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# An Enhanced Whitening Rotation Semi-Blind Channel Estimation for Massive MIMO-OFDM

Hayder AL-Salihi and Mohammad Reza Nakhai  
Centre for Telecommunications Research  
Department of Informatics, King's College London  
London, UK  
Email: ana.al-salihi@kcl.ac.uk

**Abstract**—The efficient and highly accurate channel state information at the base station is essential to achieve the potential benefits of massive multiple input multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) systems, due to limitation of the pilot contamination problem. In this paper, we investigate the whitening rotation (WR) semi-blind channel estimation algorithm for multi-cell massive MIMO to address the pilot contamination problem through semi-blind approaches of hybrid scheme of pilot and blind to reduce the number of the required pilots. We also enhance the estimation accuracy by combining the proposed estimation technique with temporal domain based channel estimation, i.e., the conventional discrete Furrier transform (DFT) based channel estimator. It has shown that the performance of the WR semi blind estimator achieves a significantly lower Mean Square error (MSE) of estimation compared to the conventional linear minimum mean square error (LMMSE). Also, the proposed scheme of the combined DFT and WR semi blind estimator is seen to have a significantly superior performance compared to LMMSE and WR semi blind estimators.

**Index Terms**—Semi-blind, channel estimation, whitening rotation algorithm, discrete Fourier transform.

## I. INTRODUCTION

Massive MIMO is promising technique to achieve 5th generation targets of peak data rates up to 10Gbit/s [1]. The major limiting factor in massive MIMO is the availability of accurate, instantaneous channel state information (CSI) at the base station. CSI is typically acquired by transmitting predefined pilot signals and estimating the channel coefficients from the received signals by applying an appropriate estimation algorithm. However, the necessary pilot reuse in cellular networks creates spatially correlated inter-cell interference, known as pilot contamination, which reduces estimation performance and spectral efficiency [1],[2].

The pilot contamination problem was analysed in [3] and it has shown that the precoding downlink signal of the base station in the serving cell contaminated the received signal of the users roaming in other cells. The authors of [4] analysed the pilot contamination problems in multi-cell massive MIMO

systems relying on a large number antennae at the base station, and demonstrated that the pilot contamination problem persisted in large-scale MIMO [5].

We want to construct a channel estimation scheme with a limited number of pilots to alleviate the potential shortcomings of the complexity of the blind scheme, while also employing statistical blind information for pilot decontamination and bandwidth efficiency. Such a scheme is semi-blind in nature since it employs both pilot and blind information [6].

MIMO channel estimation based on WR techniques has been investigated in [7]-[10], Nonetheless, to the best of the author's knowledge, the estimation accuracy performance of the WR technique has not been addressed for massive MIMO system.

In this paper, we apply the WR based channel estimation for massive MIMO-OFDM to address the pilot contamination problem. The WR technique consist of two steps: 1) estimation of a whitening matrix using information data; and 2) estimation of unitary matrix using pilots [7].

DFT-based channel estimation is a promising technique that is commonly used to improve the performance of the conventional channel estimation of the least square (LS) and the LMMSE, and it can significantly reduce noise and interference on the estimated channel coefficients. In this scheme, the channel frequency response (CFR) is first estimated by applying LMMSE or LS algorithm in the frequency domain. To improve the estimation accuracy approaches, is to perform in the transform domain by applying discrete Fourier transform (DFT) in the transform domain to get the channel impulse response (CIR) in time domain [11].

In the second part of the paper, we combine the DFT-based channel estimator with WR technique. The results show that the proposed scheme provides better performance compared with the WR based and the complex constrained cramer-rao bound (CC-CRB) reference line.

Thus far, the majority of the previous research of massive MIMO assumes the channels are independent; however, in more realistic environments, antennas are not sufficiently separated and the propagation environment does not provide a sufficient amount of rich scattering [12], so the effect of correlation should be considered.

The main contributions of this paper are summarized below.

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- We analyze the WR semi-blind channel estimator for multi-cell massive MIMO-OFDM system through theoretical analysis and computer simulation considering the pilot contamination problem. In addition, the CC-CRB for WR semi blind estimator has been plotted as a reference line.
- We develop and enhance the estimation accuracy of WR technique by combining the WR algorithm with the DFT-based estimator.
- The performance has also been measured using achievable uplink rate tool.
- The model has incorporated the effect of the channel correlation.

