to make it seem different. The kids notice a difference. They seem to feel a difference, but I think that's because we do a whole thing in one lesson. And there aren't bits coming right from other, you know, it's not a rolling programme. You know there aren't bits coming in from here, there and everywhere. We're not going off at a tangent. We're not doing this, that and the other. (Interview, March 1998)

As I have argued earlier, similarities with her own practices enabled Ursula to engage with and interpret CAME. However, at times these similarities made the differences difficult to perceive – a difficulty that I suggest was itself both painful and interesting. Thus, she had to actively convince herself of the difference. The difference that Ursula had difficulty perceiving is something more than the surface features of the lessons, because she has to convince herself *not* to do her normal investigative practices of opening the activities up and "going off at a tangent." Ursula had to convince herself about CAME's difference to ordinary closed school mathematics and of the similarities to her investigative approach. Hence, she had to convince herself of the desirability of change.

This maintenance of difference took another form, which is illustrated, in the following advice that Ursula gave to a Phase 2 teacher:

> You'll find that you start using the ideas in your other maths lessons. But don't push it. Don't force yourself to do your other maths lessons like CAME. It'll come naturally. It's really changed my teaching. ... The structure is restrictive. ... But I've noticed with myself, and you'll find it too after you've taught a few lessons, that in other lessons you start opening up your teaching a bit. (Tutor visit, January 1999)

The language that Ursula used here is certainly significant. She suggested that the teacher should not "push" or "force" CAME ideas into her other mathematics lessons. Rather the structure, here described as restrictive and earlier as closed in, has enabled her to "naturally" change her teaching. However, I suggest that Ursula was highlighting a more significant issue than the gradual nature of change. Indeed, her advice here was to keep CAME separate and to maintain a sense of difference to ordinary mathematics practice. It would seem that in advising this Phase 2 teacher to maintain this difference, Ursula was reflecting on how she herself had developed a strong identity as a CAME teacher.

I have argued that motivation to sustain change can be understood in terms of desire, thus conceiving of motivation in terms that recognise both the pleasure *and* the pain involved in the process of change. I have discussed how Ursula actively maintained this desire through her construction of an imagined future as different to her existing practices. I suggested somewhat tentatively that this desire may pre-date CAME and may have been initially generated through her involvement in the 20 days course and the opportunity this provided to challenge her own negative experiences of school mathematics.

3.4 Alexandra's and Ursula's Professional Change

In this section, I have presented Alexandra's and Ursula's professional change as one of authorship in the context of their participation and enculturation. They did not simply adopt the new practices of CAME, but rather adapted these new ideas and constructed a new identity as a CAME teacher drawing on their existing experiences. In extending their own collaborative working relationship, I argued that the form of their working relationship was a key factor in enabling this extension, despite the fact that their collaboration was not exclusively, or even primarily, mathematical. I have argued that the motivation to sustain change can be understood in terms of desire, a conception that addresses centrally the difficulty of sustaining change.

In the next section, I focus on the remaining teachers. Their professional change was, as I have already noted, much less significant than that of Alexandra and Ursula. However, in focusing on the barriers to change that these remaining four teachers experienced, I continue to explore the notion of authorship and these teachers' roles in constructing their identities.

4. Barriers to Change: Authorship Within Limited Opportunities

I now turn to examine the learning of the teachers' for whom change was least significant and focus on the barriers to change that they experienced. My contention is that, like Alexandra and Ursula, these teachers were active learners who not only sought to make sense of their engagement with CAME, but whose participation can be understood in terms of authorship.

As with Alexandra and Ursula, my intention is not to give a detailed account of these teachers' learning trajectories. Rather I use these particular cases to focus on key issues in these teachers' learning, in particular here the factors that prevented significant professional change.

I analyse Lisa's participation in most depth. Lisa, unlike Henrietta, Janice and Tony, was involved in the project throughout the fieldwork. Given her extended involvement, her limited professional change was something of a surprise. As a result, her case involves particularly interesting contrasts with those of Alexandra and Ursula.

4.1 Barriers to Lisa's Professional Change: Similarity and Difference

As I have noted earlier, Lisa's professional change in relation to Primary CAME was very limited. Her beliefs about school mathematics did not appear to change to any significant extent. Moreover, her engagement with the project was at times very limited, particularly, as I have already discussed in Chapter 4, in terms of lesson development and Phase 2 tutoring. After the first year of the project, Lisa attended fewer meetings, left many of the meetings she did attend early and made very few Phase 2 school visits. During the second and third years, she appears to have taught few, if any, further TM lessons. I note, however, that there were elements of her participation that were relatively substantial. She took an active role in Phase 2 PD sessions, leading five lesson simulations, almost as many as Alexandra. Moreover, although Lisa appeared largely uninterested in research team discussions about future lessons, she did develop a strong role within these meetings at other times.

Lisa herself attributed her reduced participation to the project "becom[ing] more disparate as time went on" and that for herself "personally more responsibilities" at school gave her less time for the project (Lisa, Questionnaire, July 2001). Given her reduced involvement, it is understandable that Lisa did experience the project as "more disparate". Moreover, other participants, teachers and academics, talked about similar feelings at various points during the fieldwork. The research team was, for example, hit by illness and personal crises during the second year and Rhoda, in particular, described the project as becoming less exciting at this point.

In terms of Lisa's school responsibilities, she did become Acting Deputy for two terms during the third and final year of the project. This certainly entailed more school commitments. Whilst Lisa did have limited time and space for change, as did one of the teachers in Clarke's (1996) study, I suggest that this does not fully explain her lack of change. She was Acting Deputy for two terms during the first year of the project, a period when she was very involved in the work of the project. Moreover, both Alexandra and Ursula, as Numeracy Consultants, had very much greater competing commitments after the first year. A more significant issue is that she did become more focused on management issues during the second and third years, leading to her promotion during the fourth year to the headship of another Outertown school.

In the next sections, I first analyse the barriers to Lisa's change, again focusing on issues of similarity and difference. Then, I discuss how she coped with these barriers, in particular her strong perception of difference.

4.1.1 Similarities Obscure Differences

In this section, building on the earlier discussion in Chapter 4, I explore how the similarities in the language and terminology of the approaches of CAME and Robert Fisher's Thinking Skills (Fisher, 1998) obscured differences in their meaning for Lisa.

It might reasonably have been expected that Lisa's previous experience developing thinking skills would have provided Lisa with resources to draw upon in order to develop CAME practices. Indeed, as I noted in Chapter 4, this was a factor in the choice of Lisa as a teacher-researcher. Strangely, however, this previous experience appeared to hinder Lisa's understanding of CAME. She was adamant from an early stage in the project's development that CAME was concerned with general thinking skills rather than specifically mathematical ones. The following comment is typical:

> It links with Robert Fisher. We're talking about developing a community of questioning. The maths content is less important than the thinking skills and the classroom culture. (Research team, January 1998)

Lisa's approach in lesson simulations was a didactic and transmissionist one (Askew et al., 1997). She relied very heavily on reading almost verbatim from the lesson notes. She avoided discussion and focused on organisational rather than mathematical issues. This in part reflected her readerly approach to mathematics teaching that I discussed in Chapter 4. In all the simulations that I observed, she did question teachers in order to elicit different ideas and strategies. In observations of Lisa's lessons, by myself and others, there were many similar instances of Lisa eliciting strategies from children. However, she did not in any of these lessons or simulations take this discussion further and ask either teachers or children to compare or contrast the ideas she had elicited.

This may in part reflect a general difficulty with mathematics, since Lisa was the only teacher of the six not to have a school qualification in mathematics. However, of more significance is that this reflects a major difference between the Thinking Skills and CAME approaches. As I discussed in Chapter 4, Thinking Skills takes a broad curriculum focus centred around the discipline of philosophy. Taking a philosophical and ethical approach, a key aim of the Thinking Skills strategy is for children to appreciate that there are a multiplicity of different ideas and perspectives to an issue. Although the interrogation of different ideas and perspectives does play a part in Thinking Skills, this appreciation is itself important. For CAME, in contrast, working within mathematics education, the sharing of different ideas and strategies is only a preparation for the more important goal of developing mathematical ideas that make explicit connections between these strategies. Hence, Lisa's elicitation of strategies was very much closer to the Thinking Skills approach.

A further factor relates to Lisa's motivation and desire. Lisa frequently referred to Robert Fisher's teaching as "inspirational":

> Robert Fisher can work with a class he doesn't know and hasn't seen before and within minutes he can get them thinking. I mean really thinking. I've seen him do it. He just sits down and gets them thinking. He's really skilled. It's inspirational. (Fieldnotes, July 2001)

In contrast, unlike Alexandra and Ursula, Lisa never referred to CAME, or any of the other participants, as inspirational. It seems likely that she experienced desire in relation to Thinking Skills. Like Ursula, she may have achieved a degree of closure to this desire. However, she certainly did not experience desire in relation to CAME. In Chapter 4, I related Lisa's lack of initial motivation and interest to differences between CAME and her mathematics teaching. Here, I suggest that her lack of motivation as desire was due in part to similarities in the terminology of the two approaches. She recognised the language used in CAME and she had ways of behaving in response to this language. She was, in Greeno's (1998) terms, attuned to these terms. Since these terms had quite a different meaning within Thinking Skills, her attunements were in many cases not appropriate to CAME. Hence, the similarity of the terms with different meanings obscured those *differences* in meaning.

Despite these similarities in terminology, I emphasise that Lisa certainly experienced CAME as very different and at times alien to her existing practices. (See, e.g., discussions in Section 4.1.2 below and in Chapter 4, Section 5.2.1.) However, it did prevent her from perceiving *how* CAME was different.

A further consequence of these similarities in the terminology of CAME and Thinking Skills was that Lisa appeared very proficient at talking *in general terms* about CAME and this enabled her to develop a very strong identity as a participant in research team meetings. In other words, her contributions to discussions gave an appearance that she had begun to engage with CAME, although in reality this participation indicated an engagement with Thinking Skills rather than with CAME. For the other participants, this apparent competence obscured her limited engagement in CAME.

4.1.2 The Different Constraints and Affordances at CAME and Beechmount

Here I focus on differences between the practices of the research team and those at Beechmount and, how in contrast to the similarities in terminology discussed above, she perceived these as insurmountable.

This discussion is set in the context of a lesson simulation of Counting Letters, a data-handling lesson. The central task in the lesson is to compare the reading difficulty of two texts using word length. The data sets are compared using range and mode. The challenge of this TM lesson is in the interpretation of these measures and the realisation that a restricted data set may lead to bias. The simulation took place at a research team meeting in May 1998 and, unless indicated otherwise, all quotations are taken from this meeting.

At the beginning of the simulation, Lisa expressed a difficulty with the challenge of the lesson: "My cognitive conflict points to the pupils' thinking" and highlighted the following comment in the lesson notes, "In this activity the content itself is not intended to stretch the pupils." (Adhami et al., 1998b, p.30). During the ensuing discussion, Lisa sought to clarify that her interpretation was correct: the mathematical content, mode and range, were at a low level in National Curriculum terms. Rhoda responded that mode was a level 2 concept. In fact, Rhoda's answer is somewhat misleading here: identifying the most frequent or most popular choice is level 2, whilst mode is level 4 and the comparison of two distributions using mode is level 5 (DfEE / QCA, 1999). Lisa's difficulty, which she expressed as cognitive conflict, reflects two issues: her expectation that TM lessons would involve mathematics that she perceived to be difficult; and, the commonplace approach to data handling in primary mathematics, in which children procedurally work out the range and calculate the various forms of averages *but* do not interpret their meaning. Amongst other issues, the discussion focused on the nature of data-handling, which

Ursula characterised as "not giving you definite answers. ... Data-handling is posing the questions."

At the end of the simulation Lisa made the following unprompted contribution suggesting that her understanding of the conceptual difficulty of the lesson had changed:

> I wouldn't look at this on the page [of the Teacher's Guide] and come up with this activity. ... I was saying to Jeremy about my cognitive conflict earlier. This is actually very accessible to a large range of children, and I now realise that the thinking agenda is at different levels for different children, although the actual maths content is at a low level.

In my early analysis of the teachers' change I interpreted this as an example of Lisa experiencing and at least partly resolving cognitive conflict with a section titled: "Understanding conceptual difficulty in low-level mathematics" (Hodgen, 1999). Indeed I discussed my analysis with Lisa at a research team meeting in June 1999 and she not only agreed with my analysis but commented further:

> Exactly. I think that's the advantage we've had over the Phase 2 teachers. That's why our understanding is so much deeper. We've had much more opportunity to talk about our problems and confusions with the lessons. Then to teach it two or three times, talk about it and each time we refine our understanding of the lesson. (Research team, June 1999)

Lisa strongly implied here that she had taught the lesson on several occasions, enabling her to develop her understanding of the lesson in ways that I had suggested in the paper. Yet, when the project ended, Lisa told me that she had never taught the lesson because she didn't like it (Fieldnotes, July 2001). This casts some doubt on my earlier interpretation of her statement above as implying that she had begun to resolve her difficulties with the lesson. Indeed, it is somewhat puzzling that Lisa should have made such a definite statement. Although there was an expectation that each teacher would teach each lesson, there was little pressure for the teachers to express positive views about a lesson and on many occasions teachers, including Lisa, expressed considerable doubts about and strong dislikes to particular lessons.

My interpretation now is that Lisa had indeed begun to resolve her difficulty with the level of challenge in the lesson. This was in part due to the level of discussion about the importance of interpretation within data-handling. However, this partial resolution was *situated* within the context of the research team. In particular, Lisa was convinced that other team members had an understanding of the thinking agenda being at a high level. Outside this community, in her own classroom, Lisa had limited resources to draw upon in order to understand this thinking agenda. Interpretation and indefinite answers to questions were quite different to the existing mathematical practices at Beechmount. Hence, back at Beechmount, the mathematical challenges within the lesson must have seemed at best vague. That Lisa never taught the lesson, thus never engaging with this difficulty *as a classroom teacher*, prevented her from addressing this issue further. However, it is important to note here that it would have been difficult for Lisa to make a different choice given the limited resources which she could draw on to make sense of the lesson. The decision not to engage was very much a passive choice, but one that was constrained in very powerful ways by her existing practices.

The constraints and affordances within the research team were quite different to those within Lisa's classroom at Beechmount and the thought experiment of the lesson simulation was very different to the reality of teaching the lesson. In particular, the idea of a lesson which did not seek "definite answers" and which required considerable organisation in the collecting of data looked very different. Like Boaler's (2000a) students, Lisa experienced the research team and her classroom as very different communities of practice. In fact, they were so different in fact that the lesson simulation, convincing in the seminar, was very unconvincing faced by the different constraints and affordances of her classroom.

In the discussion in Section 4.1.1 above, similarities obscured fundamental differences for Lisa. In the case of Counting Letters, these differences were only too apparent to Lisa. However, given the differences in the practices of CAME and Beechmount and, in particular, the different constraints and affordances within the two communities, these differences seemed insurmountable to Lisa. Indeed, although Lisa often claimed to have had little difficulty understanding the CAME approach, I suggest that her perception was largely of difference. This difference must have been very disturbing for her, since the language of CAME suggested a strong similarity to her Thinking Skills practices. She dealt with this partly by restricting her activity in the project to areas where she felt comfortable: the research team discussions; and, her largely didactic lesson simulations. However, this strategy caused further problems for her in that her identity in these settings was dependent on activity that, after the first year, she was not participating in: lesson development; and, Phase 2 tutoring. In the next section, I discuss how Lisa reconciled these issues.

4.1.3 Reconciling Difference: Identity and Narrative

In this section, I draw on Holland et al.'s (1998) metaphor of authorship to discuss how Lisa reconciled conflicting aspects of her identity. This discussion is in the context of Share an Apple, one of the fractions lessons introduced in Chapter 4.

Lisa and Ursula had arranged to team-teach Share an Apple in September 1998. I was due to observe the lesson. However, when I arrived, Lisa was unwilling to teach the lesson. As a result, Ursula and I team-taught the lesson with Lisa observing. Lisa played no part in the lesson and her part in the discussion after the lesson was merely to question the suitability of the activity as a CAME lesson.

To my surprise, in the subsequent research team meeting, Lisa referred to "our experience of team teaching" the lesson. (Research team, October 1998) Her reference to team-teach here was not an isolated one. A month later, for example, in a discussion with a Phase 2 teacher, she commented as follows:

> I've found team teaching the lessons very useful. Jeremy and I team-taught Share an Apple. It was very useful. You can bounce ideas off each other.' (Fieldnotes, November 1998)

Indeed, two years later, Lisa further reduced my role to that of an observer:

> I taught Share an Apple though. I had to have taught that one, because I did the lesson simulation. ... You remember when you and Ursula came in to observe. That was when I taught it. (Fieldnotes, July 2001)

As a researcher I found the difference between Lisa's account and both my fieldnotes and my own recollection very disconcerting. In particular, her description of "bounc[ing] ideas off each other" seemed totally at odds with my understanding of what had taken place. I checked and re-checked the fieldnotes. I began to doubt whether my fieldnotes and my recollection were accurate, although my account was supported by Ursula's recollection. One explanation of these conflicting accounts might be that Lisa's understanding of team-teaching was very different to my own, although the notion of team-teaching had been made explicit in early research team meetings. Lisa herself perceived a considerable difference between observation and team-teaching describing the latter as being "in amongst it" (Interview, March 1998). At other times she referred to team-teaching lessons with Mundher. Mundher's recollection, however, was not of team-teaching. He described Lisa's participation as follows:

> I recall that Lisa organised the groups, gave out the slips of papers with numbers roughly according to ability, was active during pair and group work, alerted me to interesting patterns some more able and less able children found, and did herself actually call on some to say their bits. (Fieldnotes, December 2001)

Whilst Lisa's involvement in this lesson appears to have been greater than my own experience of teaching Share an Apple, her participation was not what was generally understood as team-teaching within the research team.

Although Lisa's description in both cases was an exaggeration and overstatement of her participation, I find it difficult to believe that her accounts were consciously fabricated. In part, these differing accounts arise from what Hall (1996) describes as the "necessarily fictional nature" of the "narrativization of the self" involved in constructing and maintaining one's identity (p. 4). In reconciling and negotiating their differing identities, individuals are confronted with making relatively linear and coherent sense of different events and practices which are necessarily incoherent and non-linear.

For Lisa, the events described here resulted in dramatic and painful conflicts. The idea of team-teaching was very important to the project. As a research participant it was important for Lisa to have participated in team-teaching, even where this was over-stating her involvement in a particular lesson. She was, therefore, confronted with aspects of her non-participation that threatened her identity as a teacher-researcher. Hence, Lisa's strategy, not necessarily a conscious one, was to construct, or author, an alternative history. Her use of the phrase "bounce ideas off each other" is particularly significant here in that it was frequently used by Alexandra and Ursula to describe their collaboration and in using it Lisa was reinforcing her claim to have team-taught the lesson.

I emphasise that, despite her limited professional change, Lisa was, like Alexandra and Ursula, an active participant. Moreover, the process I have described here was one of co-development and Lisa's constructions were what Holland et al. (1998) describes as improvisations:

> Improvisations are the sort of impromptu actions that occur when our past, brought to the present as *habitus*, meet with a particular combination of circumstances and conditions for which we have no set response. Such improvisations are the openings by which change takes place (p. 18, original emphasis).

However, in contrast to the authorship of Alexandra and Ursula described above, Lisa's improvisations were directed at maintaining the status quo and did not involve changes to her beliefs or knowledge about school mathematics.

4.1.4 The Barriers to Lisa's Professional Change

I have argued that the barriers to Lisa's professional change were related to differences between her existing practices and those of CAME. As I discussed in Chapter 4, her zone of enactment, viewed in terms of CAME, was limited. Hence, her perception of CAME was one of difference and at times even of alienation. I have related this difference to the constraints and affordances in the two communities: CAME, and Beechmount. However, Lisa's experience of Thinking Skills added a degree of complexity to this situation: because of similarities in the terminologies of CAME and Thinking Skills, Lisa was faced with a disturbing combination of difference and familiarity. Finally, I emphasised Lisa's own active participation in coping with and negotiating conflicts through her reconstruction of events into an alternative and more acceptable version.

4.2 The Barriers of Limited Experience

In this section, I focus on two of the remaining three teachers: Janice and Tony. I have already discussed in Chapter 4 how Henrietta's inexperience led her to experience CAME as alien.

4.2.1 Janice: Limited Experience and Shame

In this section, I discuss how Janice's limited experiences were a barrier to her professional change. In doing so, I draw on Gidden's (1991) and Bibby's (2002) interpretations of Scheff's (1994) work on shame.

When Janice initially joined the research team, her participation appeared to be intense. She drew on her Phase 2 experiences to talk about her excitement and "love" of the CAME lessons. Indeed, her enthusiasm and excitement appeared to revitalise other team members. As a result, the development of new lessons received something of a boost. In particular, Janice herself proposed a lesson, Good Enough to Eat, which she presented to the research team as a lesson simulation in September 1999. This lesson focused on the identification of prime numbers. Janice had prepared a detailed scripted scenario to introduce the lesson involving an alien visiting Earth. As I have already discussed in Chapter 4, her focus here on providing a contextualisation intended to interest and motivate children characterised her wider approach to mathematics teaching. Whilst Janice may have expected praise for this activity, the reaction of the other participants, in particular the academics, was critical and focused on identifying the mathematical challenge in the lesson. This was a typical response from the research team to any new lesson. However, Janice had not experienced this previously and may have interpreted this critical reaction as wholly negative. Certainly, thereafter, she herself did not teach Good Enough to Eat again and only conducted one further trial on a new lesson.

Indeed, although Janice presented her participation in the project as a whole as substantial and intense, it was in reality somewhat limited. I have discussed in

Chapter 4 how I found no evidence for her claims of intense collaboration about CAME amongst the Brightvale teachers. Her in-school activity, apart from one formal reflection discussion, was limited to teaching the Phase 2 lessons. Again, although she was observed by another teacher, I saw no evidence of either any discussion or any team-teaching. Yet, Janice continued to express enthusiasm and "love" for CAME in research team meetings and in interviews.

Whilst Janice continued to present her CAME teaching activity as substantial, she was more open about her experiences of teaching teachers. Here, she discusses her experience of teaching a lesson simulation to the Phase 2 teachers:

> If you think too carefully about things you can really lose confidence, you know. I have to say, and I think it's something about myself, I think it has made me lose confidence a little bit. ... When I start to think about these lessons. You know sometimes I think you know I taught that well. And I have to say, I think CAME has started to make me, does that make sense, 'cos I know that when I did that lesson at the Summerside [LEA Teachers' Centre] ... when I had to explain that, it was the one with dots and it was really complicated ... do you know I thought to myself I felt really ashamed at the end of that. I thought I did that so badly. I confused everybody. And I was really disappointed with how I'd done it. ... I thought whoever's lesson it was, I've let you down. It isn't what you wanted it to be. I haven't achieved and I felt really yuck. (Interview, June 2000)

The sense of failure and vulnerability in this account is very powerful. Indeed, her experiences are expressed in considerably stronger terms than Henrietta's sense of alienation and failure, which I discussed in Chapter 4. Janice, in repeatedly declaring her "love" for CAME so powerfully and publicly, had considerably more to lose than Henrietta. Thus she experienced her "inadequate" performance as a direct threat to her identity. However, this failure and, hence, the threat to her identity was not simply an individual disappointment in herself. Fundamental to her failure here is the imagined audience of "whoever's lesson it was" and her fantasy of this unspecified other's reactions to her performance. Janice was, I suggest, experiencing what Bibby (2002) describes as a strong sense of shame. In seeing herself "suddenly through the eyes of others" (Giddens, 1991, p.65), Janice experienced a threat to her social being:

> Shame bears directly on self-identity because it is essentially anxiety about the adequacy of the narrative by means of which the individual sustains a coherent biography. (Giddens, 1991, p.65)

Janice's experience here was similar to Lisa's identity conflicts described in Section 4.1.3 above. However, unlike Lisa who constructed an alternative and more publicly acceptable history, Janice's strategy for coping with shame was a "shutting off" strategy (Bibby, 2002, p. 715). She decided that "I don't have the skills for teaching adults outside my school" (Interview, July 2000).

I contend, however, that initially Janice had at least the possibility of not only interest but also desire. Her expressions of "love" were not simply a pretence; she *did* find CAME intensely attractive. However, unlike Ursula, as described in Section 3.3 above, she was unable to create an adequate and *useful* image of CAME teaching. As a result, whilst she at least partly perceived a need to change, she was unable to imagine *how* her practices needed to change.

A major factor in Janice's limited imagination here was her own limited professional experiences. Whilst she was an experienced teacher, her experiences were restricted ones. She had taught only at Brightvale. As I have discussed in Chapter 4, Brightvale's mathematics practices were readerly and scheme-reliant. Having been Brightvale's mathematics co-ordinator for more than 10 years, Janice had played a major role in shaping these practices.

4.2.2 Tony: Overcoming Limited Experiences

Unlike, both Henrietta and Janice, Tony did not appear to experience failure and alienation, although he certainly experienced CAME as strongly different to his existing mathematical practices, which he described as "old-fashioned." In part, this was as a result of his limited engagement in the project: his school was not involved in Phase 2 of CAME; he taught very few TM lessons; he played very little part in lesson development; he played a limited role in Phase 2 tutoring. Indeed, his early participation in the research team was very much as an outsider looking in. His inexperience was a factor here and, like Henrietta, he had limited resources to draw on with which to make sense of CAME.

However, over the course of his participation in the project, Tony, like Lisa, developed a strong identity within the research team: by the summer of 2000, as the project's funded period drew to a close, Tony was contributing frequently and authoritatively in these meetings. In Lisa's case, I argued that this was because she could draw on her Thinking Skills experiences. In Tony's case, I suggest somewhat tentatively that the development of this strong identity was related to his informal sponsorship by Ursula. Unlike all the other teachers who were identified by Rhoda, Tony as a potential teacher-researcher had been identified by Ursula from outside. In an interview involving Alexandra, Ursula compared Tony to both Janice and Lisa as follows:

> Tony to me is a typical person who's ready for that sort of change. I think the reason that I sort of got talking to him in the first place was because I think he was so much like me in the fact that he said I used to hate teaching maths and now does sort of hour and a quarter lessons and really happy to do it. He's really taken on the numeracy side of things. And I think he is somebody who really wants to learn about it, whereas I think, much as I think Janice is wonderful, I don't think Janice fits that first group as well, because she's ... very into her investigations and an open way of doing things, but she's not quite such a listener. She's still quite activity based in what she does. ... Yeah, they're [Janice and Lisa] both slightly different and I think ... Tony suits our first team really, really well. But I think maybe this is maybe why Lisa's not quite so involved because she's got still a slightly different perspective. (Joint interview, May 2000)

Significantly, Ursula described Tony as a "teacher like me" in contrast to Janice and Lisa whom she described as "different". This inclusion of Tony was typical of Ursula's reflections on the research team over the final year and I suggest that this enabled Tony's greater participation. My suggestion here, as I have already noted, is somewhat speculative. However, it does suggest that such sponsorship may be a way of overcoming the barriers of limited experience.

4.2.3 Barriers to Change: Limited Experiences

For Henrietta, Janice and Tony, their limited professional experiences were a barrier to their engagement in CAME and, thus, to their professional change. Janice, despite her considerable experience as a teacher, nevertheless had limited experiences beyond Brightvale. Hence, she had limited resources to draw on with which to make sense of CAME and, in particular, despite her strong attraction to CAME, she could not perceive in what ways her practices needed to change. She experienced failure and shame. I emphasise that inexperience as a teacher per se was not itself a barrier to change. The barrier was rather the limited professional resources, or zones of enactment, that were a consequence of this inexperience. From my analysis of Tony's participation, I have tentatively suggested that sponsorship by experienced teachers may provide a way of overcoming the barrier of limited experience.

5. Discussion

At the beginning of this chapter, I raised a number of questions concerning the process of change, the motivation to sustain change and understanding differential change.

I have extended the situated approach outlined in Chapter 4 through a consideration of Holland et al.'s (1998) work on identity, thus emphasising professional change as one involving teachers as active participants. The process of enculturation and participation leaves space for teachers to invent, improvise and *author* new practices. I have developed Wenger's (1998) notion of imagination to explain how teachers can grasp how their practices need to change.

Similarities between practices enabled two of the teachers, Alexandra and Ursula, to draw on their existing, established practices as primary teachers in order to interpret and make sense of the new practices. These practices were not necessarily exclusively mathematical. The form of Alexandra and Ursula's collaboration, the particular ways of working and acting together, was useful for the engagement with CAME. Indeed, that this collaboration took place across the curriculum, and not exclusively or indeed primarily in mathematics, appeared to be at least in some senses a strength. Drawing on this case study, I suggest that teachers' practices outside mathematics may be useful in enabling teachers' development in mathematics. A key factor here was these two teachers expected, as teachers, to be faced with discussion and disagreement in the classroom and beyond and this was part of their enjoyment of teaching.

I have argued that the motivation to sustain change can be understood in terms of the powerful emotion of desire. Here I used a conception of identity drawn from the work of Lacan (Zizek, 1992), in order to locate desire in terms of the pain and difficulty in the process. Locating motivation in terms of desire has the potential to create a different vision for teacher education.

To understand differential change, I analysed the barriers to change. I explored Lisa's case in depth. For Lisa, similarities in the terminology and language of CAME and Robert Fisher's Thinking Skills obscured fundamental differences in the approaches. However, Lisa's over-riding perception of CAME was in terms of extreme difference. I discussed the situated nature of her understandings. For example, in the setting of the research team, she perceived the Counting Letters lesson as achievable. However, in the context of her classroom, the difference to her existing mathematics teaching was so great as to be insurmountable. She could not transfer her insights about one lesson beyond the confines of the research team, because the constraints and affordances in her everyday practice were so different. Faced with identity conflicts, Lisa, like Alexandra and Ursula, improvised and constructed, although her improvisations did not involve changes to her beliefs or knowledge about school mathematics.

In the case of the remaining three teachers, I argued that their limited experience was a barrier to change. Janice, for example, because of her limited experience, was unable to imagine the new practices of CAME, although they were very attractive. Indeed, she experienced the more negative force of shame rather than desire.

6. Summary

In this chapter, I have extended the situated approach developed in Chapter 4. I have emphasised the teachers as active participants and authors of their own change, drawing on the work of Holland et al. (1998). I have continued to emphasise the importance of a wide professional network in enabling professional change.

My discussion focused first on the two teachers for whom change was significant: Alexandra and Ursula. Here, I described their professional change as one involving authorship in the context of enculturation and participation. The teachers used their existing practices to make sense of and, thus, enact the new practices of CAME. Using Greeno's (1998) notions of constraints and affordances, I discussed how the two teachers were able to extend their own collaborative working relationship into CAME. Here, I found that aspects of the form of their working practices were important, despite the fact that much of their collaboration was on teaching outside mathematics.

I explored motivation and, drawing on Lacanian notions of identity, recast this as desire. This provides a way of understanding the motivation to sustain change in terms that recognise the pain and difficulty of the process.

In my discussion on the barriers to change, I focused on Lisa. I have argued that the barriers to Lisa's professional change were related to differences between her existing practices and those of CAME. I related this to differences between the constraints and affordances in the two communities. I discussed how similarities in the terminology of CAME and Thinking Skills obscured more fundamental differences in the two approaches for Lisa. I discussed how Lisa reconciled identity conflicts by reconstructing events into an alternative and more acceptable version.

In exploring the barriers to change in relation to the remaining three teachers, I discussed how limited experience rather than inexperience per se was a barrier to change. I further discussed how Janice experienced shame as a barrier to change.

Chapter 6: Reflection

1. Introduction

In this chapter I develop the analysis of the teachers' professional change by focusing on the notion of reflection.

I address the question highlighted in Chapter 1: what is the nature and role of reflection in teachers' professional change, and how such reflection can be facilitated. In doing so, I again explore the issue of motivation and here address the issue of what motivates teachers to reflect. In order to address these questions, I extend my theoretical understanding of teacher identity.

My concern here is with explicit and deep reflection by which I mean the conscious reconstruction of knowledge. Necessarily, this is associated with significant professional change. Hence, I focus on the two teachers for whom change was considerable: Alexandra and Ursula. I examine several situations where reflective activity was observed to take place, all of which are set in the context of the development of Share an Apple and Halving & Thirding, the two fractions lessons which I have already discussed in Chapters 4 and 5. A key point to note at the outset is that, despite the emphasis placed on reflection within the CAME approach, such explicit and deep reflective events were infrequent. Moreover, reflection of this nature largely took place outside formal learning situations.

The chapter is structured as follows:

- In Section 2, I provide a brief orientation locating my interest in reflection in terms of Primary CAME.
- In Section 3, I give a brief overview of the literature on reflection from which I identify several key questions to address in the chapter.
- In Section 4, I discuss and analyse several instances in which reflection took place.

- In Section 5, I discuss the themes arising from the literature in the light of these examples and link this analysis to the earlier discussion of teacher change.

2. Reflection and Professional Change in Primary CAME

In Chapter 6, I characterised the process of teacher change as one of enactment and enculturation. I emphasised, however, that this process was an active one involving the teachers' improvisation and heuristic development (Holland et al., 1998). In order to make sense of new situations and practices, the teachers drew on and adapted their existing practices. Thus, the process of change was one of the teachers interpreting new practices rather than simply adopting new practices unchanged. Despite the teachers' central interpretative role in their own change, this process of enculturation and development appeared to be largely gradual and implicit. Indeed, although the teachers perceived the process of change as at times difficult, confusing and uncomfortable, they appeared to consciously perceive their own actual change very infrequently. For example, in looking back on lesson notes written six months previously, Alexandra commented on her perception of the intangibility of her own professional change:

> It's interesting. I suppose I must be changing, because I looked at this lesson and the questioning. I'm not sure but I think it's Ursula's questioning and I like Ursula's questions. And I thought on some bits, "Well, I wouldn't necessarily do it like that. Maybe that's leading them a bit too much." (Fieldnotes, January 1999)

Her comment that "I suppose I must be changing" succinctly encapsulates both her own and Ursula's perceptions of change. In the face of the evidence of the lesson materials, she could see her practice of questioning had changed, but she could not identify a point at which her questioning had actually changed. Indeed, she felt that her changed questioning practice was how she had always taught.

