OPTIMAL FISCAL POLICY WITH ENDOGENOUS TIME PREFERENCE

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Abstract

This paper studies the role of Ramsey taxation under the assumption that the individual rate of time preference is determined by the publicly provided social level of education. We show how intertemporal complementarities of aggregate human capital can generate multiple equilibria and we examine the role of endogenous fiscal policies in equilibrium selection. Our analysis implies a lower optimal government size due to the effect of human capital on time preference.

1. Introduction

In his seminal work, Ramsey (1927) took into account agents' equilibrium reactions in forming optimal fiscal policy. This second-best approach has

been extensively revitalized in capital accumulation models of unique equilibrium and exogenous time preference (Lucas and Stokey 1983, Judd 1985, Chamley 1986, Lucas 1990, Jones, Manuelli, and Rossi 1993). The present paper introduces the role of Ramsey taxation in selecting a second-best allocation under the presence of multiple competitive equilibria generated by intertemporal complementarities of human capital in the formation of time preference. Our theoretical framework provides a new role for the conduct of optimal fiscal policy under indeterminacies and poverty traps (Ben-Gad 2003, Park and Philippopoulos 2004, Park 2009, Agénor 2010). We also analyze some interesting policy implications as the standard productive effects of optimal taxation are altered (Barro 1990, Futagami, Morita, and Shibata 1993, Glomm and Ravikumar 1997, Turnovsky 2000).

The starting point of our analysis is an endogenous growth model in which the rate of time preference depends positively on the economy-wide consumption level and negatively on the publicly provided aggregate human capital stock, which are exogenous to the agents' decisions, and we introduce in this setup optimal fiscal policy in the form of Ramsey taxation.[1] We first examine the properties of the intertemporal competitive equilibrium and we show that there can be one or two balanced growth paths (BGPs). The central mechanism that drives these results arises from two counterbalancing channels. First, a rise in human capital financed by an increase in the tax rate lowers the rate of time preference, causing savings to increase and as a result the economy can attain higher growth. This, in turn, increases the tax base, raises public expenditures on education, and hence fuels further growth. On the other hand, the rise in taxation decreases private savings, which increases the rate of time preference in the economy due to the rise in aggregate consumption and hence lowers growth. A lower growth rate, in turn, lowers the tax base that finances aggregate human capital formation leading to even higher time discounting. We show that these externalities are crucial for the steady state and the dynamics of the economy, and we establish necessary and sufficient conditions for the existence of a unique or multiple (two) BGPs.

The selection of a second-best allocation is then addressed by endogenizing fiscal policy in the context of Ramsey taxation. We demonstrate, first, how the government's objective can determine the available set of policy instruments (Atkinson and Stiglitz 1980) and, second, its importance in the implementation of additional restrictions on private decisions that can lead the decentralized economy to a unique BGP. Typically, global and local indeterminacy at the competitive equilibrium implies that the rational expectations equilibria involve random variables, which are unrelated to the economy's fundamentals and are driven by individual beliefs. However, in a second-best (Stackelberg) environment, the government can

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[1] See Section 2 for a detailed review on these assumptions.

obtain information through the agents' reaction function and consequently impose additional restrictions on the tax rate and the endowment allocation through commitment, in order to drive the economy to a unique second-best allocation. This policy is feasible under a state-dependent taxation rule that is linked to aggregate endowments and internalizes intertemporal complementarities of human capital, which fuel multiplicity of intertemporal competitive equilibria.

Some fiscal aspects of our work should be stressed in comparison with the existing literature. First, a connection can be established to earlier studies that have investigated, using models in which externalities generate multiple growth paths, the role of public policy in eliminating the poverty trap and selecting the desired competitive equilibrium (Matsuyama 1991, Boldrin 1992, Rodrik 1996).[2] However, these studies have examined how government intervention operates under exogenous taxation, without explicitly specifying the government's objective. Endogenizing government policy becomes an interesting task because the tax rate depends on the actions by private agents, which can create rather than eliminate coordination failures and strategic uncertainty, thus triggering the existence of multiple equilibria (Cooper 1999). In particular, in Park and Philippopoulos (2004) and Park (2009), multiple competitive equilibria with productive public services are the outcome of endogenous policy indeterminacy in the form of multiple tax rates. In an overlapping generations (OLG) endogenous growth model, Glomm and Ravikumar (1995) show that there may be multiple equilibrium paths when public policy, in the form of public education, is endogenous. Furthermore, Ben-Gad (2003) has shown that a sufficient degree of capital taxation in a Lucas–Uzawa endogenous growth model or a combination of factor taxation and external effects can trigger indeterminacy in the form of many (more than two) BGPs. In comparison to these studies, multiplicity can arise here for any feasible range of exogenously set tax rate, whereas endogenous Ramsey taxation is not only determinate (unique tax rate) but can also lead the market economy to the desired growth regime under a state-depended taxation rule. Regarding public policy and endogenous time preference, in related work (Dioikitopoulos and Kalyvitis 2010), we have examined the role

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[2] Although indeterminacies have widely been studied in the literature, far less is known on mechanisms in directing the economy toward a desired equilibrium. In models with a continuum of equilibria arising from the presence of animal spirits, learning can act as a selection device for choosing the rational expectations equilibrium that we can expect to observe in practice (Evans, Honkapohja, and Romer 1998, Evans and Honkapohja 1999). Ennis and Keister (2005) have presented a framework in which search frictions create a coordination problem that generates multiple Pareto-ranked equilibria and show how the desired equilibrium can be chosen using a selection mechanism based on risk dominance. Antinolfi, Azariadis, and Bullard (2007) analyze multiplicity in a model with heterogeneous agents and intertemporal complementarities between dated debt limits, which exhibits two Pareto-ranked equilibria, and show how active monetary policy can force the economy onto the optimal path.

of public capital and taxation through the endogeneity of time preference under a (static) growth-maximizing objective within a unique competitive equilibrium. By contrast, our generalized framework generates here multiple competitive equilibria and we are able to analyze the selection mechanism of the second-best allocation in the (dynamic) optimal Ramsey taxation setup.

Second, other studies have also examined the possibility of equilibrium indeterminacy with endogenous time preference. Drugeon (1996) has studied the possibility of multiple steady states when impatience depends on individual consumption and allows for increasing returns when both individual and aggregate capital stock enter the production function. In our model, multiplicity stems from the externalities generated by the effects of aggregate consumption and aggregate human capital on time preference, whereas we allow for constant returns in production through the productive role of human capital. Chen (2007) assumes that time preference depends on individual past consumption through habit formation, which forms an internal, rather than external, intertemporal complementarity resulting in multiple equilibria that arise from the interactions of consumption levels at different time periods. Recently, Agénor (2010) explores the network effects of an exogenous rise in public infrastructure, which can facilitate, through the rise in health services and patience, the shift from a low-savings poverty trap to a steady state characterized by high growth. In the present paper, we point out instead the endogenous fiscal policy impacts on the optimal dynamic individual choice, which now depends on current and lagged human capital formation decisions that can generate multiple BGPs and propagate growth effects over time.

The rest of the paper is structured as follows. Section 2 reviews related literature and Section 2 sets up and solves the optimization problem of households and firms, and studies the steady state and the dynamic properties of the decentralized economy. Sections 3 and 4 analyze the role of Ramsey taxation and growth-maximizing taxation in selecting a second-best allocation. Finally, Section 5 concludes the paper.

2. Related Literature

Over the last decades, a number of papers have relaxed the assumption of exogenous time preference. In particular, starting from Uzawa (1968), several studies have investigated the effects of individual consumption on the time preference rate; see Obstfeld (1981, 1990), Mendoza (1991), Shin and Epstein (1993), Palivos, Wang, and Zhang (1997), Drugeon (1996, 1998, 2000), Uribe (1997), Schmitt-Grohé (1998), Stern (2006), Sarkar (2007), Chen, Hsu, and Lu (2008), and Chakrabarty (2012). In turn, Epstein and Hynes (1983), Schmitt-Grohé and Uribe (2003), and Choi, Mark, and Sul (2008), among others, have endogenized the rate of time preference to aggregate consumption in variants of general equilibrium models. These models have highlighted the importance of endogenous time preference for

the dynamic patterns of consumption. Yet, very little is known about the policy aspects of endogenous time preference to fiscal policy, with the exception of Agénor (2010) who assumes that a rise in public health services lowers impatience.

