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Equilibria of Carbon Allowance Auctions: Emissions and Productivity

Lingxiao Zhao[1][0009-0005-5190-5365], Maria Polukarov[1][0000-0002-7421-3012], and Carmine Ventre[1][0000-0003-1464-1215]

King's College London, Strand, London, United Kingdom
{lingxiao.zhao,maria.polukarov,carmine.ventre}@kcl.ac.uk

Abstract. The Emissions Trading System (ETS) is a market-oriented policy aimed at regulating and diminishing greenhouse gas emissions by allocating and trading carbon allowances. Previous studies have mainly focused on dynamic model simulations, while the overall equilibrium state of ETS systems has yet to be explored. To this end, this paper proposes an empirical agent-based model to analyse European carbon allowance auctions: Within the ETS framework, energy companies adopt different strategies to interact in the primary carbon auction market. We use two different methods: partial equilibrium analysis and role-symmetric game analysis to simplify the model strategy space. We then apply the α-rank algorithm to determine the model's equilibrium strategy and conduct an in-depth analysis of the combination of these strategies. We examine carbon output levels under these conditions and find that the ETS framework effectively reduces carbon emissions across the system. We also explore the impact of different simplification methods and auction formats on the ETS market: Our results indicate that role-symmetric game analysis has better payoff performance; in addition, uniform auctions improve production efficiency, while discriminatory auctions successfully allocate resources, leading to fairer market competition.

Keywords: Emissions Trading System · Evolutionary Game Theory · Agent-based Model · α-Rank Algorithm.

1 Introduction

The Emissions Trading System (ETS) is a market-driven policy tool aimed at controlling greenhouse gas emissions by setting carbon quotas for trading [31]. The government sets an overall cap and allocates emission permits, which companies can buy or sell. Companies emitting less than their quota can sell excess permits, while those exceeding must buy more or face fines [8].

The European Emissions Trading System (EU ETS) is the world's first significant carbon emissions market, launched in 2005, the EU ETS has progressed through three complete phases and is currently in its fourth phase. The EU ETS covers about 11,000 entities in the EU, including power stations, manufacturing plants and airlines, accounting for about 45% of the EU's greenhouse gas emissions [1].

In this context, studying carbon emissions markets and ETS is necessary and timely, but also poses significant challenges for financial AI research. The system's complexity, involving multiple heterogeneous entities, makes it challenging to compute emission reductions efficiently with traditional statistical methods [23]. Moreover, analyzing corporate strategies within the carbon emissions trading system is crucial, as inefficient auction strategies can undermine ETS performance, potentially leading to market failure due to factors like bid price volatility [19], low participation [3], or low clearing prices [11].

Analyzing agent interactions in the ETS to find stable strategies and achieve emission reductions is significant for individual entities and the system. This paper models the European primary carbon emissions auction using game theory method to explore the auction strategies of energy companies. We use agent-based modelling (ABM) [21] to calculate the stable strategy under various scenarios, helping us understand agent behaviour and evaluate carbon emission levels and auction clearing prices.

Building on Tang et al.'s ABM [24], we use evolutionary dynamics algorithms to calculate the system's equilibrium strategy in the auction model, quantifying payoffs and carbon emissions reduction in the ETS. By varying auction strategies, we analyze their effectiveness in reducing emissions and examine the impact of different management methods. Our model is grounded in real industry data, underscoring the relevance and significance of our findings.

2 Related Work

Promoting greenhouse gas reduction is the core goal of ETS [6]. Since its inception, many studies have examined the policy's impact on corporate operations [15, 29]. For example, research by Chan et al. [2] shows that the promotion of the EU ETS platform has effectively reduced the material cost input of European power companies and significantly increased the income of these companies. Krass et al. [17] believe that combining taxes generated during the ETS process with subsidies and rebates can better promote the adoption of green technologies and thereby reduce greenhouse gas emissions.

Hepburn et al. [16] point out that the government's increase in auctions in the second phase of the EU ETS may improve efficiency by supporting border tax adjustments and other measures, helping to mitigate price fluctuations and enhance the long-term price signal. Further, Sarto et al. [22] study the carbon emission allocation rules of the third phase of the EU ETS and suggest that these regulations changed the way emission allowances are allocated for free to energy-intensive industries.