However, she did consider time set aside for formal reflective activity as important. In the following quote, Alexandra notes the value of reflection in terms of her tutoring work with a Phase 2 teacher:

> I think actually having that reflective time straight afterwards to talk about it and just mull it over a bit is quite, quite valuable. And actually I find it easier then because, you know, she, for me, I almost don't need to ask the questions, she's creating the input now, which is, which is good. (Interview, Alexandra, March 1999)

Whilst Alexandra highlighted the value of explicit discussion straight after teaching, she presented the actual practice of reflection as elusive and intangible. She and the teacher mulled the lesson over. Alexandra "almost" did not need to ask the questions, because the Phase 2 teacher was "creating the input." Indeed, this mirrors very closely the way she presented her own change above. Reflection, in her view, did facilitate teacher change, yet the actual change was hard to pin down.

As I noted in Chapter 2, reflection was a key component of the CAME PD approach, as promoted by the academics. Mundher highlighted the importance of reflection as follows:

> The main concern in my mind is about how to package a cycle of the professional development in such a way that there is a kind of conscious beginning, a conscious couple of steps and then a conscious reflection and this is finalised in this reflection and it [the cycle] ends with a relative equilibrium (Interview, June 1998)

I note the importance he places on the *consciousness* of the teachers' professional development and of *conscious* reflection as "finalising" the cycle, in contrast Alexandra's image of reflection as a gradual and less explicit process. His aim was for the whole cycle of teacher PD – not simply the point of reflection – to be a conscious and explicit process for the teachers. I note the emphasis he placed on reflection in finalising the cycle of professional development. Moreover, his concern was to package and formalise this process in order to facilitate this consciousness on the part of teachers. Hence, in research seminars, time was set aside to formally reflect on lessons taught. However, I actually observed such explicit reflection very infrequently and very rarely indeed within formal reflection sessions. Of course, the fact that I did not observe it does not mean that reflection did not take place. However, if reflection, understood here as a conscious and explicit transformation of knowledge, had been commonplace, it seems likely that, given the norms and culture of the seminars, the teachers would have commented on it more frequently.

Nevertheless, there were instances of what appeared to be explicit teacher reflection and, whilst these were infrequent, they did appear to be powerful and significant events in the teachers' PD. Before I examine these instances in detail, I consider briefly the literature on reflection in relation to mathematics teacher education.

3. Reflection in Mathematics Teacher Education: An Overview

Reflection is something of an ubiquitous idea within teacher education. Many commentators highlight the crucial role that reflection plays in mathematics teacher education (See, e.g., Clarke, 1994; Cooney, 1994b; Grouws & Schultz, 1996; Jaworski & Wood, 1999). However, this apparent consensus conceals some differences in meaning. Grimmett (1988), for example, lists several different conceptions of reflection:

- thoughtfulness about action - thoughtfulness that leads to conscious, deliberate moves usually taken to "apply" research findings to practice;
- deliberation and choice among competing visions of "good teaching", consideration of educational events *in context*, and anticipation of the consequences following from different lines of action, taken from these competing visions of good teaching; and,
- reconstructing experience, the end of which is the identification of a new possibility for action (p. 12, original emphasis)

I do not wish to deny the importance of thoughtful and deliberate application or consideration of competing approaches. Indeed, thoughtfulness of this kind was often evident in the teachers' deliberations. However, in this discussion, my focus is on teacher change. Hence, I am concerned with the last and strongest of these definitions: reflection as the reconstruction of experience and knowledge. I note that in the context of my analysis so far that this strong conception of reflection necessarily implies the reformation and reconstruction of identity.

The work on this strong conception of reflection draws largely on two distinct theoretical strands: on the one hand, constructivism and the work of Piaget; and, on the other, the work of Schön (1983) and his notion of a reflective professional. From a constructivist perspective, Wood and Turner-Vorbeck (1999) argue that reflection is central to teacher education. Mathematics teacher learning, they argue, "involves interpretative constructions and reconstructions in thinking through processes of reflection on the activity of self and others" (p. 174). Working within a similar tradition, the cognitive acceleration approach conceives of reflection in terms of metacognition or "becoming conscious of [one's] own thinking" (Adey & Shayer, 2002, p.6). Indeed, Adey and Shayer argue that such reflection can only take place after the action being reflected upon:

> The requirement for consciousness means that it is a process that must take place *after* a thinking act, since at that time a student is engaging in a problem-solving activity their consciousness must be devoted to that. ... The value of this type of metacognition [is] in making general thinking processes explicit, and thus more readily available for use on other occasions. (p. 6)

Schön (1983) also notes the importance of explicit reflection after the act of teaching, which he terms reflection-on-action. However, in contrast to the cognitive acceleration of Adey and Shayer, he sees reflection-*on*-action as leading to reflection-*in*-action, a deeper and more developed from of reflection taking place explicitly and consciously during the process of teaching. Nevertheless, Schön emphasises the role of reflection-on-action and thinking about action after the event as a precursor to reflection-in-action.

In practical terms, reflection remains a somewhat elusive concept. It is unclear how teachers' reflection can be facilitated or encouraged. Indeed, there is considerable evidence that enabling teachers to reflect is a far from simple task. Cooney (1994b), for example, argues, "no magical way exists to promote reflection" (p.16) Several authors (e.g., Clarke, 1994) argue that substantial time should be allocated within PD programmes for reflection, echoing the importance placed on formal reflection sessions by CAME as noted above in Section 2. However, whilst the provision of time may be a necessary condition for reflection, it is not a sufficient one. Cooney (1994b) highlights motivation, an issue that I have identified earlier. He argues that simply telling teachers that reflection is a good thing is unlikely to be universally successful: "they must see that for themselves." (p. 19) In a similar vein, Clarke (1994) argues that reflection is an active not a passive process. It is dependent on teachers perceiving a "need to become articulate, to be communicative, or to use thoughts as objects of systematic attention with their colleagues." (p. 44). Thus far, I have addressed the teachers' motivation to engage with change. However, motivation to reflect is qualitatively different to this in that it involves a reconstruction of knowledge and a recognition of change.

Given the concern with reflection *on* one's own activity, it is perhaps unsurprising that several authors use physical metaphors of distance to convey the difficulty of this process. Wood and Turner-Vorbeck (1999), for example, highlight the difficulty and complexity of "decentering." Cooney and Shealey (1997), indeed, link these physical metaphors to the notion of a teacher's willingness to reflect:

> A precondition for the act of reflection is the ability of the person to decenter and view his or her actions as a function of the context in which he or she is acting. Schön's (1983) reflective practitioner, a notion that enjoys so much credence in the field of education, cannot exist unless the individual is willing to step out of himself or herself and view his or her actions from a relativistic perspective. (p. 100)

I find the argument for reflection and the metaphor of distancing persuasive. Indeed, as I have noted above, the teachers' reflections, although infrequent, did appear to be significant events in their professional change. My aim in this chapter is to analyse several of these reflective events and to place these within the social context of teachers' identities. In doing so I address the following further questions: How does reflection actually take place? What motivates teachers to reflect? Building on the metaphor of distance, how can teachers "step outside" themselves in order to reflect on their own change?

4. Identity and Reflection

In the last chapter, I located teacher identity within different communities of practice and, thus, stressed its fragmentary and discontinuous nature. I explored how such discontinuities (if not too great) could enable teacher change through what Wenger (1998) refers to as identity reconciliation. In short, the teachers' perceived these discontinuities as tensions needing resolution, whilst overlaps and similarities between practices facilitated such resolution, by providing interest, facilitating desire and enabling teachers to adapt their existing practices.

In this chapter, I make use of a slightly different take on teacher identity, drawing partly on the work of Schifter (1996). Schifter conceives of teacher change in terms of "narratives of professional identity" (p. 2). At the same time, she stresses the plurality of teacher professional identity:

> These teachers enact multiple identities: as mathematical thinkers, as managers of classroom process, as monitors of their students' learning, as colleagues, and as members of the wider education community. "identities" in this sense – more a matter of what one does than who one thinks one is – are constructed in and realised through practices. (p. 2)

It is important to note here that, whilst Schifter sees these different aspects of identity as constructed through practices, these identities cut across the communities of practice a teacher participates in. Thus, in contrast to the notion of identity as membership of a distinct community, this conception of a teacher's identity might be as a mathematical thinker, for example, which could be enacted in a variety of distinct communities, including the classroom, planning sessions with colleagues, the wider school, community, professional communities and more. Like Schifter's teachers, the teachers in this study, at the same time as participating in the research team, were developing different aspects of their identity as primary teachers, including their identities as mathematical thinkers, for example. And, like Schifter's teachers, this went beyond the strict confines of their participation in the immediate professional development initiative. Through their participation in Primary CAME, they were developing their identities as lesson developers, as tutors, and as teacher-researchers. However, these wider aspects of their identity drew on a variety of practices and, thus, provided a link between their membership of different communities. For example, in developing an identity as a tutor, both Alexandra and Ursula drew on experiences and practices within school, as Numeracy Consultants as well as their Primary CAME tutoring work.

The distinction I make here is somewhat akin to that made by Holland et al. (1998) between figurative and positional aspects of identity that I discussed in Chapter 5.

Here, I relate positional identity to the teachers' membership of specific local communities. By figurative identity, I mean aspects of identity that cut across these local communities and, thus, reflect the teachers' participation within wider, more dispersed and plural discourse communities, which Wenger (1998) refers to as "a complex social landscape of shared practices, boundaries, peripheries, overlaps, connections, and encounters." (p. 118)

Whilst the discontinuities between the different positional aspects of the teachers' identities were crucial in how the teachers engaged with and interpreted new practices, it appeared to be these wider and more figurative aspects, and the fact that they encompassed the teachers' practices across different communities, that enabled the teachers to step outside themselves and thus reflect. Being a tutor, or a lesson developer, or indeed simply engaging with a colleague in collaborative team-teaching enabled the teachers to distance themselves from their identity as a classroom teacher or a doer of mathematics and to become aware of and reflect upon their activity and, thus, explicitly reconstruct their knowledge.

I will now develop this analysis through a discussion of four examples of reflection. (See Hodgen, 2002, Forthcoming for further discussion.) All are in the context of the fractions lessons that I introduced in Chapter 4. In the first example, I briefly give an example of reflection in the context of lesson development. In this case, reflection took place as part of rather than separate to the teacher's practices of lesson development. In the second, I explore reflection in the context of a team-teaching experience. Here, I focus on the notion of distance and suggest that the presence of another teacher was a crucial factor in locating and grounding the reflection. In the third example, I will explore the opportunities for reflection in the formal reflection sessions during the research seminars. I will suggest that whilst learning did take place, the role of explicit and conscious reflection in these sessions was limited. I will describe how reflection did take place later, but was prompted by the two teachers' experience of writing a paper for an academic audience. In the fourth and final example, I will look more closely at Alexandra's beliefs about school mathematics. The discussion focuses on an informal and impromptu discussion between Alexandra and myself that took place immediately after a Phase 2 tutoring visit. Again, in contrast to the formal "reflection" sessions, reflection did appear to take place, as part of, rather than separate to, the activity of tutoring. Moreover, the transformation of Alexandra's beliefs was in this case very significant in terms of school mathematics. I describe how reflection was enabled by the opportunity provided by the experience of tutoring to a distancing, which was itself and the ability of a teacher to "step outside" her identity as a teacher of mathematics.

4.1 Lesson Development: Reflection as Part of Practice

The writing of lesson materials was often the prompt for such reflection. For example, in drafting the final lesson materials for Halving and Thirding, Ursula commented on the earlier teaching notes, which she herself had written 18 months previously:

> They're too led. It's too much about getting an answer. The point is to get children thinking around the issues, about what fractions mean, to open up what's quite a standard school activity. These are too much like get to the answer in each episode, then go on to the next one. (Research Team, October 2000)

Thus, the experience of writing lesson materials enabled Ursula to reflect on her own change. The lesson materials provided a record of Ursula's previous thinking, thus enabling a distancing from her previous practice, in this case teaching that was "too led," whilst at the same time providing a prompt for reflection. A further factor was, I suggest, that her role here as a lesson developer enabled her to locate her previous practice within a wider perspective and provided the opportunity for Ursula to "imagine" a different teaching practice (Wenger, 1998).

Although this took place at a research seminar, I stress that the reflection was made during small group work on lesson development not in a formal reflection session. Significantly, this reflection took place as part of the "authentic activity" (Lave, 1991) of writing lesson materials rather than in a separate formal reflection session. Indeed, a second key aspect for reflection, alongside the opportunity to distance oneself, was the teachers' engagement with practice. The reflection that I observed generally took place during, or shortly after, the teachers' engagement in authentic activities, like teaching, lesson development, or tutoring, rather than in formal reflection sessions.

4.2 Team-teaching: Imagination, Engagement and Other Teachers

The team-teaching experience of Share an Apple that I discussed in Chapter 5 in relation to Lisa, provides a further example of reflection as integral to project activities. Shortly after the lesson, Lisa made comment "I don't see the value of this lesson." This in turn prompted Ursula to say: "This is a Thinking Maths lesson. It's making us think. ... It's like Picturing Numbers in that it opens up a closed activity." (Fieldnotes, October 1999) Thus, the experience enabled Ursula to link her insight about the Share an Apple lesson itself to a lesson she had up until that point expressed a very strong antipathy to, for example: "It's like what I would normally do, but closing it down." (Research Team, June 1998)

For Ursula, Picturing Numbers had signified all that was confusing about CAME. She constantly referred to the lesson when talking about the difficulties she had in grasping what was special about CAME. She described the lesson at various times in somewhat contradictory terms as "airy-fairy", "not special", "too difficult", "too easy", "not my cup of tea", "too closed", and "just what I normally do." (Interview, March 1998; Research team, December 1997, March, May & July 1998) Indeed, this was the first time she had made a positive comment about this lesson. Moreover, in highlighting the opening up of a closed activity, she placed her earlier contradictory descriptions of the lesson within a key aspect to CAME's mathematics without closure approach. (See Chapter 2.) Hence, this was I suggest a very significant reflective event in her professional change.

There were two critical factors involved in this reflection. Firstly, this lesson took place as preparation for the first Phase 2 PD session. Hence, a key concern was the issue of how to present the ideas to the Phase 2 teachers. The experience of team-teaching and the consequent movement between teaching and observation – as a tutor rather than as a teacher – enabled Ursula to step outside her identity as a teacher. Hence, she was able to both engage with the teaching and imagine how the teaching could be different. Wenger (1998) argues that this "combination of engagement and imagination" is very powerful:

> Such a practice combines the ability both to engage and to distance – to identify with an enterprise as well as to view it in context, with the eyes of an outsider. Imagination enables us to adopt other perspectives across boundaries and time ... and to explore possible futures ... [and thus] trigger new interpretations. In turn, engagement provides a place for imagination to land, to be negotiated in practice and realized into identities of participation. (p. 217)

Secondly, and perhaps more significantly, was the presence of Lisa as an observer and that Lisa's reaction was very similar to Ursula's own initial reaction to Picturing Numbers. Not only was Lisa's comment about the value of the lesson similar to Ursula's early comments, but, like Lisa, who had opted out of teaching Share an Apple, Ursula had opted out of teaching Picturing Numbers because "I just couldn't get into it" leaving the teaching to Rhoda. (Research Team, December 1997) Lisa's presence, I suggest, enabled Ursula to remember, locate and thus reflect on her previous practice.

4.3 An Audience: The Imperative to Reflect

During the first year, there were three formal "reflection" sessions during the research team seminars in which the discussion focused on the teachers' experiences of teaching the fractions lessons. As I have discussed earlier in Chapter 4, these discussions generated considerable excitement amongst the King's researchers about the lessons' potential as TM lessons.

However, despite this very positive validation of their work, both Alexandra's and Ursula's reactions to these reflection discussions were largely negative. A particular focus for the teachers' criticisms was the academics' attempts to introduce a more explicit theoretical base to the lessons. During the second reflection discussion in January 1998, the university researchers led a discussion about children's errors, strategies and misconceptions in the area of fractions informed by previous research at King's. (See, e.g., Kerslake, 1986). This discussion largely focused on distinguishing part-part and part-whole relationships, children's difficulties with fractional notation, and different meanings of fractions. A further discussion in May 1998 was focused on Mundher's mathematical background notes to the children's mathematics in both lessons, which were essentially a summary of the January 1998 discussion.

In March 1998, Ursula commented that the January discussion was "way beyond" her needs in developing the lesson:

> I mean the input we've had into Fractions so far has been an argument about, I don't know, some mathematical term or whatever between them [the academics] that hasn't been helpful to Alexandra and I in developing the lesson at all. One off chats with Mundher have, because they've been about developing the first part of the Year 5 lesson. But then what we've had actually at the meetings has not been helpful, because it's too high powered and it's not related to the task necessarily from my eyes. It's way beyond it. (Interview, March 1998)

Indeed, a year later she still remembered this discussion as a "nightmare". Similarly, during the second discussion, Alexandra commented that she felt this explicit research base was unnecessary, because "these background notes, unless you've got somebody who's particularly interested in the sort of mathematical side of it, this is too complex", a comment which Ursula had also affirmed: "Yes, far too complex, Mundher." (Research team, May 1998)

The two teachers were unspecific about what exactly was too "high powered" or "complex" and this may in part have been a defensive reaction. However, they had themselves highlighted children's difficulties with fractional notation and representation in their initial presentation of the lesson. Whilst the academics certainly focused on a more theoretical understanding of these issues, the only extra issue that the academics had added were part-part and part-whole relationships.

Therefore, it would seem that the teachers' perception of difficulty lay in relation to this, which Mundher presented as follows in his background notes:

> In both lessons the distinction is explored between the part-whole relationship expressed in fraction, such as 1/3, and the part-part ratio relationships within the whole, '1 to 2', which we at this stage avoid to express and leave implicit. It is a fact that in the 1/3 example we prefer, as adults, to relate the one part to a constructed or original total of three parts while we *see* only two parts, one of which is twice (or half) the other. It is curious that the ratio 1:2 is a more concrete representation of the outcome of finding a third of something. The fraction notation, however, is more elegant in the sense that it preserves the action, and allows easier manipulations of several fractions. A clash is evident between a perspective that looks at what is here and now, and one that is geared to subsequent use of notation. (Project memo, May 1998, Original emphasis.)

In this document, Mundher certainly took something of an idiosyncratic position on the relative difficulty of part-part and part-whole relationships and his writing style was at times dense. Moreover, in each of these reflection discussions, there was considerable disagreement between the academics about the particular difficulties children face. Nevertheless, the characterisation of ratio and proportion, or indeed any of the issues raised in these discussions, as way beyond the teachers' needs in developing this lesson is somewhat strange, and suggests that these teachers did not appreciate the importance of what Ma (1999) terms *longitudinal coherence*, or that "teachers are not limited to the knowledge that should be taught in a certain grade; rather, they have achieved a fundamental understanding of the whole elementary mathematics curriculum" (p. 122). Indeed, ratio and proportion do feature in many standard textbooks on mathematics for primary teachers (e.g., Williams & Shuard, 1994) and the topics are now introduced at Y4 in the NNS (DfEE, 1999). The ideas that Mundher described are, moreover, certainly no more challenging than the multiplication of fractions at the heart of the Whisky and Water activity.

In August 1999, 18 months later, writing a paper to present their lesson development work to an academic conference, Alexandra and Ursula took a rather different position in relation to the mathematical background. In this paper, entitled "Being a teacher and doing research," they discussed the development of the two fractions lessons, making use of a variety of project materials, including Mundher's background notes, my fieldnotes of research seminars, and draft lesson materials. Their comment about the mathematical background was as follows:

> [The agenda for] the finalised Year 5 lesson, Share an Apple (see attached background notes for clarification) [Mundher's original background notes] ... can be summarised as: meaning of fractions; notation; adding simple fractions; comparing size of fractions. ... [The Y6 lesson] naturally follows within the spiral curriculum of the CAME lessons. Like Share an Apple, it deals with exploring the distinction between the part-whole relationship expressed in a fraction, and the ratio relationships between the parts that make up the whole. (Academic paper, Alexandra & Ursula, August 1998)

Alexandra and Ursula referred explicitly to the content of the mathematical discussions, including the issue of part-part and part-whole relationships, which they had previously described as "too high powered". Moreover, they referred to Mundher's mathematical background notes as clarification: notes, I emphasise, which they had earlier described as "too complex." Significantly, I later observed Alexandra and Ursula lead discussions with primary teachers using this more explicit knowledge of children's learning difficulties. At the lesson simulation of Halving & Thirding, the Y6 lesson, in February 2000, Alexandra used their joint paper together with the academics' mathematical background notes as the basis of a presentation on children's difficulties with fractions. In June 2000, Ursula led what she described as a challenging discussion on the mathematical meanings of ratio and proportion at a National Numeracy Strategy training session. Hence, whilst their initial reactions to these ideas were negative, they did later appear to come to appreciate at least implicitly the value of what they perceived at the time to be a very challenging and mathematically difficult discussion.

Certainly, then, professional change had taken place. However, my focus in this discussion is on how they learnt it, and in particular what role, if any, reflection played in this learning. I emphasise again that, despite the emphasis placed on reflection within the CAME approach, at no point in the formal discussions did I observe the teachers explicitly reflect. Of course, the fact that I did not observe reflection taking place does not mean that such activity was completely absent. However, my observations of the teachers' frustration together with the teachers' comments suggest that little learning of a substantial or transformative nature took place within the formal reflections sessions.

In contrast, the writing of the academic paper appeared to be a prompt for the teachers to reflect. Ursula sent me a draft of the paper with the following comment:

> Another one in the morning job over a bottle of wine and plenty of disagreements! We actually looked back over Mundher's notes and found them useful. Made everything fall into place. (Ursula, Personal communication, 5 August 1999)

The imperative for a considered analysis was a critical factor in enabling significant changes in their mathematical knowledge for teaching. Holland et al.'s (1998) argument about the necessity of authorship is particularly appropriate in this case: "the world demands a response – authoring is not a choice." (p. 272) The two teachers had to produce an academic paper and an academic paper very powerfully required them to resolve some of their previous difficulties with the mathematics in the lesson. It is significant that they referred to Mundher's notes as enabling everything to "fall into place," since they had judged these earlier as "far too complex."

Although important, the writing of this joint paper was certainly not the only event in Alexandra and Ursula's professional development. The "nightmarish" research team discussions, with the written materials as an aide-memoire, provided them with the mathematical language in which to frame this subsequent reflection. However, these discussions in themselves did not produce transformations in the teachers' mathematical knowledge – rather they supplied the material, in the form of mathematical artefacts and tools, for subsequent transformative reflection. A further critical factor was that Alexandra and Ursula were engaged in this activity as teacher-researchers. This distancing enabled them to reflect on their activity as teachers whilst Mundher's notes, I suggest, provided a vivid reminder of their previous experiences. Thus, the act of writing, and the consequent need to analyse their earlier work, provided both the possibility and the necessity to step outside their identities as teachers of mathematics. However, this distancing was accompanied by a grounding in their previous experiences

A further feature of this event was the way the teachers themselves appeared to be catalysts in their own learning. Whilst not entirely spontaneous, since it was prompted by writing the paper, the teachers were themselves working together apart from the research team. Moreover, in taking place until late into the night, at Ursula's home and socially over a bottle of wine, there were very strong similarities with their existing collaborative practices of inter-linking the professional and the personal that I discussed in Chapter 5. So, whilst they were engaged in the practice of academic writing, they were distanced not only from their own initial engagement as teachers but also from their position as teacher-researchers within the research team.

4.4 Tutoring: Reminders of Oneself as a Learner

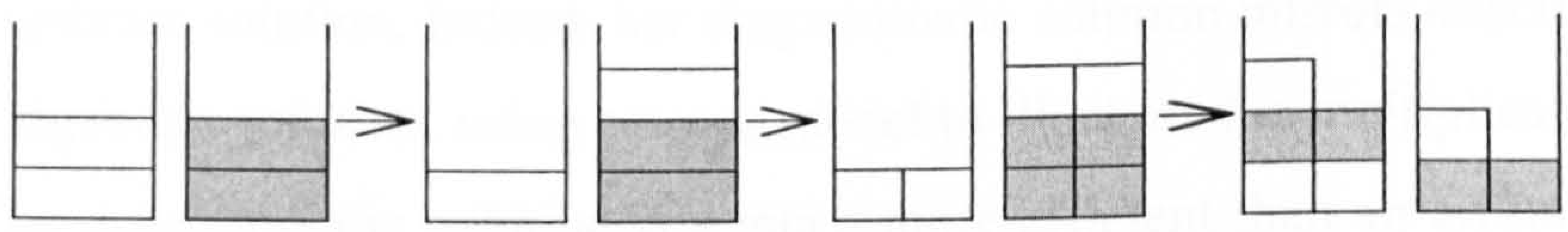
The examples that I have discussed so far involve one-off examples of reflection. In the following example, I explore a series of reflections that enabled Alexandra to begin to transform her beliefs and knowledge about school mathematics. In the main, this discussion relates to the reflection discussion at the first seminar in January 1998, to a discussion following a tutoring visit between Alexandra, another Parkway teacher and myself in January 1999, and to an interview in March 1999. The starting point for these reflections was the Whisky and Water problem, which I first discussed in Chapter 4. Given the importance of this context for this discussion, I restate the problem below:

> I have two glasses. One glass contains whisky, whilst the other contains water. If you pour half of the whisky into the water, mix it up, then pour half of that quantity back into the original whisky glass, which glass now has more whisky?

In January 1998, when the Whisky & Water problem was presented to the research team, each of the academic researchers attempted to solve the problem using an arithmetical / algebraic solution. (See, e.g., Figure 6.1.) On the other hand, Alexandra had previously solved the problem using diagrams. (See Figure 6.2.)

	Contents of Glass A	Contents of Glass B
To begin with all the whisky is in Glass A and all the Water is in Glass B	All whisky (X)	All water (Y)
Pour half whisky in Glass A into Glass B	$^1/_2 X$	$Y + {}^1/_2 X$
Pour half of the whisky & water mixture back into Glass A	$^1/_2 X + {}^1/_2 (Y + {}^1/_2 X)$	$^1/_2 (Y + {}^1/_2 X)$
Each glass contains an equal quantity of water. Glass A contains more whisky ($^3/_4$ of the total)	$^3/_4 X + {}^1/_2 Y$	$^1/_2 Y + {}^1/_4 X$

Figure 6.1: An arithmetic / algebraic solution to the Whisky & Water problem.



Step 1	Step 2	Step 3	Step 4
"I imagined the whisky [clear] and the water [shaded] divided in half."	"I poured half the whisky into the water."	"I imagined the liquid in each glass divided in half."	"I poured the mixture of whisky and water back into the original whisky glass. You can see more of the whisky is in the original whisky glass."

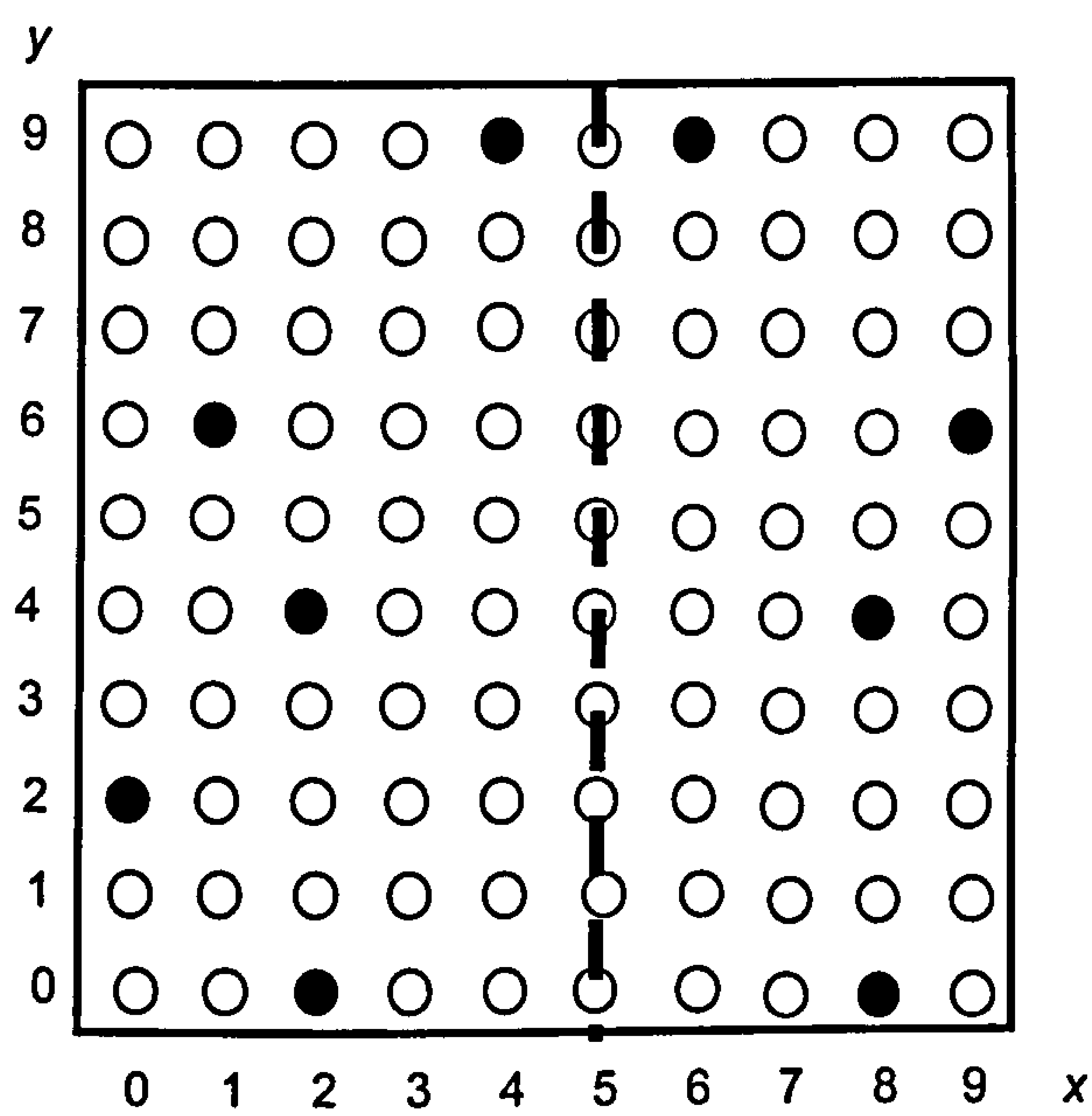
Figure 6.2: Alexandra's diagrammatic solution to the Whisky & Water problem.

When pressed to share her solution, Alexandra protested that, "It's not scientific". Her belief appeared to be that her solution, although perfectly appropriate for everyday problem solving and despite producing a convincing solution, was not truly mathematical, because it used diagrams. In short, she believed that her solution could not form the basis of a truly mathematical argument. Alexandra's belief that her pictorial solution was not mathematical was shared by some of the Phase 2 teachers. At the lesson simulation of Halving & Thirding, in February 2000, having solved the problem using a method similar to Alexandra's, a group of Phase 2 teachers spent a considerable amount of time looking for "a *mathematical* way of proving this" [emphasis added]. In the subsequent reflection session a month later, the same group of teachers returned to the topic discussing whether a child's verbal explanation of a diagrammatic solution could be classified as mathematical. All these teachers accepted the diagrammatic method as a solution to the problem, but they found it hard to accept that this method was mathematically valid.

In fact Alexandra's diagrammatic solution is both mathematically elegant and rigorous. Moreover, such solutions are a recognised element both of mathematical problem-solving (e.g., Polya, 1957) and of mathematical proof (e.g., Waring, 2000). In her solution she "imagined" that, although the liquids are mixed completely, she could still separate out the whisky and water in each glass in order to solve the problem. This is exactly the same reasoning step that is needed for an arithmetic / algebraic solution. Indeed, her diagrammatic solution mirrors exactly the arithmetic / algebraic solution, using an area model to illustrate the multiplication of fractions. The diagrammatic solution is a much more efficient than an arithmetic / algebraic solution in generating answers to a variety of related questions regarding the problem. For example, the ratio of whisky to water in each class, and hence the strength of the two mixtures, can simply be visually read off the final diagram. In contrast, the arithmetic / algebraic solution requires further manipulation to answer this second question, although the arithmetic / algebraic solution is more general in that it covers cases where the original amounts of whisky and water are different. Alexandra's solution was indeed judged a mathematically better solution in this case by the teachers and, significantly for Alexandra, by the King's researchers.

Alexandra was both pleased and excited at the positive reaction to her solution, exclaiming "Oh, yes!" (Research team, January 1998) This pleasure and excitement did not in itself result in a fundamental shift in her mathematical thinking. Indeed, she subsequently described her solution as "just my little way of doing it" (Fieldnotes, May 1998), a description which strongly suggests that she did not fully value her method as a *mathematical* solution. However, the experience did appear to provide the basis for a further reflection.

A year later, following a tutor visit to a Phase 2 school in January 1999, Alexandra appeared to experience a sudden insight about the mathematical validity of diagrams. Alexandra had taught another TM lesson, Pegboard Reflection, with the teacher observing. Pegboard Reflection is a lesson in which children explore number relations in the context of a reflection in the line $x = 5$, using pegboards to model the Cartesian co-ordinate system. (See Figure 6.3.) The transformation is then represented algebraically. Hence, like the two fractions lessons, connections are made between algebraic and diagrammatic representations, although the context in this case is relationships between numbers.