The remainder of this paper is organized as follows. The multi-cell massive MIMO-OFDM system model is presented in Section II. The WR based semi-blind estimator is addressed in Section III. DFT-based channel estimator is analyzed in Section IV. The CC-CRB for WR is analyzed in Section V. Numerical illustrations are provided in Section VI. The final conclusions are drawn in Section VII. .

The following notation is adopted throughout the paper: Capital and small letters represent matrices and vectors respectively. The superscripts  $(\cdot)^H$  and  $(\cdot)^{-1}$  denote the conjugate transpose and the inverse operation, respectively.  $tr(\cdot)$  denotes the trace operation, and  $\mathbf{I}$  denotes the identity matrix (of appropriate dimensions). The Frobenius norm of a matrix  $\mathbf{X}$  are denoted by  $\|\mathbf{X}\|_F$ . The  $E\{\cdot\}$  has been employed to denote expectation with regard to all random variables within the brackets. A Gaussian stochastic variable  $\mathbf{o}$  is denoted by  $\mathbf{o} \sim CN(r, q)$ , where  $r$  is the mean and  $q$  is the variance. A circularly symmetric complex Gaussian stochastic vector  $\mathbf{x}$  is denoted by  $\mathbf{x} \sim CN(\mathbf{x}, \mathbf{Q})$ , where  $\mathbf{x}$  is the mean and  $\mathbf{Q}$  is the covariance matrix.

## II. SYSTEM MODEL

Following the system model of [13] with modification, we consider a multi-cell Massive MIMO OFDM system with  $C$  cells, as shown in Fig. 1. Each cell is comprised of  $M$  antennas at the base station and  $N_t$  single antennas users equally spaced on a circle of radius 100 m, and the system applies time division duplex (TDD) mode to exploit channel reciprocity.

The uplink channel is used for pilot-based channel estimation and the received signal at the base station can be expressed as

$$\mathbf{y} = \mathbf{x}_p \mathbf{H} + \mathbf{n}, \quad (1)$$

where  $\mathbf{x}_p$  is a pilot signal that is used for channel estimation and the term  $\mathbf{n}$  is an ergodic process that consists of independent receiver noise  $\mathbf{n}_{noise} \sim CN(0, \zeta_{BS}^2 \mathbf{I})$ , where  $\zeta_{BS}^2$  is the noise variance, as well as the potential interference  $\mathbf{n}_{interf} \sim CN(0, \mathbf{S})$  from other simultaneous transmissions, where  $\mathbf{S}$  is the covariance matrix during pilot transmission,  $\mathbf{S} = E\{\mathbf{n}_{interf} \mathbf{n}_{interf}^H\}$ , and  $\mathbf{H} \sim CN(0, \mathbf{R})$  is the channel matrix between users and  $M$  antennas at the base station,

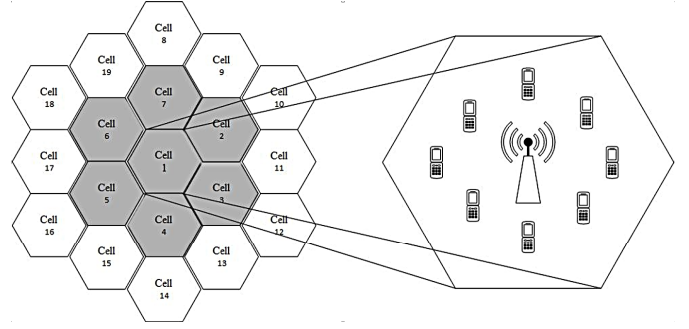


Fig. 1. Illustration of the system model of a multi-cell multi-user massive MIMO that consists of 19 cells; each cell contains nine UEs that are equally spaced on a circle with a radius of 100 m [13].

where  $\mathbf{R}$  is the channel covariance matrix and can be given as  $\mathbf{R} = E\{\mathbf{h}\mathbf{h}^H\}$ , and the average power is  $P^{UE} = E\{|\mathbf{x}_p|^2\}$  [13].