Advances in computer technology have popularized ABM in carbon market research, providing deeper insights into agent behaviours under various policies [23]. Cong and Wei's [5] agent-based carbon allowance auction model (CAAM) contributed to EU ETS analysis and was expanded by Tang et al. [24] to study government-firm interactions, highlighting its positive impact on China's carbon reduction. Wei et al. [25] also used ABM to investigate enterprise compliance strategies under the ETS, and revealed a non-monotonic "L-shaped" carbon price trend and highlighted the current ETS penalty mechanism's inefficiency.

Finally, the spread of evolutionary game theory from its biological roots has become a crucial tool in economic analysis, particularly in evolutionary economics. Illustrating its applications to energy markets, the Nie et al.'s [20] study in public transportation demonstrates how different carbon tax and subsidy combinations affect corporate decision-making, particularly in adopting new energy buses. Chen et al. [4] apply the theory to analyse manufacturers' responses to carbon tax and subsidy scenarios and show that dynamic approaches are more effective for encouraging low-carbon production.

3 Game Model

This section details our agent-based model for ETS, designed to dissect the dynamics of the primary auction market and accurately forecast market-clearing prices with authentic market data. In this market, firms can only purchase carbon emission quotas from the government. The model delves into agents' interaction in the carbon allowance auction system during the fourth phase of EU ETS (2021-2030) and studies differences in their decision-making across different industries from a bottom-up perspective. This offers advice for the European Commission on the policy development for the next phase of EU ETS.

We extend the Tang et al.'s [24] model, paying particular attention to the behaviours of energy companies that use different raw materials for power generation in the primary carbon emissions auction platform. This initial allowance market is the most essential part of the ETS design [30, 16]. In the ETS, the government serves as policy architect, carbon quota provider, and auctioneer, while energy firms, mainly non-renewable, actively bid in such auctions. For simplicity, our model does not account for the distribution of free allowances.

3.1 Model Parameters

First, we set the model's fundamental parameters, which remain constant when varying agent strategies in the simulation.

Model Parameters We collected data from the European Energy Exchange (EEX) [7], which indicates that an average of 18 agents participated in each auction in 2021 at the start of the EU ETS's fourth phase. The reports also detail the total carbon emissions and the number of auctions in 2021. Accordingly, we set the number of auction participants, N, at 18 and schedule annual auctions. The initial carbon emissions allocation, E_{t_0}, is based on 2021's total carbon emissions divided by the year's auction count, representing the carbon emission quotas the government plans to sell in the first auction. This approach aligns our model with the EEX's actual trading environment, enabling a realistic simulation of auction mechanisms and strategies in the EU ETS.

Agent Parameters Each agent is assigned an initial expected carbon price, pv_{i,t_0}, and submits a first bid, bid_{i,t_0}, randomly selected from [0, pv_{i,t_0}]. The expected prices follow a normal distribution around the 2021 EU ETS average transaction price, adjusted by a factor of 1.5. This approach introduces randomness while reflecting realistic market expectations. In addition, we use the initial allocation of carbon emission, E_{t_0}, multiplied by the cover ratio, to obtain the expected overall emissions of the agents participating in the auction. The cover ratio is the ratio of agents' total demand emissions to the government's available auction emissions. We then use the Dirichlet distribution to randomly assign the total expected emissions among agents in the auction, generating their initial expected emissions, v_{i,t_0}.

3.2 Auction Process

Allowance auctions on the EU ETS platform can take several forms, including two primary variants of sealed-bid auctions: discriminatory or uniform-price formats. The main distinction between these formats lies in the payment structure for the winning bidders.

Auction Formats In the uniform-price auction, all winning bidders pay a uniform price, which is determined by the auction's clearing price at time t. Within this framework, each agent will announce its bidding price, $bid_{i,t}$, and the needed carbon permits, $v_{i,t}$. All the agents are ranked according to the bidding price, from high to low (w.l.o.g., bidder i is the one with the i-th highest bid). Accordingly, when the cumulative quantity of permits reaches the total allowance supply, E_t, the clearing price of the market is expressed as $pc_t = bid_{i=m,t}$, where m represents the last agent that successfully bid and received all or some of the required permits. The payment price, $bid'_{i,t}$, and the corresponding purchase volume, $v'_{i,t}$, for each agent i at step t are calculated as follows:

$$bid'_{i,t} = \begin{cases} pc_t = bid_{m,t}, & i \leq m \\ 0, & i > m \end{cases}$$

$$v'_{i,t} = \begin{cases} v_{i,t}, & \text{for } bid_{i,t} > bid_{m,t} \\ v_{i,t} \cdot \frac{E_t - \sum_{j=1}^{w-1} v_{j,t}}{\sum_{j=w}^{m} v_{j,t}}, & \text{for } bid_{i,t} = bid_{w,t} \wedge m = i = w \\ 0, & \text{for } bid_{i,t} < bid_{m,t} \end{cases}$$

Each successful agent's payment for carbon emissions equal to the equilibrium or clearing price of the auction. On the other hand, carbon allowances are allocated based on the bid hierarchy. If the supply falls short and multiple agents, w, place bids equal to the final successful bid, the remaining items will be proportionally allocated among these agents.