Reflection in $x = 5$

(4,9)	→	(6,9)
(1,6)	→	(9,6)
(2,4)	→	(8,4)
(2,0)	→	(8,0)
(8,0)	→	(2,0)
(0,2)	→	(10,2)
(x, y)	→	(10 - x, y)

Note: The points where $x = 0$ and $y = 0$ are included on the pegboard to emphasise that these represent the x- and y-axes. Additionally, the reflection of any point where x = 0 cannot be placed on the pegboard.

Figure 6.3: The Pegboard Reflection activity.

After the lesson, Alexandra had had a long discussion with the Phase 2 teacher in which she had to justify the context of the co-ordinate system in a CAME lesson in response to the teacher asking: "What's difficult about co-ordinates?" A particular focus of Alexandra's response was to emphasise "counting the zero" in identifying the co-ordinates of a point. These issues had themselves been discussed during the reflection sessions at research seminars. Alexandra and I then returned to her own school, Parkway, to discuss the visit.

In the Parkway staffroom, Alexandra initiated a discussion with another Parkway teacher about number lines and their mental images of numbers with: "You know, the way I picture numbers is in steps. Steps of 1 up to 20, then steps of 10 up to 100, then steps of 100." I then suggested that this linked to her discussion with the Phase 2 teacher about the co-ordinate system. Alexandra responded as follows:

> But it's different isn't it. On the number line you're counting steps, but with the co-ordinates you're counting the zero, aren't you. So it's different. You're counting steps on the number line and you're counting points with the co-ordinates [Long pause] No, it isn't. They're the same thing really. I've just realised that. Counting the zero means you're counting the steps. ... Co-ordinates are like a 2D number line. (Fieldnotes, January 1999)

This appeared to be a very intense experience for Alexandra. My fieldnotes record it as follows: "It felt like ideas slotting into place there and then. ... like an 'ah-ha' moment, where this suddenly occurred to Alexandra." Indeed, this was one of the few times that I observed any of the teachers experience a conscious and explicit revelation of this type.

Whilst she expressed this in a slightly clumsy way, the connection Alexandra made between number lines and the co-ordinate system is a very significant one, since as she recognised here Cartesian co-ordinates are formed by two perpendicular number lines. Although the immediate prompt for this was my comment, Alexandra's discussion with the Phase 2 teacher was I suggest more crucial. The Phase 2 teacher had confronted her with a problem for which she had no set response, yet as a CAME tutor, she expected herself to be able to respond. However, in constructing her response, she drew on earlier research seminar discussions. This link between Cartesian co-ordinates and the number line had been made very explicitly during these seminars. Indeed, Mundher had earlier introduced a preparation activity in which points on a single number line were reflected in the point $x = 5$. (Draft lesson materials, September 1998) Yet, despite these prolonged discussions in which she took an active part, it appeared that she had not fully grasped this connection until this reflection.

What appeared to be crucial to Alexandra making the connection for herself, was the necessity to justify the challenge of the lesson in her role *as* a tutor. In her role as a tutor, she had been forced to communicate articulately with the Phase 2 teacher requiring her to justify the importance of co-ordinates. Whilst this discussion was not focused on the Whisky and Water problem, this problem appears to have prepared the ground for her insight about the validity of mathematical diagrams. In this case, the discussion about children's understandings appears to have created a need to resolve issues within her own learning of mathematics. As a tutor, she was able to step outside and reflect on her identity as a learner and doer of mathematics. In addition, it seems likely that, as in Ursula's Share an Apple experience discussed above, the presence of the Phase 2 teacher helped Alexandra to vividly remember and thus engage with her earlier experiences in the research seminars. Hence, as in the reflections discussed earlier, the distancing from herself as a teacher, afforded by her identity as a tutor, was grounded by a concrete reminder of her previous engagement.

This first reflection itself prompted Alexandra to reflect further:

Alexandra:	Thinking about that it was something no-one really made clear to me at school. You know that something like quadratic equations have a spatial meaning. No-one made the connections between the spatial and the number system.
Jeremy:	A bit like Whisky and Water.
Alexandra:	Yes, like at school we just did fractions using fraction notation, you know using the procedure to multiply and add fractions. No-one ever made it clear that diagrams were just as mathematical.

(Fieldnotes, January 1999)

Here Alexandra linked her earlier insight into the co-ordinate system to the grander notion of linking spatial and numerical, or arithmetic / algebraic, representations. Indeed, in invoking the iconic notion of quadratic equations, she made the link to algebra very clear. School experiences of learning mathematics were very important to Alexandra. In making sense of teaching, she often used anecdotes from her own school experiences. Indeed, she often referred to the absence of a connectionist approach in her own school mathematics. However, up until this point, her references to the notion of connections were largely general and unspecific. When prompted to make a connection with the Whisky and Water problem, she linked her diagrammatic solution very explicitly to the standard procedures for the multiplication of fractions. Her comment that diagrams are "just as mathematical" is very different to her earlier description of this as "just my little way of doing it." In contrast to her earlier pleasure at her diagrammatic solution being judged acceptable by 'experts', here she appeared to understand the mathematical validity of diagrammatic solutions for herself. Thus, at least in relation to this area, she had moved from orientation of external authority in mathematics to one of personal author/ity (Povey et al., 1999).

A further reflection took place at an interview in March 1999, when I asked Alexandra to comment on an earlier and less developed version of the above analysis, which particularly focused on the mathematical validity of her diagrammatic solution. She responded as follows:

> I would say now, Jeremy, it's using one preferred learning style to achieve an outcome and it's partly about that, isn't it ... I'm going off at a tangent now, but do you remember when we were talking about number lines and I was explaining my convoluted number line that I had in my head. It never occurred, I know this sounds really stupid and pathetic, but ... I'd never thought about the fact that you'd have a number line in your head or [another teacher] wouldn't be able to visual a number line in her head and ... those shared experiences or lack of experiences depending on your particular learning style. It's made ... me think more ... about ... the intellectual processes that kids go through to get somewhere. (Interview, March 1999)

Although much less intense than the earlier reflection, this emphasises the shift she has made. Indeed, her link back to the discussion about images of number evokes the beginnings of her revelation, which was not present in my earlier analysis. Moreover, she expressed this in terms of learning styles, an area in which she felt herself to be pedagogically strong. Thus, she embedded these new beliefs by interpreting them through her existing practices.

Alexandra's 'new' mathematical knowledge, in terms of specific concepts and skills, is in a sense relatively small. She has not learnt to use diagrammatic solutions, since she could do these previously. Moreover, during the development of lessons, she demonstrated on many occasions an arithmetical proficiency that would suggest she would have been able to successfully perform the arithmetic / algebraic solution used by the academic researchers. However, in terms of her beliefs about school mathematics, the shift in her thinking is highly significant. She appears to be developing what Cobb, Boufi, McClain and Whitenack (1997) refer to as a mathematizing orientation. Her shift towards Povey's (1997) sense of author/ity in mathematics is considerable. Alexandra views her own invented and informal solution as just as valid as the arithmetical / algebraic solution. Knowledge about how mathematical ideas can be represented or are judged valid is crucial to the teaching of mathematics (Yackel & Cobb, 1996). Indeed, as Wagner and Parker (1993), for example, note algebra and geometry are often taught completely separately. Without an understanding of the validity of diagrams in mathematical argument, it is difficult to see how a teacher could promote a connected understanding for children. Moreover, in terms of Ma's (1999) *profound* mathematical understanding, she has developed an understanding of the importance of *multiple perspectives*: an appreciation of the "different facets of an idea and various approaches to a solution, as well as their advantages and disadvantages" (p. 122). She does this both in pedagogical terms, through the link with learning styles, and mathematically, through her recognition of the validity of mathematical diagrams. In addition, through the connection she makes to quadratic equations, she appears to be developing an understanding of the importance of longitudinal coherence, a knowledge of the whole curriculum going beyond the primary curriculum, which I identified as an important aspect of CAME's mathematics without closure in Chapter 2.

Nevertheless, this does raise a question as to whether these changes to Alexandra's beliefs and orientations towards school mathematics were matched with a corresponding increase in the depth of her understanding of mathematical skills and concepts. I address this question in the next chapter, Chapter 7.

5. Discussion

At the beginning of this chapter in Section 3, I raised several questions in relation to reflection: How does reflection actually take place? What motivates teachers to reflect? Building on the metaphor of distance, how can teachers "step outside" themselves in order to reflect on their own change? In this section, I address these by drawing out themes from the examples discussed above.

In all the examples that I have analysed, the teachers were faced with situations to which they had no choice but to respond to an external demand. In relation to the writing of the lesson materials, for example, Ursula was forced to re-evaluate her earlier leading approach to teaching in order to move on and revise the materials. In the example of the academic writing, both Alexandra and Ursula had no choice but to reconcile and resolve their earlier difficulties with the mathematics, since an academic position required this. In each of these cases, the teachers reflected not because they chose to in isolation, but rather because circumstances required them to. I stress, however, that this is not a deterministic analysis with teachers solely governed by wider forces and structures. Indeed, in each of these cases the teachers had authoring and improvisational choices, and the teachers were themselves catalysts in their reflections. For example, in Section 4.4, it was Alexandra herself who raised her own mental images of number. As in the quote from Holland et al. (1998): "the world demands a response – authoring is not a choice" (p. 272) however, the response itself, whilst certainly constrained, is far from determined absolutely. Reflection was certainly not determined in these situations. In Chapter 5, for example I discussed how Lisa, when faced with demands in relation to Share an Apple, constructed an alternative history that reconciled without addressing her dilemmas. This certainly involved reconstruction, but this was not a reconstruction of her knowledge.

A key feature in differentiating reflection from the more gradual processes of change and development through enculturation that I discussed in Chapter 5, was that these circumstances required the teachers to look across lessons and contexts and thus take a broader view. However, this reflective activity had important features in common with these processes of enculturation, improvisation and authoring. Indeed, reflection, whilst characterised by sudden insights or clear cut breaks with past practice, was dependent on the teachers drawing on existing practices in order to interpret and make sense of new practices.

In the examples, I have described how distancing was enabled by these teachers' different and developed roles within the project. Key here was not simply the teachers' engagement with CAME, but the depth of this engagement. Here, I refer to the teachers' development of wider, substantive, figurative identities that cut across their membership of particular communities: their identities as tutors, lesson-developers and researchers. Through these different roles, the teachers were able to step outside their identity as a teacher or as a learner of mathematics, for example, and thus, "decenter" and distance themselves.

This distancing was accompanied by vivid reminders of the teachers' own early engagement and attempts to make sense. These reminders took the form of written notes, lesson materials or the presence of another teacher engaging in ways similar to the teachers' own previous engagement. Indeed, whilst I rarely observed reflective activity occur during formal team reflection sessions, these sessions did often provide the material for subsequent and significant reflection. Reminders enabled a further distancing. Alongside the distance afforded by their different identities, these reminders enabled the teachers to distance themselves from a previous self and treat this previous self as a conscious object of reflection. This combination of distance and proximity enabled the teachers to "imagine" different practices, whilst at the same time "anchoring" these "imagined futures" in terms of their past experiences.

In this analysis, I have emphasised the importance of the teachers' identities. However, I note that these identities were themselves fostered through Alexandra's and Ursula's participation in different but related communities of practice. In other words, it was the breadth and *depth* of their zones of enactment – and thus their multiple but inter-related identities that enabled reflection and change. Teacher change then is not so much a matter of extolling teachers to reflect, but rather one of nurturing teachers' rich participation in a variety of settings: as teachers, learners of mathematics, curriculum makers, tutors and researchers.

6. Summary

In this chapter, I have explored reflection, understood as the conscious reconstruction of knowledge. Given my interest in this strong definition of reflection, my discussion centred on Alexandra and Ursula, the two teachers for whom I found change to have been considerable. I discussed four examples of reflection, all in the context of the fractions lessons first discussed in Chapter 4. Although I observed reflection to take place infrequently, these reflections were nevertheless important events in these two teachers' professional change.

In a brief review of the literature on teacher reflection, I highlighted two further issues: teachers' motivation to reflect; and the metaphor of distancing. In my analysis, I drew on Schifter's (1996) notion of multiple identities in order to emphasise aspects of the teachers' identities that cut across their participation in local communities: their identities as teachers, tutors, learners of mathematics, lesson developers and researchers.

I found that teachers' motivation to reflect was rooted in the teachers' social circumstances. They did not *choose* to reflect in isolation. Rather, reflection took place in situations where the teachers were compelled to respond, although the teachers themselves appeared to be catalysts in making such responses reflective. Reflection, in terms of conscious and explicit change, was facilitated by a distancing, which itself was fostered by these teachers' multiple identities within and beyond the project: as class teachers, tutors, lesson developers, mathematics learners and researchers. In their identities as tutors, for example, the teachers were able to step outside and reflect upon their identities as teachers or learners of mathematics. This reflection was itself not an individual activity, but was rather a collaborative and social one in which the teachers engaged with their previous selves and imagined new practices, in the context of their authentic activity as tutors, lesson developers or teacher-researchers. Formal reflection sessions, whilst not often the context for reflection, were crucial in providing teachers with vivid reminders of their previous activity and engagement.

In these examples, the changes to the teachers' knowledge were largely in terms of the teachers' beliefs and orientations towards school mathematics. I, therefore, raised the question of whether the changes discussed here were accompanied by equivalent developments in the teachers' mathematical knowledge. I address this question in Chapter 7.

Chapter 7: Mathematical Knowledge

1. Introduction

In this chapter I focus on the teachers' mathematical knowledge.

Principally, I address two of the questions outlined in Chapter 1: the interrelationship of teachers' beliefs and knowledge about mathematics; and, the issue of differential change amongst the teachers. In doing so, I discuss the further questions raised in Chapter 5: firstly, in terms of the teachers for whom change was significant, were the changes in their beliefs and orientations towards school mathematics mirrored by a similar change to their mathematical knowledge; and, secondly, in relation to the other teachers, did these teachers' mathematical knowledge change, despite the less significant changes to their beliefs. I note my interest is in the extent to which the teachers' mathematical knowledge developed from a procedural understanding towards a principled understanding.

I focus on two of the teachers: Alexandra, for whom belief change was significant: and Janice, for whom belief change was less significant. I discuss their mathematical knowledge mainly by drawing on an analysis of the mathematics interviews that were conducted with these two teachers in July and December 2002, respectively. These interviews explored the teachers' knowledge of specific concepts in the area of multiplicative reasoning.

My aim here is to use these case studies to begin to develop a more general understanding of the nature of changing primary teachers' mathematical knowledge. In particular, I explore the barriers to change in this area. I note, therefore, that in exploring the two teachers' subject knowledge I produce a broadly indicative account rather than a comprehensive and detailed analysis.

The structure of the chapter is as follows:

- In Section 2, I discuss the issue of mathematical knowledge for teaching, giving a brief overview of the literature in this area.
- In Section 3, I discuss several methodological issues, expanding on the earlier discussion in Chapter 3. In particular, I discuss my focus on multiplicative reasoning, my choice of these two teachers, and my reasons for not interviewing the other teachers.
- In Section 4, I analyse the two teachers' mathematical knowledge as evidenced in the mathematics interviews, together with the ways in which the teacher knew this knowledge.
- In Section 5, I discuss more speculatively the barriers to change in this area highlighted by my analysis.

2. Mathematical Knowledge for Teaching: An Overview

In this section, I review the literature on teacher knowledge of mathematics in order to develop my own position on the nature of a teacherly knowledge of mathematics, which I distinguish from the knowledge needed to practice mathematics.

It appears self-evident that teachers should know about mathematics in order to teach it effectively. However, as I noted in Chapter 1, teacher knowledge in mathematics is an area of some controversy. There is general agreement that teachers need to know about mathematics and, indeed, that broadly teachers need to know more mathematics than they do already. There is evidence that poor subject knowledge in mathematics has a negative impact on teaching (e.g., Bennett & Turner-Bisset, 1993; McDiarmid, Ball, & Anderson, 1989; Rowland et al., 2000). There is considerably less consensus on what constitutes the mathematical knowledge necessary for teaching. Some have argued that improving teachers' knowledge of mathematics per se will lead to better teaching (e.g., Alexander et al., 1992). However, the evidence base in this area is somewhat equivocal. Several studies, for example, have found no link between teachers' mathematical knowledge as measured in terms of academic mathematical qualifications and effective teaching (Askew et al., 1997; Begle, 1968, 1979; Eisenberg, 1977). What is clear is that the connection between teacher knowledge and teaching outcomes is neither simple nor straightforward and that further research in this area is needed.

To deal with this problem, research has focused on exploring the nature of teacher knowledge in mathematics. One strand of this research has been to link mathematical knowledge for teaching to ways of knowing in the discipline of mathematics. Lampert (1986), for example, distinguishes between *procedural* and *principled* knowledge of mathematics. Procedural knowledge is a rule guided "knowing that" and concerns mathematical procedures and their use to compute correct answers. Principled knowledge on the other hand is a wider and more conceptual "knowing how" and includes the knowledge of mathematical concepts that enable the construction of procedures for solving mathematical problems. Lampert's distinction has similarities with has similarities to Skemp's (1976) distinction between instrumental and relational understandings, Prestage and Perks' (2001) learner-knowledge and teacher-knowledge, and Thompson's calculational and conceptual orientations (Thompson et al., 1994). These analyses point to a strong link between what are commonly thought of as beliefs about mathematics and knowledge of mathematical concepts (See, e.g., Askew, 1999). One approach has been to explore teachers' conceptions of mathematics as a discipline and, in particular, draw on Lakatos's (1976) notion of fallibilism in mathematics. (See, e.g., Ernest, 1998; Lerman, 1990). I referred earlier in Chapter 5, to the work of both Cooney (1994a) and Povey (1997) in relation to teachers' conceptions of authority in mathematics. Povey sees external authority as relating to Cobb, Wood, Yackel and McNeal's (1992) conception of traditional *school maths* with a focus on procedural knowledge. Author/ity, she sees as linked to Cobb, Wood et al.'s conception of *inquiry maths* with a focus on principled knowledge. Similarly, Cooney and Shealey (1997) relate these positions to Ernest's (1991) development of Lakatos's work, linking these positions to absolutism and fallibilism, respectively.

Increasingly, however, researchers have been arguing that mathematical knowledge for teaching is distinct and different to the knowledge necessary to practice mathematics. Much of this work builds on Shulman's (1986) notion of *pedagogical* *content knowledge* which "goes beyond the subject per se to the dimension of subject knowledge *for teaching* ... the particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9, original emphasis). (See, e.g., Askew et al., 1997; Ball, 1991; Carré & Ernest, 1993; Ernest, 1989, for developments of the notion of pedagogical content knowledge in mathematics education.)

In her study of elementary mathematics teachers in China and the US, Ma (1999) provides a useful analysis clarifying the principled nature of pedagogical content knowledge in primary mathematics, which she refers to as *profound understanding of fundamental mathematics*. This profound understanding consists of four interrelated elements: knowledge about the *basic ideas* underlying the mathematical curriculum, or what might be thought of as closest to mathematics subject knowledge; *connectedness* between simple and more fundamental ideas; consideration of *multiple perspectives* and approaches to mathematical ideas; and, knowledge of the entire elementary, or primary, mathematical curriculum and its *longitudinal coherence*. To exemplify these aspects of teacher knowledge, I use the notion of fractions, an area that will be one focus of the analysis later in this chapter, although I emphasise Ma's elements are not intended as a taxonomy. Basic ideas includes an understanding of fractions in terms of division and as rational numbers; connectedness includes not simply knowledge of how to convert vulgar to decimal fractions and vice versa, but of how and why these representations are equivalent; multiple perspectives includes different meanings of fractions together with models which exemplify these meanings; and, longitudinal coherence includes knowledge of aspects of mathematics not normally taught within primary, for example, the division by fractions algorithm together with a knowledge of how and why this procedure works. Ma's work has many resonances in the literature: for example, Askew et al.'s (1997) connectionist orientation, and Aubrey's (1997) finding that teacher subject knowledge has a crucial effect on teaching in early years mathematics education.

In relation to primary mathematics, pedagogical content knowledge refers to a different knowledge *about* the mathematics of the primary school curriculum rather than knowing *more* mathematics. Clearly primary teachers do not need a knowledge of undergraduate mathematics. However, my argument is that pedagogical content knowledge in primary mathematics does include an in-depth and explicit knowledge of at least some concepts that to a professional mathematician, for example, are certainly basic ideas but which may only be implicitly known.

To illustrate this difference, I use the example of a presentation at the British Congress of Mathematics Education by Saunders (1999), a mathematician rather than a mathematics educator. Saunders commented on several mathematical errors contained in the Teacher Training Agency's needs assessment materials in mathematics (TTA, 1998a, 1998b). These materials are aimed at enabling KS2 teachers and schools to assess their training needs in relation to mathematics subject knowledge. The examples that Saunders commented on were taken from a section entitled "Mathematical Argument," which focused on a teacher's " knowledge of the mathematics involved ... and an ability to help children set out a convincing argument to justify their results" (TTA, 1998b, p. 22). One of the statements that he discussed was similar to the pocket money question, the starting point for the development of the two fractions lessons discussed in Chapter 4:

> Is this statement always, sometimes or never true? ... A half is greater than a third. (TTA, 1998b, p. 22)

The diagnostic feedback on this was as follows:

> Sometimes true.
> If we are talking about the numbers 1/2 and 1/3 then clearly that statement is true. However if we are talking about the operations to half and to third (i.e. x 1/2, x 1/3 or divide by two, divide by three), then it is possible that a third is greater than a half. For example, a half of 12 is six, whereas a third of 30 is 10. (TTA, 1998a, p. 15)

Saunders commented on this as follows:

> Very clever, I'm sure, but this is supposed to be helping people, not playing tricks. Note that the use of English is also unprofessional. The verb is to "to halve", not "to half", and "to third" is an obsolete form not included in most dictionaries. I've never understood why it is considered necessary to invent special words for schools which are

> neither the correct mathematical terms nor the words in everyday use. (Saunders, 1999, p. 3)

In a sense, Saunders' argument that this is "playing tricks" is absolutely correct and, in the strict terms in which the problem is stated, a half is always greater than a third. Moreover, his criticism of the use of English is technically correct, if somewhat pedantic. Yet, this is a common primary classroom activity, which is used to promote argument along the very lines mapped out in the TTA commentary.

One strength of this question relates to learners' understandings of fractions, in particular many children's limited understanding of what a fraction can mean and understanding a fraction in terms of division. Moreover, it touches on the common misconception that multiplication always makes things bigger. In asking whether the statement is "always, sometimes or never true" learners are asked to question these limited understandings. Here, the ambiguity and imprecise understanding of language are crucial. The ambiguity allows for different learners to interpret the statement in different ways. Conversely, less ambiguity would provide less room for differences in interpretation. Knowledge of learners' naïve and limited understandings, together with strategies that challenge them, are clearly important for teaching. That such mathematics-specific knowledge forms an important and desirable element of teacher-knowledge would be something I would expect Saunders to agree with.

A second strength again lies in the ambiguity of the statement. The statement as posed will appear initially as obvious to some learners (although for others the comparative size of fractions as numbers is far from obvious.) The problem as set encourages distrust in the obvious solution, allowing for the generation of counter-examples, an important aspect of mathematical argument and proof. This hits at the heart of Saunders' conception of teacher knowledge in mathematics. Of course, for an academic mathematician, this mathematical content is trivial, obvious and uninteresting, and simply playing tricks with self-evident basic mathematical facts. Yet, much of the primary curriculum, the reasons for children's difficulties, and mathematical contexts which promote discussion would be in a similar way trivial and obvious to a professional mathematician. Hersh (1998) argues that mathematics' "most salient feature is the uniquely high consensus it attains" (p. 249). A teacherly knowledge of mathematics includes a knowledge of ways of unpicking and deconstructing the universally accepted facts of mathematics in order to enable children to access this consensus for themselves rather than simply accepting it, and, thus, to view mathematical knowledge as potentially fallible (Lakatos, 1976) and develop Povey's (1997) author/ity as a knower of mathematics. Certainly mathematicians generally know these things in relation to their own mathematical practices and, moreover, know these things intuitively. Moreover, a mathematician *could* know these things in relation to primary mathematics, but knowing them would be irrelevant to their day-to-day mathematical practices.

Of course, to a degree, the differences that I highlight here are subtle ones that reflect central differences in the practices of mathematics educators and mathematicians. Whilst these communities share the context of mathematics and elements of their practices are aligned, they are nevertheless different discourse communities engaged in different enterprises.

The nature of pedagogical content knowledge is itself, however, something of a contested idea within the education research community. McNamara (1991), for example, argues that there is no clear distinction between subject knowledge and pedagogical content knowledge. That this is the case does not, I believe, reduce the concept's usefulness. Indeed, Corbin and Campbell (2001) argue that pedagogical content knowledge is most useful as a metaphor that locates teacher knowledge embedded within the complex and unpredictable practice of teaching. Another critique is epitomised by Brown and McIntyre (1991), who argue that much of a teachers' knowledge is tacit, craft knowledge that cannot be codified as theoretical abstract knowledge. Certainly, there is merit in these arguments and, indeed, the argument of this thesis is underpinned by understandings of knowledge as situated. Nevertheless, there is, I contend, a role for considering knowledge abstractly. Teachers' day-to-day practices include talking *about* mathematical knowledge not simply doing mathematics, both in terms of talk with children in class and with other teachers in planning. This is all the more the case for teachers whose practices extend far beyond the classroom, as tutors, curriculum developers or Numeracy Consultants, for example. There is another side to Brown and McIntyre's (1991) argument in that they are arguing that the knowledge of an expert teacher is more intuitive than that of a novice. Indeed, knowing and using mathematics in the complex world of the classroom is in a very real sense more difficult than knowing mathematics in the relatively simple setting of a test. For example, knowing the equivalence of 0.2 and $^1/_5$ in answer to a closed test question is different from using this knowledge to respond to an unanticipated comment from a child during a whole class discussion.

Addressing themselves to the issue of characterising teacher knowledge of mathematics, Prestage and Perks (2001) draw on the work of Aubrey (1997) to argue that teacher-knowledge, in contrast to the learner-knowledge needed to pass mathematics examinations, is one which cannot be developed only through classroom practice. Indeed, echoing Povey's (1997) author/ity of self and reason, they argue teacher-knowledge requires,

> an explicit intellectual re-working by teachers of their subject knowledge beyond that gained from classroom practice. This classification acknowledges that more than practice might be used to develop subject knowledge and infers a deliberate "standing back" from the classroom situation in order to re-think aspects of subject knowledge to plan for teaching. (p. 103-4)

However, Prestage and Perks argue that the mathematics knowledge of most mathematics teachers is narrowly situated in classroom practice, and, thus, remains at the level of learner-knowledge. That few teachers attain this more developed level of teacher-knowledge is a point that is supported by Ma's (1999) research and the findings of Askew and Millett (2001).

I have argued that a teacherly knowledge of mathematics is a principled knowledge reflecting both the mathematical concepts that are known and the ways in which they are known. I have argued that teacher knowledge is distinct to the knowledge required to practice mathematics. In this chapter, I analyse the teachers' knowledge of specific mathematical concepts using Ma's categories and examine the ways this mathematics was known by using the teachers' beliefs and orientations towards these in order to explore the teachers identities in relation to mathematics.

3. Setting Up the Case Study: Some Methodological Issues

3.1 Why These Two Teachers?

I focus on two of the teachers, Alexandra and Janice: Alexandra as a teacher who changed significantly; and, Janice as a teacher for whom change was less significant. In addition, as I discussed in Chapter 4, both were experienced teachers who had engaged with the project, unlike Henrietta and Tony, who had experienced CAME as alien. Moreover, both Alexandra and Janice had led NNS training sessions aimed specifically at improving other teachers' subject knowledge.

Given the significant changes to her beliefs and orientations towards mathematics, I hypothesised that Alexandra's mathematics knowledge would have also changed considerably. On the other hand, I hypothesised Janice's subject knowledge would have changed less significantly, although, given her frequent claims to "love" mathematics, I did expect her knowledge to be procedurally strong.

My intention was to interview Ursula as well as Alexandra and Janice. However, although she had been more than willing to be interviewed generally and had at times gone out of her way to arrange convenient times, Ursula proved very elusive when I tried to arrange a mathematics interview. Although she never actually refused to participate in the interview and several times indicated her willing to do the interview, I eventually decided that her reluctance did constitute at least an unconscious refusal to participate and did not pursue her for the interview.

Of the three remaining teachers, Henrietta had already left the project when I decided to carry out these interviews and would have been impossible to interview. The other two teachers, Lisa, and to a lesser extent Tony, had been very unwilling to be interviewed. It seemed exceedingly unlikely that I would be able to arrange an interview with either teacher specifically focusing on their mathematical knowledge.

3.2 Why Multiplicative Reasoning?

I chose to focus on the teachers' understanding of multiplicative reasoning for a number of reasons. Multiplicative reasoning was an area which the Primary CAME research team considered in some detail. Although a substantial part of this work and discussion took place prior to Janice joining the project, she had still been involved as a Phase 2 teacher. In terms of lesson development, Alexandra, working principally with Ursula, had developed two lessons focusing on fractions, whilst Janice had developed a lesson exploring factorisation and prime numbers. In Chapters 4, 5 and 6, I discussed how the two fractions lessons had been the context for significant shifts in both Alexandra's and Ursula's developing beliefs about school mathematics. My decision here was influenced by the research literature promoting the value of lesson development in terms of teacher education (e.g., Clarke et al., 1996; Lewis, 2000; Stigler & Stevenson, 1991).

One element of multiplicative reasoning, ratio and proportion, or FDPRP (Fractions, decimals, percentages, ratio and proportion), has been consistently highlighted in HMI / OfSTED reviews of mathematics teaching and learning as a particular weakness in teacher subject knowledge and, hence as a key area for NNS inset. (OfSTED, 1999, 2000, 2001.) This NNS focus on FDPRP resulted in a further reason in that the LNRP Focus 2 project was focusing on teachers' knowledge of multiplicative reasoning and had already developed an interview structure which I could adapt and use (Askew & Millett, 2001). (In fact, the LNRP focus 2 mathematics interview was similar to the pilot mathematics interview that I conducted in April 1998, since both were developed from the interview schedule used in Bibby (2001).) Focusing on this area would provide further possibilities for contrasts and comparisons within LNRP more widely, which, with Bibby, I have begun to develop elsewhere (Bibby & Hodgen, 2002). Indeed, in addition to their roles as CAME tutors, I had been able to observe both teachers delivering NNS training sessions in the area of multiplicative reasoning, in Alexandra's and Ursula's cases as Numeracy Consultants and in Janice's case as mathematics co-ordinator of Brightvale School.

There is a significant amount of research literature in relation to primary or elementary teachers' knowledge of multiplicative reasoning which could inform my analysis, although much of this is in the context of pre-service teachers (e.g., Graeber, Tirosh, & Glover, 1989; Ma, 1999; Simon & Blume, 1994; Tirosh, Fischbein, Graeber, & Wilson, 1999; Zazkis & Campbell, 1996). Indeed, this literature was used in formulating several of the problems posed in the interview.

The interview focused both on the teachers' ability to successfully solve the problems, their learner-knowledge, and their wider understanding of the underlying mathematics, their teacher-knowledge. Although the teachers used some procedures normally covered in KS3, the questions themselves largely related to aspects of mathematics within the KS1 and 2 Framework for teaching mathematics (DfEE, 1999). I did, however, add one question about simultaneous equations, because, during other interviews, all the teachers' referred to this and algebra as "difficult" mathematics. Additionally, the interview raised aspects of the teachers' beliefs and attitudes towards mathematics, both through the direct questions that I asked and in terms of issues raised by the teachers themselves.

4. Alexandra's and Janice's Mathematical Knowledge

In this section, I analyse Alexandra's and Janice's mathematical knowledge drawing principally on the mathematical interviews that I conducted with them. These interviews were conducted in December and July 2000, respectively, and quotations in this section are taken from these interviews unless otherwise stated. The mathematical problems used in these interviews can be found in Appendix K. Rather than analyse each teachers' mathematical knowledge separately, I discuss their knowledge in relation to Ma's (1999) aspects. I, then, focus on their ways of knowing highlighting the fragility of their knowledge, their beliefs and identity in relation to the discipline, and aspects of their anxiety about mathematics.

I make extensive reference to the Framework for Teaching Mathematics (DfEE, 1999). I do this to emphasise that the mathematical understandings under discussion here are within the primary curriculum.

4.1 Knowledge of Specific Mathematical Concepts

When I conducted the mathematics interviews, I was surprised that Alexandra's subject knowledge appeared to be relatively weak. I have discussed in Chapters 6 and 7 how Alexandra had become more mathematically orientated. Yet these changed beliefs about mathematics and doing mathematics seemed not to have impacted deeply on her understanding of specific mathematical concepts. Moreover, although both teachers could successfully carry out many of the procedural calculations involved in the problems, they did appear to have great difficulty over several of the questions, particularly when they touched on the more conceptual aspects. Janice, also, made some fundamental errors in the standard division algorithm that she did not correct, suggesting that her knowledge was not as procedurally strong as I had expected it to be.

There were several common features in their difficulties, which I discuss below, although they faltered over slightly different questions. For example, whilst Alexandra had some considerable difficulty with identifying the factors of $3^2 \times 5^2 \times 7$, Janice had no difficulty with this. Conversely Alexandra had no difficulty identifying the factors of 513,252, although Janice experienced considerable difficulties with this problem.

My aim in the following analysis is to provide a very broad picture of their knowledge. In particular, I use Ma's (1999) aspects of profound understanding to assess the extent to which their knowledge was procedural and / or principled. As I discussed earlier, these aspects are not a taxonomy and there are many overlaps between the sections in the following analysis.