The transmitted signal for each antenna in time domain can be expressed as

$$\mathbf{x}_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{x}_p[k] e^{2j\pi nk/N}, \quad (2)$$

supposing the discrete-time CIR can be given as

$$\mathbf{h}[n] = \sum_{l=0}^{L-1} \alpha_l \sigma[n - \tau_L], \quad (3)$$

where  $\alpha_l$  is the complex path gain of  $l$ th path,  $\tau_L$  is the delay spread of  $l$ th path, and  $L$  is the length of the CIR. And the channel frequency response given by [14] and [15]

$$\mathbf{H}[k] = \sum_{l=0}^{L-1} \alpha_l e^{-2j\pi lk/N}. \quad (4)$$

### A. Uplink LMMSE Channel Estimation

The estimated channel using the conventional (LMMSE) estimator can be expressed as [13] and [16]

$$\hat{\mathbf{h}} = \mathbf{x}_p^* \mathbf{R} (\sqrt{P^{UE}} + \mathbf{S} + \zeta_{BS}^2 \mathbf{I})^{-1} \mathbf{y}, \quad (5)$$

which completed entirely in the frequency domain.

## III. WHITENING ROTATION BASED SEMI-BLIND MIMO CHANNEL ESTIMATION ALGORITHM

Based on the WR algorithm proposed and analysed in [7]-[10], the channel matrix  $\mathbf{H}$  can be decomposed as

$$\mathbf{H}_L = \mathbf{W} \mathbf{Q}^H, \quad (6)$$

where  $\mathbf{W}$  is a whitening matrix that is given by  $\mathbf{W} = \mathbf{P}\mathbf{\Sigma}$  and  $\mathbf{Q}$  is a unitary rotation matrix, thereafter, the concern is how to estimate the whitening matrix  $\mathbf{W}$  and rotation matrix  $\mathbf{Q}$ , which will be presented as follows.

#### A. Estimation of $\mathbf{W}$ from User Data

The whitening matrix can be estimated blindly by applying the singular value decomposition approaches (SVD) to  $\mathbf{H}$  as

$$\hat{\mathbf{H}} = \mathbf{U}_W \mathbf{\Xi}_W \mathbf{V}_W^H \quad (7)$$

and

$$\hat{\mathbf{W}} = \mathbf{U}_W \mathbf{\Xi}_{\tilde{W}}, \quad (8)$$

where  $\mathbf{\Xi}_{\tilde{W}}$  consists of the  $N_t$  largest singular values of  $\mathbf{\Xi}_W$  [7]-[10].

#### B. Estimation of $\mathbf{Q}$ from Pilots

The unitary matrix  $\mathbf{Q}$  can be estimated from the pilot symbol sequence  $x_p$ . The LS function for the estimation problem of the unitary matrix  $\mathbf{Q}$  can be written as

$$\min \|\mathbf{y} - \mathbf{W}\mathbf{Q}^H \mathbf{x}_p\|^2, \quad (9)$$

where  $\mathbf{Q}^H \mathbf{Q} = \mathbf{I}_{N_T}$ .

By letting

$$\mathbf{M} = \mathbf{W}^H \mathbf{y} \mathbf{x}_p^H, \quad (10)$$

the rotation matrix  $\mathbf{Q}$  can be calculated by

$$\hat{\mathbf{Q}} = \mathbf{V}_Q \mathbf{U}_Q^H, \quad (11)$$

where  $\mathbf{U}_Q$  and  $\mathbf{V}_Q$  are obtained from an SVD of  $\mathbf{M}$  as

$$\mathbf{M} = \mathbf{U}_Q \mathbf{\Xi}_Q \mathbf{V}_Q^H. \quad (12)$$

Having estimated  $\mathbf{W}$  and  $\mathbf{Q}$  matrices, the channel matrix  $\mathbf{H}_L$  can be given from (8) [7]-[10].