The main underlying idea in the present paper is that agents are less impatient in a more educated surrounding environment. This point goes back to Strotz (1956), who had noticed that discount functions are formed by teaching and social environments, and was reraised by Becker and Mulligan (1997), who argued that schooling and other social activities in "future-oriented capital" focus agents' attention to the future. Doepke and Zilibotti (2008) explored the role of parental time invested in patience to develop a theory of preference formation and explain the historical reversals in economic fortunes. Perhaps the most prominent illustration of the fiscal policy aspects associated with the effects of education on patience concerns the causes of the high savings rate observed in postwar Japan. Horioka (1990) and Sheldon (1997, 1998) have attributed this behavior, among other factors, to an array of public policies implemented through educational programs that promoted the virtues of patience and thrift.

Existing empirical evidence suggests that education strongly affects patience by rendering agents less impulsive to choices that tend to overweight rewards in close temporal proximity. Fuchs (1982) was the first study that attempted to investigate empirically the association between time preference and education, and showed that there is a positive link between patience and years of schooling. Lawrance (1991) has found that nonwhite families without a college education have time preference rates that are about seven percentage points higher than those of white. Similarly, Harrison, Lau, and Williams (2002) have shown on a sample of Danish households that highly educated adults have subjective discount rates that are roughly two-thirds compared to those who are less educated. Khwaja, Silverman, and Sloan (2007) have found that the years of education affect negatively the degree of impulsivity defined as the measure of an individual's ability to set goals and to exercise self-control. Recently, Meier and Sprenger (2010) and Perez-Arce (2011) report that college education is significantly associated with time preference and Bauer and Chytilova (2009, 2010) estimate that an additional year of schooling in Ugandan villages lowered significantly the discount rate.

We close this short review on the endogeneity of time preference by noting that an indirect channel capturing the impact of human capital on impatience may stem from income and wealth, which also guarantees a stationary rate of time preference; see Schumacher (2009), Agénor (2010), and Strulik (2012) for recent theoretical contributions that have adopted this assumption. In empirical studies, Hausman (1979) and Samwick (1998) have found that discount rates are inversely related to income level, and Horowitz (1991) and Pender (1996) have reported that discount rates decline with wealth.

3. The Competitive Decentralized Equilibrium

3.1. The Basic Model

Consider an economy with a constant number of infinitely lived agents that consume a single good. We assume that the rate of time preference, ρ, is not a positive constant, as in standard growth theory, but is endogenously determined by aggregate consumption, C, and aggregate human capital, H. Each household seeks to maximize intertemporal discounted utility given by

$$\int_0^\infty u(c) \exp\left[-\int_0^t \rho(C_s, H_s)\, ds \right] dt, \tag{1}$$

with instantaneous utility function of the form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where $0 < \sigma \leq 1$, subject to the initial asset endowment $A(0) > 0$ and the income resource constraint

$$\dot{A} = rA + w - c, \tag{2}$$

where A denotes per capita financial assets, c denotes per capita consumption, and r and w denote the market interest rate and the wage rate, respectively.[3]

The time preference function has the following properties:

ASSUMPTION 1: $\rho(C, H) \geq \check{\rho} > 0.$

ASSUMPTION 2: $\rho(C, H) = \rho(\frac{C}{H})$ $with$ $\rho'(\cdot) \geq 0.$

Assumption 1 shows that the rate of time preference is positive, implying that there exists a lower bound denoted by $\check{\rho}$. By Assumption 2, the rate of time preference depends positively on the ratio of aggregate consumption to aggregate human capital. This assumption follows a strand of the literature that has linked the rate of time preference to social factors taken as external by agents (Epstein and Hynes 1983, Schmitt-Grohé and Uribe 2003, Choi *et al.* 2008, Agénor 2010, Dioikitopoulos and Kalyvitis 2010). In particular, we assume that a higher level of the economy-wide average consumption raises individual impatience (Epstein and Hynes 1983, Schmitt-Grohé and Uribe 2003, Choi *et al.* 2008). Intuitively, as the economy gets richer and consumes more in the aggregate, each individual wanting to "keep up with the Joneses" becomes more impatient to consume. In addition, we assume that the higher the human capital stock in the economy, the more patient is

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[3] Throughout the paper, the time subscript t is omitted for simplicity of notation. Note that positive felicity is guaranteed only for $0 < \sigma < 1$. We also include here the logarithmic utility case ($\sigma = 1$) to allow for comparisons in our simulations with this extensively used specification.

the agent and willing to forego current consumption (Becker and Mulligan 1997). Assumption 2 implies homogeneity of the rate of time preference to the ratio of consumption to human capital, which is required for the rate of time preference to be bounded at the steady state (Palivos *et al.* 1997) and for the utility function to be consistent with balanced growth (Boyd 1990, Dolmas 1996).[4]

In the supply side of the economy, there exists a continuum of perfectly competitive homogenous firms, normalized to unity, that seek to maximize profits. Each firm i uses physical capital, K_i, and labor, L_i, under the following production technology:

$$Y_i = K_i^a (h L_i)^{1-a}, \tag{3}$$

where $0 < a < 1$ denotes the share of physical capital in the production function, Y_i denotes individual output, and h denotes labor productivity. The latter depends linearly on the average human capital stock and is given by

$$h = \frac{H}{L}, \tag{4}$$

where L denotes the aggregate labor force. Equation (4) is in the spirit of Arrow (1962) and Romer (1986), and captures the idea that knowledge is a public good that all firms can access at zero cost.[5]

Following among others Glomm and Ravikumar (1997) and Blankenau, Simpson, and Tomljanovich (2007), we assume that human capital is provided by the public sector and serves as an input in the production function. The law of motion for the human capital stock is then given by

$$\dot{H} = v I_H - \delta_H H, \tag{5}$$

where I_H denotes public expenditures on education, δ_H denotes the human capital depreciation rate, and v is a scale parameter capturing the technology of education. Following Turnovsky (1996, 2000), we assume that the government sets its expenditures as a fixed fraction of output and imposes a flat tax rate on output, τ, to finance spending on human capital according to a balanced budget policy given by[6]

$$I_H = \tau Y. \tag{6}$$

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[4] Note that although Assumption 2 may imply a causal relation, the dynamics of our general equilibrium setup are able to capture the endogeneity between human capital and time preference. Interestingly, this dynamic feedback propagates a source of multiple growth equilibria discussed later on.

[5] Notice that a richer formulation of labor productivity could allow for an extra parameter capturing diminishing returns, e.g., in the form of congestion as in Eicher and Turnovsky (2000) and Dioikitopoulos and Kalyvitis (2008). This, however, would not affect the main results derived later on.

[6] We do not consider private funding of education as the focus of the paper is on the effects of fiscal policies on individual time preference through the education channel. It is also straightforward to show that we could obtain the same results if the flat tax

Finally, the standard law of motion for the physical capital stock is given by

$$\dot{K}_i = I_i - \delta_K K_i, \tag{7}$$

where I_i denotes investment in physical capital and δ_K denotes the physical capital depreciation rate.

3.2. The Reduced Model and Balanced Growth

We can now define the competitive decentralized equilibrium (CDE) of the economy in order to analyze its properties.

DEFINITION 1:   *The CDE of the economy is defined for the exogenous policy instruments τ, factor prices r, w, and aggregate allocations K, H, I_H, L, and C, such that*
    i) individuals solve their intertemporal utility maximization problem by choosing c and A, given τ and factor prices;
    ii) firms choose L_i and K_i in order to maximize their profits, given factor prices and aggregate allocations;
    iii) all markets clear and in the capital market $A = \frac{K}{L}$ (per capita assets held by agents equal capital stock per capita); and
    iv) the government budget constraint holds.
    The CDE is then defined by (i)–(iii) under the aggregation conditions $\int_0^1 K_i = K$, $\int_0^1 L_i = L$.