4.1.1 Basic Ideas

In terms of their knowledge of basic ideas, Alexandra could successfully answer all the questions performing most of the necessary calculational procedures correctly, although on several questions this took a considerable amount of time and whilst solving the problems she made several mistakes which she corrected during the interview. Several of these mistakes appeared to indicate a limited understanding of the concepts underlying these procedures. For example, she initially interpreted the question 1 $^3/_4 \div {}^1/_2$ as division by 2, a common misconception that was shared by many of the teachers in Ma's (1999) study. She did, however, indicate some awareness of her limited understanding referring to division by fractions as follows: "If I was doing that the way I was taught to do it, I would just turn that all upside down. And I have real problems with this idea of division by fractions." However, she was unable to carry out this procedure and solved the question using two methods: initially, by converting to decimals mentally and using a calculator; and, shortly after by a repeated subtraction method.

As I have already noted, Janice was generally less successful with the problems as a whole. She made more mistakes, made errors that she did not correct, and could not complete two of the problems during the interview. She could not carry out the standard short division procedure correctly, an issue that I discuss further in Section 4.2.1 below.

They both made errors in carrying out the procedures. Some of these appeared to be trivial, such as mis-remembering a multiplication fact, which they quickly self-corrected. I have ignored these types of errors in the analysis. However, other mistakes appeared to indicate that they had a limited grasp of the concepts underlying these procedures.

4.1.2 Connectedness

Both teachers understood equivalence of decimal and vulgar fractions and, both could readily indicate which numbers from a list were equivalent to $^1/_5$. However, both teachers had a strong preference for one representation when calculating or interpreting fractions.

Janice appeared to find any of the questions involving decimals difficult and commented on her dislike of decimals. For example, in the following question, 0.5×0.2, she rejected using a calculator, saying she preferred fractions. Hence, she converted this to a calculation involving vulgar fractions, $^1/_2 \times {}^2/_{10}$, then solved the resulting calculation by "multiplying top and bottom and cancelling", although she actually cancelled before multiplying the numerators and denominators. Having solved this problem procedurally and observing the answer to be a tenth, she realised that it could be read as "a half of two tenths." She did not, however, convert the vulgar fraction, $^{1}/_{10}$, back to its decimal fraction equivalent. For 3 ÷ 0.75, she converted to fractions and performed the following procedure quickly and without any apparent difficulty: $3 \div {}^{3}/_{4} = {}^{3}/_{1} \div {}^{3}/_{4} = {}^{3}/_{1} \times {}^{4}/_{3} = {}^{12}/_{3}$. However, she did not simplify $^{12}/_{3}$ to 4.

Alexandra, on the other hand as I noted 4.1.1 above, perceived vulgar fractions as more difficult than decimal fractions. Rather than use the "turn upside down and multiply" procedure for fractional division, she preferred to convert vulgar fractions to decimals and use either a calculator or the standard multiplication algorithm. She also used a standard multiplication algorithm to solve the question above, 0.5 × 0.2, as in Figure 7.1, commenting on how she knew where to place the decimal point in the product: "There are two decimal places in the question, so there must be two decimal places in the answer." This, together with her inclusion of the multiplication by zero, strongly suggests that her understanding of this method is certainly heavily reliant on procedural rather than principled knowledge.

```
    0 . 5
×   0 . 2
---------
    1   0
0   0   0
---------
0 . 1   0
```

Figure 7.1: Alexandra's procedure for solving 0.5 × 0.2

Although Alexandra read the answer correctly as 0.1 and used the same form as in the question, she did not notice, as Janice did, that this could be read as a tenth or that the calculation was equivalent to either of the relatively simple "half of two tenths" or "half of a fifth." Hence, she appeared to have no strategy to check or make sense the result of this calculation procedure. Indeed, she could not generate an illustration of this problem. Whilst she did not get this problem "wrong", her knowledge did appear to be partial and limited.

As I noted in Chapter 6, Alexandra found the notion of a connectionist a very powerful idea. Indeed, throughout the mathematics interview she referred to the importance of making connections clear for children. However, her approach to this problem would suggest that her actual knowledge of connections, and in particular her knowledge of the concepts underlying the equivalence of these representations was weak. Indeed, her connections appeared to be one-way connections: she recognised fairly easily the decimal equivalents to simple unitary vulgar fractions, but did not *spontaneously* recognise the vulgar fraction equivalents of simple decimals, knowledge which appears in Y5 of the Mathematics Framework (DfEE, 1999, Y456 examples, p. 31).

Janice was proficient at performing the standard procedures for multiplying and dividing fractions, unlike Alexandra. However, this procedure appeared to obscure understanding that she felt should be obvious for her and it was only the answer, $^{1}/_{10}$, that prompted her recognition of $^{1}/_{2} \times {}^{2}/_{10}$ as "a half of two tenths". It would seem that Janice knew $^{1}/_{10}$ to be half of $^{2}/_{10}$ and she understood the operation of $^{1}/_{2} \times$ as "half of." She did not recognise either $^{2}/_{10}$ or 0.2 as $^{1}/_{5}$ in the context of this problem. However, she had described this specific knowledge of equivalences of $^{1}/_{5}$ as "our bread and butter stuff" earlier in the interview. Her lack of recognition suggests strongly that, like Alexandra, her knowledge of equivalence of vulgar and decimal fractions was weak. Indeed, like Alexandra, her connections appeared to be one-way connections, although in Janice's case these connections were in the "opposite" direction to those of Alexandra.

4.1.3 Multiple Perspectives

I asked the teachers to provide illustrations in the form of a story, diagram or picture to use with children for the following questions: 0.5×0.2; $3 \div 0.75$; and, $1\,^{3}/_{4} \div {}^{1}/_{2}$. In asking these questions, I was interested in the ease with which the two teachers could generate a variety of appropriate and pedagogically useful illustrations, and in the range of different meanings of multiplication and division that they drew upon. The Mathematics Framework outlines three understandings of multiplication: repeated addition, scaling, and describing an array for multiplication (DfEE, 1999, p. 47). The description of the multiplicative models is, however, very heavily orientated towards the development of calculational strategies, and the scaling model is only described once. Only two understandings are provided for division:

sharing and grouping (DfEE, 1999, p. 47). Although a link is drawn between repeated subtraction and grouping, the understanding in the Framework of both of these division models appears to be actually closely related to repeated subtraction. Ma (1999) takes a more conceptual and less explicitly calculational approach, describing three models of division: measurement; partitive; and, factors and product, which broadly include the multiplicative understandings of repeated addition, scaling and array in the Mathematics Framework. Using the problem $1\ ^3/_4 \div ^1/_2$, she exemplifies these as follows: the measurement model as "finding how many $^1/_2$s there are in $1\ ^3/_4$, and, finding how many times $1\ ^3/_4$ is of $^1/_2$" (p. 72); the partitive model as "finding a number such that $^1/_2$ of it is $1\ ^3/_4$" (p. 74); and, the factors and product, or area model, as "finding a factor that multiplied by $^1/_2$ will make $1\ ^3/_4$" (p. 76). Although there are other more developed categorisations of multiplication and division problems (e.g., Anghileri, 2001), I will for the purposes of this discussion use Ma's models, both because of their simplicity and because of their close similarity to the multiplicative models used in the Mathematics Framework.

Given Alexandra's experiences in developing the two fractions lessons discussed in previous chapters, and her experiences of teaching the multiplicative models on NNS courses (e.g., DfEE, 2000), I had expected her, at least in terms of multiplication, to demonstrate something approaching the sophisticated understanding of models shown by one of the teachers from Ma's study:

> The equation of $1\ ^3/_4 \div ^1/_2 =$ can be represented from different perspectives. For instance, we can say, here is $1\ ^3/_4$ kg of sugar and we want to wrap it into packs of $^1/_2$ kg each. How many packs can we wrap? Also, we can say that here we have two packs of sugar, one of white sugar and the other of brown sugar. The white sugar is $1\ ^3/_4$ kg and the brown sugar is $^1/_2$ kg. How many times is the weight of white sugar that of brown sugar? Still, we can say that here is some sugar on the table that weighs $1\ ^3/_4$ kg; it is $^1/_2$ of all the sugar we now have at home, so how much sugar do we have at home? All three stories are about sugar, and all of them represent $1\ ^3/_4 \div\ ^1/_2$. But the numerical models they illustrate are not the same. I would put the three stories on the board and invite my students to compare the different meanings they represent. After the discussion I would ask them to try to make up their own story problems to represent the different models of division by fractions. (Ma, 1999, p. 80)

However, both Alexandra and Janice found the generation of any models extremely difficult and required considerable support and prompting to tackle these questions. Indeed, unlike the teacher from Ma's study who integrated the interconnections between these different models within a teaching approach, both teachers appeared to find the question itself extremely surprising and novel and both asked me, with apparent disbelief, if I could do it.

Alexandra provided a single story for just two of the three problems. The first, for 3 ÷ 0.75, was "how many lots of seventy five pence can you get from three pounds." Reflecting her preference for decimal fractions, she found this relatively straightforward after I had suggested thinking about contexts involving measures. However, she had considerable difficulty producing the following story for $1\ ^3/_4 \div {}^1/_2$:

> If you said that was one, and that was three quarters you'd get three halves and half a half out of it. But that's not very helpful is it? ... One, OK, that's one and three quarters, so you can get one, two, three. Three halves out of it. And half of a half.

This story certainly provides an illustration of how $1\ ^3/_4 \div {}^1/_2 = 3\ ^1/_2$. However, it is simply a re-statement of the problem in terms of repeated subtraction and, moreover, unlike her previous money story, it is set within mathematics. Indeed, this measurement model, which Alexandra used to illustrate both problems, reflects the only occurrence of division by fractions in the Mathematics Framework: "How many halves in $3\ ^1/_2$?" (DfEE, 1999, Y456 examples, p. 25). However, Alexandra had developed the two fractions lessons with the specific aim of enabling children to develop a range of models for the representation of fractions. In fact, the Halving & Thirding lesson had used both measurement and area representations for the multiplication of fractions, an aspect of the lesson which she herself had highlighted several times during the lesson simulation to Phase 2 teachers (PD session, February 2000). In her solution to the Whisky & Water problem, she had herself generated an area model to illustrate the multiplication of fractions. (See Chapter 6, Figure 6.2.) It is somewhat surprising that, given these fairly intense lesson development experiences, together with her experiences as a Numeracy Consultant, she could not transfer the area model to division by fractions or, more significantly, to the multiplication of decimal fractions. Indeed, she was unable to provide an illustration of 0.5×0.2. More surprising still is her reaction to being asked to think of models. The use of models in the form of pictures and stories is at the heart of making mathematics accessible to children. Indeed, I had observed Alexandra emphasise different meanings of multiplication and division, including repeated subtraction / addition and the area / array models, and the need to understand children's different ways of seeing algebraic relationships during NNS training sessions (NNS 5 day training, June & October 2000). I did not observe her discuss multiplication in terms of scaling during any of these sessions. The scaling model of multiplication did, however, feature in two of the secondary CAME lessons she trialled.

Like Alexandra, Janice also provided a single illustration for just two of the problems: her reformulated $^1/_2 \times {}^2/_{10}$, and $1\ ^3/_4 \div {}^1/_2$. Again, her failure to provide models for the problems involving decimal fractions emphasises further her preference for vulgar fractions discussed in Section 4.1.2 above. Her first story, for $^1/_2 \times {}^2/_{10}$, was as follows:

> If it's two tenths, there's two of my fingers. Two of my fingers are broken, and then half of them get better [LAUGHS] and I've only got one bad finger.

Whilst this is mathematically correct, it seems unlikely to be a pedagogically useful example. Moreover, although $^2/_{10}$ appears in her story, the situation illustrated is actually closer to $^1/_2 \times 2$ rather than $^1/_2 \times {}^2/_{10}$. Indeed, Janice herself suggested it was not a good example. However after debating with herself about whether she could draw or do something visually, she concluded that she could not do better than this and judged it "good enough."

For $1\ ^3/_4 \div {}^1/_2$, she had considerable difficulty generating a story. Her first suggestion was one which interpreted the question as division by 2, the same misconception that Alexandra displayed above. However, Janice self-corrected this saying the answer to that question could not be $3\ ^1/_2$. She commented as follows on her difficulty:

> I'm bounded by my getting to this [$1\,^3/_4 \div {}^1/_2 = 3\,^1/_2$] when really just saying two lots of that [$1\,^3/_4$] gives you the answer.

In short, in interpreting $1\,^3/_4 \div {}^1/_2 = 3\,^1/_2$ as $1\,^3/_4 \times 2 = 3\,^1/_2$, she saw the problem in terms of multiplication rather than division. Whilst this shows an understanding of the connectedness of multiplication and division, it also suggests that her understanding of multiple ways of conceptualising the problem was limited. A further difficulty appeared to relate to her description of multiplication as "odd" because "dividing by fractions makes things bigger." Indeed, when I suggested that she might find it helpful to think about a story for $1\,^3/_4 \div 4$, she said: "That's no use. That's a whole number. What story would you make it where you could explain why a fraction makes the answer bigger?" This strongly suggests that Janice saw division by fractions, understood as numbers between 0 and 1, as separate to division by whole numbers.

The solution she eventually came up with was prompted by my suggestion to think in terms of measures. She began by repeating the error of interpreting division by $^1/_2$ as division by 2:

> Somebody's had half of their life. And they are one and three quarters years old. So they've had that bit of it, they've had half. No, that's still multiply isn't it? I'm still multiplying. But - if they have another half of their life how old will they be? ... Three and a half. But then I'm cheating. I'm using the answer, sorry.

The story is mathematically correct and, unlike her previous story, is one that could be pedagogically useful if the presentation were tidied up. Janice appeared to find the generation of a story difficult in part due to the weaknesses in her knowledge identified above. Firstly, her knowledge of the basic ideas involved was weak and insecure. Despite "knowing" that division by $^1/_2$ was not equivalent to division by 2, she repeatedly made this error. Her repeated self-correction indicates that her knowledge here was partial and lacked fluency. Secondly, her knowledge of mathematical connections in this area was weak. Her understanding of division by fractions appeared to be separate to her understanding of division more generally. Hence, she could not extend or adapt exemplifications of division by whole numbers to the case of fractions.

A further issue relates to her reference to "using the answer" as "cheating." Indeed, I referred in Chapter 4 to Janice's use of stories to make mathematics problems interesting to children, although often these stories appeared to be of more relevance to Janice rather than the children. I suggest this indicates that Janice did not approach this problem entirely from a teacherly perspective in that she looked at the problem, $1\ {}^{3}/_{4} \div {}^{1}/_{2}$, as needing contextualisation in order to provide interest and motivation, rather than looking for a context that would demonstrate different meanings of the problem.

Finally, Janice, like Alexandra, provided two stories that illustrated the same model, although in Janice's case this was the partitive model for division and the corresponding scaling model of multiplication. Of course, this does not mean that Janice did not know other models. However, the difficulty that she encountered generating these stories does suggest that she lacked an intuitive familiarity with these and different models of multiplication / division.

The difficulty that both teachers had in providing models or illustrations that they might use when teaching children reflects the findings of other studies of primary teachers' subject knowledge (e.g., see Askew & Millett, 2001; Ma, 1999). This was despite both teachers' experiences in CAME. The difficulty appears to have been related in part to the weakness of their mathematics knowledge in relation to basic ideas and connectedness. Further, Alexandra's failure to draw on her experiences of developing the fractions lessons suggests that her knowledge was narrowly situated. She "knew" about different models for the multiplication of fractions in the supported and structured context of lesson development and, as a tutor during INSET sessions, when such knowledge was explicitly part of her role. However, she had not generalised this knowledge sufficiently to enable her to easily generate such models in different contexts.

4.1.4 Longitudinal Coherence

Throughout this analysis, I have pointed to aspects of the primary mathematics curriculum for which these two teachers' knowledge was at best partial and weak.

These weaknesses were, moreover, not confined to the higher end of the curriculum. For example, children are expected to have covered and to understand the different models of multiplication, as repeated addition, an array, or scaling, by the end of Y3, since these models drop out of the understanding multiplication strand thereafter (DfEE, 1999, Y123 examples, p. 47; Y456 examples, p. 52-53). Clearly, adapting such understanding to the context of fractions is conceptually more difficult. However, in the roles of Numeracy Consultant and Mathematics Co-ordinator, both of these teachers might be expected to be able to draw on such sophisticated understandings in order to help and advise other teachers.

Generally, the two teachers appeared to regard that the questions covered aspects of the mathematics curriculum that primary teachers should know. Janice did, however, query the question involving the division of fractions as "not something we do in primary school," although, as I noted above, this does appear in the Framework at Y6 as a word problem (DfEE, 1999, Y456 examples, p. 25). She is correct that the formal procedure for this does not appear within the primary curriculum. However, the introduction of the formal procedure is intended to take place after children have developed some understanding in previous years, including Y6. Her understanding of the problem as simply involving the procedure further emphasises her limited understanding of the longitudinal coherence of the mathematics curriculum. I note also that, as I discussed in Chapter 2, ideas of longitudinal coherence in the form of sowing seeds for the future development of the big ideas in mathematics was a feature of the CAME approach.

A further feature of both teachers' knowledge of the primary curriculum is their perception of what they regarded as basic concepts. For example, they both regarded the equivalence of fractions, decimals and percentages as basic primary teacher knowledge. Indeed, Alexandra said that it would be "a bit concerning" if a primary teacher did not know this. This appears in Y5 at the top end of the primary curriculum (DfEE, 1999, Y456 examples, pp. 21, 31, 33). Yet, the knowledge of different meanings of multiplication, which, as I noted above, appears earlier in the Framework, was something that both teachers felt to be not only difficult but also unnecessary to primary teaching. It is perhaps unsurprising that they regarded this more conceptual knowledge as more difficult than the more procedural and factual knowledge of equivalence. A further issue here is that they appeared to perceive a huge gulf between the knowledge they themselves felt to be secure and the knowledge they felt to be weak: what they knew they felt to be very easy; what they did not know they felt to be very difficult.

4.2 Problems of Context

There was a further weakness in the teachers' mathematical knowledge that, whilst related to this procedurality, goes beyond simply a procedural way of knowing. Their knowledge appeared to be easily disrupted by the context of the mathematics interview. I refer to knowledge the teachers themselves expected to "know" securely, but which considerably less secure than they expected it to be. In particular, this knowledge was less secure in non-routine contexts. I explore this in relation to Janice's use of a procedure for division, and both teachers' difficulties with problems set in "real-life" contexts.

4.2.1 Using a Standard Written Procedure for Division

Janice made a series of unusual and only partially corrected errors in using a standard contracted procedure for division when answering the following question:

> True or false. There is a multiple of 7 between 6226 and 6231.

Janice mis-read the question that I actually gave to her and answered the following question: Is there a multiple of 7 between 6226 and 6221? [6223 is a multiple of 7.] She may have simply mis-read the question. It is significant that she did not appear to notice this mis-reading, although I did not point it out to her. She addressed this question by dividing 6226 by 7 to find the remainder.

Janice's solution is outlined below in Figure 7.2. I outline her solution in some detail since her errors were unusual. After using a calculator and failing to interpret the display, she made two attempts, both incorrect, to divide 6226 by 7 using a standard procedure. She, then, incorrectly worked out the remainder and gave an incorrect solution. Finally, she attempted without success to explain how she had used the remainder to solve the problem. I have annotated her solution to highlight all these stages in her solution.

1	**Calculator:**	Janice correctly divided 6226 by 7 using a calculator.
2		She appeared to be unable to interpret the display of **889.4285714**
3		*"Oh, no, I'd rather do [paper and pencil] because I don't want to use decimals."*
4	**Attempt 1:**	She incorrectly performed the standard pencil and paper division algorithm for 6226 divided by 7.
5		**888 r8** **7)6226.0**
6		*"Seven eights are fifty six ... seven eights are fifty six, seven eights are fifty six."*
7	**Attempt 2:**	She "corrected" her previous calculation.
8		**889.4r2** **7)6226.^{3}0**
9		*"What am I doing? So seven eights are fifty six. - - - I can't work it out. Oh it's a nine, isn't it? That's because I used a calculator."*
10		She corrected the quotient to 889.
11		*"Seven nines are, that's sixty six. Seven nines are sixty three, that's a three."*
12		She added a carry 3 to the dividend.
13		*"Four sevens are twenty eight. I'm much better than this, I don't know why I'm getting confused. Seven nines are sixty three, three left over, seven fours are twenty eight, remainder two. And it carries on."*
14		She "corrected" the quotient to 889.4 remainder 2.
15	**Remainder:**	She worked out the "remainder."
16		She added the .4 to the remainder of 2.
17		*"So remainder six."*
18	**Solution:**	She used the "remainder" of 6 to calculate whether there was a multiple of 7 between 6226 and 6221.
19		*"So, if that's remainder six, I've got to take six off that."*
20		She wrote 6226 – 6 = 6220.
21		*"So ... six, two, two, nought ... That's the multiple of seven."*
22		*"So there are no multiples of 7 between 6226 and 6221."*
23	**Explanation:**	She attempted to explain her use of the remainder to solve the problem, and appeared to get extremely confused.
24		*"Now I'm feeling frustrated because I know this is simple and I'm doing it wrong."*

[Note: 6226 ÷ 7 = 889 remainder 3. Hence, 6223 is a multiple of 7. [6226 = (889 × 7) + 3)]

Figure 7.2: Janice's calculations for 6226 ÷ 7.

The significant features are as follows:

Firstly, she was unable to use the calculator display, 889.42 … , to generate the remainder. This reflects the difficulties she had with decimal fractions that I discussed above. [Lines 1 – 3]

Her first attempt at the pencil and paper method contained an error, which appeared to be as a result of the repeated mantra of "seven eights are fifty-six." She recognised and corrected this error. [Lines 4 – 6]

In correcting this first error, Janice made a similar error to her repeated mantra above: she carried on "too far" in her calculation producing a quotient of 889.4 remainder 2, a quotient which has little meaning mathematically. This was a much more significant error. The digit "4" correctly here represents $^4/_{10}$, whilst the "remainder 2" actually represents $^2/_{70}$ (or $^2/_{10} \div 7$), although it seems highly unlikely that Janice understood this. It seems more likely that, although Janice was unable to interpret the calculator display in relation to the problem, she did expect the calculator to be correct. Hence, having seen .42 on the calculator display, she expected to encounter 4 and 2 in the quotient. [Lines 7 – 14]

She added the 4 and the 2 to get a "remainder" of 6. This sum, and hence her "remainder" of 6, has no mathematical meaning. Indeed, in adding these numbers, Janice was in effect adding the numerators of two fractions without regard for the denominators, a common error in children's reasoning about fractions (Kerslake, 1986). In this specific example, it seems likely that Janice did this in order to transform 4 remainder 2 into an "acceptable" remainder. [Lines 15 – 17]

She used this "remainder" to calculate whether there was a multiple of 7 between 6226 and 6221. Although her answer is incorrect here, it is significant that the calculation she performed here was correct. This calculation demonstrates an understanding of the meaning of the remainder as the difference between the dividend and the nearest lower multiple of the divisor. Indeed, what Janice got "right" here was in many ways "harder" and less routine than what she got wrong: carrying out a routine division procedure to produce a remainder. [Lines 18 – 22]

Finally, she became confused in attempting to explain how the pencil and paper algorithm works and how she has used the remainder to calculate that 6220 is a "multiple" of 7. Here, she appeared to experience directly the fragility of her knowledge in her frustration that she could not explain what she had done. [Lines 23 – 24]

The errors that Janice made here were unusual ones, which suggest that she had a limited understanding of the division algorithm. However, although her knowledge of the division algorithm was procedural, this was a procedure she had used frequently throughout her teaching career and was one she felt herself to be proficient at. Moreover she had explained and taught the procedure *and* how it worked to the teachers in her school as part of the NNS school training. It seems likely that in other circumstances Janice would have been able to carry out the procedure here correctly. However, the combination of the mathematics interview, her own inability to interpret the calculator display, her belief in the calculator as an arithmetical "authority," and the unusual nature of the problem served to throw Janice off course. This was compounded by her acknowledged difficulties with decimals.

Janice's knowledge here was not robust enough to withstand the different constraints and affordances of a new and non-routine situation (Greeno, 1998). The procedural nature of Janice's understanding here coupled with her weak and partial mathematical knowledge limited the ways in which she could respond to such constraints and affordances. Hence, fragility is a consequence of the more general weakness in her mathematical knowledge and is a reflection of the difficulties she had in adapting her knowledge into new and non-routine situations. However, the least routine element of the problem, the use of the remainder to identify the nearest lower multiple of 7, Janice could do. It was the most routine aspect that she had difficulty with. Indeed, I suggest that, whilst the non-routine nature of the problem itself was an issue here, the non-routine nature of the setting, the mathematics interview, coupled with Janice's beliefs about authority in mathematics as external, were more important factors in these errors.

4.2.2 Real-life Contexts

Two of the ten problems were presented in a "real-life" context. Although both teachers had at other times stressed their belief in the importance of real-life contexts for teaching, both teachers encountered considerable problems with both of these problems. The first of these problems involved ratio and enlargement:

> True or false? From a plane, a field 90m by 100m looks more 'square' than one 950m by 1000m.

Both teachers did eventually provide a correct mathematical answer. However, both also queried the "reality" of the context. Clearly there can be an argument as to the pedagogical value of any real-life context and the extent to which it is a useful vehicle for teaching a particular mathematical concept. However, the teachers here questioned the extent to which the mathematics *could* model the situation. Indeed, the teachers' problems appeared to be less about the pedagogic value of the context and more about their own difficulties with the mathematical ideas involved. In particular, although they both recognised the problem mathematically as a multiplicative problem involving ratio, they *felt* the real-life situation to be better represented additively. Alexandra, for example, expressed her difficulties as follows:

> Nine tenths and nine and a half tenths, yeah. Mmm. But it's still a bigger distance. Mmm. I know it's proportional. [PAUSE] Oh, this is very interesting isn't it? I know that's nine tenths and that's nine and a half. But I still think, because of the larger numbers, the distance ... I think it would be that one [950m x 1000m], but it's hard, isn't it? I think. [PAUSE] I actually think, I still say, that they look the same. ... I'm still going to go by the fact that a hundred metres, oh, I don't know. [LONG PAUSE] Ten metres and fifty metres I think. Yeah, I'm still going to stick with my answer [that both rectangles would look the same]

Although Janice knew "nine and a half tenths" to be closer to 1 than nine tenths, she still doubted her answer, because 50m is a greater distance than 10 m. Her doubt concerned the relative size of the numbers involved, although she justified this by implying that the mathematics does not represent the context.

Both teachers identified the intended mathematics in this problem, with Janice describing it as proportional and Alexandra identifying it as ratio. They both could solve the mathematical problem correctly. The problem for both teachers, and Janice particularly, was that they appeared not to have fully accepted the problem as one involving ratio and proportion. Indeed, the real-life context here highlighted their difficulties in *intuitively* knowing this problem as multiplicative rather than additive.

The second real-life problem was as follows:

> How would you check the price of a bag of fruit with your calculator if the fruit cost £1.68 per kilogram, and your bag weighed 0.86 kilograms?
> [Solution: 1.68 x 0.86 = 1.4448 giving a price of £1.44]

Both teachers found this problem extremely difficult. In common with many of the teachers in the LNRP focus 2 sample (Askew & Millett, 2001), both initially approached the problem using division [1.68 ÷ 0.86], reflecting the common misconception that "division makes things smaller." Alexandra was, with considerable support and prompting, able to solve the problem correctly.

Janice, on the other hand, found both the wording of this question and the metric context extremely confusing. She argued that she had not been told the weight of the bag and commented that she did not "think well" in kilograms. Indeed, she was unable to solve the problem despite several assurances that the weight of the bag was negligible. She did later e-mail me the following mathematically correct but inaccurate solution:

> 168p=100%
> x = 86%
> I find 1% by dividing =1.68 and then multiplying by 86.
> After rounding my answer £1.46
> (Janice, Personal communication, 20 July 2000).

I note that this solution was accurate enough for the context of supermarket shopping. Nevertheless the inaccuracy is of interest and was, I suggest, caused in part by her reluctance to use a calculator discussed above. Hence, I suggest she rounded 1.68 to 1.7 in order to make her pencil and paper calculation easier.

As with the division algorithm above, the simple calculation involved here was knowledge Janice expected to know, but which was disturbed by the setting of the mathematics coupled with the real-life context. Indeed, she commented as follows in her e-mail: "I thought it was asking me to work out the weight of the fruit after I had deducted the weight of the bag. This was of course ridiculous. Why would the bag be so heavy" (E-mail communication, July 2000).

The teachers' difficulties with these real-life problems certainly reflected the weaknesses in their mathematical knowledge and mirror the difficulties that children have with such problems. Of further significance is that they were thrown by the context. Both teachers perceived one of CAME's strengths to be the way in which mathematics was placed in such contexts, although they both appeared to view this as a way of motivating and interesting children, rather than as a way of using such situations to illustrate mathematical ideas or on engaging with mathematical modelling. Their performance on these questions would suggest that teacher education needs to pay greater attention to these aspects of problem-solving.

4.2.3 The Fragility of Alexandra's and Janice's Mathematical Knowledge

Much has been written about the greater difficulty of non-routine problems over mathematics of a routine nature (e.g., Brown, 1994; Grouws, 1990). Indeed, McLeod (1992) links this difficulty to beliefs about mathematics as procedural, and particularly a common belief that mathematics problems should be solved quickly. It is no surprise then that these two teachers found these problems more challenging than the others. However, the level of difficulty they had and, in particular, the apparent ease with which aspects of their knowledge were disturbed. I emphasise that the mathematics disrupted here was, unlike the representation of the multiplication of fractions, mathematics that both teachers felt was secure. In other words, their mathematical knowledge was fragile.

4.3 The Procedurality of Alexandra's and Janice's Mathematical Knowledge

Both teachers' mathematical knowledge had significant weaknesses in relation to all of Ma's (1999) aspects of teacher knowledge. Indeed, they appeared to share at least some of the misconceptions and limited understandings of children (See, e.g., Hart, 1981; Kerslake, 1986). Their knowledge was largely bounded by procedures and weaknesses in terms of their understanding of basic ideas and connections compounded weaknesses in other areas. In short, judged against the criteria I set out in Section 2, their knowledge was neither teacherly nor principled. Moreover, their knowledge was fragile: aspects of their knowledge they felt to be secure were easily disrupted by non-routine or real-life contexts.

Although in many ways Alexandra's knowledge was less fragmented and more teacherly than Janice's, it was still very procedural. Indeed, her knowledge of some of these procedures appeared in places to be insecure. Moreover, her experiences of working with conceptual ideas in the CAME fractions lessons, did not appear to have impacted deeply on her knowledge in this area.

It is certainly possible that the teachers' knowledge might have appeared to be "better" outside the test situation. Certainly, the constraints and affordances of a test situation are very different from settings in which these teachers might use this knowledge in teaching children or other teachers. Indeed, I have observed that Alexandra appeared to know at least some of this knowledge in different situations, although these were settings where she was working with others, either Ursula or Mundher, and she had access to lesson or course guidance. Nevertheless, given the overall weakness of both teachers' knowledge across all of Ma's (1999) categories, it seems likely that their knowledge would have been equally compartmentalised in the more complex environment of a classroom.

4.4 Beliefs and Identity

For both Alexandra and Janice, the mathematics interview appeared to involve very high stakes. Both exhibited strong reactions of frustration, discomfort and pain during the mathematics interview. Their comments included: "Go away while I work this out"; "This is horrible"; "I'm not finding this comfortable" (Alexandra); and, ; "This is so painful"; "I know that's stupid. That is really stupid."; "That's appalling"; "Yes. That is really bad. I'm going to go home and cry." (Janice).

Alexandra, both prior and subsequent to the interview, consistently referred to it as "the test". During the interview she said "you are not going to go and tell Jane [Outertown's primary advisor and Alexandra's boss] I can't do this anymore." Whilst this comment was made amid laughter, there was a serious message behind it, as is clear from a later comment:

> It's [the thesis] about us, isn't it. What are you going to say about us? Are you going to say I shouldn't be a Numeracy Consultant. Because you know. And deep down that's what I think. (Fieldnotes, July 2001)

It is significant that at no point did Alexandra doubt her credentials as a teacher. She was certainly self-critical of her teaching at times, but in doing so she did not question her fundamental ability as a teacher. Indeed, her perception was that her authority as a Numeracy Consultant rested primarily on her strengths as a generalist teacher. Yet, this was something of a double-edged sword and did not prevent Alexandra from doubting her mathematical ability. As a consultant advising on mathematics teaching and teaching mathematics subject knowledge to teachers, she was aware of gaps and uncertainties in her subject knowledge. In short, she believed her subject knowledge to be at least in some ways deeply inadequate and was concerned about this being found out or exposed.

Although Janice's position as a primary teacher and mathematics co-ordinator was not as dependent upon her mathematical knowledge as that of Alexandra, she nevertheless saw the experience as one in which her mathematical performance was under scrutiny. At the end of the interview, she asked: "Did I pass?". Indeed, throughout the interview she sought constant reassurance: "Is that how you'd have done it?"; "Was that better?"; "Thank you for telling me the answer. [WHISPERED] Thank you." She took away two of the problems for which she was dissatisfied with her solution: "This is going to annoy me. ... I should be doing it in a better way. I shall go home and play around with this." In fact, she e-mailed a correct solution to one of these problems that evening. This is particularly significant in that Janice sent me an e-mail on just two occasions during the two years she was involved in this research as a teacher-researcher, although I e-mailed her on several occasions.

This issue of threat was evident in both Alexandra's and Janice's performance during the interview itself. For example, both teachers gave vague responses to several of the problems and I had to push both to provide more specific responses. There were long and at times difficult pauses. They constantly referred to their dissatisfaction with their performance. They got confused at times and made fairly trivial mistakes. Both asked me asked me to ratify their solutions. Although all of these things occurred on occasion during other interviews, they took place throughout these mathematics interviews.