In the discriminative-price auction, winning firms need to pay different carbon allowance prices. These prices, $bid'_{i,t}$, are directly contingent upon the individual bid prices, $bid_{i,t}$, put forth by each firm, and the weighted average of all successful transactions gives the clearing price.

Auction Cost In both cases, winning agents need to pay the transaction price to the government. In the uniform-price auction, each agent's cost, $Ccost_{i,t,uni}$, equals the clearing price pc_t multiplied by the obtained emission permits $v'_{i,t}$. In the discriminative-price auction, a firm's cost, $Ccost_{i,t,dis}$, is the product of its payment price $bid'_{i,t}$ and the allocated allowances volume $v'_{i,t}$.

3.3 Production Process

After having acquired carbon permits in the auction, agents will use different energy sources to generate electricity. Each agent will generate income and pay production costs (and, possibly, penalties) during this process. For brevity, other expenditures on power generation are ignored.

Production Cost The production cost, denoted as $Ecost_{i,t}$, represents the cost incurred by an agent for energy generated. Our framework categorises electricity-generating agents into three classes: Oil, Gas, and Coal. Specifically, each agent's unit cost of generated electricity is computed based on two key factors: the 2021 average energy price and the energy conversion coefficient pertinent to each energy source. For each agent, the production cost calculation formula is as follows:

$$Ecost_{i,t} = v_{i,t}/\ f_{type} \cdot R_{type}. \tag{1}$$

In this context, R_{type} signifies different raw material prices agents required to generate 1Mwh of electricity in 2021. The f_{type} denotes the ratio of the carbon dioxide emissions the corresponding energy source generates to produce 1Mwh of electricity [18]. Additionally, $v_{i,t}$ represents each agent's anticipated carbon dioxide emissions at stage t, encapsulating their expected production level.

Production Income At each stage t, each agent will perform power generation activities based on its expected carbon emissions and sell the generated electricity to the government or other consumers. The calculation formula for production income is as follows:

$$Income_{i,t} = v_{i,t}/f_{type} \cdot P_{i,ele}. \tag{2}$$

In the equation, $P_{i,ele}$ represents the electricity sales price of agent i in euros/MWh. Before the simulation, we take the 2021 European electricity prices as a normal distribution of the mean (excluding the extreme 5% values on both sides) and generate a random price for each agent [9]. This price will not change over time.

Penalties As for the penalties, if the actual carbon emissions produced during the agent production process exceed the total carbon volume allowed by auction $v'_{i,t}$, i.e. if $v_{i,t} > v'_{i,t}$, agents should be punished for such illegal carbon emissions:

$$Penalty_{i,t} = \begin{cases} 0, & v_{i,t} - v'_{i,t} \leq 0 \\ \xi \cdot (v_{i,t} - v'_{i,t}), & v_{i,t} - v'_{i,t} > 0 \end{cases}$$

The parameter ξ is the penalty per unit. According to EU ETS, the EU will impose a fine of a fixed amount per ton of carbon dioxide that exceeds the allowed emissions.

Payoff Finally, at each stage t, each agent i receives its current payoff, $u_{i,t}$, for auction participation and power generation:

$$u_{i,t} = Income_{i,t} - Ecost_{i,t} - Ccost_{i,t} - Penalty_{i,t}.$$

Based on the current payoff, each agent will adjust the parameters in the next auction and production activities stage.

3.4 Strategies Decision Process

In the current model, we define three different strategies regarding the firm's preference for uncertainty and volatility associated with market prices according to the following principles inspired by [10, 5, 24]:

Risk-seeking Strategy: $bid_{i,t+1} = pc_t + \frac{3}{4}(pv_{i,t} - pc_t)$,
Risk-neutral Strategy: $bid_{i,t+1} = pc_t + \frac{1}{2}(pv_{i,t} - pc_t)$,
Risk-averse Strategy: $bid_{i,t+1} = pc_t + \frac{1}{4}(pv_{i,t} - pc_t)$.