#### C. Estimation Accuracy Analysis

In this section, we derive an expression for the MSE of the WR semi blind channel estimation for multi-cell massive MIMO-OFDM systems based on the analysis of [9]. The channel estimation error is related to the error caused by the estimated  $\hat{\mathbf{Q}}$ , which is due to noise and pilot contamination embedded in  $\hat{\mathbf{M}}$ . It can be calculated as follows:

$$\hat{\mathbf{M}} = \mathbf{W}^H \mathbf{y} \mathbf{x}_p^H, \quad (13)$$

$$= \mathbf{W}^H (\mathbf{h}_l \mathbf{x}_p + \sum_{i=1, i \neq l}^L \mathbf{h}_i + \mathbf{n}) \mathbf{x}_p^H, \quad (14)$$

$$= \mathbf{W}^H \hat{\mathbf{W}} \hat{\mathbf{Q}}^H + (\sum_{i=1, i \neq l}^L \mathbf{h}_i + \mathbf{n}) \mathbf{W}^H \mathbf{x}_p^H. \quad (15)$$

Thus, the accuracy performance of the WR semi-blind channel estimation is determined by the error between  $\mathbf{M}$  and  $\hat{\mathbf{M}}$ , namely  $e = \mathbf{M} - \hat{\mathbf{M}}$ , where .So, the error  $e$  can be given as

$$e = (\sum_{i=1, i \neq l}^L \mathbf{h}_i + \mathbf{n}) \mathbf{W}^H \mathbf{x}_p^H. \quad (16)$$

The error correlation is given as:

$$\mathbf{R}_e = E\{e e^H\}. \quad (17)$$

Based on assuming the training signal has a unit power so the term  $E\{\mathbf{x}_p \mathbf{x}_p^H\} = \sqrt{SNR}$ , the term  $E\{\mathbf{W} \mathbf{W}^H\} = E\{\mathbf{\Sigma}^H \mathbf{\Sigma}\} = E\{\|\mathbf{\Sigma}\|_F^2\}$  and the component  $\sum_{i=1, i \neq l}^L \mathbf{h}_i$  is the covariance matrix of pilot transmission  $\mathbf{S}$ , so the error correlation can be expressed as

$$\mathbf{R}_e = \frac{E\{\|\mathbf{\Sigma}\|_F^2\}}{SNR}, \quad (18)$$

Finally the MSE of WR semi-blind estimation can expressed as

$$\begin{aligned} MSE &= \|\hat{\mathbf{M}} - \mathbf{M}\|_F^2 = \|e\|_F^2 = tr\{\mathbf{R}_e\} \\ &= \frac{E\{\sum_i^m \lambda_i\} (\zeta_{BS} + \mathbf{S})}{SNR}, \end{aligned} \quad (19)$$

where  $\lambda_i$  is the eigenvalue components [7]-[10].

## IV. DFT-BASED CHANNEL ESTIMATOR

To improve the LMMSE estimation accuracy, the DFT-based channel estimation algorithm has been proposed to reduce the noise and pilot contamination components in the transfer domain by exploiting the property of OFDM systems, since the channel impulse response (CIR) is much less than the symbol duration. So after IDFT is turned into time domain, DFT-based channel estimation algorithm remains as in the samples in cyclic prefix (CP). This will reduce the noise power that exists only outside of the CIR part. The basic block diagram of the DFT-based estimation is shown in Fig. 2. The  $n$ th estimated sample of CIR can be expressed with LMMSE estimation

$$\begin{aligned} \hat{\mathbf{H}}[n] &= IDFT_N \{\hat{\mathbf{H}}_{LMMSE}[K]\}, \\ &= \sqrt{\frac{1}{N}} \sum_{k=0}^N \hat{\mathbf{H}}_{LMMSE}[K] \cdot e^{-j \frac{\pi k n}{N}}, \\ &= \hat{\mathbf{h}}[n] + \hat{\mathbf{n}}[n], \end{aligned} \quad (20)$$