Overall, the reactions of both teachers display a strong orientation towards Povey et al.'s (1999) external authority in mathematics. In Janice's case this accords with my other findings. However, I have argued in Chapter 6 how Alexandra had developed aspects of a personal author/ity in mathematics. Her performance here strongly suggests that her development of this orientation was somewhat insecure. It also adds weight to the suggestion of Thompson (1984) that changes to teachers beliefs about mathematics may precede changes in mathematical knowledge.

4.4.1 The Possibility of Desire

In Chapter 5, I discussed the issue of desire as a motivating force for change in terms of mathematics teaching. As I noted in this earlier discussion, an advantage of this approach was that it located the motivation for change in relation to the difficulty and pain of the process. I tentatively suggest that desire provides a way of understanding the attraction of mathematics.

The pain that Alexandra and Janice experienced during the mathematics interview has resonances elsewhere in the discipline of mathematics. Doing mathematics is at times a painful experience, even for successful mathematicians. (See, e.g., the case of Richard Borcherds, the 1998 Fields Medallist, described in Singh, 1998). Indeed, part of the attraction of doing mathematics is this combination of frustration and pleasure. (See, e.g., Polya, 1957.)

Alexandra did appear to experience the possibility of desire for mathematics. Unlike the other teachers, she often expressed pleasure when doing mathematics in seminars, albeit these were infrequent occasions and her pleasure was largely associated with external approval. She expressed a strong interest in taking mathematics further "to see how far I could go" (July 2000) and borrowed Burton (1984) and Polya (1957) to pursue this on her own. However, she appeared to take this no further. Ultimately, I suggest, unlike her experiences in developing and changing her beliefs about teaching, this was an individual and isolated approach to learning mathematics, and it was difficult for her to generate or maintain a desire for mathematics.

5. Discussion

Both teachers' mathematical knowledge in the area of multiplicative relations was procedural and fragile. Given my analysis of this procedurality above, it seems likely that any changes to their mathematical knowledge were at best very small. This is surprising in relation to both teachers, because both had had very much more professional development in mathematics education than most primary teachers, and, aside from masters level, it is difficult to see what "more" they could get in the current climate. It is particularly surprising in Alexandra's case. She had not only developed the two fractions lessons in this area; she had written an academic paper about teaching the multiplication of fractions; she had taught this on CAME PD and on NNS courses; and, she attended NNS Numeracy Consultant sessions about fractions and multiplication.

I did only conduct these interviews with two of the teachers. In particular, as I discussed in Section 3.1 above, I was not able to interview Ursula, the other teacher for whom belief change was significant. Given Ursula's reluctance to do the mathematics interview and her unwillingness to be observed performing mathematics in other settings, it seems likely that she had many similar difficulties to Alexandra and Janice. However, given the lack of data, I can only speculate on this point.

It is clear from this study that change in this area is very difficult. In this section, I suggest some reasons for these teachers' difficulties.

Firstly, these teachers' mathematical knowledge was situated. It does seem likely that faced with these problems in other situations both teachers would have "known" more and performed "better." Indeed, Alexandra did "know" about different representations for the multiplication of fractions when she led the lesson simulation of Halving & Thirding in February 2000. Moreover, it is likely that Alexandra, for example, would have appeared to "know" more about models of multiplication in the context of running an NNS course, although I note, as an aside, these course are heavily scripted. This points to their mathematical knowledge as being rigidly situated in limited and particular contexts. This would suggest that teacher education in primary mathematics should be addressed towards enabling teachers to adapt, transform and thus "transfer" their mathematical knowledge into a greater range of settings. Boaler's (1997) work with secondary school students would suggest that doing more open-ended mathematics might be one way of achieving this.

Secondly, although CAME sought to develop teachers' subject knowledge, the academics deliberate strategy was to tackle such issues gently. This was a trojan horse approach in which through teaching the lessons and engaging with children on challenging mathematics, the teachers' own subject knowledge would itself develop. This approach certainly recognised the difficulties and anxieties that primary teachers encounter in relation to mathematics. However, this was not a motivating strategy and it was not a strategy that acknowledged and engaged with this anxiety and pain. I suggest that teacher education in primary mathematics should address teachers' subject knowledge directly, whilst acknowledging these difficulty of the process and exploring ways of generating mathematical desire.

Thirdly, these teachers' subject knowledge was intricately inter-related with aspects of their identities. Bibby and I have discussed elsewhere (Bibby & Hodgen, 2002) how the teachers in this and another study appeared to be stuck in the past as secondary school learners in terms of their mathematics knowledge. We argued that this was a barrier to developing a teacherly understanding of mathematics. I suggest that teacher education in primary mathematics needs to find ways of enabling teachers to overcome these issues of identity.

6. Summary

In this chapter, I have explored the issue of the teachers' mathematical knowledge. I did this using the cases of two of the teachers, Alexandra and Janice, and analysing their knowledge through a mathematical interview focused on multiplicative relations. This research tool was adapted from a structured interview used in two other studies (Askew & Millett, 2001; Bibby, 2001).

I reviewed the literature on mathematics teacher knowledge highlighting differences between a teacherly knowledge of mathematics and the knowledge required to do mathematics.

I analysed the teachers' knowledge using Ma's (1999) criteria for a profound understanding of elementary mathematics: basic ideas, connectedness, multiple perspectives, and longitudinal coherence. I found the teachers' knowledge to be procedural, fragile and at times insecure.

Both teachers exhibited a strong orientation towards external authority in mathematics during the interview. In Alexandra's case, this was especially surprising, since elsewhere I had found her to have developed an orientation towards author/ity.

I discussed reasons for weaknesses in these teachers' mathematical knowledge and suggested possible strategies for teacher education to address

Chapter 8: Review and Discussion

1. Introduction

In this chapter I review the argument as whole and discuss the contribution and implications of this study.

As with any work of this sort, the approach has at times been wide-ranging. Hence, in this chapter I draw out and discuss themes that cut across the arguments in individual chapters.

The structure of this chapter is as follows:

- In Section 2, I outline briefly the contribution of this thesis, although this is intended as an overview and orientation for the reader since these ideas are developed throughout this chapter.
- In Section 3, I review the research as a whole, pulling out several themes that run through the earlier discussions.
- In Section 4, I discuss the generalizabilty of the claims I make in this thesis.
- In Section 5, I discuss the limitations of this study.
- In Section 6, I discuss what these findings mean in a broader sense. In this section, I address several of the issues that I have highlighted for discussion during the course of earlier chapters.
- In Section 7, I discuss briefly how I have addressed the research questions and raise questions for further research.
- Finally, in Section 8, I make some brief concluding remarks.

2. The Contribution of This Study: Overview and Orientation

The study first and foremost makes a theoretical contribution to understanding the nature and processes of teacher change in primary mathematics. By focusing on a small group of specially chosen teachers involved in an accelerated professional development setting (Nolder, 1992) over an extended period, this study is aimed at exploring the possibilities for professional change in primary mathematics teacher education. The fact that, even given such a favourable environment with a group of willing volunteers, only two of the six teachers changed their beliefs to any significant extent is itself of significance and of even more interest was that this belief change was not accompanied by proportionate changes in the teachers' mathematical knowledge. This was therefore an ideal environment in which to study both the processes of change and the barriers to change. A major aspect of the research was focused on the teachers' development as tutors and, indeed, two of the teachers became Numeracy Consultants during the course of the fieldwork. Hence, the study also contributes to understanding the professional development of practitioner-based teacher educators, a current policy focus of the NNS professional development approach.

I have extended and developed theories of situated learning both empirically and theoretically. Empirically, I have demonstrated how the approach can be used to understand primary teacher change over an extended period of time. Theoretically, I have emphasised the role of individual teacher agency in conjunction with social structure in the process of change, thus introducing a more heterogeneous and differentiated picture of change than is apparent in much of the situated literature. To do this I have drawn on both cultural and psychoanalytic theories of identity in order to conceptualise teacher change as one involving teachers' authorship and imagination. Alongside this, I have used Boaler's (2000a) notions of similarity and difference as a starting point from which to develop analytic tools to examine the ways in which teachers can make sense of new ideas and adapt or transform, and thus transfer, their existing practices and knowledge into new situations. In doing so, I have extended our understanding of the role and nature of both reflection and motivation in teacher change.

I set out to explore the relationship between beliefs and knowledge. Here, I have found that substantial changes in beliefs about school mathematics are not necessarily accompanied by changes to a teacher's knowledge of specific mathematical concepts. Indeed, this study suggests that developing a deep and principled knowledge of mathematics is a very difficult process. I have also addressed the nature of teachers' difficulties in this area. To do so, I have extended the approaches of Ma (1999) and others by relating teachers' mathematical knowledge to theories of identity.

Some of what I am saying here is not wholly new. It is well established, for example, that the professional change of mathematics teachers is a difficult and complex process, although my study does suggest that PD in primary mathematics is an even more difficult process than the literature suggests. I have also added both empirical and theoretical depth to knowledge in this area in studying professional change in depth over an extended period. Moreover, this study is directed at better understanding the *nature* of these difficulties: in what ways is professional change difficult; and, in what ways can these difficulties be overcome?

Some aspects to this study are particular to primary education. The study is concerned with a group of generalist teachers who had not studied school mathematics beyond GCSE equivalent. Mathematics is, moreover, a subject that generates negative emotions for many primary teachers, which was certainly an issue in relation to the teachers' knowledge of specific mathematical concepts. Nevertheless, the theoretical approach and the understandings that I have developed concerning motivation, transfer and reflection are I believe applicable beyond the confines of primary mathematics teaching. Indeed, mathematics has frequently been used as a paradigm case from which to study education or teaching and learning generally. In a recent review concerned with the general applicability of situated learning theories to teacher education, for example, Putnam and Borko (2000) almost wholly draw on examples on examples from mathematics teacher education, the majority of which are set within the primary sector.

3. Review of The Argument

In this section, I review the research and give an overview of the theoretical approach that I have developed. I note that I provide an introductory guide to the structure and content of the thesis in Chapter 1 and I do not intend to repeat myself here. Rather, in this section, my intention is to give an overview of the study drawing together the argument as a whole and to pull out key themes that cut across the discussion in individual chapters. In doing so, I review the analytic tools that I developed linking these to the literature and the empirical imperatives that led to their development.

The overall aim of this study was to explore the ways in which teachers' beliefs and knowledge change and develop in the context of a professional development initiative in primary mathematics. I have largely focused on teachers' beliefs in relation to three areas: their orientations towards authority in mathematics drawing on Povey's (1997) work (see Chapter 4); their orientations towards mathematics as a connected discipline drawing on the work of Askew et al. (1997); and, the extent to which they developed an understanding of *mathematics without closure*, a set of ideas about school mathematics promoted by the Primary CAME project (see Chapter 2). In terms of the teachers' knowledge of mathematics, I have examined the extent to which they developed a principled understanding of mathematics drawing on the work of Lampert (1986), Ma (1999) and others. (See Chapter 7.)

The fieldwork for this study was conducted over a four-year period from November 1997 until July 2001. I used a qualitative methodology, taking the role of a participant observer and drawing on both ethnographic (Charmaz, 2000; Strauss & Corbin, 1998) and social constructivist approaches (Kvale, 1996). (See Chapter 3.)

The research focused on the professional development of six teachers participating as teacher-researchers in the Primary CAME project, a joint venture between King's College London and Outertown LEA. The six teachers were: Alexandra and Ursula, initially class teachers at Parkway School, who during the course of the fieldwork became Numeracy Consultants for Outertown LEA; Henrietta and Lisa from Beechmount School; Janice from Brightvale School; and, Tony from Meadowside

School. Alexandra, Lisa and Ursula were involved for the entire four years. (See Chapter 2.)

Although for the purposes of my research this was an opportunistic sample, the six teachers in this study were specially chosen in order to facilitate the Primary CAME project's development. These teachers were identified at the outset of the project as having sufficiently supportive schools and professional experience in order to enable them to develop as CAME teacher-researchers fairly rapidly. Yet, as I noted in Section 2 above, despite this background and the extended and extensive professional development programme, only two of the teachers, Alexandra and Ursula, changed their beliefs about school mathematics to a significant extent. This differential change amongst the teachers has been one of the foci for my research both in examining the issue of differential change per se and as a point of contrast from which to explore the process of professional change more generally. Moreover, I found that this belief change had not been accompanied by changes in mathematical knowledge. The issue of mathematical knowledge is the focus of Chapter 7. Even for the two teachers who changed, the process of change was a slow and somewhat opaque process in which I rarely observed explicit learning. Hence, a central problem for the study was to "uncover" this process of learning.

In Chapters 4 and 5, I developed an approach to understanding teacher change which draws on and extends the situated theories of Lave and Wenger (1991), Wenger (1998), Boaler (1997) and others. Using this approach, I located teachers' personal resources within a social context and conceive of their learning as one of participation and enculturation. Hence, I analysed as social resources the individual factors that Earl et al. (2000) identify, namely individual capacity and motivation, thus avoiding a conception of individual teachers as in deficit (Brown & McIntyre, 1991). An advantage of this approach is that it focuses on discontinuities between settings, or communities of practice, and problematises the notion of the transfer of knowledge between different communities. Hence, the situated approach starts with the assumption that change and development is not a simple process. Drawing on the work of Boaler, I used ideas of *similarity* and *difference* to explore the extent to which the teachers were able to draw on their existing practices in order to make sense of new practices and, thus, overcome the discontinuities between these practices and engage as *legitimate peripheral participants* in the project.

My argument is focused on interpreting the teachers' individual capacity in terms of the teachers' social resources rather than in terms of individual ability. Again using the notions of similarity and difference, I examined the extent to which the teachers could draw on, adapt or transform existing practices in order to make sense of and improvise practice in the new context of CAME. I focused on the extent to which similarities and differences between the teachers' existing practices and those of CAME either enabled or restricted the teachers' participation and discussed how the new ideas of CAME were mediated through the teachers' wider professional networks, or *zones of enactment* (Spillane, 1999). I argued that, whilst a wide zone of enactment was a necessary factor in facilitating change, it was not a sufficient one. Crucial factors in facilitating substantial change were the interconnections between their existing professional practices and those of CAME and the extent to which these sets of practices overlapped. I argued that the *form* of these practices, particular ways of working and knowing, are as important as their *content.* In Chapter 5, I discuss how the two teachers who did change were able to extend and make use of their existing collaborative relationship, much of which was enacted in non-mathematical curriculum contexts, in becoming CAME teacher-researchers.

A weakness in these situated approaches is that much of the literature is focused on the community as a whole and as a result the depiction of individual change within communities is homogenous and somewhat undifferentiated. Whilst Wenger (1998) has attempted to address these concerns in his discussion of individuals' multi-membership of many communities and the role of imagination in learning, his account is at times speculative and generally lacks empirical foundation. In order to address this weakness, therefore, I have drawn on Holland et al.'s (1998) work on identity in order to emphasise the role of teachers as co-authors in their own change. Thus, I conceive of *all* the teachers, not just those who changed, as actively making sense of CAME. (See Chapter 5.)

In Chapter 4, I identified teacher motivation as key factor in teacher change, highlighting two aspects: initial motivation and motivation to sustain change.

Motivation is a further weak area in Lave and Wenger's (1991) work. Drawing loosely on Marxist theory, they identify two forms of motivation: *use value* and *exchange value*. These broadly correspond to the familiar intrinsic and extrinsic forms of motivation (e.g., Middleton & Spanias, 1999). Lave and Wenger's argument is that *authentic* and *undistorted* learning focused on facilitating an individual's central participation in a community necessarily engenders use values, or intrinsic motivations, rather than exchange values or external motivations. Again, this analysis lacks differentiation and fails to explain the different patterns of individual change. Moreover, it does not specifically address the difficult issue of sustaining change: what motivates teachers to continue with change despite this process being at times painful and difficult.

Whilst all the teachers had at the project's inception indicated their willingness to participate, this willingness did not appear to be sufficient to enable them to engage with CAME. I discussed how, for the two teachers who changed, a combination of similarities and differences between their existing mathematics teaching practices and those of CAME created *interest* and thus the motivation to initially engage with change. In broad terms, the crucial factor in generating this interest was that existing and new practices were similar but not too similar. This close combination of similarity and difference enabled the teachers who did change to perceive not only a *need* for change but also *how* their existing mathematics practices needed to change. (See Chapter 4.)

However, whilst initial motivation was a crucial factor in teachers' initial engagement with new ideas, *interest* does not explain the teachers' motivation to continue with a change process that is at times extremely painful. I found that the two teachers, who did change, expressed this motivation in strongly emotive terms. To understand this, I drew on Lacan's psychoanalytic approach to identity as interpreted by Zizek (1992), Brown and Jones (2001) and others. Using this, I recast the teachers' motivation to change in terms of *desire*. I discussed how, for Ursula, the desirable image a new identity as a changed mathematics teacher provided a *compulsion* to engage with the ideas of CAME despite this being at times a painful process. Holland et al.'s (1998) notion of authorship is key here in that Ursula actively maintained this desire through her construction of this imagined and *only partially grasped* future as a different teacher. (See Chapter 5.)

Reflection is a widespread and much accepted idea in teacher education. Indeed, it forms a key element of the CAME approach. Yet, I observed explicit reflection to take place infrequently. Here, I take reflection to mean the conscious reconstruction of knowledge. I explored the issue of reflection using the case of the two teachers who changed significantly. For these two teachers, there were reflection events that, whilst infrequent, appeared to be significant events in their learning. Again, here I addressed the issue of motivation, drawing on Holland et al.'s (1998) notion of identity as authoring. I placed the motivation to reflect in terms of compulsion. Reflection took place in social settings which demanded a response from the teacher. Using Schifter's (1996) conception of multiple identities I found that the teachers' engagement in multiple roles provided opportunities for the teachers to reflect by enabling them to step outside themselves. (See Chapter 7.)

I discussed the barriers to change in Chapters 4 and 5, using the ideas of similarity and difference. I argued, using the case of Lisa, that surface similarities between existing and new practices could obscure differences between practice and, hence, both obscure the need to change and fail to generate interest in change. For Janice, who did appear to experience both interest and desire in CAME, the difference between her existing practices and those of CAME were too great to overcome. Faced with difficult and painful experiences, she experienced *shame*, a strongly negative emotion that inhibited her ability to imagine and construct her new identity (Giddens, 1991). Using the cases of both Henrietta and Lisa, I described how too great a difference between practices could offer limited *potential*, or capacity, for change leaving teachers unable to make sense of CAME. I argued that limited experience, rather than inexperience per se, was a barrier to change. The two inexperienced teachers in this study, Henrietta and Tony, had, as a consequence of their inexperience, relatively limited zones of enactment and this was a barrier to their professional change.

I analysed two of the teachers' mathematical knowledge in relation to multiplicative reasoning. I focused on two of the experienced teachers. I found both teachers' knowledge to be not only procedural but also *fragile*. Their knowledge was easily upset by the setting of a mathematics interview. It was a surprise that the significant changes in Alexandra's beliefs were not accompanied by similar changes in knowledge, particularly as the interview explored mathematical concepts she had previously shown some proficiency with in developing lessons and leading professional development sessions. Indeed, in the setting of the mathematics interview Alexandra's beliefs appeared to revert to an orientation of external authority.

4. The Generalizability of This Study

In this section I address the generalizability of this research. The claims I am making to generalizability relate to the theoretical approach that I have developed together with the understandings of individual capacity, motivation, reflection and beliefs and knowledge that arise from it. One important factor then is that this qualitative case study of a small and somewhat unique group of teachers does not stand alone. The study is a contribution, along with other studies in the field, to the development and elaboration of theory relating to teacher education in general and to primary mathematics teacher education in particular. In doing so, my study adds to and extends the developing body of literature exploring situated approaches to teaching and learning. Hence, one aspect of the generalizability of my research rests on the extent to which it resonates with and expands this wider literature base both empirically and theoretically.

My claims for generalizability do not rest on the typicality of this case: . Rather they rest on the *atypicality* of this particular case. Although the sample was an opportunistic one in that these particular teachers were chosen by others, the sampling process was nevertheless both theoretical and purposive (Silverman, 2000). The teachers and setting here were a special case. The teachers were unusual for primary teachers in the time and commitment that they devoted to an initiative in mathematics education. Alexandra, Ursula, and later Tony, were even more unusual for primary teachers in that they became, as Numeracy Consultants, LEA primary mathematics specialists. The CAME professional development initiative was unusual in its intensity, in the access it provided for rich deliberations (Spillane,

1999) with academics *as* peers, and in the opportunity it provided for the teachers to develop strong identities not simply as mathematics teachers but also as curriculum developers and tutors or teacher educators. This professional development met to a very high degree all of the principles for mathematics teacher professional development identified by Clarke (1994). Moreover, the coincident timing of CAME alongside the implementation of the NNS provided an further degree of intensity to the teachers' professional development. Finally the access that I had to these teachers was unusual in that I studied their professional change over a four-year period, for large parts of which I met with them on a fortnightly basis. Hence, this was what Mitchell (1984) calls a telling case, "in which the particular circumstances surrounding a case, serve to make previously obscure theoretical relationships suddenly apparent" (p. 239).

Given the uniqueness of this context, I had expected the teachers' professional change as a whole to be significantly greater than it was. I expected that *all* the teachers would have changed and for this change to extend to fundamental aspects of their mathematical knowledge. In short, I expected this to be an exemplification of *what could be* (Kvale, 1996), a case from which to explore the possibilities of how primary teachers could develop as mathematics teachers with a profound understanding of mathematics (Ma, 1999) which they used to inform their teaching. It was a surprise then that the teachers' professional change was considerably less significant than I had expected and that the case taken as a whole was much less exceptional than I had hypothesised. As a result, the focus of my study shifted somewhat and, whilst a central concern remained on the processes of professional change, I also considered the barriers to change. Whilst this study is certainly less of a visionary picture than I expected at the outset, this more grounded setting involving the contrasting degrees of change, was closer to Kvale's (1996) *what may be*. Thus it provides a theoretical approach that is perhaps more immediately and pragmatically useful to the generality of teacher education practices in primary mathematics.

However, as Stake (2000) argues, the difficulty in case study research is not that generalisation is difficult, but rather that it is *too easy* to over-generalise. Hence, throughout the thesis, I have sought to validate my analysis through rich and thick descriptions and, thus, "by specifying the supporting evidence and making the judgements explicit, the researcher can allow readers to judge the soundness of the generalization claim" (Kvale, 1996, p. 233) Ultimately then, the generalizability, as well as the validity, of the claims that I make rest on the extent to which the argument and findings resonate with, convince, and are *useful* to the interested reader, both those in the research community and those concerned with policy and practice in teacher education.

5. Limitations to This Research

There are a number of limitations to this study. Firstly, this research involves a small sample of teachers. Whilst this enabled me to investigate the teachers' professional change in considerable depth, it is nevertheless important to emphasise that the empirical findings of this study are not *directly* applicable to other settings. I am not claiming that these six teachers typify primary teachers as a whole. The interest in this case lies in these teachers' atypicality and the opportunity afforded by this special case to explore the sometimes opaque processes of teacher change.

Pragmatically, as a single researcher with limited resources, I bounded the case study in several ways. I limited the aspects of the teachers' professional lives that I studied. I restricted myself largely to the teachers' mathematical development and even here my study was strongly focused on the teachers' involvement in Primary CAME. My investigation of their ordinary mathematics teaching, their relationships with teachers not involved in CAME, and their involvement in the NNS was limited. Moreover, my research is focused on the teachers' beliefs and knowledge about mathematics. I have not considered in any depth the changes to their classroom practices.

I have not considered issues of race and gender. These are important issues. For example, the academics were all male, whereas the teachers were predominantly female. In the context of the academics' high status mathematics knowledge, such a balance inevitably has consequences, which I do not explore in this thesis. Henrietta was the only black teacher in the research team. I have not investigated the extent to which this issue was important.

I have considerably less data on both Lisa and Henrietta. In Lisa's case, it proved difficult to arrange an individual interview with her after the first year. Whilst I took steps to address this problem (See Chapter 3), this remains a limitation. In retrospect, for example, it would have been valuable to explore whether Lisa's transformation in terms of management was significant and in what ways her development in this area affected her development as a mathematics teacher.

Henrietta left the project after the first year. My analysis has focused on the implications of her inexperience as a barrier to her professional change. However, over the lifetime of the project she would have developed into a considerably more experienced teacher. It would have been interesting to explore the extent to which a more extended participation in CAME would have enabled her to develop a more extensive zone of enactment, and whether this would have enabled her to overcome her perceptions of CAME as alien. In this regard, the other inexperienced teacher, Tony, was also involved in the project for a limited period.

5.1 What Would I Do Differently?

This is an exploratory study. Hence, I did not start the research with the theoretical approach that I present here. Indeed, my ideas about teacher education have developed considerably over the course of the research. Inevitably, I made decisions that, with the benefit of hindsight, I would have changed. However, as a research apprenticeship, a PhD inevitably involves many such blind alleys. Whilst I stand by the research process, there are nevertheless, in addition to the limitations outlined above, aspects of the research that, in retrospect, I would have done differently.

In exploring multiplicative relations, I deliberately chose a "difficult" area of mathematical knowledge. Given the difficulty the two teachers had in this area, I would have widened my gaze to examine other aspects of their knowledge. I would also have conducted mathematical interviews at an earlier stage in the fieldwork. This would have enabled me not only to track the teachers' changing mathematical knowledge, but also to explore the extent to which their difficulties in terms of multiplicative relations were reflected in less "difficult" areas of primary mathematics.

My understanding of motivation as desire arose late in the analytic process. As a result, I had no opportunity to investigate this issue further with the teachers. Whilst I did explore the teachers' mathematical histories during interviews, this did not directly examine the teachers' conceptions of desire. Had this arisen earlier, it would have been extremely valuable to explore the ways in which this mathematics teaching was conceived as an object of desire.

6. Implications

In this section I discuss the implications of this study, which I present in the form of a series of interrelated propositions.

Professional change in primary mathematics is difficult

My study adds further weight to the substantial body of evidence on the difficulty of teacher change and to Thompson's (1984) view that changes in beliefs precede changes in practices more generally. Further I have suggested that the process of change may be even more difficult than the existing literature suggests. In particular, I have argued that changing teachers' mathematical knowledge is very difficult. I suggest that it is important for teacher education initiatives to recognise these difficulties.

Professional change is facilitated by teachers' rich participation in a wide range of settings

I found that the teachers' participation across a variety of professional activities, and, hence, the extent of their zones of enactment, were key factors in facilitating professional change. The two teachers for whom professional change was significant were involved in professional activities beyond their school, had a close collaborative working relationship and were already developing an investigative approach to teaching mathematics. However, I found that wide participation was not sufficient for professional change. A further factor was the depth and range of interconnections, overlaps, between these teachers' existing practices and those of CAME. In this regard, I found that teachers' activity beyond mathematics could be a resource for professional development.

Limited experience was a barrier to change. Although the professional change of both the inexperienced teachers in this study was limited, this was due not to their inexperience itself but rather to their limited zones of enactment resulting from this inexperience. I suggest that one valuable focus for initial teacher education would be to focus on developing teachers' wider professional identities as in the work of Ponte and Brunheira (2001). One of the inexperienced teachers, Tony, did, however, develop a strong identity within the research team. I suggest that the sponsorship and mentoring of less experienced teachers by more experienced teachers may be a way of enabling inexperienced teachers' wider engagement.

Professional development can be facilitated by the teachers' activity in multiple roles as teachers, lesson developers, tutors and researchers

I found professional change to be facilitated by the teachers' participation in a variety of roles as teachers, lesson developers, tutors and researchers. Indeed, teachers' active participation as authors of their own professional change was a significant factor.

In relation to reflection, this rich participation provided opportunities for reflection as well as the possibility of distancing in order to reflect on one's identity. As I observed in Chapter 6, teacher change does not occur from telling teachers to reflect. Reflection is not a spontaneous and individual activity, but rather results from situations in which teachers recognised an imperative to respond. Activity in different roles provided such an opportunity. I observe that facilitating such activity is itself difficult. Indeed, despite the intense professional development opportunity afforded by CAME, only two of the teachers appeared to develop strong identities in all these areas.

Facilitating reflection requires more than the provision of reflection sessions

I found reflection, the conscious reconstruction of knowledge, to be an infrequent yet significant event in teacher professional change. Although reflection rarely occurred in formal reflection sessions, discussion at reflection sessions did provide the basis for reflection at other points in time. A valuable focus for reflection sessions would be on generating such reminders to facilitate future reflection either in the form of written materials or through teacher educators' interventions.

Professional development requires differentiation to account for the differences between teachers' social resources, or zones of enactment

I found the differences in the teachers' zones of enactment in this study to be a very significant factor. Teacher education needs to attend to such differences between teachers and, thus, differentiate the professional development. I have observed that these differences can be somewhat opaque. In this study, the extent of such differences between teachers was not apparent at the time. This suggests that teacher educators need to pay particular attention to this aspect. A valuable focus for teacher education would be on developing teachers' zones of enactment.

Recognition of similarities and differences between new and existing approaches was an important factor in these teachers' professional change, both in generating the initial motivation for change, and in enabling the teachers to adapt existing practices. Enabling teachers to recognise these similarities and differences would be a valuable focus for teacher education.

In one sense the gulf between the teachers' zones of enactment in this study was huge. Alexandra and Ursula's existing close collaborative relationship and the investigative approach to mathematics at Parkway were particularly significant factors here. Yet, as I briefly discussed in Chapter 4, Lisa's participation might have been transformed if Jenny, the Beechmount mathematics co-ordinator, rather than Henrietta, had been involved in the research team. Lisa had a working relationship with Jenny and Jenny had begun to introduce a more investigative approach to mathematics at Beechmount. Hence, whilst Clarke's (1994) principle that professional development should be focused on groups of teachers has merit, this study suggests that teacher education need to pay particular attention to the particular make-up of such groups.

Teachers' professional change may be limited by the professional space they have to engage with change

Although Lisa's professional change in relation to mathematics was limited, she did appear to change considerably in the area of management. During her participation in Primary CAME, she was also attending a range of performance management courses and implementing this within her school. Indeed, although this was an area which interested both Alexandra and Ursula, Lisa was significantly more successful in this area gaining a promotion to a headship. Lisa was able to attend meetings, and to participate (albeit to a limited degree) in CAME. Her participation was very extensive in comparison to most primary teachers' participation in mathematics initiatives. However, I suggest that one additional factor in Lisa's limited professional change in mathematics may have been that she had limited professional "space" to engage with CAME.

Professional development initiatives in primary mathematics need to attend to teachers' mathematical knowledge

The limited change of the teachers' mathematical knowledge in this study was a surprise. I suggest that one factor in this lack of change may have been the CAME strategy of deliberately treating mathematics gently. Hence, I suggest that an important factor in primary mathematics teacher education is to address issues of teachers' mathematical knowledge directly.

The teachers in this study had particularly significant difficulties in relation to "real-life" and non-routine contexts. I suggest that emphasising the use of real-life contexts to illustrate and represent mathematical ideas and engaging directly with processes of mathematical modelling would be valuable strategies in teacher education.

Mathematical desire appears to be an important factor in the teachers' change

I found teachers' emotional relationships with mathematics to be highly significant in their professional change. I emphasise that this significance was both positive and negative. Desire was a powerful motivating force in enabling teacher to continue with professional change despite the difficulty and pain involved. On the other hand, one of the teachers experienced a strong negative sense of shame, which appeared to be an insurmountable barrier to her change. Mathematics involved powerful emotions for all these teachers. This study suggests that acknowledging these issues and finding ways of enabling teachers to generate desire in relation to mathematics are important issues for teacher education.

7. Issues for Further Research

I have addressed the research questions outlined in Chapter 1 in various ways. I have developed a theoretical approach drawing on and extending situated theories using a range of conceptions of identity. This approach emphasises both the role of teachers themselves as active meaning-makers and the importance of their professional and social context in mediating their understandings of primary school mathematics. In doing so, I have located the personal and individual factors of teachers' capacity and motivation to change in social terms. I have explored the inter-relationship between beliefs and knowledge about mathematics, finding in common with many other studies this to be a very complex relationship. In particular, I have found enabling teachers to develop a principled understanding of mathematics to be extremely difficult. I have investigated the issue of reflection finding that both the opportunity and the motivation to reflect are related to the richness and depth of teachers' participation in a variety of social contexts. The differential change of the teachers in this study has been a central theme within this thesis and, using this, I have discussed the nature of the barriers to change that these teachers experienced.

Nevertheless my study generates further research questions. Firstly, I have developed a theoretical approach to understanding the process of teacher change in primary mathematics, extending and developing existing theories. I did not, however, focus in depth on the teachers' developing classroom practices. This raises two questions: To what extent is this theoretical approach useful in a wider context both to researchers and practitioners? In what ways can this approach be extended to encompass teachers' classroom practices in addition to their beliefs and knowledge?

The study was set within a somewhat unusual context. The six teachers had access to an unusually extended and extensive professional development opportunity in primary mathematics, an experience which would be impossible to replicate for the vast majority of primary teachers. Hence, I raise the following questions: In what ways are the insights gained here applicable to professional development in mathematics for primary teachers generally? How can teachers' rich participation be facilitated for the generality of primary teachers? In terms of motivation, how teacher educators can help teachers to develop feelings of desire in relation to mathematics and mathematics teaching?

I have found the interrelationship of teachers' beliefs and knowledge about mathematics to be complex and for teachers' mathematical knowledge to be particularly difficult to change. Yet, Ma's (1999) study demonstrates that it is possible for primary teachers to develop a profound understanding of primary mathematics, although the teachers in her study were specialists. My study raises the important question of how this can be achieved with generalist primary teachers.