The different strategies explore agents' attitudes to market uncertainty, with risk seekers adjusting bids for better payoffs and risk-averse firms opting for safer bids near the clearing price. These strategic distinctions influence not only individual bidding behaviors but also impact overall market dynamics. Firms can navigate market fluctuations by aligning bidding strategies with their risk tolerance.

According to the firm's strategy, each agent will regenerate the auction bid, $bid_{i,t+1}$, for the next stage. Each agent does not change the strategy it uses during the simulation. Also, the new personal value, $pv_{i,t+1}$, for each agent is derived using data from the previous stage, calculated as:

$$pv_{i,t+1} = (Income_{i,t} - Ecost_{i,t})/v_{i,t}.$$

Furthermore, new output volumes $v_{i,t+1}$ are determined based on the firm's payoff changes, adhering to the formula:

$$v_{i,t+1} = \begin{cases} v_{i,t} \cdot (1 + \Delta x_{t+1}), & \text{if } u_{i,t} - u_{i,t-1} > \delta \cdot u_{i,t} \\ v_{i,t}, & \text{if} |u_{i,t} - u_{i,t-1}| \leq \delta \cdot u_{i,t} \\ v_{i,t}/(1 + \Delta x_{t+1}), & \text{if } u_{i,t} - u_{i,t-1} < -\delta \cdot u_{i,t} \end{cases}$$

where $u_{i,t}$ and $v_{i,t+1}$ represent the payoff and output volume of enterprise i at time t, and Δx_t and δ represent two parameters for the output adjustment and a price increment, which are set to 50% and 20% respectively.

Additionally, considering that the total carbon emissions in each stage decrease at a decay rate of λ, the new total supply of carbon emissions for auction is calculated as follows:

$$E_{t+1} = (1 - \lambda) \cdot E_t.$$

After updating the data required for the next stage of the auction, the simulation of the model was repeated until the end of the fourth stage of the EU ETS, going through a total of 10 steps in 10 years. Ultimately, each agent i will get its final payoff, represented by $u_{i,t10}$.

Our goal is to study how the carbon auction platform can achieve steady reductions in carbon emissions by setting different auction methods and carbon emissions policies. According to the EU ETS report [13, 12], the emission cap in the fourth stage continues to decrease yearly, with an annual linear decay rate of 2.2%. At the same time, the EU ETS imposes a fine of 100 euros per ton for carbon emissions that exceed the allowed amount.

In our research, we set various simulation variables to assess the impact of the different parameters. Initially, we examine two auction models: uniform price and discriminative price. We also investigate the effects of two emission cap decay rates, specifically at 2.2% and 3.2%, along with three penalty factors set at 100, 200, and 500, as illustrated in Table 1. We utilise an agent-based model to compute the payoff matrix and apply the α-rank method to solve the game. This method calculates the set of stable strategies within the system, which enables us to determine the system's equilibrium solutions under various scenarios and evaluate the allocation efficiency of the auction model, amongst other things.

Table 1: ETS policy scenarios and auction designs considered.

Auction Format	**Decay Rate (%)**	**Penalty (€)**
Uniform, Discriminative	2.2, 3.2	100, 200, 500

4 Equilibrium

4.1 Background on α-Rank

We consider N agents game, each of whom can play $\mathcal{S}_i$ strategies and receives a payoff $M_i : \prod_{i=1}^{N} S_i \to \mathbb{R}$. Let $\mathcal{S}_{Joint} = \prod_{i=1}^{N} S_i$ be the space of the joint strategy profile. The α-rank of the game is calculated by defining an irreducible Markov chain whose nodes are pure strategy profiles in $\mathcal{S}_{Joint}$. We define the Markov probability transition matrix as C. Consider any two joint strategy profiles $a, b \in \mathcal{S}_{Joint}$, that differ in only one individual strategy for the i^{th} agent, the probability of transitioning from a to b which varies only in player i 's strategy is:

$$C_{a,b} = \begin{cases} \eta \frac{1-\exp(-\alpha(M_i(a)-M_i(b))}{1-\exp(-\alpha m(M_i(a)-M_i(b)))} & \text{if } a \neq b \\ \frac{\eta}{m} & \text{otherwise,} \end{cases}$$

where $\eta = \frac{1}{\sum_{i=1}^{N}(S_i-1)}$, $\alpha \geq 0$, $m \in \mathbb{N}$ are hyperparameters to be chosen. Also, $C_{a,a} = 1 - \sum_{b \neq a} C_{a,b}$ ensures that transition probabilities are valid, and $C_{a,v} = 0$ for all v that differ from a in more than a single player's strategy. We denote π as the unique stationary distribution of the chain C as $\alpha \to \infty$ [21]. The score of the strategies profile can be calculated via the stationary distribution π of this Markov chain. The higher the score, the better the stability of the strategies profile.