The per capita growth rate of consumption in the CDE is given by

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}[r - \rho(\cdot)]. \tag{8}$$

The first-order conditions of the firms' profit maximization problem are given by $r = (1 - \tau)a(\frac{K_i}{hL_i})^{a-1} - \delta_k$ and $w = (1 - \tau)(1 - a)(\frac{K_i}{L_i})^a h^{1-a}$, and state that the marginal productivity of capital and labor have to equal respective factor prices. Using the conditions for homogenous and symmetric firms $L_i = L$ and $K_i = K$, and assuming for the rest of the paper without loss of generality that $\delta_K = \delta_H = \delta$, the growth rates of aggregate consumption, aggregate physical and human capital stocks are given by the following equations:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left[a(1 - \tau)\left(\frac{K}{H}\right)^{a-1} - \rho(\cdot) - \delta\right], \tag{9}$$

$$\frac{\dot{K}}{K} = (1 - \tau)\left(\frac{K}{H}\right)^{a-1} - \frac{C}{K} - \delta, \tag{10}$$

rate was imposed on labor and capital income because of the Cobb–Douglas production technology.

$$\frac{\dot{H}}{H} = v\tau \left(\frac{K}{H}\right)^{a} - \delta. \tag{11}$$

The transversality condition for this problem is given by

$$\lim_{t \to \infty} \frac{K(t)}{C(t)^{\sigma}} \exp\left\{ -\int_{0}^{t} \rho\left(\frac{C(s)}{H(s)}\right) ds \right\} = 0. \tag{12}$$

Equations (9)–(11) summarize the dynamics of our economy. At the BGP consumption, physical and human capital grow at the same rate, $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = g^{CDE}$. The BGP of the economy can be derived by defining the auxiliary stationary variables, $\omega \equiv \frac{C}{K}$ and $z \equiv \frac{K}{H}$.[7] It is straightforward to show that the dynamics of (9)–(11) are equivalent to the dynamics of the following system of equations:

$$\frac{\dot{\omega}}{\omega} = \left(\frac{a}{\sigma} - 1\right)(1-\tau)z^{a-1} + \omega - \frac{1}{\sigma}\rho(z\omega) - \left(\frac{1}{\sigma} - 1\right)\delta, \tag{13}$$

$$\frac{\dot{z}}{z} = (1-\tau)z^{a-1} - \omega - v\tau z^{a}. \tag{14}$$

The following proposition determines the properties (existence and uniqueness) of the BGP at which $\frac{\dot{\omega}}{\omega} = \frac{\dot{z}}{z} = 0$.

PROPOSITION 1 (Properties of BGP): *The growth rate of the economy at the BGP with endogenous time preference to the ratio of aggregate consumption to human capital, for given parameter values and tax rate, is given by*

$$g^{CDE} = v\tau\bar{z}^{a} - \delta,$$

provided that there exists $\bar{z} > 0$: $\Phi(\bar{z}) = (\frac{a}{\sigma})(1-\tau)\bar{z}^{a-1} - v\tau\bar{z}^{a} - \frac{1}{\sigma}\rho(\bar{z} \cdot \bar{\omega}(\bar{z})) - (\frac{1}{\sigma} - 1)\delta = 0$ and $\bar{\omega}(\bar{z}) = (1-\tau)\bar{z}^{a-1} - v\tau\bar{z}^{a} > 0$, where $\bar{\omega}$ and $\bar{z}$ are the steady-state values of ω and z, respectively. We distinguish the following cases:

Case 1: *A sufficient condition for the existence of a unique well-defined physical to human capital ratio, which corresponds to a positive growth rate, is $(\frac{a}{\sigma} - 1) (1-\tau)^{a}\tau^{1-a}v^{1-a} - (\frac{1}{\sigma} - 1)\delta < \frac{1}{\sigma}\check{\rho}$.*

Case 2: *A necessary condition for the existence of two well-defined physical to human capital ratios, which correspond to two positive growth rates, is $(\frac{a}{\sigma} - 1) (1-\tau)^{a}\tau^{1-a}v^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$. This condition is also sufficient if $\rho''(.) \leq 0$ and $-\frac{1}{\sigma}\rho'(0) > (\frac{a\tau v^{2}}{1-\tau})((\frac{a-1}{\sigma}) - 1)$.*

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[7] Note that relative to standard endogenous growth models, the time preference function is an implicit function, which is time varying toward the BGP with a well-defined equilibrium if it exists; see, e.g., Palivos *et al.* (1997). For the verification of the transversality condition in infinite time horizon problems, see Michel (1982).

Proof: See Appendix A.   ■

COROLLARY 1 (Ranking of BGPs): *In the case of multiplicity with two growth rates ranked $g_1 > g_2$, it follows that $\rho_1 < \rho_2$, $\bar{\omega}_1 < \bar{\omega}_2$, $\bar{z}_1 > \bar{z}_2$.*

Proof: See Appendix A.   ■

Proposition 1 states that when the rate of time preference in the economy depends on the ratio of aggregate consumption to human capital, there can be a unique or multiple (two) BGPs. Hence, although the instantaneous utility and production technology functions satisfy the standard concavity assumptions, the existence of a unique positive steady-state growth rate is not guaranteed under the assumption that the aggregate human capital affects the impatience rate of agents. Corollary 1 shows that in the competitive equilibrium with high growth, g_1, the rate of time preference is lower, $\rho_1 < \rho_2$, consumption to physical capital ratio is lower, $\bar{\omega}_1 < \bar{\omega}_2$, and the physical to human capital ratio, $\bar{z}_1$, is higher, $\bar{z}_1 > \bar{z}_2$.

The central mechanism that drives multiplicity stems from the impacts of aggregate consumption and aggregate human capital on time preference, which fuel two counterbalancing channels. Consider the construction of a BGP through a rise in the tax rate in order to finance human capital accumulation. This decreases private savings and increases the rate of time preference in the economy due to the rise in aggregate consumption, thus lowering growth. A lower growth rate, in turn, lowers the tax base that finances public investment in human capital leading to even higher time discounting. On the other hand, by increasing the tax rate, the government increases the aggregate level of human capital expenditures in the economy. Thus, the rate of time preference falls, savings propensity increases and the economy can attain higher growth, which, in turn, increases the tax base and raises public expenditures on education and growth. In other words, apart from the standard relation between human capital and growth through the production function, there is an intertemporal complementarity between aggregate human capital, time preference, and growth through the Euler equation (8), setting off a virtuous growth cycle.[8]

The final outcome will depend on the structural parameters of the economy. Case 2 of Proposition 1 (multiplicity of BGPs) is more likely to arise when the inverse of the intertemporal elasticity of substitution, σ, is sufficiently low and the elasticity of human capital in the production function is sufficiently high. For instance, assuming a zero depreciation rate of human capital, it is straightforward to show that when $\sigma < a$, the second channel dominates for any tax rate. This happens because the standard

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[8] Drugeon (1998) analyzes extensively the theoretical implications of the effects of individual and aggregate consumption on time preference.

dynamic mechanism of intertemporal substitutability between the savings rate and the rate of return on capital that preserves a unique BGP fails, thus giving rise to indeterminacy.

3.3. Transitional Dynamics and Stability Analysis

In this subsection, we examine the relation between savings, the return on physical capital and growth, along with the complementarities of human capital on intertemporal utility. To this end, we analyze the transitional dynamics and local stability of the market economy, which are determined by the two-dimensional system of Equations (13) and (14). In matrix notation, we can write

$$
\begin{bmatrix} \dot{\omega} \\ \dot{z} \end{bmatrix} = \Xi \begin{bmatrix} \omega - \bar{\omega} \\ z - \bar{z} \end{bmatrix},
$$

where $\Xi \equiv [\begin{smallmatrix} (1 - \frac{\rho'(\cdot)\bar{z}}{\sigma})\bar{\omega} & [(\frac{a}{\sigma} - 1)(1-\tau)(a-1)\bar{z}^{a-2} - \frac{1}{\sigma}\rho'(\cdot)\bar{\omega}]\bar{\omega} \\ -\bar{z} & [(1-\tau)(a-1)\bar{z}^{a-1} - va\tau\bar{z}^a] \end{smallmatrix}]$. After some algebra, we obtain that the determinant, J, and the trace, Ω, of the above system are given by

$$
J = \bar{\omega}\bar{z}\left[\underbrace{-\frac{a}{\sigma}(1-\tau)(1-a)\bar{z}^{a-2} - va\tau\bar{z}^{a-1}}_{<0} - \frac{\rho'(\cdot)\bar{z}^{a-1}}{\sigma}[(1-\tau)a - v\tau(1+a)\bar{z}] \right],
$$

$$
\Omega = a(1-\tau)\bar{z}^{a-1} - v\tau(1+a)\bar{z}^a - \frac{\rho'(\cdot)\bar{\omega}\bar{z}}{\sigma}.
$$

The sign of J is ambiguous and depends on the parameters of the economy and the endogeneity of the rate of time preference. Under a constant rate of time preference, $\rho'(\cdot) = 0$, the standard result of a unique growth rate and a steady-state ratio of physical to human capital stock that is saddle-path stable is obtained. However, when the rate of time preference is endogenous, the local dynamics of the economy are nontrivial.