I have discussed ways in which reflection was facilitated. However, the infrequency of observed reflective events suggests that reflection was far from guaranteed. Indeed, I found teachers themselves to be catalysts in promoting reflection. This raises the continuing question of how reflection can be more effectively facilitated.

Finally, my discussion of differential change raises the following interrelated questions: How can professional development in primary mathematics be differentiated in order to compensate for the differences between teachers' zones of enactment? How can the development of teachers' rich and broad zones of enactment be facilitated? In what other ways can the barriers to change that I have identified be overcome?

8. Concluding Remarks

As I have noted above, although my study contributes to understanding the nature of teacher change in primary mathematics, it leaves much still to be learnt. In addressing each of the research questions, I have highlighted further empirical and theoretical work to do. Given the exploratory nature of this research and the fragmentary nature of the field of primary mathematics teacher education, this was to be expected. Hence, my study contributes to this emerging field to which I hope to contribute further in the future.

In Chapter 1, I stated that my aim was not to provide a definitive answer to the research questions but rather to contribute to the developing empirical and theoretical knowledge base within the area of mathematics teacher education generally and primary mathematics in particular. I feel that this aim has been achieved in my study.

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Appendix A: Lesson notes

In this Appendix I reproduce three of the Thinking Maths lessons discussed in this thesis: the two fractions lessons developed by Alexandra and Ursula, Share an Apple and Halving and Thirding; and the lesson developed by Lisa, Gardens. These lessons are taken from the Pre-publication draft of the Primary CAME materials and are reproduced with permission of the project director, David Johnson, on behalf of the Primary CAME research team.

1. Share an Apple

Background Notes

An activity to explores pupils' understanding of the basic part-whole relations through spoken language and demonstration on objects, and how that is represented in the fraction notation.

It then requires the implicit or explicit use of equivalent fractions through the need to compare the size of simple fractions and their complements to 1.

The activity is in two episodes, each with a whole class introduction, pair or small group work, and sharing phases. It ends with a reflection phase which should be conducted regardless of how far the class has progressed.

Episode 1: Meaning of unitary fractions and their notations

Children first rehearse the meanings of 'half' (then a quarter and eighth) through demonstrations on objects, focusing on the equality of parts. They then work independently on meaningfully halving and quartering a book, a glass of water and a coin. Children then rehearse the convention of writing fractions, describing the 'top' and 'bottom' numbers and the partition line, allowing some pupils to reconstruct that convention for themselves. Being confined to halves and quarters (and possibly eighths, for which the 'th' should be highlighted) they should be able to compare fractions and to find which two unitary fractions make 1, or is less or more than 1.

Episode 2: More fractions and their sums

Children approach the 'one third' practically and as a mental activity This is intuitively accessible but requires careful handling on paper and in language, especially when combined with 'two thirds' and comparison of size with earlier simpler fractions. They work on formulating justifications of comparisons of two fractions and deciding whether their sum is more, equal or less than 1. Their reasoning lines are then shared, with the implicit ideas of equivalent fractions made explicit where feasible.

Reflection

Children look back at their work. They verbalise for themselves the meaning of a fraction as a mental image in response to language use and convention of notation on the page. They may talk about how different people would best manipulate and combine fractions on the page.

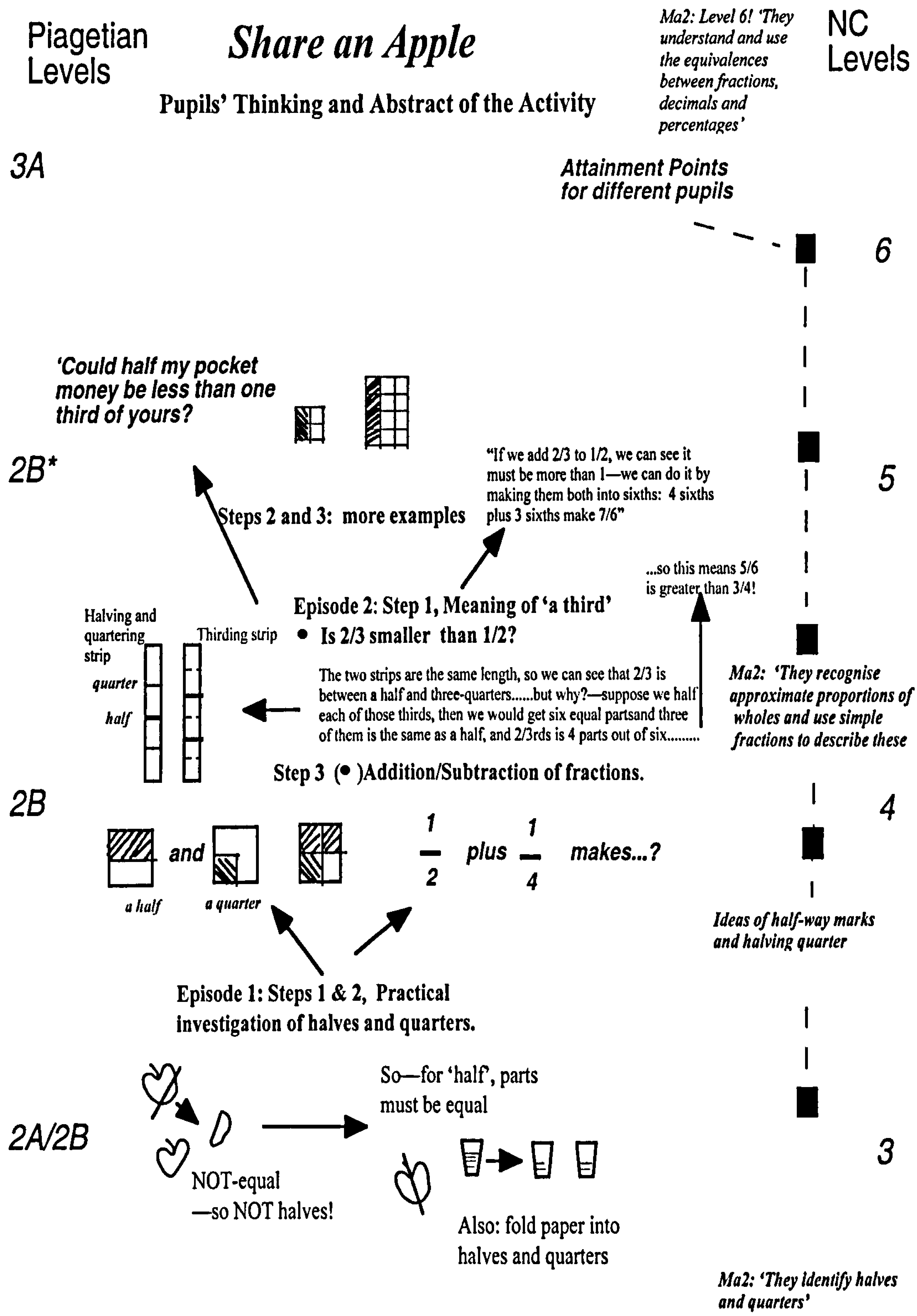
Before you teach

Remember that the intention is to address existing misconceptions and clarify the connections in this extremely confusing topic for children. Even the issue of equal parts is important since in practice it is nearly impossible to achieve in continuous quantities, but it is absolutely exact as a mathematical idea in the head, or on paper!

Encourage any expression and clarification of difficulty or insights at any level. For example ideas about 1/4 may include 'one over 4', 'a fourth', ' a quarter', 'one slash four', 'one out of the 4 equal parts', 'the 1 is still inside the 4', or others.

Share an Apple

Pupils' Thinking and Abstract of the Activity



Share an Apple: Teaching Notes

CAME Aims
- Exploration of part-whole relationships involving known simple fractions - halves and quarters.
- Focus on comparing sizes of fractions - involves intuitive ideas on equivalent fractions.

Resources:
- An apple
- A knife
- A glass
- Strips of A3 paper
- String
- Blu tac
- Paper for children to write on
- Notesheet

Organisation:
- Near ability pairs on mixed tables

Vocabulary:
Half, quarter, three quarters, third
Equal parts
part
whole
denominator
numerator

National Curriculum Reference:
Fractions - comparison, equivalence, notation
Reasoning and justification

Whole class preparation: (about 10 mins) **1**
- *How could we cut this apple into halves?* Elicit answers agree the best answer and then cut apple.
- Draw what you have done on the board. *How do we know these are halves?*
- *How would you write half in figures?*
- *How could we cut it into quarters? What is important to remember about each part?* (equal parts)
- *How do we write a quarter? What do the two numbers mean to you?* Discuss this.

Paired work: (about 10 mins) **2**

- Teacher holds up a glass of apple juice, £1 coin and a book.
- *In pairs record how you would find:*
  half / quarter of a glass of apple juice
  half / quarter of £1
  half / quarter of a book.
- *Are there different methods for doing this? Check with others on your table. Would you agree that others could guarantee finding a half or a quarter? Which do they*

Paired work: (about 10 mins) **4**

- Give out strips of paper.
- *How would you find a third of the strip?*
- *How many parts will the strip be divided into?*
- *What about the sizes of these parts?*
- *Is 1/3 bigger or smaller than 1/2? How do you know?*
- *If you fold your strip in half again, how many sections do you think you will have?*
- Given pairs of fractions children justify and prove which one is bigger. Why? Together do they add up to more or less than 1 whole? (5/6, 7/8, or 3/4, 4/5 etc)

Class sharing: (about 10 mins) **2**

- Report back on how they worked out these problems.
- Emphasis notion of a quarter being half of a half.
- Ask how they would find three quarters of £1.
- Ask children to demonstrate notation on board. Discuss meaning of notation. 1/4 + 1/4 + 1/4. *Which part of the fraction are you adding? Why?*
- 1/2 + 1/4. *How does this work?*

Class sharing: (about 10 mins) **5**

- Children report back on their findings and demonstrate their answers using strips of paper. Record on the board.
- *I have two children, would it be possible for half my daughter's pocket money to be less than one third of my sons?*

2. Halving and Thirding

Background Notes

An activity intended to extend children's use of fractions towards ratios and proportions. Starting from *fractions* of *fractions* handled as a grid on their notesheets they are then given a paint mixing problem which they can model for themselves on the page, and they have to use mathematics to compare depths of the colour green in two mixtures of blue and yellow.

This lesson is about argument and mathematizing everyday numeracy. There are three episodes, each differently challenging but mutually supportive. The first can be primarily conducted in whole-class discussion mode, but for each of the two others the reflection and sharing phase is very important, so each needs its own introduction and children's construction phases.

Episode 1 – Working on halving halves

The context is familiar: there is water in a jar (say half a litre), and half of it is poured into a similar jar. So there is half of the original now in each. Class discussion should focus on the different descriptions and connections between these. This is demonstrated physically or as a visualisation.

Episode 2 – Thirding a half and halving a third on a grid

Although a familiar context in terms of "school mathematics", the challenge occurs through the questions asked. Given nine 36 cell grids children shade squares to show (a) two successive operations of halving, and (b) two successive operations of thirding. The key aim of this episode is the integration of the use of both words and fraction symbols to express what is happening in each operation on the notesheet and to discuss the effect and usage of operations using children's natural mathematical language.

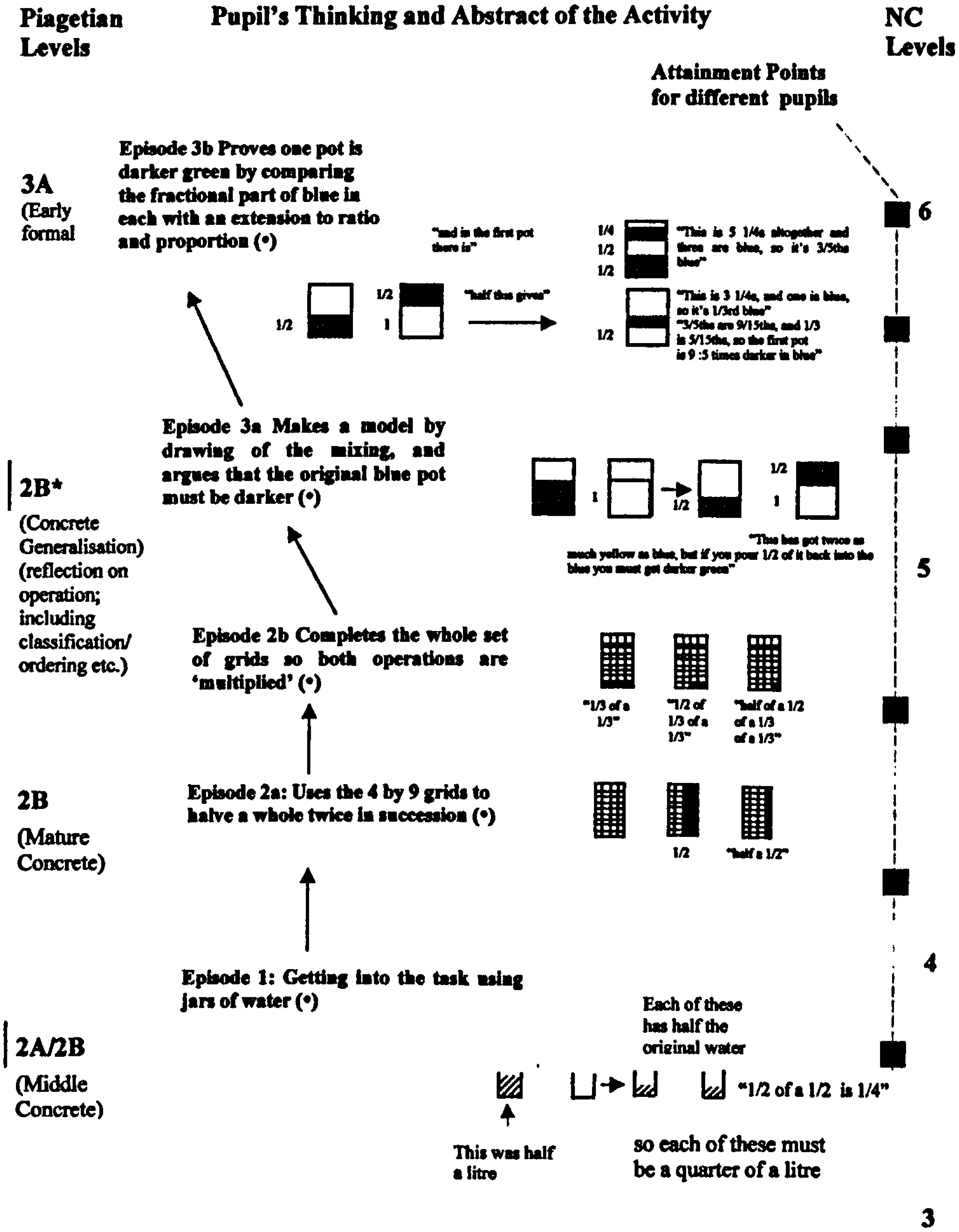
Episode 3 – Using various ways of symbolising fractions to solve a proportions problem

We know that blue mixed with yellow makes green, and the more blue the deeper the green. But if you mix them once, and then halve the mixture and pour half back, how do the greens compare in depth? From episodes 1 and 2 there are various strategies which different children will use to attempt the task. The aim here is less to solve the problem than to share on the board the different strategies, symbolised in different ways, and to discuss the difficulties that arise. The pupils have to construct fractions of fractions. They use images, diagrams and rationalise intuition to clarify for themselves ideas of ratio and proportion. Children simultaneously consider multiple images and their links.

Before you teach

Episode 3 is very challenging, though very rewarding and effective. There are several different levels of positive achievement of understanding within the three episodes, so you need to monitor and decide how far to take the class. With some a rich class sharing of ways of handling episode 2 might have occurred towards the end of an hour, and this would be quite enough. How much time you decide to give episode 3 would depend on how many of the children were in some way or another tackling it without giving. It is also worthwhile to leave the question "hanging" in the air.

Halving and thirding:



Halving and Thirding: Teaching Notes

CAME Aims
- To develop understanding of fractions, ratio and proportion in a variety of familiar contexts.
- To link language to meaning and make connections.

Resources:
- Jug about half full with coloured water
- A mug (if demonstrating practically)
- 15 A3 copies of Halving and Thirding notesheet
- Blu Tac

Organisation:
- Near ability pairs on mixed ability tables

Vocabulary:

full	empty
half full	half empty
half of a half	quarter
halving	dividing by 2
add	more
of	lots of
times	

National Curriculum Reference:
Fractions, ratio and proportion
Number operations
Problem solving and reasoning

Whole class preparation: (about 10 mins)1
- *I've got water in this jug and I pour half of it into this mug.* Demonstrate with a jug and a mug or mime and *imagine that I have a jug half full.*
- *What can you tell me about the jug or the mug?* Half of a half ... half take away a quarter ... half divided by two ... half empty ... half full. Collecting as many of the suggestions in sentences, words, pictures and numbers/symbols (i.e. "If we take half of a half we get a quarter" or "1/2 - 1/4 = 1/4").
- *Are there any things on the board, which you think, mean the same thing?*
- Explore connections between the different descriptions.

Paired work: (about 10 mins) **2**

- Give out notesheet 1.
- *Here are pictures of a bar of chocolate.*
- *You will need to quickly shade in the pictures to show what is happening.*
- *Going across you will need to shade half, then half again.* Point out that they can only shade where the lines are, as you would break a bar of chocolate, and not make up lines of your own.
- *Going down you need to shade a third and then a third again.*
- Remind them about the recording that you did on the board for the jug and mug. *You should record what is happening in words and numbers/symbols like we did before.*

Paired work: (about 15 mins) **4**

- *Imagine that I have got 2 pots of paint in my hands. They are not full but they do have the same amount of paint.* Mime actions.
- *One pot is blue and the other pot is yellow.*
- *I pour half of the blue paint into the yellow paint and stir it up completely.*
- *Then I pour half of it back into the first tin and stir it up completely.*
- *Are the pots the same colour? Why or why not? Can you prove it?*

Class discussion: (about 10 mins) **3**

- *How did you decide where to shade?* Some children will do this intuitively by spatial awareness and others will need to count squares.
- Stick notesheet 1 on the board with Blue Tac and collect feedback on one or two particular pictures where children have a lot of description.
- *What have you got to describe the top right picture?* Start with contributions from the least able children.
- *The centre picture is described as 1/3 of a 1/2, has anyone else got a different description?* 1/2 of a 1/3 ... 1/6 ... 6/36 .... *In what ways are these the same?* Explore difference between. *When would you use each one? When would you say it one way or another?*
- *What about the bottom middle picture?*

Class discussion: (about 10 mins)

5

- Collect feedback, asking children to demonstrate diagrams, number explanations on the board. Let the children discuss each other's arguments.
- *How much darker is it?* Accept general responses - eg, almost 'twice as dark' - leading to a ratio of 9 to 5 (see diagram).

3. Gardens

Background notes

An activity to recognise and represent a simple two step numeric pattern in visual settings. Children move from verbal to algebraic and tabular representations of the pattern, moving then to graph if appropriate. They recognise that similar patterns exist in different settings.

This activity is in three episodes, each with an introduction, construction and sharing phases. It ends with a reflection phase that should be conducted however far the pupils have reached in the activity.

Episode 1: Recognising and describing a numeric pattern in a setting
Children first handle a two step pattern of lamps and branches on different trees. They attempt to describe in words then symbols based on generalised number, and tabulate results.

Episode 2: Describing a similar pattern and comparing two patterns
Children try similarly to describe a pattern of coloured and white beads in words then in symbols. They then find what is common and what is different between the two patterns of the branches and lamps on the one hand, and the coloured and white beads on the other.

Episode 3: Comparing graphs of two patterns
Children plot points on graph to represent the two patterns and compare with other representations. They try to describe the two line graphs in terms of their slope/incline/stepwise rise and their starting points and link these features with their verbal expressions, (and exceptionally with the algebra)

Reflection
Children look back at their work and discuss the advantages and disadvantages of the different representations.

Before you teach
Unlike many of the Thinking Maths activities which get harder as the lesson continues, this one oscillates up and down in difficulty, with the peaks at the points where the concept of a *variable* is introduced (treating '3b' as meaning 'three time the number of branches' and '2b' as 'twice the number of coloured beads', where t and b can vary. You need to decide how much to labour these steps with your class, or how much to confine your children's discussion to the other questions on the worksheets, and the Tables and Graphs (where the '3' and the '2' are connected with the steepness of the lines).

Gardens

Pupils' Thinking and Abstract of Activity

Piagetian Levels | **Attainment Points for different pupils** | **NC Levels**

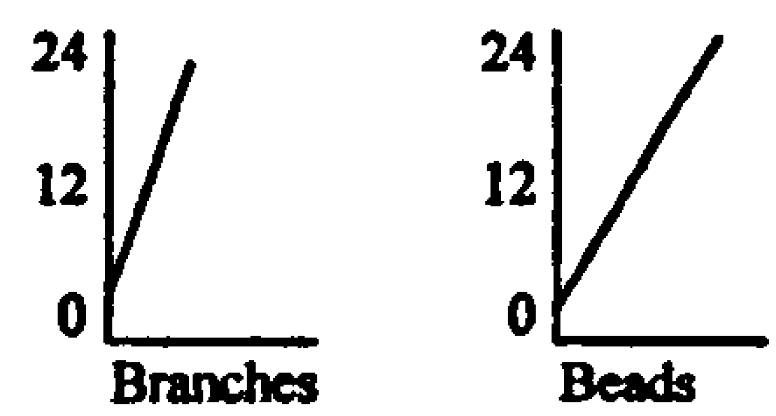
3A (Early formal

Creates insight by comparing Words and Symbols v Tables v Graphs.

"The branches line is steeper AND the branches line starts at 2, but the beads at 1".

6

Produce graphs of the two relationships.



2B* (Concrete Generalisation) (reflection on operation; including classification/ ordering etc.)

Compare Tables in terms of similarities and differences.

Produce tables of Branches and Lamps, Black and White Beads.

Black	0	1	2	3	4	5	6
White	1	3	5	7	9	11	13

5

Branches	0	1	2	3	4	5	6
Lamps	2	5	8	11	14	17	20

2B (Mature Concrete)

Find a way of handling the symbols in the relation:
$l = 3b + 2$ or $w = 2b + 1$
This needs a lot of talking through. How the symbols connect with the words. Children want to read it as 'three branches' or 'two black beads'.

4

Produce word equation for number of lamps (or Number of White Beads).

"Number of lamps = Three times the number of branches add two".

2A/2B (Middle Concrete)

Recognising and describing the pattern of branches and lamps.

3

Gardens: Teaching Notes

CAME Aims
- To be able to express relationships between 2 variables in different ways, using words, words and symbols, symbols, tables and graphs.

Resources:
- Notesheets 1 & 2
- Notesheet 3 (1 between 2)
- Paper
- Large prepared tables and graphs for class sharing
- Pencils, rubbers and rulers

Organisation:
- Working in near ability pairs.
- Draw trees from notesheet with branches and trees but no lamps onto board before lesson.

Vocabulary:

pattern	adding
doubling	
symbol	multiplying
trebling	
algebra	
letters as symbols	
plot	
steepness	slope
intercept	y axis

National Curriculum Reference:
Algebra
Solving real life problems
Reasoning and justification

Whole class preparation: (about 15-20mins) **1**
- Refer children to drawing on board. Add lamps to first branch.
- *What do you notice about the lamps, branches and trees?* 2 lamps at the beginning of each tree, groups of 3, etc. Repeat for next tree.
- Children work on rest of notesheet 1 questions. (5mins)
- Share ideas for each part of notesheet.
- *How could we write this in symbols?* eg $3b + 2 = l$ Compare with relationship expressed in words.
- Draw up a table of the results on the board as a class.
- Look at the adding 3 pattern. How does this compare with the $3b + 2 = l$?

Group work: (about 10 mins) **2**

- Using second notesheet begin pattern and discussion on the board.
- Children work in pairs to suggest further patterns.
- Suggest number of white beads for 100, 17, 333 coloured beads.
- Children describe the relationship.
- Discuss and record as a class eg $w = 2c + 1$.

Paired work: (about 10 mins) **4**

- Give out notesheet 3, one for each pair.
- Children work in pairs to plot graphs and then in groups to discuss findings.

Class sharing: (about 10 mins) **3**

- Fill in the table on the board together (see notesheet 2).
- *What did you notice that was similar about the trees and the paving slabs?* Record the responses on the board.
- *What did you notice that was different?* Record the responses on the board.

Class sharing: (about 15 mins) **5**

- Discuss and compare 2 graphs eg *What is similar and different about the 2 patterns? Starting points, gradient, both straight line. What would happen is you took intermediate point etc.?*
- *What are the advantages/disadvantages of each?*

Appendix B: Phase 2 Teachers and Schools

Ashwood

Joanne	Autumn '98 – Spring '99	Maths Co-ordinator. Supporting role to class teachers. Promoted to Numeracy Consultant post.
Nina	98/99 & 99/00.	Stopped attending Spring '00.
Valerie	98/99	Maternity leave from Autumn '99.

Beechmount

Jenny	98/99 & 99/00	

Brightvale

Glenda	99/00	Left for another teaching post.
Janice	98/99 & 99/00	Maths Co-ordinator. Supporting role to class teachers. Joined Research Team Summer '99.
June	98/99 & 99/00	
Karen	98/99	Left for another teaching post. Casual attendance. Most lessons taught by Janice.
Meg	Summer '99	Supply teacher Summer '99. Casual attendance.
Ruth	98/99	Left for another teaching post. Casual attendance.
Sandra	98/99 & 99/00	
Tracy	99/00	Casual attendance. Most lessons taught by Janice.

Cheston

Frances	99/00	
Tom	98/99	Left teaching.

Greenbank

Danielle	99/00	
Gudhreer	98/99 & 99/00	
Win	98/99	Promoted to management post in another school.

Heathmead

Barbara	99/00	
Doreen	98/99	Supply teacher. Casual attendance.
Faride	98/99 & Autumn '99	Left teaching December '99.
Grace	98/99 & 99/00	
Hazel	Spring - Summer '00	

Oakham

Nicola	98/99 & 99/00	Promoted to Outertown Numeracy Consultant Summer '00
Sarah	98/99 & 99/00	

Parkway

Enid	98/99 & 99/00	
Oliver	98/99 & 99/00	

Roseway

Hanife	99/00	Casual attendance.
Ivan	98/99	Left for another teaching post.
John	98/99 & 99/00	

Appendix C: Notes of a research team meeting

The following notes refer to a two day meeting over a Friday and Saturday at the beginning of the second year. These notes take the form of a summary with some brief reflection comments followed by a section from a transcript of the Saturday meeting. This section records a discussion about CAME and open / closed approaches to teaching mathematics.

1. Meeting Summary

Primary CAME Central Research Team Meeting 18th and 19th September 1998
At Outertown Professional Development Centre

Meeting Summary

Fieldnotes book 3 (p.6-23). Sessions taped (selections transcribed).

18th September 1998:

Present: Rhoda, Michael, David, Alexandra, Mundher, Ursula, Jeremy, Lisa.
Apologies: Henrietta

Agenda handout (prepared by Rhoda):

10	Discussion of Draft Booklet and lessons taught
11.15	Discussion of testing and Phase 2 schools
	Planning of first INSET session for new schools
2	New ideas for lessons
3.15	Agree agenda for Saturday

Saturday tasks:
- Plan INSET for 6 Oct and 1Dec 1998
- Plan focus for school visits
- Timetable for testing
- Further lessons for Y5
- Lessons for Y6
- Draft booklet

Agenda on board slightly different (written up by Mundher):

Discussion of draft booklet
Lessons taught
11 Coffee / tea –
Testing
Phase 2
INSET for new schools
New lessons - Mundher
Agree agenda for tomorrow

Mundher started by asking for 10 minutes discussion at the beginning of the meeting. In lieu of P-CAME notes 1, he presented his thinking on the tension between the research and the development aspects of the project (i.e. the production of finalised materials v. thinking about new material v. data collection and analysis). Focused on the peer tutoring role (including the interpretation of observation as inspection rather than supportive & team-teaching). Discussion was much longer than 10 minutes.

Ursula and Rhoda circulated their lesson observations on the Phase 2 teachers (observations of ordinary maths lessons rather than CAME lessons). They gave a report-back on these. Ursula commented that the one NQT was the closest to CAME. Some discussion about the peer-tutor role.

Discussion about the draft Teacher's Guide. Rhoda led this. David proposed keeping it as draft. Lisa commented that this would allow ownership of the document by the next cohort.

David outlined the research programme. 2 phases - 1. Leverhulme research extending CAME into primary (development of lessons etc) and my research -all focusing on teacher-researchers and 2. The implementation of the programme with the Phase 2 schools (and the pre & post tests).
There is a lack of qualitative data. He proposed the teacher-researchers make a tape of their feelings / interpretations (rather than writing up notes) after each visit. He will provide tape recorders. There was a long discussion on this. Teacher-researchers not keen (body language issues! - see fieldnotes comment.)

Lunch

(Ursula, Rhoda not present for the new lessons discussion.)

Mundher presented ideas for new lessons (together with a 'model for generating new lessons' involving identification of the maths potential (lower and upper ends) and the process.

Lesson development ideas:

Size of bag (Alternative to Design a desk.) Mundher gave an informal lesson simulation. Maths involved - data handling and median. David in particular was not keen. Mundher commented that, 'I'm being shot down as usual.'
Bookshelf (GAIM) Problem solving exercise involving a limited amount of wood to make a bookshelf.
Frames. Picture with a mat and a frame.

Michael outlined the pre and post testing for Phase 2 schools

Immediate reflections:

1. Lisa wasn't at all involved in or engaged by the lesson development discussion – is there a reason for this? Find a way to ask her informally.

2. Interesting discussion at the beginning prompted by Mundher's ideas about tensions. There seems to be a sense of the teachers moving into their new role as tutors. Listen to and transcribe discussion about tutoring.

19th September 1998

Present: David, Michael, Mundher, Ursula, Alexandra, Rhoda, Jeremy
Apologies: Lisa, Henrietta.

Agenda: Reflections of lessons taught recently

Further lessons for Y5 - finalise next 6
Plan INSET for 6 Oct and 1 Dec
Plan focus of school visits - role of peer tutors
Lessons for Y6

Reflections on lessons.
Alexandra reported on Length of Words.
Ursula reported on Design a desk. Need pre-lesson work on measuring skills. Key challenges for children: which average is best, ways of reducing the data, introduction of range.
Rhoda reported on Robots.

Discussion about the next 6 lessons for Y5. Discussion about new names for some of the activities. Agreed a provisional order for the next 6 lessons. Issues to be addressed the range of maths domains; the lack of new material; bridging between CAME lessons not addressed. Also discussion about possible Y6 lessons. David possibility of developing one of the Numeracy Lessons: Area method of multiplication into a CAME lesson.

Discussion about the Professional Development for the next phase of teachers, reflecting back on the first year's work. Issues about openness and closure and the distinction between TM and general maths lessons.

Lunch.

Planned structure of INSET day.

Mundher led simulation on contrast between telling and discovery approaches to teaching. Rhoda commented on the importance of saying this is what we do in the classroom in relation to the type of questioning etc.

Immediate reflections

1. Transcribe and code discussions of open / closed mathematics. Could pick up in interviews and ask for specific examples.

2. Lisa wasn't at the Saturday meeting. There is something of a gulf developing between the teachers at the two schools.

3. Teachers do appear to switch off at times during the discussions.

4. Rhoda trying to impose a timetable on the group – she'd produced a typed agenda for the first time.

5. I agreed to lead one of the activities at the PD session on 6th October. Bit worried about doing this – does this compromise my role as a researcher? Think about & talk to David.

2. Section from Research Team Meeting Transcript

The following discussion took place as part of a discussion on the teachers' roles as tutors to the Phase 2 group on 19 September. This discussion was transcribed in detail. This section is one part of a longer transcript of the meeting.

Mundher: Is it a time now to step back and reflect on what we have done, to reflect on the process of developing these lessons? If I think back about a year ago to this time in October, and we'd just started this work and we had not really planned things properly, but we know the general direction. I'm talking about the research team. We'd planned the direction, but we haven't actually, we don't really know the details. And we throw things at you. [LAUGHTER] And Ursula was very upset because she didn't really know what we were talking about for while.

Ursula: I still don't. [LAUGHS]

Alexandra: And now she's a numeracy consultant.

Mundher: So among the things we're talking about, we're choosing lessons, we're thinking about lessons, we've done and we've observed and there was discussion and there was a lot of written observations of lessons. And as well as what we did, is there some lesson about lessons for us in this process of choosing the right lessons.

Rhoda: I feel that we haven't developed enough new material from the teaching point of view, you know what people are doing in the classrooms, as I would have liked. I feel we've been too reliant on this [The Secondary Teachers Guide] and what's already in place.

Mundher: Is that bad or good?

Alexandra: Also new lessons have tended to come from investigative materials we've found. We're not developing anything new, perhaps there is nothing new to develop.

Rhoda: No, I don't think there is.

Michael: There's also not generating new Thinking Maths lessons, but there's also the whole business of bridging which we haven't addressed all that much. How do you take both some of the agenda of the Thinking Maths lessons and also the style of handling Thinking Maths lessons, how do you now look at the rest of you maths teaching and relate the one to the other? That's

Rhoda: I think the philosophy of it as a lot clearer now. I feel a lot happier.

Alexandra: Maybe that will come, maybe the idea of new lessons will come easier this year, because you've got to bear in mind that for us it was very new and we were very

Ursula: Confused.

Alexandra: Confused? Well yeah and a bit nervous about it all. But now we're at the stage where we actually want to have somebody in the classroom so that we can discuss it and things like that. So I think we've had to grow in terms of confidence with the original material and now we're maybe more at the stage when we're able to look at other materials and say well that could be a thinking maths lesson. We're more confident now about what could be a year 5 lesson or what could be a year 6 lesson as well.

Ursula: No, I don't think it becomes any clearer what could be a thinking maths lesson or what could be a maths lesson, because I think it affects your practice so that your whole practice has changed.