4.2 Strategy Configuration

Using the α-rank algorithm in multi-agent games requires substantial computational power and time due to its NP-hardness [28]. As agent numbers increase, the payoff matrix grows exponentially, significantly increasing computational complexity. Therefore, we simplify the model's strategy space with two methods: ***partial equilibrium analysis*** and ***role-symmetric game analysis.***

– ***Partial Equilibrium Analysis***
  In the partial equilibrium analysis method (PEA), we categorize the 18 agents into two distinct groups. The first group consists of 9 agents, each employing a strategy in the auction, evenly distributed across three energy types: oil, gas, and coal, with each energy type represented by three companies. The second group, also comprising nine agents, is designated as the "environmental" group. These agents maintain consistent bidding behaviour throughout the simulation, adhering to a risk-neutral strategy without altering their bids [14]. To minimize the influence of the environmental grouping on the game dynamics, we simulate this group one million times, using the average results as environmental data to ensure a stable market analysis background.
  This approach simplifies the complex game into an asymmetric game involving nine experimental agents. The model's strategy configuration space, S_p, is a 3^9 strategy combination profile, meaning each of the nine agents has three strategies from the strategy set $s = [\text{Risk-seeking}, \text{Risk-neutral}, \text{Risk-averse}]$.

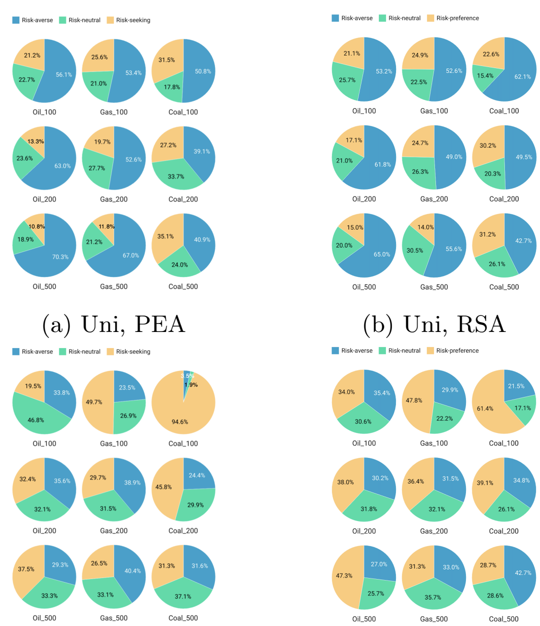
– ***Role-symmetric Game Analysis***
  In the role-symmetric game analysis method (RSA), we partially replace the original complex asymmetric game with a part-symmetric game to reduce the strategy space of the original game [27]. Specifically, in each ETS simulation, each agent's initial unit expected price pv_{i,t_0} comes from the same prior distribution, namely, the normal distribution related to the real market transaction price. We believe that simplifying the strategy configuration space in this case can ensure that the interests of agents are as unaffected as possible by the arrangements of other agents.
  In the ETS model, the agents in our study scenario are divided into three roles according to their power generation mode. Each type of firm has six agents, and all agents in each role are considered homogeneous. The strategy combination number of strategy configuration space, S_r, is greatly simplified compared to the original strategy space.

After simplifying the strategy space, we use Empirical Game Theory Analysis (EGTA) to compute the equilibrium strategy for the ETS model [26]. This process involves initializing the market, defining strategies, simulating combinations, and recording payoffs. The results form a payoff matrix, from which the equilibrium strategy is calculated using the α-Rank algorithm. Agents then retrade based on this profile, and market performance and payoff changes are recorded.

Specifically, we simulated the ETS model for each strategy combination in the configuration spaces S_p and S_r, calculating each agent's payoff at each step under different strategy combinations. After the simulation, we applied a 1% discount rate to the payoffs at each step, discounting them according to the time elapsed before the end of the simulation. We then summed the discounted payoffs for each agent and created the payoff matrix. To fully capture all possible scenarios, we used random data in each simulation, covering all scenarios and simulating each one 100 times.