PROPOSITION 2 (Local Stability): *Under Assumptions 1 and 2, any competitive equilibrium, $\bar{z}(\cdot) \equiv \bar{z}(a, \tau, v, \sigma, \delta) > 0$, is locally stable for any parameter value and policy instrument in its assumed domain.*

Proof. See Appendix B.   ∎

COROLLARY 2 (Type of Stability): *Following Proposition 2, the local dynamics are described as follows:*

Case 1   (Saddle-path):   If $\rho'(\cdot) < \xi(\cdot)$,   where $\xi(\cdot) \equiv \xi(a, \tau, v, \sigma, \delta) \equiv \frac{\frac{a}{\sigma}(1-\tau)(1-a)\bar{z}(\cdot)^{a-2} - va\tau\bar{z}(\cdot)^{a-1}}{[(1-\tau)a - v\tau(1+a)\bar{z}(\cdot)]} \in R$ is an implicit function of parameters, then the equilibrium is saddle-path stable.

Case 2   *(Other types of stability): If* $\rho'(\cdot) > \xi(\cdot)$, *then the type of stability depends on* $\gamma(\cdot) \equiv \gamma(a, \tau, v, \sigma, \delta) \equiv \Omega(a, \tau, v, \sigma, \delta)^2 - 4J(a, \tau, v, \sigma, \delta)$, *where*

   (i)   *If* $\gamma(\cdot) > 0$, *the equilibrium is a stable node.*
  (ii)   *If* $\gamma(\cdot) < 0$, *the equilibrium is a stable focus.*
 (iii)   *If* $\gamma(\cdot) = 0$, *all trajectories are closed orbits (center).*

   In the case of multiple competitive equilibria, there exist set of parameter values such that case 1 and case 2 hold simultaneously.

Proof: See Appendix B.   ■

Proposition 2 shows that under the assumption of endogenous time preference of aggregate consumption to human capital ratio, any BGP as defined in Proposition 1 is locally stable. Corollary 2 to Proposition 2 shows that global indeterminacy can result to local indeterminacy in the sense that there can exist a continuum of ways toward a stable equilibrium, which is locally determinate (saddle-path) if the slope of the impatience function is sufficiently lower than a threshold level of parameters. Otherwise, the case of many paths in the neighborhood of a stable competitive equilibrium cannot be excluded.

The main message of the stability analysis of this section is that multiple competitive equilibria under endogenous time preference derived in Proposition 1 are stable, and hence both are meaningful. In our framework, time preference is endogenous to two arguments, aggregate human capital that provides a source of instability and aggregate consumption that stabilizes the economy. Intuitively, human capital positively affects the growth rate, through two complementing channels, namely, the increase in the productivity of the economy and the increase in patience and savings. Both sources move in the same direction and generate a dynamic complementarity that fuels growth. At the same time, consumption externalities affect growth negatively through the Euler equation. When the economy grows during the transition, the rise in aggregate consumption slows down growth and stabilizes the economy toward the steady-state ratio, $\bar{z}$, with a constant BGP.

As shown in the proof of Corollary 2, for the same parameter values, two BGPs can exist: a low one that is saddle-path stable and a high one that is locally indeterminate (stable node).[9] Notably, the type of stability also depends on the slope of the time preference function, as reflected in cases 1 and 2 of Corollary 2. Case 1 shows that when the slope of time preference is relatively low, the type of stability is saddle-path as the dynamics of our model follow those of the model with exogenous time preference. A sufficiently high slope of time preference activates the dynamic

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[9] Notice that our numerical results in subsequent tables rely on stable equilibria.

complementarities described above and the dynamics toward the steady state become nontrivial.

4. Time Preference and Ramsey Taxation

In this section, we endogenize fiscal policy and examine the second-best selection mechanism of the governments' objective in the context of Ramsey taxation. In the current setup, there exists a range of the initial endowments of the aggregate physical and human capital stocks in the CDE under which the economy will exhibit multiplicity for any tax rate. We examine here if, and how, the government's objective can impose restrictions and lead to an initial endowment allocation that solves the indeterminacy problem.

DEFINITION 2: *Ramsey taxation is given under Definition 1 when (i) the government chooses the tax rate and aggregate allocations in order to maximize the welfare of the economy by taking into account the aggregate optimality conditions of the CDE, and (ii) the government budget constraint and the feasibility and technological conditions are met.*

The government seeks to maximize welfare of the economy subject to the outcome of the decentralized equilibrium summarized by (9)–(11). The Hamiltonian of this problem is given by

$$\Lambda^R = u(C)e^{-\Delta} + \frac{1}{\sigma}\tilde{\mu}_C C\left[a(1-\tau)\left(\frac{K}{H}\right)^{a-1} - \rho(\cdot) - \delta\right]$$

$$+ \tilde{\mu}_K[(1-\tau)K^a(H)^{1-a} - C - \delta K] + \tilde{\mu}_H[\upsilon\tau K^a H^{1-a} - \delta H] + \tilde{\mu}_\rho[\rho(\cdot)],$$

where $\tilde{\mu}_C$, $\tilde{\mu}_K$, and $\tilde{\mu}_H$ are the dynamic multipliers associated with (9), (10), and (11), respectively, and $\Delta \equiv \int_0^t \rho(C_s, H_s)\,ds$.

The first-order conditions of the Ramsey problem include the constraints (9)–(11) and the optimality conditions with respect to C, H, K, and τ:

$$\dot{\tilde{\mu}}_C = -[C^{-\sigma}]e^{-\Delta} - \frac{\tilde{\mu}_C}{\sigma}\left[a(1-\tau)\left(\frac{K}{H}\right)^{a-1} - \rho(\cdot) - \delta\right]$$

$$+ \left[\frac{\tilde{\mu}_C C}{\sigma} - \tilde{\mu}_\rho\right]\rho'(\cdot)\frac{1}{H} + \tilde{\mu}_K, \tag{15}$$

$$\dot{\tilde{\mu}}_K = \frac{\tilde{\mu}_C C}{\sigma}\left[a(1-a)(1-\tau)\left(\frac{K}{H}\right)^{a-1}K^{-1}\right]$$

$$- \tilde{\mu}_K\left[a(1-\tau)\left(\frac{K}{H}\right)^{a-1} - \delta\right] - \tilde{\mu}_H\upsilon\tau a\left(\frac{K}{H}\right)^{a-1}, \tag{16}$$

$$\dot{\tilde{\mu}}_H = -\frac{\tilde{\mu}_C C}{\sigma}\left[a(1-a)(1-\tau)\left(\frac{K}{H}\right)^a K^{-1} + \rho'(\cdot)\frac{C}{H^2}\right],$$

$$-\tilde{\mu}_K(1-a)(1-\tau)\left(\frac{K}{H}\right)^a - \tilde{\mu}_H\left[v(1-a)\tau\left(\frac{K}{H}\right)^a - \delta\right] \quad (17)$$

$$+\tilde{\mu}_\rho \rho'(\cdot)\frac{C}{H^2}$$

$$\frac{\tilde{\mu}_C aC}{\sigma}\left(\frac{K}{H}\right)^{a-1} + (\tilde{\mu}_K - v\tilde{\mu}_H)\left(\frac{K}{H}\right)^a H = 0, \tag{18}$$

$$\dot{\tilde{\mu}}_\rho = \frac{C^{1-\sigma}}{1-\sigma}e^{-\Delta}, \tag{19}$$

$$C^{1-\sigma}e^{-\Delta} + \tilde{\mu}_C \dot{C} + \tilde{\mu}_K \dot{K} + \tilde{\mu}_H \dot{H} + \tilde{\mu}_\rho[\rho(\cdot)] = 0. \tag{20}$$

Equations (15)–(19), the optimality condition for the Hamiltonian $\lim_{t\to\infty} \Lambda^R = 0$ as given by (20), and Equations (9)–(11) characterize the solution of the Ramsey problem. As the system of equations is analytically intractable, we focus our analysis on the tax rate and growth rate at the steady state, as in Chamley (1986) and other related papers. Notice that, according to Definition 2, the Ramsey problem is a Stackelberg equilibrium in which the government announces the tax schedule through a commitment technology, and then the households reacts. Hence, the government chooses among the competitive equilibria to maximize welfare using distortionary taxation and, given Proposition 2, the competitive equilibrium chosen is locally stable for any tax schedule.