Alexandra: Yeah.

Mundher: But it's important to make the distinction. I mean I knew that this would happen. What happens is that you forget what you were before. So you are continuously living in the present, but the problem is we do need to stop and think what's happened. I mean especially in your case, for example, which is quite important, because this is exactly what happened to myself. I mean when I started with Michael, I don't really know what I am doing. And we do the lessons and they are successful and we have written them up. But I became to be clearer from a particular lesson that went well and then it became clearer in that particular lesson that went well. But the overallness, of the overall confusion remained for a long time. Because I mean now after we've done this as Rhoda is saying, it is clearer, the philosophy is clearer. I think it's only clearer because we've done the lesson actually and we've done the lessons as they're done.

Rhoda: Yeah, which is why, as Alexandra said, maybe we needed to do that.

Mundher: But I'm interested in what Ursula can say about the original confusion. What was the confusion and in what way is the confusion still there or less?

Ursula: To begin with the confusion with me was the lessons seem relatively closed. You seem to be closing down on children on when you know I understand from courses, the 20 day course and looking at investigative ways of doing things where you're trying to open things up all the time. The big confusion for me with CAME is you're closing things down for children and narrowing it down all the time. Whereas now I think in any maths lesson, I think I see the questions in two ways and I can choose now which one I want to do. I can quite happily close down the children and focus in a little more. Or I can open it up wider. So I've got more the repertoire of both things, I think. I mean I can't do it very effectively, but I think I can do it. That's the difference.

Michael: There's a difference between closure

Ursula: I don't mean closing the maths down. I mean closing the focus, narrowing it, narrowing.

Michael: as being there is one single aim of this. There's a difference between that the process of confining people's attention.

Ursula: Well, there are times when you need to say to a child, "OK. So if you've found this out then, let's go wider than it. What else can we associate with it?" ' Which is the way that I used to do things, but I never used to do things like, "OK. Let's focus this down, down, down" '

Michael: But the other person - It's not just a question of, isn't it saying, "Look. I want you to go deeper in this, but in this direction? I want you to go deeper but in this direction rather than wider doing different sorts of applications of the same." You're focusing attention and saying, "Look. There's more in this if you look at this aspect of it or that aspect of it, if you think about it or you work more in it. Can you find that?" That sounds like narrowing, but not the same sense as closure, I think.

Ursula: I don't mean closed.

Mundher: Framing or constraining.

Michael: Yes. Constraining.

Mundher: I think that that's a big issue, because it's important for the good teachers who are in investigative mode to, to cater for them, to recognise that for them that actually their difficulty will be in closing or restricting the thing. While for the teachers who are in the didactic mode, this is types of teachers, their difficulty will be in opening it up. And the opening up of things is really about what are the typical ways of opening it up, like where else do you see it. But I don't know that these general questions are very useful, not within this. What did you mean the philosophy is clearer now?

Rhoda: It is, it is clearer - in that you're trying to develop children's discussion, their explanations and in fact to some extent what they're saying doesn't really matter.

Mundher: The details don't matter.

Rhoda: No, I mean it does matter, but it's, the most important thing is that they're verbalising their thoughts.

Michael: [Unclear]

Ursula: It's almost giving them the answer, because you stop them at that point where you will draw out from the whole class eventually, you eventually draw out the answer you were looking for at that particular moment. So for some kids it's like, it's like, I don't know, it's like just giving them the answer and then moving them on a bit, which is quite hard to come to terms really with when you're used to trying to make them all write it out for themselves.

Mundher: Because the answer's not important, you give them the answer and focus them on the process.

Ursula: Mmm.

Alexandra: Mmm. - But the structure's clearer as well for me. The you know the separate bits.

Rhoda: But it comes, in some of the lessons it's easier to see than others.

Alexandra: Yes.

Rhoda: I mean Roofs is a classic one, - but - oh what have I just done. Robots is not, it's not, I mean it takes quite a bit of - of unpicking what are the key things there that you're trying to get out of the lesson.

Ursula: My lesson was like that it didn't naturally fall into those slots.

Rhoda: Yeah.

Mundher: How difficult is it or easy is it to distinguish between two parts of the agenda, the cognitive part and the process part. I mean what you're talking about now is actually the social part. It's the ways of learning relative to. That's why you're saying the discussion and the talk and the actual content of the talk is not as important if you know it's not so important. And the content part is actually the content, the substance of the discussion which are important, where the answer is not important, but the beyond the answer, the reasoning is important.

Rhoda: Yeah.

Mundher: So you notice the problem which is actually difficult to talk about unless you split it into two bits. You know this is the thinking part and this is the social or the interactive part, you know the cognitive and the

Rhoda: But it's like the one Michael has got on the board. You know , the answer's 28. I mean that's quite simple. Most people will accept that's 28 without help. Yeah but why? Why? And looking at it in a whole range of ways.

Mundher: So this is what you meant by the answer's not important?

Rhoda: Yeah.

Mundher: So the answer's not important because of the reasoning which is important but also because of the process of talking which is different from the reasoning. The reasoning happens in the minds of children. The talking

Rhoda: But the talking being

Mundher: The talking helps

Rhoda: supports the reasoning.

Ursula & Alexandra inaudible discussion.

Rhoda: There's a phrase that something about thoughts of worlds or worlds of thought

Alexandra: We're just saying it isn't in a sense that dissimilar to what's coming up as I understand in the numeracy framework with mental maths. Talking about how you get to the same things as other people. It's not just the answer. And in a sense this is doing it.

Ursula: But this will still push, in many cases this will still push people.

Alexandra: Yeah.

Rhoda: But you see the influences in terms of numeracy strategy, the major influence is obviously Anita Straker, but you know Margaret was on the committee and

Mundher: So it's influencing it

Rhoda: Yeah. Yesterday the group that I did we were talking about the definition of numeracy and how that has evolved. I mean look everyone's talking to each other. I mean it's built on the TTA stuff that, that Mike did and it's all kind of linked in. Everyone talks to each other in the maths field. And then it probably seems similar in Outertown, 'cos I'm the one who's telling everybody and so I've been influenced by this

Alexandra: And the 20 day courses and so on

Appendix D: Primary CAME Test Data

In this Appendix I briefly summarise the Primary CAME pre-post tests results. For further details and analysis see Adhami (2002).

1. Phase 1 Schools' Results 1997 – 1999

The initial design for the Phase 1 schools was to have one teachers from Y5 and one from Y6 in each of the two schools. These teachers' classes were initial designated as experimental classes, with the other Y5 class in the school designated as a control for the purposes of the post-tests in 1999. However, the lessons were actually trialled in both Y5 classes. As a result there were no control classes in this preliminary work.

The results on the two tests, Proportional Reasoning Test 1 and the Mathematics Reasoning Test, are summarised below. The results on both tests were encouraging in that levels of reasoning had greatly improved in the post tests. However, although these results suggest a positive effect, without control classes for statistical comparison these results are only descriptive.

Proportional Reasoning Test 1: Spatial relations

During the first year of the project, in Autumn 1997, each of the Y5 classes in the Phase 1 schools were given a Pre-test of PRT 1: Spatial Relations, one of the Piagetian tests used in the CSMS survey of cognitive development for which National age norms had been developed. This test involves the children only drawing their answers to questions involving demonstrations using jam-jars imagined half-full of water, at different inclinations, a plumb-line, and also two drawing tasks involving a house and trees on the side of a hill and another involving perspective looking down an imaginary avenue of trees. In June 1999 the classes were given the same test to see whether there was any evidence of cognitive change in the children. Table D.1 summarises the data for these schools:

	Means*		CSMS Percentiles			
School	Pre-test 97	Post-test 99	Pre-test 97	Post-test 99	%le Gain	Effect-size (S.Ds)
Parkway	4.69	5.76	46.5	63.6	17.1	0.83
Beechmount	4.78	5.54	46.4	55.0	8.6	0.42

[Note: *The test means are on a Piagetian scale where 2A/2B = 4 to 5, 2B = 5 to 6 & 2B* = 6 to 7. The percentiles shown in Table D.1 are population norms derived from the CSMS survey of 14,000 children between the ages of 10 and 16 (Shayer, Küchemann, & Wylam, 1976).]

Table D.1: Summary of Proportional Reasoning Test 1 data in Phase 1 schools.

Mathematics Reasoning Test

In September 1997, each of the Y5 classes in the Phase 1 schools were given a pre-test of the Mathematics Reasoning Test. In July 1999, the classes were given the same test to see whether there was any evidence of changes in the children's mathematical reasoning. The results are summarised in the stem and leaf diagrams below.

The Mathematics Reasoning Test is based on work at the University of Wisconsin by Collis, Romberg and others and previously used in the UK in the ImpacT study (Watson, 1993). There is a stem followed by three items with each of the items corresponding to a higher level of reasoning based on the SOLO taxonomy – the A items are unistructural, B multistructural and C relational (Biggs & Collis, 1982).

According to the Wisconsin work it is reasonable to work with total scores for individuals using these to provide an indication as to some overall Solo level for the class.

Stem and leaf diagrams showing pre-test (September 1997) and post-test (July 1999) for four classes in the two Phase 1 schools

Parkway School (5A/6A) Class teacher Alexandra in Y5 and Y6.
(Some variation in spelling of names pre- and post-test.)

Pre-test (N=30)

21 - 24																	
17 - 20	17	19	20														
13 - 16	14	16															
9 - 12	9	9	9	10	10	10	10	10	11	11	11	11	11	11	12	12	
5 - 8	7	8															
1 - 4																	

Post-test (N=29)

21 - 24	21	21	21														
17 - 20	17	17	17	18	18	19											
13 - 16	13	13	13	13	14	14	14	15	15	16	16	16	16	16	16		
9 - 12	10	11	12	12													
5 - 8	8																
1 - 4																	

Parkway School (5E/6O) Class teacher Enid in Y5 and Oliver in Y6.
(Some variation in spelling of names pre- and post-test.)

Pre-test (N=30)

21 - 24																	
17 - 20	18																
13 - 16	13	13	16														
9 - 12	9	9	9	9	9	9	10	11	11	11	12						
5 - 8	5	5	6	6	6	7	7	7	7	8	8	8	8				
1 - 4	3	4															

Post-test (N=30)

21 - 24	21	21															
17 - 20	17	18	18	19	19	19	20	20	20								
13 - 16	13	13	14	15	15	15	15	16	16	16	16						
9 - 12	10	11	11	11	12												
5 - 8	5	6	8														
1 - 4																	

Beechmount School (5H/6L) Class teacher Henrietta in Y5 and Lisa in Y6.

Pre-test (N=28)

21 - 24																	
17 - 20																	
13 - 16	13	15															
9 - 12	9	9	9	10	11	11											
5 - 8	5	5	5	6	6	6	6	7	7	7	7	7	8	8	8		
1 - 4	4	4	4	4	4												

Post-test (N=28)

21 - 24																	
17 - 20	17	17	17	19													
13 - 16	13	13	13	13	14	14	14	16									
9 - 12	9	9	9	9	10	11	12	12	12								
5 - 8	6	6	7	8	8	8											
1 - 4	4																

Beechmount School (5J/6X) Class teacher Jenny in Y5 and another teacher in Y6. (Some changes to class register pre-test to post-test.)

Pre-test (N=26)

21 - 24																	
17 - 20																	
13 - 16	13	13	14														
9 - 12	9	9	11	11													
5 - 8	5	6	6	6	6	6	6	6	7	7	7	8	8	8	8	8	8
1 - 4	4	4															

Post-test (N=28)

21 - 24																	
17 - 20	17	18	18	19													
13 - 16	13	14	15	15	15	16	16	16									
9 - 12	9	9	10	11	11	12	12	12									
5 - 8	6	8	8	8													
1 - 4	1																

2. Phase 2 Schools' Results 1998 - 2000

Testing schedule: The research design involved the use of three Control schools, giving six classes (N $\approx$ 180) for comparison with the 11 classes on the Main Study who would be receiving the TM intervention. As with the Laboratory schools previously, all classes were given PRT I, Spatial Relations as pre-test in September 1998 in anticipation of using the same test as post-test in June 2000. This was to have a general measure of the children's cognitive ability by which to assess any gains over the two-year period over and above what would be expected as the children mature. These tests would then enable the cognitive gains achieved by the main study classes to be compared with the control classes, with due allowance made for differences in initial abilities of the children in each class. They would also allow the year 2000 Key Stage 2 results of the classes to be compared, again in relation to the different initial abilities of the children in control and main study classes.

The results are shown in Table D.2.

Table D.2 gives the PRT I Pre-Post test data for all classes. The overall comparison between Experimental and Control classes is certainly statistically significant, with a mean effect-size of 0.26 standard deviations. The effect-sizes were computed by subtracting the mean Control school change from the Pre-Post test change of each school, and dividing this difference by the standard deviation of the whole sample at Pre-test (± 20.53%). Four of the Main study classes have shown substantial gains, and with one exception their general trend is positive compared with the control school classes.

Classes	Effect-size (S.D.s)	*p*	Pre-%le	Post-%le	Change
Controls					
C1A	-0.17	ns	65.3	61.9	-3.4
C1B	-0.15	ns	71.1	68.1	-3
C2A	0.06	ns	60.4	61.6	1.2
C2B	-0.05	ns	59.7	58.7	-1
C3A	0.18	ns	61.2	64.8	3.6
C3B	-0.02	ns	63.0	62.6	-0.4
Mean	**-0.02**		**63.45**	**62.94**	**-0.51**
Experimentals					
M1	0.47	<.01	59.2	68.4	**9.2**
M2A	0.56	<.01	57.6	68.5	**10.9**
M2B	0.18	ns	63	66.2	3.2
M2C	0.21	ns	57.2	61	3.8
M3A	0.08	ns	56.4	57.6	1.2
M3B	0.48	<.01	50.5	59.8	**9.3**
M3C	-0.14	ns	59.3	56.0	-3.3
M4	0.18	ns	52.6	52.9	0.3
M5	0.04	ns	66.4	69.5	3.1
M6A	0.16	ns	58.5	61.3	2.8
M6B	0.36	<.05	55.0	61.8	**6.8**
Mean	**0.23**		**57.79**	**62.09**	**4.30**
E -C	**0.26σ**				**4.81**

Overall t-test. E - C, t = 2.7, p = 0.0082

Table D.2: Changes in CSMS percentiles for Control and Experimental classes over 20 months

Appendix E: Biographies of the six teachers

1. Alexandra

Alexandra was a Y5 teacher at Parkway during 1997/8, when she joined the research team. She was a Y6 teacher until April 1999, when she became a part-time Numeracy Consultant for Outertown LEA and for Westerly LEA, a neighbouring local authority. From April 2000, she was a full-time Numeracy Consultant for Outertown LEA.

Alexandra had a B.Ed. (Middle Years) with a subject focus in French, having first obtained a Cert Ed. She completed her initial training in the mid 1970s. Although she was trained to teach in the middle school years, most of her teaching had been in primary schools, including both KS1 and KS2. She had taught in schools in a variety of LEAs. She had taught at Parkway since 1990. She was a Language Co-ordinator and saw herself as a language specialist. When she joined the primary CAME research team, she was in the process of completing a Diploma in Language Education. She had some experience of delivering training for the Outertown Advisory Service. When Ursula started teaching at Parkway as a NQT in 1992, Alexandra was her induction mentor. They worked in parallel classes for three years and had planned mathematics week at Parkway together.

Alexandra had a C pass in O-level Mathematics. She had undertaken extended Inset in mathematics education including Rhoda's course on teaching mathematically able children.

When she first heard about the CAME project, she was "fighting to be involved" both because she wanted the chance to work on a research project and because she was keen to collaborate with Ursula. Alexandra was the most active of the Phase 1 teachers in terms of lesson trials (25) and tutor visits (15).

2. Henrietta

Henrietta was a Y5 teacher at Beechmount during 1997/8, when she joined the research team. She went on maternity leave from July 1998 and formally left teaching in April 1999.

Henrietta had a B.Ed. (Primary). She completed her training in 1994. The first year of the primary CAME project was her third year of teaching and her second year at Beechmount. She had no subject co-ordination role at Beechmount. Henrietta had not worked directly with Lisa prior to the research team.

Henrietta had a GCSE in mathematics. She had not attended any Inset in mathematics during her teaching career.

Henrietta was very quiet compared to the other teachers in research team meetings. Henrietta taught 8 lesson trials.

3. Janice

Janice was a Y3 teacher at Brightvale, when she joined the project. She had previously been involved as a Phase 2 teacher, supporting Y5 and Y6 class teachers at her school. In September 2001, she was promoted to Acting Deputy Headteacher at Brightvale.

Janice had a Cert Ed teaching qualification, which she completed in the mid-1970s. She had taught at Brightvale for all 25 years of her teaching career. Hence, all her teaching was in KS2. She had been Mathematics Co-ordinator for 10 years. During her participation in Primary CAME, she was also undertaking initial teacher education mentor training.

Janice had an A pass at O-level in mathematics. She had undertaken several extended INSET courses in mathematics education, including a 20 days GEST course. She ran NNS booster training in mathematics for Outertown LEA.

Janice appeared very excited about Primary CAME. She often talked about her liking for lesson development and initiated one lesson. She taught a large number of Phase 2 lessons, but only 4 lesson trials.

4. Lisa

Lisa was a Y6 teacher at Beechmount during 1997/8, when she joined the project. From September 1997 until April 1998, she was also Acting Deputy at the school. She continued as a Y6 teacher throughout the project. Throughout this time, she was also a member of the school's senior management team, latterly with responsibility for the school's performance management systems. From September 1999 until April 2000, she was again Acting Deputy at the school. In April 2001, she moved to become Headteacher of another Outertown school.

Lisa had a Cert. Ed. (Middle Years) with subject foci in English and French. She had taught at all school levels from Nursery to KS4, although most of her teaching was at the primary level. She had taught in Oxford, Central London as well as several Outertown schools. Lisa was the Art Co-ordinator at Beechmount. When she joined the project, she was in the process of completing an extended course on teaching thinking skills.

Lisa gained her teaching certificate prior to the requirement for an O-level in mathematics and she had no academic mathematics qualification at any level. Primary CAME was the first form of external training that she had undertaken in mathematics education.

In the first few meetings, Lisa was very keen to contribute to discussion. However, over the first year, she became much quieter and often said very little throughout a day long meeting. In the third year she attended very few research team meetings. She taught 11 lesson trials, all in the first year, and carried out 2 tutor visits to Phase 2 schools.

5. Tony

Tony was a Y6 teacher at Meadowside School in June 1999 when he joined the project. In September 2001, he was promoted to Acting Deputy Headteacher at his school. In January 2002, he became a Numeracy Consultant for a LEA outside London.

Tony had a PGCE. Meadowside was his second school. His class teaching experience was exclusively in Y6. Tony was the school Mathematics Co-ordinator and had been involved in the Outertown Numeracy Project.

Tony had a GCSE in mathematics. Aside from NNS training, he had not been involved in any extended INSET in mathematics education. She ran NNS booster training in mathematics for Outertown LEA.

Tony's involvement in Primary CAME outside research team meetings was relatively limited. He taught 4 TM lessons of which 2 were lesson trials. He did not initiate any new lessons, although he was involved in the drafting of lesson materials. He made only 3 peer-tutoring visits 2 of which he left early.

6. Ursula

Ursula was a Y6 teacher at Parkway during 1997/8. In September 1998 she became a Numeracy Consultant for Outertown LEA. In April 2001, she became Deputy Headteacher at another Outertown school.

Ursula had a B Ed with a subject focus in drama. She completed her initial training in 1991. Parkway was her first school. She taught at Parkway for for 6 years. When Ursula started teaching at Parkway as a NQT in 1992, Alexandra was her induction mentor. They worked in parallel classes for three years and had planned mathematics week at Parkway together. At the project's inception, Ursula was Parkway's Mathematics Co-ordinator and was in the process of developing a new scheme of work. Prior to this she had been the school's Information and Communication Technology Co-ordinator.

Ursula had an A pass at O-level in mathematics. She had begun an A/S level course, although she had decided not to complete this. She had taken a GEST 20 days mathematics course run by King's College with Outertown LEA. She was an occasional participant in Rhoda's BEAM writing group.

When she began working as a Numeracy Consultant, Ursula saw herself as a mathematics specialist. However, she said that she missed classroom teaching and latterly rejected the mathematics specialist label. Ursula was very involved in lesson development and initiated 4 lessons. She taught 15 lesson trials.

Appendix F: Research memos

I reproduce here four research memos. The first two, 10 and 14 November 1998, discuss my own participation in the research. The third, 4 March 1999, records some methodological issues raised by my reading of Miles and Huberman (1984). The fourth, August 1999, records a discussion of two issues arising from an interview with Alexandra..

1. Memo 10th November 1998

This research business is very difficult. Today I got to the school before Ursula and really got quite involved in Faride's lesson. I'm sure this was the right decision in terms of the CPD [continuing professional development], but was it right for my research? My lesson notes are certainly somewhat partial (and there were some interesting things in terms of conversations that I had with Faride, Ursula had with Faride and all three of us, of which I've recorded very little). But I'm worried that I am not really finding out about the tutor role if I'm doing. Maybe I am, but I'm certainly feeling that the issues are somewhat muddled. On the positive side it does enable me to say things like 'My first CAME lesson went completely wrong' and to develop a dialogue about the lesson. These discussions whilst teaching do seem to be very interesting 'critical moments' and it may be that the only way in to them is through being an active participant.

2. Memo: 14th November 1998

I've been a bit worried about involvement (or Participant observation with a capital P as David calls it). I got very involved in Faride's lesson. Faride was getting bogged down and Ursula hadn't arrived yet. I led a bit of the whole class discussion stressing the need for children to generate and use rules (rather than the teacher judging and using them). I'm sure that this was right for the teacher's development. BUT Is this contaminating my data? Well, I think not. It's certainly changing the sort of data I get - data as an interested participant rather then disinterested observer. However, it does have the advantage that I build up a relationship with the teachers. I've become very aware that they really don't know what I'm doing in their classrooms. I also feel that I misjudged things at Greenbank and should have become more involved. It is really difficult for teachers to be observed by one person let alone two. I think by just observing I made this observation worse (more of a disinterested judgement, whereas the intention is to develop a supportive, trusting, 'interested' relationship between teacher and tutor). ... Having thought about Greenbank, I think I do need to contribute to (but not lead) these discussions. In order to keep reflecting on my position as researcher, I think I must record as fully as possible my involvement and where possible seek participant validation for this. As part of this, I will share lesson summaries (my emergent thoughts about the teachers) with the tutors. I think I am doing this anyway and can't do otherwise, so this will be a record of this involvement. ... I'll also back off a bit to let the teacher-researchers establish the tutoring relationship on their own as least for this term. I'd better discuss this with David, but every other visit seems about right.

3. Memo: 4th March 1999

Rigorous research. I had a quick look at Miles and Huberman last night. The big question it raised for me is AM I BEING RIGOROUS ENOUGH?
Specifically, am I recording enough in my fieldnotes and am I typing enough up (so that others can see). Spradley refers to this as an expanded account (do it quickly etc...). In thinking about the methodology, I think I've perhaps been too concerned with the big philosophical issues (not that these aren't important) and I also need to emphasise the detail (also as a way of making sense of the philosophical). Perhaps one way tackle this would be to work through the activities in Spradley (as a way of exploring the methodology).

The Spradley book is good on interviewing techniques. In a sense, much of it is "obvious" but there's a danger in rejecting it then as simple or of not seeing the subtleties beneath this obvious core. The three types of questions seem important to remember upfront (descriptive, structural, contrast) but also the ways of emphasising the strangeness of situations (rather than translating quickly into the familiar).

Also:
Ask about use rather than meaning
Looking from interviewee's point of view
How would you say that to ....
What would you do in a certain situation
Describe a particular situation (it might help here to ask teacher-researchers to describe a classroom I haven't been in - of course this has disadvantages - I didn't really get very far into their 'good' lessons at the last set of interviews).
Repeat explanations and questions (this is different to ordinary speech)
Restate what they say
What questions would you ask?

I'm interviewing Ursula today and I'll try to keep some of these things in mind for that.

[This memo refer to Miles and Huberman (1984) and Spradley (1979).]

4. Memo: 4th August 1999

Second Nature
Alexandra said that second nature is about forgetting. She gave example of a way of understanding management styles: "First you've got unconscious incompetence, then conscious incompetence, then conscious competence, then unconscious competence. And it's the middle bit that is most uncomfortable, but you do forget the struggles that you went through in order to reach that unconscious competence."
Ursula said, "That bit's exactly right about less able children and less able teachers." pointing to [somebody like Rhoda who's got bags and bags of mathematical knowledge it seems to me, her mathematical knowledge's at a very different level to mine, might not necessarily think of - putting in the questions that a less confident teacher - might want to ask because it, it almost comes as, as second nature].

Alexandra talked about layers of knowledge building up on each other and meshing [as in her own quote given to her.]

Ownership and Balance

Alexandra said that Gudhreer had been "quite wary to begin with. She felt her maths was on show. And she was quite defensive." Alexandra's initial observation visit "The first time I went in didn't help, particularly when she asked for help and I didn't realise that she was talking to me." "Now she's had some very successful lessons and team taught with me a bit, it's different. She's got more ownership. The relationship's more balanced. She's moved a long way." Ursula said "She must have moved a long way, because that first lesson I observed she was teaching something that was mathematically incorrect. And she didn't like dialogue coming back to her from the kids. She wanted to teach them something that she knew and they didn't." I said that wasn't what I'd taken from Ursula's notes. Ursula: "That was because I was sending the notes back to her." Jeremy: "I took that she'd done a lot of work with the children on working relationships." Ursula agreed, but emphasised that Gudhreer was teaching something that was mathematically incorrect and that she hadn't worked it through herself. She'd taken it straight from the textbook, but hadn't understood it.

Alexandra said that balance is the same as bouncing off each other, but as she was saying it she said it is also different. The second phase teachers don't have the same amount if "time to develop the kind of relationships that we did. They don't have the same level of involvement that we have." "Our relationship [Ursula and Alexandra] is very special." "But it is about relationships and the teacher taking risks." She referred to research into reading at KS2 in Outertown which has found that the important things are building up relationships and the teacher showing she can take risks, particularly emotional risks." Ursula said, "Like the Numeracy Strategy, for it to work, you've [as Numeracy Consultant] got to put yourself on the line and teach the class. It won't be a perfect lesson. There isn't such a thing as a perfect lesson, but it's important that it's not perfect. Then you can work with the teacher." Alexandra said that some teachers would use that to criticise, but Ursula thought relatively few would. Ursula went on, "And once you've asked a child a question yourself, you've got experience of the child when it comes to talking with the teacher. ... And the kids know that you're a teacher and not just someone who come in to observe. ... It's the same for the teachers and the kids. The learning's just the same."

She said that "Sometimes Gudhreer's dialogue sounds very hard. She's always quite abrupt, but there's something underlying it. The children respond. It's almost like she's an old style teacher who's changing."

Alexandra agreed with my suggestion that balance is similar to bouncing off each other but much less intense.

We talked about the bouncing off each other in the research team. Alexandra agreed that it's not just about more heads, but that different people bring different things and that the bouncing off each other enables some re-remembering of the things that have become second nature.

Alexandra said that Mundher had changed he's got more understanding of the primary classroom and "He's moved form primary children can't do that to why aren't they doing that in the secondary schools."

Ursula agreed that bouncing off each other is very similar to her description of a good maths lesson. We talked about bouncing off conveying an intensity and a passion in the relationships. Alexandra again stressed that classroom culture is key.

Appendix G: Schedule of data

1. Summary of Data Collected

The following tables summarise the main data set (Table G.1), the secondary data set (Table G.2) and lesson observations by others (Table G.2). In addition to the data outlined here I made a large number of fieldnotes. In addition, I have used project memos, drafts of lessons and e-mail communications which are not detailed here.

	97/8	98/9	99/00	00/01	Total
Individual interviews	4	2	6		12
Alexandra	*1*	*1*	*1*		*3*
Henrietta	*1*				*1*
Janice			*2*		*2*
Lisa	*1*				*1*
Tony			*2*		*2*
Ursula	*1*	*1*	*1*		*3*
Mathematics interview			1	1	2
Alexandra				*1*	*1*
Janice			*1*		*1*
Joint interview			1		1
Alexandra, Ursula			*1*		*1*
Group interviews			2		2
Alexandra, Janice, Lisa, Ursula			*1*		*1*
Alexandra, Janice, Lisa, Tony, Ursula			*1*		*1*
Questionnaire for Lisa				1	1
Research seminar observations	14	13	11	2	40
Phase 2 PD session observations		6	6		12
NNS PD session observations		1	2	2	5
Alexandra		*1*	*1*	*2*	*4*
Janice			*1*		*1*
Ursula			*1*	*2*	*3*

[Note: There were 41 research team meetings during 98/9. I was not present at one of these meetings. The NNS PD sessions for Alexandra and Ursula in 98/99 and 99/00 are double counted.]

Table G.1: Summary of main data set

	97/8	98/9	99/00	00/01	Total
Interviews	3	1	5		9
Mundher	*1*				*1*
Rhoda	*1*	*1*			*2*
Phase 2 teachers			*5*		*5*
Mathematics interview pilot	*1*				*1*
Lesson observations	5	20	16		41
Alexandra	*1*	*5*	*4*		*10*
Janice			*5*		*5*
Henrietta	*1*				*1*
Lisa	*1*				*1*
Tony			*1*		*1*
Ursula	*2*	*1*			*3*
Other research team members		*3*	*1*		*4*
Phase 2 teachers		*11*	*5*		*16*
Taped reflections of Phase 2 visits		2			2
Alexandra		*1*			*1*
Lisa		*1*			*1*
Phase 2 tutor visits		8			8

Table G.2: Summary of the secondary data set.

	97/8	98/9	99/00	00/01	Total
Lesson observations	6	20	16		41
Henrietta (observed by Michael)	*2*	*5*	*4*		*10*
Lisa (observed by Michael)			*5*		*5*
Alexandra / Ursula (joint observation / team-teaching)	*1*				*1*
Janice	*1*				*1*

Table G.3: Lesson observations by others.

2. Interview Dates

Teacher	Date	Comments
Individual Interviews		
Alexandra	27/3/98	Interview 1
Alexandra	30/3/99	Interview 2
Alexandra	18/7/00	Interview 3
Alexandra	18/12/00	Mathematics Interview
Henrietta	10/3/98	Interview 1
Janice	10/11/99	Interview 1a
Janice	21/6/00	Interview 3
Janice	20/7/00	Mathematics Interview
Lisa	31/3/98	Interview 1
Tony	4/11/99	Interview 1a
Tony	13/7/00	Interview 3
Ursula	25/3/98	Interview 1
Ursula	4/3/99	Interview 2
Ursula	19/7/00	Interview 3
Joint interview		
Alexandra, Ursula	12/5/00	
Group Interviews		
Alexandra, Janice, Lisa, Ursula	17/3/00	
Alexandra, Janice, Lisa, Tony, Ursula	30/6/00	

Table G.4: Interviews with the six teachers

3. Research Team Meetings

In Table G.5 below, I summarise the dates of the research team meetings. The venue for these meetings was either at the Outertown Teachers' Centre or at King's College.

Date	Venue	Comments
Year 1 (97/98)		
14/11/97	Outertown	
21/11/97	Outertown	
5/12/97	Outertown	
12/12/97	Outertown	
16/1/98	Outertown	
30/1/98	King's	
9/2/98	King's	
6/3/98	King's	
20/3/98	King's	
1/5/98	Parkway / Outertown	
22/5/98	King's	
5/6/98	Beechmount / Outertown	
26/6/98	King's	
10/7/98	Outertown	
	14 meetings	
Year 2 (98/9)		
18/9/98	Outertown	
19/9/98	Outertown	
6/10/98	Outertown	a.m. only (p.m. PD)
1/12/98	Outertown	a.m. only (p.m. PD)
8/1/99	Outertown	
9/1/99	Outertown	
26/1/99	Outertown	a.m. only (p.m. PD)
26/2/99	Outertown	
9/3/99	Outertown	a.m. only (p.m. PD)
23/4/999	Outertown	Not present - no meeting notes
27/4/99	Outertown	a.m. only (p.m. PD)
18/6/99	Outertown	
19/6/99	Outertown	
29/6/99	Outertown	a.m. only (p.m. PD)
	14 meetings	

Year 3 (99/00)		
10/9/99	Outertown	
11/9/99	Outertown	
29/9/99	Outertown	a.m. only (p.m. PD)
19/11/99	Outertown	
25/11/99	Outertown	a.m. only (p.m. PD)
14/1/00	Outertown	
15/1/00	Outertown	
17/3/00	Outertown	
26/5/00	Outertown	
30/6/00	Outertown	
1/7/00	Outertown	
	11 meetings	
Year 4 (00/01)		
20/10/00	Outertown	
5/1/01	Outertown	
	2 meetings	
Total	41 meetings	

Table G.5: Dates of research team meetings.

4. Primary CAME PD Sessions

Date	Venue
Year 2 (98/99)	
6/10/98	Outertown
1/12/98	Outertown
26/1/99	Outertown
9/3/99	Outertown
27/4/99	Outertown
29/6/99	Outertown
Year 3 (99/00)	
29/9/99	Outertown
25/11/99	Outertown
3/2/00	Outertown / Phase 2 schools
23/3/00	Outertown / Phase 2 schools
6/6/00	Outertown / Phase 2 schools
20/7/00	Outertown
	12 meetings

Table G.6: Dates of Primary CAME PD sessions.

The venue for the main session was the Outertown Teachers' Centre. For 3 meetings, the afternoon session took place in several Phase 2 schools.