4.3 Equilibrium Strategies

After simplifying the game strategy space with the two methods described above, we employed the α-Rank algorithm to estimate the approximate equilibrium of the model. Specifically, we calculated the weight of each strategy in various scenarios, with each simulation scenario containing a different set of strategy weights. When multiple profiles had the same highest score, the average weight of each feature across all stable strategies was provided. Following a large number of simulations, we collected the average results in the equilibrium state and defined the weight in the strategy combination as the probability of different types of agents adopting each strategy.



(a) Uni, PEA

(b) Uni, RSA

(c) Dis, PEA

(d) Dis, RSA

Fig. 1: Equilibrium distribution under different auction methods and analysis approaches

Figures 1a and 1b show the strategy probability distributions obtained by the two simplified methods when the uniform auction model is used, and the carbon supply decay rate is 2.2%. From the figure, we can see that when the penalty is 100, oil and gas agents significantly tend to adopt risk-averse strategies, and more than half of the agents prefer this conservative bidding method. As the penalty coefficient increases, the strategies of these two types of entities tend to be more conservative, especially those of oil agents. This pattern implies that oil industry agents may opt to curtail production by lowering auction bids, thereby minimising the potential for more losses due to the higher penalties. On the contrary, the coal agents' actions show a more balanced distribution across risk strategies. Although they tend to use risk-seeking strategies, they do not increase significantly with the increase in the penalty.

Additionally, as the penalty coefficient increases, the strategy probability distribution under the RSA method follows a similar trend to that of the PEA method, but its changes are less drastic. This may be because the PEA method does not have to follow the homogeneity assumption, leading to more sensitive and diverse responses from each agent. In contrast, the RSA method, constrained by the homogeneity assumption, restricts agents of the same type from adjusting within a smaller strategy space, resulting in less overall change in the strategy probability distribution.

Shifting the focus to the discriminative auctions, the data presents a contrasting scenario. As shown in Figures 1c and 1d, the risk-seeking strategies are more pronounced, particularly in the coal sector at the 100 penalty level with the PEA method, where a striking majority of 95% prefer risk-taking. This trend suggests that higher potential bid rewards in a discriminative auction entice agents to adopt more aggressive strategies despite penalty risks.

When comparing the two auction types, we find that the clearing price mechanism of uniform auctions leads to a greater preference for risk-averse strategies. By contrast, discriminatory auctions encourage riskier practices, especially in the coal industry, because winning bids determine the price. Agents appear willing to accept the possibility of higher penalties to ensure greater personal rewards.

5 Effects of ETS Framework

To gain insights into the ETS model's impact, our experiment concentrated on several key aspects. Firstly, we analysed the payoffs achieved by various types of agents. Secondly, we examined the bid prices within the auction market. Lastly, we focused on assessing the carbon emission levels of the participating agents. During the experiments, agents were required to utilise the equilibrium strategy during the ETS simulation.

5.1 Agents' Payoff in the ETS System

Within our ETS model, each agent category has three distinct firms. To facilitate a more effective comparison across different types of agents, we employ the average payoff of the three agents within each field for our analyses. Initially, we delve into the impact of variations in the carbon emission supply decay rate on different types of energy companies.

In our analysis, it was intriguing to observe that the agent's payoff volatility is markedly higher in discriminative auctions compared to uniform auctions. This pronounced fluctuation manifests in the final payoff and throughout the simulation process. To quantify this, we calculated the average R^2 values of the agents' payoff curves over time for both auction formats.

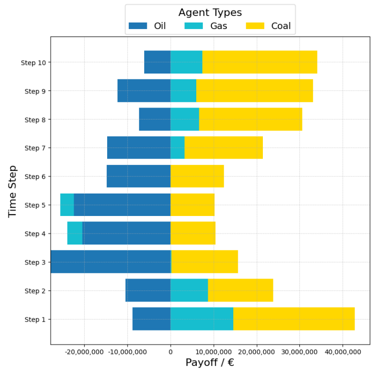
Table 2: Performance of R^2 under different scenarios

Simplification Method	**Auction Format**	$\mathbf{R^2}$
PEA	Uniform	0.91
PEA	Discriminative	0.79
RSA	Uniform	0.95
RSA	Discriminative	0.83