After some algebra, the long-run allocation of the Ramsey environment is characterized by the following system of equations:

$$\tilde{\tau} = (1-a) - \frac{\rho(\tilde{\omega}\tilde{z})\tilde{z}^{-a}}{v}, \tag{21}$$

$$\left(\frac{a}{\sigma} - 1\right)(1-\tilde{\tau})\tilde{z}^{a-1} + \tilde{\omega} - \frac{1}{\sigma}\rho(\tilde{\omega}\tilde{z}) - \left(\frac{1}{\sigma} - 1\right)\delta = 0, \tag{22}$$

$$(1-\tilde{\tau})\tilde{z}^{a-1} - \tilde{\omega} - v\tilde{\tau}\tilde{z}^a = 0. \tag{23}$$

The system of Equations (21)–(23) yields the Ramsey tax rate, $\tilde{\tau}$, and $\tilde{\omega}$ and $\tilde{z}$ as functions of the parameters, provided that the tax rate determined by (21) is feasible. The following numerical example illustrates the outcome of the economy under the Ramsey allocation.[10]

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[10] Example 1 uses a linear time preference function for computational tractability. A concave function would also satisfy our assumptions. In the online Companion Appendix to

EXAMPLE 1:   *Consider a linear time preference function, $\rho(\frac{C}{H}) = b * (\frac{C}{H}) + \check{\rho}$, that satisfies Assumptions 1 and 2 with parameter values $a = 0.35$, $\delta = 0.01$, $b = 0.5$, $\check{\rho} = 0.005$, $v = 0.1$, and $\sigma = 0.2$. Under the Ramsey allocation, we find a unique second-best allocation with the growth rate given by $g^R = 0.1$ corresponding to a tax rate given by $\tilde{\tau} = 0.597$.*

Example 1 shows that for parameter values under which the competitive equilibrium exhibits multiple BGPs, the government attains a unique BGP by implementing the appropriate allocation and restrictions in the Ramsey environment. Intuitively, this happens because the government uses the allocation of aggregate endowments and an associated tax rate to select a stable second-best regime and attain welfare maximization. Formally, this is accomplished through the state-dependent taxation rule, given by Equation (21).

Concerning standard literature, global and local indeterminacy at the CDE implies that the rational expectations equilibria involve random variables, which are unrelated to the economy's fundamentals and are driven by individual beliefs (Benhabib and Farmer 1994, Benhabib and Perli 1994). In the current setup, the government selects a second-best regime through the endogenous allocation of endowments and the choice of a feasible tax rate in a dynamic environment. This selection is feasible since intertemporal complementarities, which fuel multiplicity and are external to the agents in the CDE, are internalized under Ramsey taxation. Hence, the government obtains information through the agents' reaction function and consequently imposes restrictions on the tax rate and endowment allocation through commitment in order to drive the CDE to a unique second-best allocation.

5. Growth-Maximizing Fiscal Policy and Comparative Statics

In this section, we analyze growth-maximizing fiscal policy rules. Modern growth theory has shown particular interest in growth-enhancing policies, as the understanding of the forces of economic growth is crucial in order to identify the relative merits and synergies of government interventions. Moreover, the growth rate is usually the main measurable objective of the government. Although earlier papers, like Barro (1990), have mostly considered welfare and growth-maximizing policies under a unified perspective, subsequent studies have emphasized the role of growth maximization as an independent policy target.[11]

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the paper, we derive the detailed system of equations under Ramsey taxation and we analyze the properties of the second-best allocation. We also report a detailed set of additional numerical results.

[11] See Economides, Park, and Philippopoulos (2007) and Dioikitopoulos and Kalyvitis (2010) for a similar approach.

DEFINITION 3:   *A growth-maximizing (GM) allocation is given under Definition 1 when (i) the government chooses the tax rate and aggregate allocations in order to maximize the growth rate of the economy by taking into account the aggregate optimality conditions of the CDE, and (ii) the government budget constraint and the feasibility and technological conditions are met.*

The government seeks to maximize the growth rate of the economy, g, given by

$$\max_{z,\tau} g = v\tau z^a - \delta$$

subject to the CDE response summarized by $(\frac{a}{\sigma})(1-\tau)z^{a-1} - v\tau z^a - \frac{1}{\sigma}\rho(\omega(z)z) - (\frac{1}{\sigma}-1)\delta = 0$ and $\omega(z) = (1-\tau)z^{a-1} - v\tau z^a$.

The first-order conditions with respect to z and τ are

$$av\,\hat{\tau}\hat{z}^{a-1} + \left(\frac{a}{\sigma}\right)(a-1)\hat{\lambda}(1-\hat{\tau})\hat{z}^{a-2} - \hat{\lambda}v\hat{\tau}a\hat{z}^{a-1}$$

$$-\frac{1}{\sigma}\hat{\lambda}[a(1-\hat{\tau})\hat{z}^{a-1} - v(a+1)\hat{\tau}\hat{z}^a]\rho'(\cdot) = 0, \tag{24}$$

$$v\hat{z}^a - \left(\frac{a}{\sigma}\right)\hat{\lambda}\hat{z}^{a-1} - v\hat{\lambda}\hat{z}^a + \frac{1}{\sigma}\hat{\lambda}[\hat{z}^a + v\hat{z}^{a+1}]\rho'(\cdot) = 0, \tag{25}$$

where $\hat{\lambda}$ is the associated Lagrange multiplier, and $\hat{z}$ and $\hat{\tau}$ are the GM values of z and τ, respectively. Solving (25) for $\hat{\lambda}$ and substituting in (24), we can obtain the following system of equations that characterize the GM policy rules:

$$\hat{\tau} = \frac{a(1-a+\rho'(\cdot)\hat{z})}{a + v\rho'(\cdot)\hat{z}^2} > 0, \tag{26}$$

$$\left(\frac{a}{\sigma}\right)(1-\hat{\tau})\hat{z}^{a-1} - v\hat{\tau}\hat{z}^a - \frac{1}{\sigma}\rho(\omega(\hat{z})\hat{z}) - \left(\frac{1}{\sigma}-1\right)\delta = 0. \tag{27}$$

Equation (26) yields the GM tax rate, $\hat{\tau}$. Since the problem is a static one, the dynamics of the economy follow those of the competitive equilibrium and are locally stable. Notice that when the rate of time preference is constant $\left(\rho'(\cdot)=0\right)$, the government has to implement a marginal tax rate that is equal to the elasticity of publicly provided human capital in the production function, $\tau = (1-a)$, as in Barro (1990), Futagami *et al.* (1993), and Glomm and Ravikumar (1997). However, under endogenous time preference $\left(\rho'(\cdot)\neq 0\right)$, the GM tax rate can be lower or higher than the elasticity of human capital in the production function since the tax policy also depends on demand-driven parameters.