5. NNS PD Sessions

Date	Teachers	Comments
7/6/99	Alexandra	Westerly NNS Introductory INSET for schools
22/6/00	Alexandra / Ursula	Outertown NNS 5 day training
26/6/00	Janice	Brightvale NNS INSET
5/10/00	Alexandra / Ursula	Outertown NNS 5 day training
18/10/00	Alexandra / Ursula	Outertown NNS 5 day training

Table G.7: Dates of NNS PD Sessions observed.

Appendix H: Interviews

In this Appendix I reproduce the interview schedules for the semi-structured interviews. These took the form of a few broad questions together with a series of prompts. The questions and prompts were slightly modified for the different teachers. In particular, I used quotes from previous interviews and research team seminars as prompts and to enable participant validation.

Interview 1 (Alexandra, Henrietta, Lisa, Ursula)

This interview was conducted with the initial group of teachers in March 1998.

I'd like to begin by asking you about some lessons that you've given.
- Tell me about a (non-CAME) maths lesson which you feel went particularly well.
  - In what ways was this lesson successful?
  - Aims for children's learning
  - Children's initial understandings
  - Differentiation
  - Are there ways you could improve this lesson for next year?
- Tell me about the best CAME lesson you've given to date.
  - In what ways was this lesson a success?
  - Contrast with other CAME lessons.
  - Contrast with non-CAME lessons (e.g. last one you've described)
  - Similarities and differences with the successful lesson you've just described?

Now I'd like to move on to talk about the CAME meetings.
- I want to ask you what you thought of one of the CAME Central Research Team meetings last term. It was on the 5th December at the Teachers' Centre.
- [Reminder card]
  - Can you tell me your thoughts and feelings about this meeting?
  - Did you find this meeting useful?
  - Were there parts you liked and parts that you found less useful?
  - How could you have improved this meeting?
  - Use the discussion to ask about other Central Research Team meetings.
- Normally these Central Team meetings have included a number of different features - a review of lessons already delivered, lesson simulations, the development of new lessons, CAME theory.
- [Prompt card]
  - Could you tell me about your reactions to these different elements?
  - How do the CAME ideas relate to your own ideas about mathematics teaching and learning?

Do you think these ideas are useful for mathematics teaching and learning generally?
Probe balance between theory and practice.
Do you think this framework encourages discussion and reflection outside of the meetings?

Think of another teacher that you know who's maybe not the strongest maths teacher. If they were going to give the CAME lessons, how would you help them to do this?

How accessible is CAME? In what ways could you make it more accessible without losing the key features?
Would the approach help weaker maths teachers?

Now I'd like to think about teaching and learning mathematics more generally.

Tell me about a child in your class who is (or is becoming) a successful mathematician.

Personal / demographic details
Contrast with someone who is a less successful mathematician.

If you were given the chance to create the ideal primary maths classroom, what would it be like?

What would be the ideal way of working in such a classroom?
Is it the same for all children?
Differentiation?
What do you feel are the barriers to achieving such good practice?

Interview 2 (Alexandra, Ursula)

This interview was conducted with Alexandra and Ursula in March 1999. I planned also to use this interview with Lisa, although this was not possible.

Looking back over the last 18 months of P-CAME, can you tell me about your thoughts and feelings about what happened?

successes
frustrations?
own professional change as teacher (and beyond)
how did it happen?
Relationship with others - Rhoda / Mundher / other teacher-researchers and university researchers
significant factors in your own PD
what do you see as important elements of the PD?
Describe them - what happens? how do you use?
elements of PD simulations / team teaching / theory / reflection / lesson development / anything else?
openness / closure
importance of practice in the classroom
relationship with Numeracy Strategy
views about teaching / mathematics

Move on to peer-tutoring. Can you tell me about one of the teachers that you're working with and how you've worked with her?

Comparisons with others
Comparisons with own experiences
Early days - but successes, frustrations etc
Refer back to responses to last area.
Team teaching - describe - how does it happen? Is it always the same?
elements of PD simulations / team teaching / theory / reflection / lesson development / anything else?
openness / closure
importance of practice in the classroom
views about teaching / mathematics

Interview 3 (Alexandra, Janice, Tony, Ursula)

This interview was conducted with Alexandra, Janice, Tony and Ursula in June and July 2000.

Can you tell me about your own maths learning at school and beyond?

High points
Lessons that stick in your memory
Teachers
Good teaching / bad teaching
Easy maths / hard maths
Other children?
College / ITE
Links to teaching now

Over past 3 years how do you think you've changed as a teacher? I'm particularly interested in maths teaching.

How do you know? Specifics?
Mathematical knowledge?
Knowledge of teaching?
As a teacher of teachers?
What has been important about CAME for you?
CAME versus other things
Order the important things in your PD
Numeracy Consultant training – probing questions?
And the research team?

Teachers you've worked with – how do you know they've changed?

Prompts?
What do you look for?
Becoming a Learner
Risk-taking
Mathematical knowledge
Connectionist?

Nature of maths – mathematics without closure?
What's changed about other teacher-researchers as a teacher / a teacher of teachers?
Teachers who haven't changed?

Reflecting on Share an Apple / Halving and Thirding
What happened?
Why fractions?
What's difficult about fractions, decimals etc ...?
Links, connections ?
Possibly concept mapping?
Accessible to all?
Challenge, struggle?
What about teachers' notes?
Different sections?
Piagetian things?
Anything important not tackled in these lessons?
Do you remember the meeting at King's where we talked about it?
What do other teachers think?

Interview 1a (Janice, Tony)

This interview was conducted with Janice and Tony in November 1999. It was a modification of the first two interviews conducted with the initial group of teachers.

Tell me a bit about yourself.
Why primary teaching? How long?
What training? Good training / less good training? Anything particular to maths?
How did you get involved in CAME? Interest in maths - or more general?
Reflections on involvement in development team?

Could you describe a good maths lesson (not CAME) that you've taught?
What made it good?
And a good CAME lesson? Similarities and differences?
Key features of CAME? Theory important?
Numeracy Strategy? Similarities and differences?
What's important about maths?

Peer tutoring experiences / working with others?
Links to own experiences?
Team teaching - have you had a chance? Is it always the same?
Elements of PD simulations / team teaching / theory / reflection / lesson development / anything else?
openness / closure
Teachers' views about teaching / mathematics

Joint Interview (Alexandra and Ursula)

This interview was conducted jointly with Alexandra and Ursula in May 2000.

Can you tell me about your relationship as teachers?

When did you start working together ... was it always close ... what's important about it ... has it changed ... is it different being Numeracy Consultants to being teachers?

You've both talked to me about the 20 days course. Can you tell me some more? Was this course special or did this happen with others?

Thinking about yourselves as teachers of teachers, what's been important in your development?

How important has CAME been ... for you ... in terms of the NNS?

Has your mathematical knowledge changed? In what ways? What has influenced this change?

And how important is a teacher's knowledge of mathematics ... or thinking about mathematics?

Thinking about the Phase 2 group of teachers, to what extent have they changed as a result of CAME?

You've both talked about teachers taking risks or letting go of control and teachers seeing themselves as learners as key factors in teacher change. Can you tell me some more about this?

Group Interview 1 (Alexandra, Janice, Lisa, Ursula)

This interview was conducted with Alexandra, Janice, Lisa, Ursula in March 2000.

Working with Phase 2 teachers either PD, or tutors or whatever, what's been important?

Has there been change in their teaching? How do you know?
What's changed?
Knowledge about teaching? Mathematics?
Mathematical knowledge?
Classroom culture?
Collaboration?
Difference between teachers?
What's helped / hindered it?
What would you do differently?
Compare with NNS?

And yourselves, as teachers of teachers, what's been important in your development?

This group?
Development of lessons?
Working together?

Mundher? Others?
Phase 2 teachers?

Group Interview 2 (Alexandra, Janice, Lisa, Tony, Ursula)

This interview was conducted with Alexandra, Janice, Lisa, Ursula in March 2000.

You talked last time about the importance of two things in teacher change of development

teachers seeing themselves as learners and learning with the children
and teachers taking risks in relation to their teaching

What has enabled you to do these things?

How important is mathematical knowledge

or knowledge about maths teaching?
or views of mathematics?

We also talked about collaborating – in this group and with others in your school

is there a difference between collaboration and working together?
Good collaboration and bad collaboration?
How do you get teachers to collaborate on maths?

What has this group been like to work in?

Appendix I: Transcription Notation

-	short pause
italics	emphasis by speaker
?	questioning tone
[lower case]	comments in lower case within brackets provide clarification
[UPPER CASE]	comments in uppercase within brackets indicate laughter or a significant pause
(inaudible)	parentheses are used to indicate words that are either wholly or partially inaudible on the tape
...	ellipsis indicates words omitted

Appendix J: Excerpt from Transcribed Interview

This is an excerpt from the transcript to the group interview with Alexandra, Janice, Lisa and Ursula on 17 March 2000.

Jeremy: What I was saying, Ursula, is em I'm kind of getting to the end of my research and I'm sort of thinking about finishing collecting data and one of the things to do with you was think about as a group - what's happened, what's happened with the - the phase 2 teachers and what's been important, whether they've changed and secondly what's been important from CAME in terms of em - you developing as teachers-trainers or peer-tutors and I know there's other things that have happened apart from CAME. OK? - So I guess the first thing is - has there been change in the phase 2 teachers? - Have they changed their teaching?

Alexandra: I think in some cases they've built on good skills anyway and developed but em I think - it's given people the confidence actually in their maths teaching

Jeremy: Are there people where that's happened and people where that hasn't happened?

Ursula: There's certainly people where it hasn't happened - very much

Alexandra: Yeah.

Janice: But I think as well as that there are people, mentioning that other person, they think they've changed. I think even if you've only changed a little bit, maybe even changed their views. Because I remember one particular teacher when she started was very negative about CAME and doing the lessons and in fact she's very positive.

Alexandra: Yeah, but

Janice: Enjoys it, enjoys teaching the lessons and says she's getting a lot out of coming

Alexandra: That's a degree of change, isn't it?

Janice: Yes, it is. That's difficult to measure. -

Jeremy: Well, how do you know if people have changed? -

Ursula: You don't really. I, I learnt in terms of the fact that I went to do the observations. Well, when I first started I observed the teachers. So I observed them doing a normal lesson. And I think I can probably predict changes in their normal teaching from the people that I'd observed. Unfortunately the person who was changing most for me has left. - -

Alexandra: Yeah. I mean the people, well one person really, in particular, has actually commented upon the critical [unclear - evidence?], but then she was also doing the, she's been part of cohort 1 in numeracy as well and so there's been a big emphasis for her in terms of her maths change. It's a bit hard in some places to divorce it.

Janice: You're actually saying you've observed people.

Ursula: I did. Right at the very, very beginning, for the first teachers coming on board, Rhoda and I did a set of observations.

Janice: Then you've got something, you've got something. Because one of my problems has been that whenever I've actually been in, they've always been watching me. I've always been the one that they've had a chance to see. And I haven't actually observed a great deal, other people - But that's my fault, isn't it, really. -

Jeremy: So what's happened in terms of the people who've changed? What's -

Ursula: Janice, do we call you a phase 2 teacher? I know you're part of this group, but you came on board at about the same time as phase 2, didn't you?

Janice: yes

Ursula: as well as Tony

Janice: No, Tony wasn't part of the group was he?

Ursula: No, he wasn't part of the group. He became involved

Janice: straight into this group

Ursula: But I think it's the difference between being part of this group and then being part of that other group in terms of - I suppose your knowledge and your expertise and what you expect and - what comes out of it, 'cos Tony's also grown phenomenally as a maths teacher.

Jeremy: So what's different about this group to the phase 2 teachers?

Ursula: More involved, I suppose -

Janice: We get more chance to reflect, don't we. When the other teachers come, although it's always built into the agenda and they get time to feed back on lessons, that part always seems to be -

Ursula: tokenistic

Janice: very tokenistic, yes, just. I think that's one of the things where we've made a mistake really, because - we get a long time to reflect, don't we?

Alexandra: But it's a different sort of reflection, isn't it? They're reflecting on lessons that are, that are, are given, whereas we

Ursula: wrote the lessons

Janice: But they should still have more time, I think, to say how it's gone and how they feel about it, 'cos we always seem to be rushing on to give them the next lesson

Alexandra: Mmm

Ursula: And really, they only come once a term and for a lot of them it's, I don't know, it's tuning in, isn't it? You have one day out of the classroom. I remember what it was like to come on a course. You've got a day out of the classroom and you take time tune in -

Alexandra: Well, in a sense you've [Janice] done both, haven't you? So, I mean you're quite well placed to say what the difference is.

Janice: I feel much more involved. I mean obviously I suppose. But you do feel more of a part of it and - yeah, I think, I think I do feel we should leave more time for them to feedback on how it's gone and talk about it. -

Jeremy: I mean would that make it less tokenistic - if it had more time?

Janice: I mean there isn't the time there. That's the problem.

Ursula: I think it's too spread out. I think if you really want to develop things, you really need to do it closer together. So there isn't a term's gap between people doing something and then being asked to talk about it when it's a really hazy memory

Janice: And I mean we were all given thinking diaries but - nobody really uses them

Ursula: I don't think anybody has the time to

Janice: No

Ursula: because this is just like for most people it's a very tiny part of inset, it's a minute little part of everything else that they do.

Jeremy: Whereas for you it hasn't been?

Lisa: I think you have to go back to the beginning when we first started and we were developing the lessons. And we spent all our time on development of the lessons, whereas now we're sharing the development of the lessons, well, as far as they go, with the phase 2 teachers. So we've cut the time in half or even less, 'cos there is much less time on the development of lessons, which has very much come out this afternoon

Alexandra: But the other thing that we had at the beginning that was, I think very involving were things like the luxury of spending, and I don't think the phase 2 teachers have had it certainly not the new ones, was all the time we spent on you know the nature of CAME lessons and discussing that and I think that was valuable as well.

Lisa: Mmmm.

Alexandra: Because it's not easy to come to

Ursula: The other thing we did a lot of was, which we've only just come back to in this group, was the team teaching

Alexandra: Yeah

Ursula: And it's only in changing that afternoon that we've come back to team-teaching

Alexandra: Yeah

Lisa: And just teaching the same lesson two or three times and we talked about it and how we can develop it and now we're just rushing through with well shall we change that word, you know what do you think about the timing, which compared with what we were doing in the first group is a nonsense. When you think what quality time we gave to the original lessons and the lack of time that we're able to give to these subsequent lessons, there's no comparison

Ursula: But the experience that we got is very different to being a peer-tutor. I mean we got such, I mean Alexandra and I used to teach together all the time and then we swapped or we'd use each other's classes

Lisa: Mmm

Ursula: which is very different to going in on a one to one with somebody

Lisa: and that came out very much

Ursula: 'cos they don't know you and they haven't got a reason for knowing you apart from they are part of this course

Lisa: Yeah and I mean that came out very much from what you said about someone who didn't want to be observed, which is to me missing the whole point of you know what this was all about

Alexandra: Also I think I mean you built up you know that you [JH] would come or Mundher and team teach alongside and it felt just the same really. You know you're kind of working, you're working to people's suggestions and things like that em

Lisa: No threat basically and it would appear from what, what's been said that there's some of the phase 2 teachers who still find it quite threatening

Alexandra: No, the thing is that

Lisa: and what they're saying is no I'm not having anyone in my room

Alexandra: But the way it was done was very much, it was a kind of condensed period wasn't it? We did a lot of that - in, in a fairly short period of time, whereas with this as you say, it's all very spread out [Others join in with this]. You can't, well you can do

Janice: It's about relationships though [Unclear contribution - several people speaking]

Alexandra: It's about relationships, yeah. It is. [Unclear contribution - several people speaking]

Ursula: But it is the same as what we do. If we have quite an intense period of time in school. Say we're working there for 3 weeks or something. You get to know the people in that school that week and you have far more influence than if you go in perhaps one day every half term

Alexandra: Yeah, yeah

Ursula: where people just see you as oh you can come in today sort of thing

Jeremy: But given, so given the phase 2 has been a lot less intense, has been, how come some of the teachers have changed?

Ursula: I think it was the teachers that were very open to changing or to wanting to improve in the first place. We've got some very reflective teachers on the course and we've got some teachers who are quite happy in what they do and they don't question the fact that they could perhaps get better

Alexandra: Yeah, maybe

Ursula: That's not a criticism of them. It's just that they're quite happy in what they're doing and they're quite secure in what they're doing and they don't see that - no

Alexandra: And maybe it's about also teachers whose style of teaching is quite close to the kind of style of teaching that this is promoting

Lisa: and if their style of teaching is quite close to this style, then - they're teachers who are likely to want to you know better their practice anyway but I mean part of the continuous professional development is to try and improve the practice of those normally wouldn't make changes

Alexandra: but on the other hand, again I'm thinking about somebody who - was like that but had a real thing, hated maths teaching, hated teaching maths so probably would have felt she wasn't particularly a good maths teacher

Ursula: I think it's, I think I agree with that because I've done some CAME lessons with teachers that I work with on the numeracy side of things and where I've done a CAME lesson they've been, those teachers have really moved forward and have actually done some CAME lessons themselves as well and they see a difference between the two lesson but they wanted to know what the difference is and they've taken on board some of those differences. And those ones are the ones who really want to learn as well, those particular people.

Lisa: Mmm - So part of it is a willingness to want to move on or.

Ursula: It's being a reflective teacher, but not all teachers are.

Appendix K: Mathematics Interview

This interview was conducted with Janice in July 2000 and with Alexandra in December 2000. The interview began with a question relating to a NNS PD session that I had observed the teachers teach. The mathematics questions were taken from the LNRP interview schedule (Askew & Millett, 2001) with the addition of the final simultaneous equations problem.

How you feel about these problems and how you would go about solving them.

True or false?
From a plane, a field 90m by 100m looks more 'square' than one 950m by 1000m.

There is no multiple of 7 between 7001 and 7005

Which of the above numbers (2, 3, 4, 5, 6, 7, 9, 11) are factors of ...
63 165 513,252 $3^2 \times 5^2 \times 7$

Which of these numbers are equivalent to 1/5 ?

One fifth	20%	1.5	3/7
1/5	0.5	3/15	1 ÷ 5
five tenths	a fifth	5/20	0.2

How would you convert 3/5 to a decimal?
How would you convert 1/7 to a decimal?

How would you check the price of a bag of fruit with your calculator if the fruit cost £1.68 per kilogram, and your bag weighed 0.86 kilograms?

How would you solve this problem?
0.5 x 0.2
What would you say would be a good story, diagram or model for 0.5 x 0.2?

How would you solve this problem?
3 ÷ 0.75?
What would you say would be a good story, diagram or model for 3 ÷ 0.75?

How would you solve this problem?
$1\ ^{3}/_{4} \div {}^{1}/_{2} =$
What would you say would be a good story, diagram or model for $1\ ^{3}/_{4} \div {}^{1}/_{2} = ?$

How would you solve this problem?
$3x + y = 4$
$x + 2y = 3$

Appendix L: Codes Used and Example of Coded Data

1. Codes

Using / adapting ideas
Opportunity
Modelling
Imitating
Risk
Learning from each other
Learning with children
Learning by doing
Bouncing ideas
Stories
Teaching twice

Tutoring
Lesson development

Team-teaching
Collaboration
Relationships
Reflection
Planning

Inclusion / exclusion

Sticking to lesson agenda
Open
Closure
Freedom
Focused

NNS
Other CPD
ITE
Own school learning

Gut reaction
Feeling

Like
Love
Hate
Uncertainty
Inspiration
Motivation
Interest
Vulnerable
Struggle

Similarity
Difference
Existing teaching

Pace
Differentiation
Children's collaboration
Fun
Questioning
Individual

Experts
Right answers
Just my own thoughts

Connections
Justify
Mathematical concepts
Big ideas

CAME lessons
CAME theory
CAME teaching approach
Cognitive conflict
Children's ideas
Classroom culture

Beliefs – maths
Beliefs – teaching
Metaphors of learning
Generalist teaching

2. Example of Coded Data

This example of coding is taken from the interview with Lisa on 31 March 1998.

CAME lessons

schools, something like that and those who had worked out the formula were able to do that.

Closure

Jeremy: And were there different sorts of formulas?

Lisa: Em ... I think those who actually got to the formula, from what I can remember, it was the same formula. I don't remember there being different formulas.

Children's Ideas

Jeremy: OK. You've talked about the differences in children being up at the board, but were there other similarities and differences with other CAME lessons?

Lisa: With other CAME lessons or with other lessons.

Differences

Jeremy: With other CAME lessons.

Lisa: Oh, with other CAME lessons. Em ...I'll have a look. [Looks through the Teacher's Guide] ... I mean the similarities would be the, the whole class discussion and contributions.

Similarity

Em, something I picked up on how you can make sure you get a good spread of children making contributions and to value, em, the contribution of the less able ... as opposed to not wanting to necessarily in some other work, if you were doing a whole class lesson, taking the risk of asking them, em, and to put them in that sort of position of taking the risk. Whereas in this sort of work I feel that you can give them the opportunity to take a risk, because their contribution will be valued regardless of what they have to say.

Differentiation

Risk

Jeremy: How do you organise that in a CAME lesson?

Lisa: In a CAME lesson I get the opportunity to walk around and pick up what's going on, but also to, em, not feel restricted in who I ask so much and use that as part of the .. the teaching that we can, we can listen to all different points of view as to what people have got to say.

Freedom
Opportunity
Feeling

Jeremy: Now you've got the book open at Number Operations. Are there contrasts between Number Operations, you've done that twice, haven't you?

Lisa: Em, no, just trying to think.

Jeremy: Or you've done it as two lessons.

Lisa: Number Operations? No, I think I only did that once.

Jeremy: Did you do the last bit?

Lisa: No, I used Number Operations 4 ... Let me have a look. Just trying to think. Oh, I think I did the first bit with negative numbers, which was very useful, 'cos the children were finding the understanding of negative numbers .. difficult to comprehend.

Math'l ideas

Struggle

So that worked out well with Number Operations 1.

Success

I didn't do Number Operations 2 or 3. We did go on and have a look at Number Operations 4, but I chose to take out the decimal points. One of the reasons why I took out the decimal numbers was because it was whole class and working from level 2 to level 5, I wanted to, I wanted everyone to have access to it and I think the minute the decimals were there that would have

Math'l ideas

National Curr.

Differentiation

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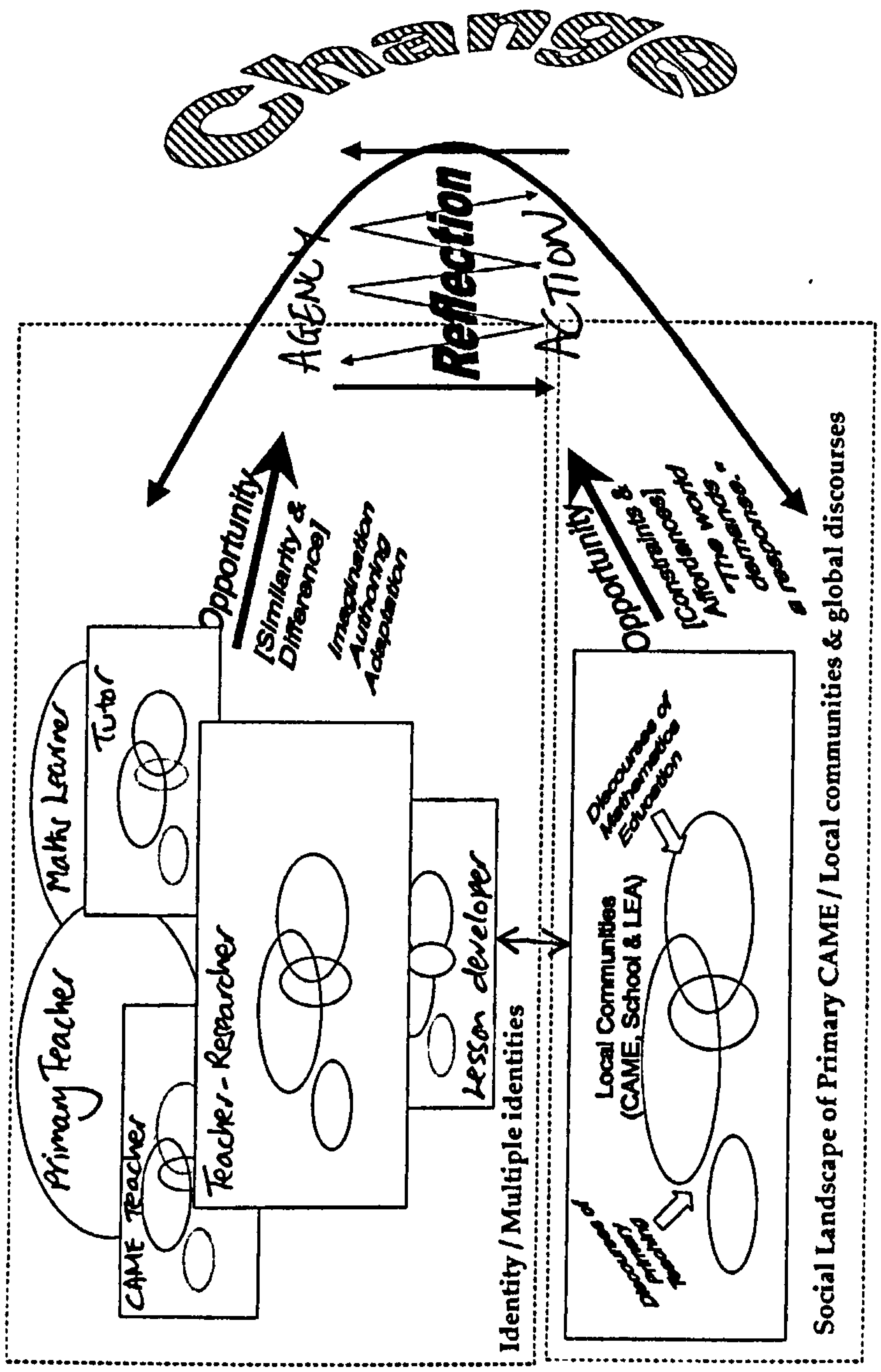
Appendix M: Formative analytic diagrams

The diagram below is an example of one of the diagrams that I used to develop my analysis. This diagram was used to connect the various analytic tools that I developed.



Appendix N: The teachers' schools

In this Appendix I give an overview of the teachers' schools: Beechmount, Parkway, Brightvale, and Meadowside. Table N.1 shows a comparison of the schools' achievement in mathematics measured by KS2 results.

	1997	1998	1999	2000
National Average	62%	58%	69%	72%
Outertown Average	61%	57%	68%	70%
Parkway	69%	77%	75%	81%
Beechmount	44%	43%	44%	50%
Brightvale Girls	48%	66%	64%	73%
Meadowside	70%	100%	86%	78%

Table N.1: Percentages of pupils achieving level 4 and above in end of Key Stage 2 mathematics tests 1997-2000 at the CAME Phase 1 teachers' schools compared to national and Outertown averages.

1. Beechmount Junior School

Beechmount was a junior (KS2) school situated in park in a residential inner-city area in the north of Outertown. The school shared a site with Beechmount Infant School and was surrounded by a large park. The 1998 OfSTED report considered there was "a degree of social and economic disadvantage socially" and, according to the National Statistics Office neighbourhood statistics, the school is located in an area of significant deprivation. A high proportion of the pupils (47% in 1997/8) were from ethnic minority backgrounds. The school had a substantial degree of pupil mobility.

After extensive building work to increase the number of classrooms, the school accommodation was very good. The school was expanding from 2 form to 3 form entry. During 1997/8, at the beginning of the project, Y3 and Y4 were 3 form entry, whilst by the third and final year of the project, all four years were 3 form entry. The school had a number of meeting and resource rooms. The staffroom was large, although on my visits to the school it did not appear to be well used. During my visits to the school, I was never introduced to any teachers who were not directly involved in the CAME project.

The OFSTED report in 1998 stated that, although the KS2 tests results were generally in line with national averages, results in mathematics were below average (44% of children achieved level 4 and above compared to a national average of 62% in 1997). The OFSTED report considered the attainment of children on entry to the school to be broadly in line with national averages, although below average in mathematics. However, by the end of KS2, pupils' achievement in mathematics was well below average. OFSTED judged that expectations in mathematics were too low and mathematics teaching was overly reliant on one commercial scheme. As can be seen from Table N.1, Beechmount's KS2 mathematics test results were static over the period 1997 – 1999, then rose slightly in 2000, but was still well below national averages. The school received intensive support from the Outertown Numeracy team during 1998/9, 1999/2000 and 2000/1.

In 1998/9, mathematics ability setting was introduced throughout the school, following a trial in Y6. CAME lessons were, however, taught in mixed ability classes. Beechmount joined the primary CAME project as a laboratory school in Autumn 1997. Two teachers, Henrietta and Lisa, were members of the research team. Jenny, the deputy and mathematics coordinator, joined the project as a Phase 2 teacher in Autumn 1998. Henrietta left teaching in Autumn 1997.

2. Parkway School

Parkway was a 2 form entry junior (KS2) school situated in a residential area just south of the town centre of Outertown. The school was a church school. The school shared a site with a Parkway Infant School and had access to substantial playing fields on adjacent land. A high proportion of the pupils (33%) were from ethnic minority backgrounds. According to the National Statistics Office neighbourhood statistics, the school is located in a relatively deprived area.

The school was due to expand to 3 form entry from Autumn 2000 and there was substantial building work to the school's accommodation between 1998 and 2000. The school's common areas were cramped, although classrooms were of a reasonable size. The staffroom was relatively small, although it appeared to be well used. The headteacher appeared supportive of the project and gave his office for some meetings involving members of the project team. Other meetings took place in classrooms at breaktime or in the staffroom.

The OFSTED report of 1997 concluded that standards in mathematics were high. Pupil's attainment was well above average, which built on what OFSTED judged to be an intake of children whose attainment was above the national average on entry to the school. By the end of KS2, the children's attainment in mathematics was well above average. The OFSTED report highlighted investigational activities in mathematics as a strength. As can be see from Table N.1, the KS2 mathematics test results of children achieving level 4 or above were higher than the national and Outertown averages over the period 1997 – 2000. Results rose overall during the period, although there was a slight fall in 1999.

Mathematics was taught in mixed ability classes. Parkway joined the primary CAME project as a laboratory school in Autumn 1997. Two teachers, Alexandra and Ursula, were members of the research team. Two further teachers, Enid and Oliver, joined the project as Phase 2 teachers in Autumn 1998. Ursula left Parkway to become a Numeracy Consultant for the Outertown Advisory Service in Autumn 1998. Alexandra left Parkway to become a Numeracy Consultant in Summer 1999.

3. Brightvale Girls School

Brightvale was a 3-form entry single sex junior (KS2) school. The school shared a large inner-city walled site with two other schools: Brightvale Boys and Brightvale Infants. The school was situated in a residential area in the north of Outertown. Households in the area were judged in the 1998 OfSTED report to be broadly similar to the national average, economically and socially, although, according to the National Statistics Office neighbourhood statistics, the school is located in a relatively deprived area. A high proportion of the children were from ethnic minority backgrounds. In 1998, 41% of the children had a first language other than English.

The girls school was accommodated in a large self-contained turn of the century school building. The school had a welcoming atmosphere. For example, pairs of girls were appointed as monitors to greet visitors arriving before the start of school. On my first visit, the headteacher, who was not directly involved in the Primary CAME project, personally welcomed me to the school. There was a large and busy staffroom, which doubled as a resource and workroom for the teachers. Classrooms were situated along two corridors and there appeared to be frequent interaction between teachers, both during and between lessons.

The OFSTED report conducted in Spring 1998 identified mathematics teaching and learning as a weakness with pupils achievement being lower than the national average, although results overall were in line with national averages. OFSTED judged the work in mathematics was often "too easy" and that children were required to do insufficient work. Progression from year to year was also judged to be a problem. Janice, as mathematics coordinator, was, however, praised as being "enthusiastic and knowledgeable". As can be seen from Table N.1, the schools' KS2 mathematics test results of 48% level 4 and above in 1997 were well below the national average of 62%, although the attainment of pupils at intake was judged in the OFSTED report to be broadly in line with national averages. However, KS2 mathematics test results improved dramatically and in 1998, 1999 and 2000 were broadly in line with national averages. The school has not been identified by the Outertown numeracy team as requiring intensive support, although the school did choose to send two NQTs on the optional 3 day training offered by the numeracy team in summer 2000.

Children were largely taught in mathematics ability sets, although CAME lessons were taught in the mixed ability class groups. During 2000, the school began to introduce a new commercial scheme for mathematics in order to meet the requirements of the NNS.

Brightvale joined the Primary CAME project as a Phase 2 school in Autumn 1998. In summer 1999, Janice, the mathematics co-ordinator, joined the research team. Several lessons were subsequently trialled at the school. Over the two years, 1998/9 and 1999/2000, eight teachers from the school were involved in the project. CAME lessons were generally taught with two teachers present. Supply cover was provided through mathematics coordinator release time or by the headteacher.

4. Meadowside Primary School

Meadowside was a 1 form entry primary (KS1 & KS2) school with an attached nursery class. The school was situated in an estate of privately owned housing within an affluent area in the south of Outertown.

The school's accommodation was open plan and classroom space was relatively small. The school was surrounded by substantial open space. The staffroom was very small, although it appeared to be well used. During my visits to the school, I was never introduced to any teachers apart from Tony.

The OFSTED report of 1997 concluded that standards in mathematics teaching and learning were high. OFSTED reported that "based on the Outertown entry profile, the school receives a mix ability intake with the majority of pupils achieving average levels of attainment with a smaller proportion working both above and below". As can be seen from Table N.1, the percentage of children achieving level 4 or above in the KS2 mathematics tests was well above the national average. Although results were high, in 1997 the school had identified mental mathematics as a priority in its development plan. As a result, Meadowside volunteered to take part in the Outertown Numeracy Pilot during 1998/9.

Mathematics was taught in mixed ability classes. Tony, the Y6 teacher, joined the research team in Summer 1999.