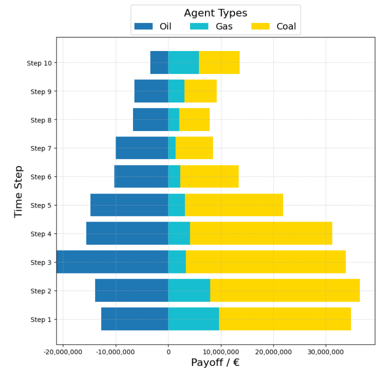
Table 2 shows the changing trend of the model yield curve under different scenarios as the emission decay rate increases. Under the PEA method, the $R^2_{uni,partial}$ is greater than $R^2_{dis,partial}$, signifying a more stable payoff trend. This suggests that changes in the decay rate in uniform auctions primarily lead to an overall shift in the payoff curve without significantly altering its trajectory. However, in discriminative auctions, not only does the payoff curve shift, but its trend also varies, resulting in more pronounced changes and fluctuations during the auction process. Furthermore, the R^2 of different auction formats in the RSA scenario is greater than that in the PEA method. This also shows that the RSA method is more stable with parameter emission decay rate changes.

We averaged the payoff data across various decay rates to further examine the influence of different auction formats on agent payoff. Figure 2 illustrates the agent payoff trend changes as the simulation progresses under each auction format and game simplification method. Firstly, due to constraints in raw material prices, the payoffs for oil agents are consistently negative. In contrast, coal agents achieve substantial profits owing to their lower material costs. Secondly, in the uniform auction, we observe that all agent types' payoffs initially decrease and then rise, indicating that this method improved production efficiency and reduced carbon emissions. Conversely, in the discriminative auction, agents' payoffs approach zero by the end, which shows effective resource allocation and reduced profit margins due to market competition.

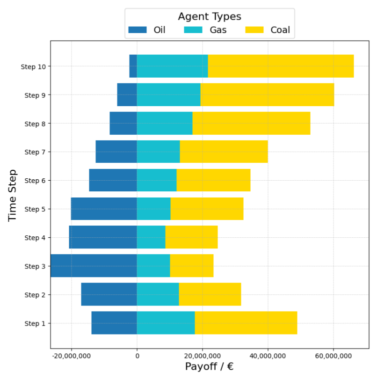
After further comparing the payoff data under PEA and RSA, we found that the overall payoff of RSA is higher than that of PEA. This may be because PEA involves solving complex games with multiple interacting parties, which increases computational complexity and may lead to lower payoffs due to approximate solutions or local optimal solutions. RSA simplifies the internal structure of each agent type, reducing computational complexity, making it easier to achieve the global optimal solution, and consequently obtaining higher overall payoffs.



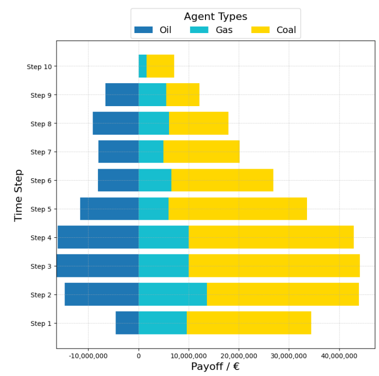
(a) Uni, PEA



(b) Dis, PEA



(c) Uni, RSA



(d) Dis, RSA

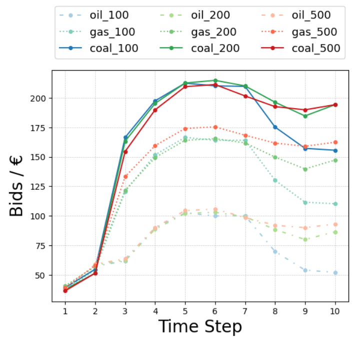
Fig. 2: Payoff over time for each type of agent under different auction methods and analysis approaches

5.2 Bids and Clearing Price in Auction Market

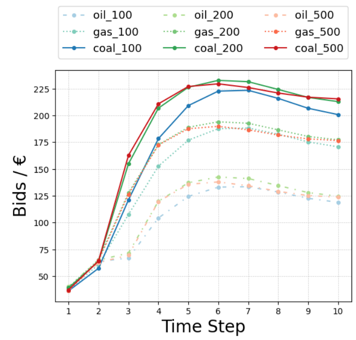
In the ETS model, analysing the changes in bids of different types of agents in the carbon emission auction market under market equilibrium can help us better study the interactions in the model. We present the data graphically to visualise this correlation, plotting the agent's bidding curve under different scenarios.

Figure 3 reveals notable differences in the bidding behaviour of various agents throughout the auction process. We observe that coal agents consistently place the highest bids in both auction formats, whereas bids from oil agents are comparatively lower. However, in the early stages of the simulation, the differences between the bids of different types of agents are not very significant.