To highlight these points, we provide some numerical examples for a range of parameter values to check the selection of the GM allocation and

Table 1: Changes in b and GM allocation

b	g^{GM}	$\hat{\tau}$	$\hat{z}$	$\hat{\omega}$	$\frac{\hat{C}}{H}$	$\hat{\rho}$
			Multiplicity in CDE ($\sigma = 0.2$)			
0.1	0.102	0.569	6.859	0.011	0.077	0.01270
0.4	0.106	0.641	5.479	0.002	0.013	0.01029
0.7	0.107	0.645	5.426	0.001	0.007	0.00996
			Uniqueness in CDE ($\sigma = 1$)			
0.1	0.062	0.727	0.973	0.205	0.199	0.024
0.4	0.051	0.809	0.445	0.260	0.116	0.051
0.7	0.045	0.843	0.297	0.288	0.085	0.065

Notes: $a = 0.35$, $\delta = 0.01$, $v = 0.1$, $\check{\rho} = 0.005$, $b = 0.5$.

how the comparative statics evolve. For comparison purposes, we illustrate below these analytical results using the parameter values of Example 1 for which the CDE exhibits multiplicity.

EXAMPLE 2:   *Consider a linear time preference function, $\rho(\frac{C}{H}) = b * (\frac{C}{H}) + \check{\rho}$, that satisfies Assumptions 1 and 2 with parameter values $a = 0.35$, $\delta = 0.01$, $b = 0.5$, $\check{\rho} = 0.005$, $v = 0.1$, and $\sigma = 0.2$. The GM tax rate is given by $\hat{\tau} = 0.64$ with respective growth rate $g^{GM} = 0.11$, physical to human capital ratio $\hat{z} = 5.45$, rate of time preference $\hat{\rho} = 0.01$, consumption to physical capital ratio $\hat{\omega} = 0.02$ and consumption to human capital ratio $\frac{\hat{C}}{H} = 0.10$.*

Example 2 shows that under the parameter values that produce multiplicity in the CDE, the GM allocation can act as a selection device and impose the allocation restrictions and the tax rate that guarantee a unique BGP. Notice that in the GM allocation, the "high-growth" BGP is selected, a result that is consistent with the government's objective.[12]

The slope of impatience function, b, is crucial in terms of the qualitative response of the economy. Table 1 shows the response of the economy to changes in b. The upper panel of Table 1 indicates that if we set parameter values where the CDE is characterized by multiple BGPs, the selection of the GM allocation is not affected by changes in b. In turn, the lower panel of Table 1 checks the response of the Barro (1990) taxation rule to changes in b and, in conjunction with the upper panel, provides a picture of the response of the endogenous allocation of the GM allocation problem to changes in the slope of impatience function. In particular, an increase in b leads to an increase in the tax rate and to a decrease in the physical to human capital stock ratio, whereas the effects on the rate of time preference

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[12] For the parameter values used in Example 4 and the growth maximizing tax rate $\hat{\tau} = 0.643$, the CDE gives two equilibrium growth rates $\bar{g}_1 = 0.046$ and $\bar{g}_2 = 0.106$.

Table 2: Changes in σ and b and summarized GM allocation responses

	g^{GM}	$\hat{\tau}$	$\hat{z}$	$\hat{\omega}$	$\frac{\hat{C}}{H}$	$\hat{\rho}$
increase in σ for low σ	$(+)$	$(+)$	$(-)$	$(-)$	$(-)$	$(-)$
increase in σ for high σ	$(-)$	$(-)$	$(-)$	$(+)$	$(+)$	$(+)$
increase in b for $\sigma = 0.2$	$(+)$	$(+)$	$(-)$	$(-)$	$(-)$	$(-)$
increase in b for $\sigma = 1$	$(-)$	$(+)$	$(-)$	$(+)$	$(-)$	$(+)$

and the growth rate are ambiguous and depend on the level of the intertemporal elasticity of substitution. Intuitively, an increase in the slope of the impatience function increases *ceteris paribus* the rate of time preference, which lowers savings and capital accumulation and, in turn, decreases the physical to human capital ratio. Also, by the Euler equation, an increase in the rate of time preference lowers the growth rate and the tax base of the economy, and generates an endogenous increase in the tax rate to finance public expenditures at the BGP. In turn, the endogenous increase in the tax rate activates the previously analyzed mechanism. For a sufficiently high level of the intertemporal elasticity of substitution (e.g., $\sigma = 1$), the rise in the tax rate increases consumption more than human capital expenditures and reinforces the initial increase in the rate of time preference leading to an additional decrease in the growth rate of the economy. In contrast, when σ is sufficiently low (e.g., $\sigma = 0.2$), an increase in the tax rate increases human capital expenditures leading to an increase in the growth rate of the economy that counteracts the initial decrease. Also, in the latter case, the increase in the tax rate lowers the consumption to human capital ratio, since human capital expenditures increase more than consumption for low σ, leading to lower rate of time preference and counteracting the first-order increase. As summarized in Table 2, for low values of σ, the response of the rate of time preference to the decrease in the consumption to human capital ratio, $\frac{C}{H}$, is high and dominates the initial exogenous increase of the rate of time preference caused by b, whereas for high values of σ, the initial increase in ρ dominates its endogenous decrease driven by $\frac{C}{H}$.

6. Concluding Remarks

This paper studied the macroeconomic implications of the endogeneity of time preference to aggregate human capital provided by the public sector. We derived the long-run behavior of the economy and analyzed the impact of fiscal policy. The main findings are that multiple BGPs emerge in the decentralized economy and that second-best (Ramsey and growth-maximizing) taxation can act as a selection device in order to lead the economy to a desired BGP.

An equivalent way to analyze the impacts of public policy on individual patience and, in turn, on incentives to save would be through expenditures on public health. As discussed in Agénor (2010), healthier individuals are less myopic and tend to value the future more, an effect that works through the standard "life expectancy" channel emphasized in OLG models with endogenous lifetimes or mortality rates (Blanchard 1985). Our analytical results offer some novel policy implications for economic performance as it is argued that active public policies in sectors like education and health are crucial in boosting growth, particularly in countries that face development traps. Given that countries with similar structural characteristics often seem to display divergent economic behavior, our findings suggest an additional generating mechanism of "low-growth" in the long run. This stems from the linkage between endogenous time discounting and productive fiscal policy, with the latter now operating through the demand, rather than the supply, side of the economy by forming the patience of consumers. In turn, our results on the role of second-best fiscal policy in driving the economy to a "high-growth" path, albeit highly stylized, indicate the importance of active policymaking in determining the long-run performance of the economy through individual patience by enhancing education, health, or other "future-oriented" policies.

Appendix A

Proof of Proposition 1 and Corollary 1

The method will be to separate function $\Phi(z)$ in two functions and find their intersection to solve it. We define $\Gamma(z) \equiv (\frac{a}{\sigma})(1-\tau)(z)^{a-1} - v(z)^a \tau - (\frac{1}{\sigma} - 1)\delta$ and $\Lambda(z) \equiv \frac{1}{\sigma}\rho(z \cdot \omega(z))$. Both $\Gamma(z)$ and $\Lambda(z)$ are continuous in z. In order for $\omega(z) > 0$ to hold, we must have $z < \frac{1-\tau}{v\tau}$.

Equation $\Gamma(z)$ has the following properties:

(1) $\lim_{z \to 0} \Gamma(z) = +\infty$, $\lim_{z \to \frac{1-\tau}{v\tau}} \Gamma(z) = (\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta$.

(2) $\frac{\partial \Gamma(z)}{\partial z} < 0$, $\frac{\partial^2 \Gamma(z)}{\partial z^2} > 0$.

From the properties of $\Gamma(z)$, it follows that it is a strictly decreasing and convex function in its domain, starts from $+\infty$, and ends at $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta$.

Equation $\Lambda(z)$ has the following properties:

(1) $\lim_{z \to 0} \Lambda(z) = \frac{1}{\sigma}\rho(0) = \frac{1}{\sigma}\check{\rho}$, $\lim_{z \to \frac{1-\tau}{\tau}} \Lambda(z) = \frac{1}{\sigma}\rho(0) = \frac{1}{\sigma}\check{\rho}$.