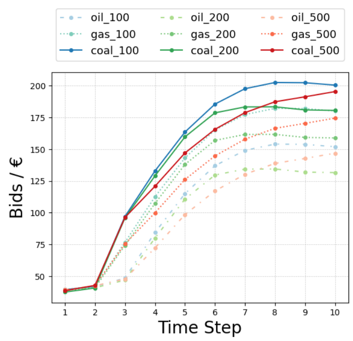
Additionally, under PEA, agents' bidding behavior shows distinct patterns in the two auction formats. In the uniform auction, bidding curves of the same agent type under different penalties are similar initially but diverge over time.



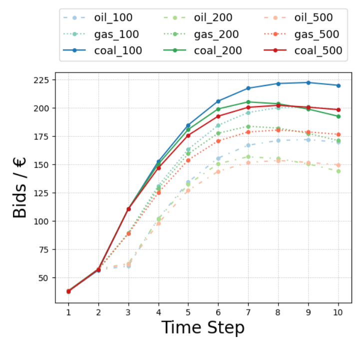
(a) Uni, PEA



(b) Dis, PEA



(c) Uni, RSA



(d) Dis, RSA

Fig. 3: Bids evolution over time for each agent under different auction methods and analysis approaches

Conversely, in the discriminative auction, bidding curves show marked variation initially but gradually converge. These observations corroborate previous findings: bidding curves under various penalties converge and stabilize in the discriminative auction, indicating a late-phase equilibrium. In contrast, in the uniform auction, bidding curves diverge in the later stages, suggesting agents adapt their bids to optimize payoffs based on generation efficiency. On the other hand, the figures of RSA bid curves in different auction modes are more similar, and the differences between the bids of different types of agents are smaller than those in PEA.

5.3 Carbon Emissions Analysis

One of the most essential tasks of the ETS model is effectively reducing carbon emissions in the system. To study it more deeply, we calculated the proportion of emission reductions for different types of agents in the simulated final stage compared with the starting stage under the two auction formats, as shown in Table 3.

Table 3: Proportion of agent emission changes under different auction forms

	PEA		RSA	
	Uniform	**Discriminative**	**Uniform**	**Discriminative**
Total	-17.48%	-8.20%	-11.45%	-6.29%
Oil	-51.28%	-25.03%	-34.48%	-17.63%
Gas	-24.77%	30.83%	-22.86%	25.25%
Coal	24.09%	-30.61%	25.07%	-23.74%

We can find that both auction methods can effectively reduce the agent's overall carbon emissions in the model. Oil power generation companies have the best emission reduction effects, while natural gas and coal power generation companies have very different emission reduction effects under two auction models. On the other hand, we found that the overall emission reduction of the ETS model in the PEA method is more significant than that of the RSA method. This difference in emission reduction leads to a lower income for the agent in the PEA method, which leads to a smaller payoff result for the ETS model.

However, from (1) and (2), the agent's power generation strongly correlates with its emissions, so reducing carbon emissions will also reduce fossil fuel power generation. While the government can stabilise the power market's supply using various auction formats and penalty values, only by advancing clean energy technologies to supplant conventional energy sources can achieve both carbon emission reduction and power price stability.

6 Conclusions

This paper presents an agent-based model to study the dynamics of the EU ETS primary carbon emissions auction market, using data from various energy sector industries. The model employs two strategy space simplification methods and the α-rank algorithm to calculate the equilibrium strategy. It examines the impact of these simplification methods and auction formats on agents' strategic decisions and bidding patterns. Additionally, the ETS model integrates auction and production processes, comparing scenarios with different penalty coefficients and supply decay rates.

Our findings indicate that agents prefer risk-averse strategies in uniform auctions and risk-preferring strategies in discriminative auctions. These findings are valid under both PEA and RSA methods, indicating their robustness.

We also discover that uniform auctions increase production efficiency and reduce carbon emissions, while discriminative auctions effectively allocate resources and foster balanced market competition. The bidding result shows that uniform auction bidding curves diverge over time, and discriminative auction curves converge. These insights can help regulators select auction formats to enhance ETS market efficiency.

Finally, our experiments show variations in carbon emissions across different model scenarios. Most agents successfully reduced emissions, indicating the ETS framework can lower carbon emissions while maintaining stable payoffs. However, reducing production emissions may negatively impact electricity markets, necessitating government efforts to transition fossil fuel power generation to clean energy.

Overall, our results provide valuable insights for policymakers, especially the European Commission, in shaping future EU ETS policies. Future work could extend the model to include a secondary auction market to assess allocation efficiency and explore interactions among agents from different industrial sectors.

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