(2) $\frac{\partial \Lambda(z)}{\partial z} = \frac{1}{\sigma}\rho'(.)[a(1-\tau)z^{a-1} - v\tau(1+a)z^a]$. We have $\frac{\partial \Lambda(z)}{\partial z} > 0$ for $a(1-\tau)z^{a-1} - v(1+a)\tau z^a > 0 \Rightarrow z < \frac{a(1-\tau)}{v(1+a)\tau}$ and $\frac{\partial \Lambda(z)}{\partial z} < 0$ for $z > \frac{a(1-\tau)}{v(1+a)\tau}$. Thus, $\Lambda(z)$ has a maximum at $z = \frac{a(1-\tau)}{v(1+a)\tau}$.

From the properties of $\Lambda(z)$, it follows that it is an inverse U-shaped curve starting from $\frac{1}{\sigma}\check{\rho}$ and ending at $\frac{1}{\sigma}\check{\rho}$.

Assuming equilibrium existence, from the properties of $\Lambda(z)$ and $\Gamma(z)$, it follows that there exist one or two positive balanced growth rates. For low values of z, since $+\infty > \frac{1}{\sigma}\check{\rho}$, we get that $\Gamma(z)$ lies above $\Lambda(z)$. Also, for the upper bound value of z, $\Gamma(z) = (\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta$ and $\Lambda(z) = \frac{1}{\sigma}\check{\rho}$. Since both functions are continuous, if $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta < \frac{1}{\sigma}\check{\rho}$, which means that $\Gamma(z)$ starts above and ends below $\Lambda(z)$, implying that $\Gamma(z)$ will cross $\Lambda(z)$ once and there will exist a unique balanced growth rate. Thus, $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta < \frac{1}{\sigma}\check{\rho}$ is a sufficient parametric condition for a unique balanced growth rate.

If $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$, then there can exist two balanced growth rates because $\Lambda(z)$ is an inverse U-shaped curve, while $\Gamma(z)$ strictly monotone and decreasing, so $\Gamma(z)$ can cross $\Lambda(z)$ at most two times. Thus, $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$ is a necessary parametric condition for multiplicity.

In order for this condition to be sufficient, we need to find the parametric condition under which $\Lambda(z)$ cannot be tangent to $\Gamma(z)$. If they are tangent, since $\Gamma(z)$ is always decreasing, this has to be at the region where $\Lambda(z)$ is decreasing, i.e., $z > \frac{a(1-\tau)}{v(1+a)\tau}$. In other words, we need to prove that there cannot be an intersection of the first derivatives of $\Lambda(z)$ and $\Gamma(z)$, $\frac{\partial \Lambda(z)}{\partial z} \neq \frac{\partial \Gamma(z)}{\partial z}$, for $z > \frac{a(1-\tau)}{v(1+a)\tau}$.

Equation $\frac{\partial \Lambda(z)}{\partial z}$ has the following properties: $\lim_{z \to \frac{a(1-\tau)}{v(1+a)\tau}} \frac{\partial \Lambda(z)}{\partial z} = 0$, $\lim_{z \to \frac{1-\tau}{v\tau}} \frac{\partial \Lambda(z)}{\partial z} = -\frac{\tau}{\sigma}\rho'(0)(\frac{1-\tau}{v\tau})^a$, $\frac{\partial^2 \Lambda(z)}{\partial z^2} = \frac{1}{\sigma}\rho''(.)[a(1-\tau)z^{a-1} - v\tau(1+a)z^a]^2 + \frac{1}{\sigma}\rho'(.)[a(a-1)(1-\tau)z^{a-2} - v\tau a(1+a)z^{a-1}] < 0$ for $\rho''(.) \leq 0$. Thus, for $\rho''(.) \leq 0$, $\frac{\partial \Lambda(z)}{\partial z}$ is a monotonically decreasing function starting from 0 and ending at $-\frac{\tau}{\sigma}\rho'(0)(\frac{1-\tau}{v\tau})^a$.

Equation $\frac{\partial \Gamma(z)}{\partial z}$ has the following properties: $\lim_{z \to \frac{a(1-\tau)}{v(1+a)\tau}} \frac{\partial \Gamma(z)}{\partial z} = \tau(\frac{a(1-\tau)}{v(1+a)\tau})^{a-1} v[(a-1)(\frac{1+a}{\sigma}) - a]$, $\lim_{z \to \frac{1-\tau}{v\tau}} \frac{\partial \Gamma(z)}{\partial z} = a\tau(\frac{1-\tau}{v\tau})^{a-1} v[(\frac{a-1}{\sigma}) - 1]$, $\frac{\partial^2 \Gamma(z)}{\partial z^2} = a(a-1)z^{a-2}[(\frac{a-2}{\sigma})(\frac{1-\tau}{z}) - \tau] > 0$. Thus, $\frac{\partial \Gamma(z)}{\partial z}$ is an increasing function starting from $\tau(\frac{a(1-\tau)}{(1+a)\tau})^{a-1} v[(a-1)(\frac{1+a}{\sigma}) - a]$ and ending at $a\tau(\frac{1-\tau}{v\tau})^{a-1} v[(\frac{a-1}{\sigma}) - 1]$.

Then, since $\frac{\partial \Lambda(z)}{\partial z}$ starts above $\frac{\partial \Gamma(z)}{\partial z}$, $\lim_{z \to \frac{a(1-\tau)}{(1+a)\tau}} \frac{\partial \Lambda(z)}{\partial z} > \lim_{z \to \frac{a(1-\tau)}{(1+a)\tau}} \frac{\partial \Gamma(z)}{\partial z}$, and both functions are monotone, then a sufficient condition for non-intersection is that $\frac{\partial \Lambda(z)}{\partial z}$ ends above $\frac{\partial \Gamma(z)}{\partial z}$, that is, $\lim_{z \to \frac{1-\tau}{v\tau}} \frac{\partial \Lambda(z)}{\partial z} > \lim_{z \to \frac{1-\tau}{v\tau}} \frac{\partial \Gamma(z)}{\partial z}$. This happens if $-\frac{\tau}{\sigma}\rho'(0)(\frac{1-\tau}{v\tau})^a > a\tau(\frac{1-\tau}{v\tau})^{a-1} v(\frac{a-1}{\sigma} - 1) \Rightarrow -\frac{1}{\sigma}\rho'(0) > (\frac{va\tau}{1-\tau})v(\frac{a-1}{\sigma} - 1)$. Thus, if $\rho''(.) \leq 0$ and $-\frac{1}{\sigma}\rho'(0) > (\frac{a\tau v^2}{1-\tau})(\frac{a-1}{\sigma} - 1)$, condition $(\frac{a}{\sigma} - 1)(1 - \tau)^a \tau^{1-a} v^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$ is sufficient for the presence of two positive balanced growth rates. (Figure A1
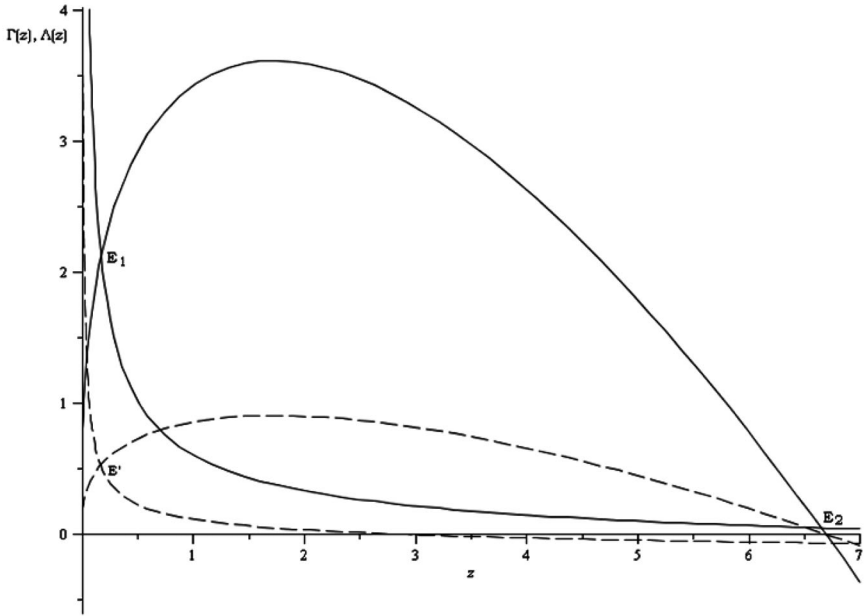
Figure A1: Uniqueness and multiplicity of equibrium. Notes: (1) Parameter values: $a = 0.35$, $\delta = 0.01$, $v = 0.1$, $\rho = 0.005$, $b = 2$, and $\tau = 0.6$. (2) The solid lines correspond to $\sigma = 0.2$ where multiple balanced growth rates exist (E_1 and E_2). The dashed lines correspond to $\sigma = 0.8$ where there exists a unique equilibrium (E').

depicts two examples with parametric values that correspond to uniqueness or multiplicity and the corresponding shapes of $\Lambda(z)$ and $\Gamma(z)$.)   ∎

According to Proposition 1, in the case of multiple balanced growth rates $\Lambda(z)$ and $\Gamma(z)$ intersect twice, for $\bar{z}_1$ and $\bar{z}_2$. Let those two balanced growth rates ranked as $\bar{z}_1 > \bar{z}_2$. To find the corresponding ranking of $\bar{\omega}_1$ and $\bar{\omega}_2$, we solve (14) for $\bar{\omega}$ in the steady state, and we take the derivative with respect to z, $\frac{\partial \bar{\omega}}{\partial z} = (a-1)(1-\tau)\bar{z}^{a-2} - av\tau\bar{z}^{a-1} < 0$. Thus, $\bar{\omega}$ is a strictly decreasing function of $\bar{z}$, so $\bar{z}_1 > \bar{z}_2 \Rightarrow \bar{\omega}_1 < \bar{\omega}_2$. To find the ranking of g_1 and g_2, we take the derivative of g with respect to z, $\frac{\partial g}{\partial z} = v\tau a\bar{z}^{a-1} > 0$. Thus, g is an increasing function of z, so $\bar{z}_1 > \bar{z}_2 \Rightarrow g_1 > g_2$. The ranking for the rate of time preference, $\rho(\bar{z} \cdot \omega(\bar{z})) = \sigma\Lambda(\bar{z})$, which is a nonmonotonic function of z , comes from the analysis above. As $\Gamma(z)$ lies above $\Lambda(z)$ and is monotonically decreasing, it cannot cross twice $\Lambda(z)$ in its increasing part. Then, $\bar{z}_1 > \bar{z}_2 \Rightarrow \Lambda(\bar{z}_1) < \Lambda(\bar{z}_2) \Rightarrow \sigma\Lambda(\bar{z}_1) < \sigma\Lambda(\bar{z}_2) \Rightarrow \rho_1 < \rho_2$. So, in case of two balanced growth rates with high growth, g_1, and low growth, g_2, the endogenous variables are ranked as $\rho_1 < \rho_2$, $\bar{\omega}_1 < \bar{\omega}_2$, $z_1 > z_2$.   ∎

Appendix B

Proof of Proposition 2 and Corollary 2

The method will be to evaluate the determinant and the trace of the linearized dynamical system. We will consider a well-defined steady state, i.e., $\bar{\omega} > 0$, $\bar{z} > 0$, as the one considered in Proposition 1. Given the implicit functions for the rate of time preference and the nonlinear system of equations for the determination of the steady state, our endogenous variables, $\bar{z}(a, \tau, v, \sigma, \delta)$, $\bar{\omega}(a, \tau, v, \sigma, \delta)$, are treated as functions of the economy parameters.

Then, for $\bar{\omega}(\cdot) > 0$ and $\bar{z}(\cdot) > 0$, the first two arguments inside the parenthesis of the determinant of the matrix are always negative for the assumed values of the parameters and policy instruments (recall $a \in (0, 1)$ and $\tau \in (0, 1)$). But regarding the third part, although $\frac{\rho'(\cdot)\bar{z}^{a-1}}{\sigma} > 0$, the sign of $[(1 - \tau)a - v\tau(1 + a)\bar{z}]$ depends on the value of the parameters . Then, for parameter values that the steady state, $\bar{z}^l(\cdot)$, is lower than a parametric threshold $\bar{z}^T \equiv \frac{a(1-\tau)}{v(1+a)\tau}$, then $-\frac{\rho'(\cdot)\bar{z}^{a-1}}{\sigma}[(1 - \tau)a - v\tau(1 + a)\bar{z}] < 0$. Thus, for parameter values where $\bar{z}^l(\cdot) < \bar{z}^T$, the determinant of the matrix is negative, and, in turn, the balanced growth rate of the economy is stable. Then, we need to show what happens for parameter values that result to steady-state value, $\bar{z}$, above the threshold, $\bar{z}^h(\cdot) > \bar{z}^T$. In this case, the determinant of the matrix can be positive, so we need to consider the trace. In particular, if the determinant of the matrix is positive, then we need to show that the trace is negative to prove stability for $\bar{z}^h(\cdot) > \bar{z}^T$. The trace is $(1 - \frac{\rho'(\cdot)\bar{z}}{\sigma})\bar{\omega} + [(1 - \tau)(a - 1)\bar{z}^{a-1} - va\tau\bar{z}^a]$, where from (14) in the steady state, we substitute for $\bar{\omega} = (1 - \tau)z^{a-1} - v\tau z^a$, and after some algebra, we obtain: $trace = a(1 - \tau)\bar{z}^{a-1} - v\tau(1 + a)\bar{z}^a - \frac{\rho'(\cdot)\bar{\omega}\bar{z}}{\sigma}$. The sign of the third part is negative, $(- \frac{\rho'(\cdot)\bar{\omega}\bar{z}}{\sigma} < 0$ as $\rho'(\cdot) > 0$ by Assumption 2) and $a(1 - \tau)\bar{z}^{a-1} - v\tau(1 + a)\bar{z}^a$ depends on the value of $\bar{z}$. For $\bar{z}^h(\cdot) > \bar{z}^T \equiv \frac{a(1-\tau)}{v(1+a)\tau}$, the trace is negative. Thus, the steady state is stable.

To sum up, for $0 < \bar{z}(\cdot) < \bar{z}^T$, the determinant is positive and the steady state is stable (in particular, saddle path). For, $\bar{z}(\cdot) > \bar{z}^T$, the steady state can be either saddle-path stable (negative determinant) or stable (negative trace) with indeterminate type of stability that will be analyzed later on. Thus, for any parameter value in the assumed domain, and Assumptions 1 and 2, a well-defined steady state, $\bar{\omega}(\cdot) > 0$, $\bar{z}(\cdot) > 0$, will be always stable. ∎

Proof of Corollary 1 that analyzes the type of stability comes straightforward from the analysis of the Jacobian and the discriminant of the characteristic equation of Ξ, γ, and the theorems of two-dimensional dynamical systems in continuous time. To show that there exists a set of parameters that case 1 and case 2 hold simultaneously, we consider a linear time preference function, $\rho(\frac{C}{H}) = b * (\frac{C}{H}) + \check{\rho}$, that satisfies Assumptions 1

and 2 with parameter values $a = 0.35$, $\delta = 0.01$, $b = 0.5$, $\check{\rho} = 0.005$, $v = 0.1$, $\tau = 0.4$, and $\sigma = 0.2$. There are ratios of physical to human capital, a low one, $\bar{z}_1 = 0.6789$ that corresponds to a high consumption to physical capital ratio, $\bar{\omega}_1 = 0.7367$, relatively high rate of time preference, $\bar{\rho}_1 = 0.255$, and relatively low growth, $g_1^{CDE} = 0.025$, and a high one, $\bar{z}_2 = 14.94$, which corresponds to a low consumption to physical capital ratio, $\bar{\omega}_2 = 0.0003$, relatively low rate of time preference, $\bar{\rho}_2 = 0.007$, and relatively high growth, $g_2^{CDE} = 0.093$. In the low balanced growth rate, the determinant of Ξ is negative, $J = -0.93$, and the low balanced growth rate displays saddle-path stability ($\rho'(\cdot) = 0.5 < \xi_1 = 7.35$) and case 1 holds. In the high balanced growth rate, the determinant is positive, $J = 0.00127$, the trace is negative, $\Omega = -0.1158$ and the type of stability is a node as, $\gamma = 0.0083 > 0$ and case 2 holds ($\rho'(\cdot) = 0.5 > \xi_2 = -0.09$).   ∎

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