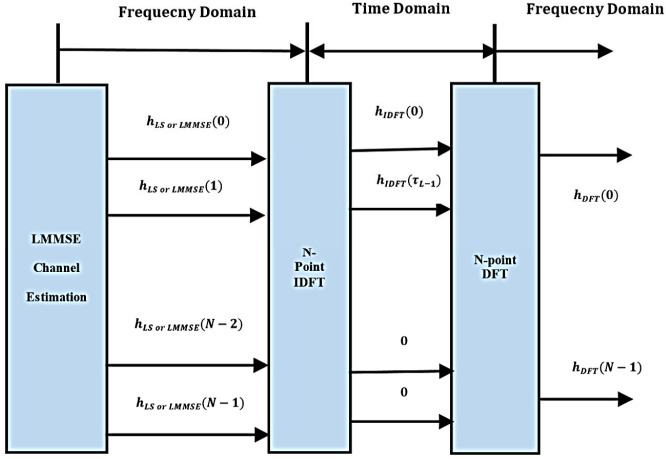


Fig. 2. DFT-based channel estimation [19]

where  $\hat{w}[n]$  is the AWGN component in the transfer domain,  $IDFT_N\{\hat{\mathbf{H}}\}$  indicates N-point IDFT, and  $\hat{\mathbf{h}}[n]$  is the transform domain impulse response that can be expressed as

$$\hat{\mathbf{h}}[n] = \sqrt{N} \sum_{l=0}^{L-1} \alpha_l e^{j\pi n k / N} \sigma[n - \tau_L]. \quad (21)$$

Because the estimated CIR from LMMSE has most of its power concentrated in a few first samples, the CIR is typically limited to the time delay  $\tau_{L-1}$ , which is less than the guard interval and much smaller compared with the number of subcarrier  $N$ . Consequently, the CIR can be described as

$$\hat{\mathbf{h}}_n = \begin{cases} IDFT_N\{\mathbf{H}[K]\}, & 0 \leq n \leq \tau_{L-1} \\ 0, & \tau_{L-1} \leq n \leq N \end{cases},$$

by doing so, more noise and interference are cancelled, and intended channel information is retained. Then, DFT operation is carried out to recover the channel responses into frequency domain:

$$\hat{\mathbf{H}}_{DFT}[K] = DFT_N\{\hat{\mathbf{h}}[n]\} \quad (22)$$

From the above, it can be seen that compared to LS, the DFT based channel estimation method makes use of IDFT/DFT to suppress noise and interference as much as possible in time domain [14],[17] and [18].

## V. CC-CRB FOR WR BASED ESTIMATOR FOR MASSIVE MIMO-OFDM SYSTSEM

In this section, we will apply the CC-CRB of the WR based semi-blind channel estimation of the massive MIMO in correlated fading channel. The CC-CRB on the covariance of any estimator  $\hat{\boldsymbol{\theta}}$  can be given as  $E\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^H\} \geq J^{-1}(\boldsymbol{\theta})$ ,

where  $J(\boldsymbol{\theta})$  is the Fisher information matrix (FIM) corresponding to the observation  $f$ , and can be expressed as

$$J(\boldsymbol{\theta}) = E\left(\frac{\partial}{\partial \boldsymbol{\theta}} \log l(\boldsymbol{\theta}, f)\right) \left(\frac{\partial}{\partial \boldsymbol{\theta}} \log l(\boldsymbol{\theta}, f)\right)^T, \quad (23)$$

where  $l(\boldsymbol{\theta}, f)$  is the likelihood function corresponding to the observation  $f$ , parametrized by  $\boldsymbol{\theta}$ . So the lower bound on the MSE of the estimation of  $\boldsymbol{\theta}$  is given as  $E\{\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|_2^2\} \geq \text{tr}(J^{-1}(\boldsymbol{\theta}))$  [9] and [16].

Based on the assumptions in Section II that the term  $\mathbf{n}$  consists of independent receiver noise  $\mathbf{n}_{noise}$  as well as potential interference  $\mathbf{n}_{interf}$  from other simultaneous transmissions. Thus,  $\mathbf{J}$  can be expressed as  $J = \frac{E\{P^{UE}\}}{\zeta_{BS}^2 + S^2} \mathbf{I}_{2MN_T * 2MN_T}$  from the result in [7] and [20], the estimation of the CC-CRB of the relative MSE of the constructed estimate  $\hat{\mathbf{h}}$  of CC-CRB for massive MIMO can be expressed as

$$MSE_{CC-CRB} = \frac{((\frac{\zeta_{BS}}{N})^2) + (\frac{S}{N})^2 N_t^2}{2P^{UE}L}. \quad (24)$$

## VI. ACHIEVABLE UPLINK RATE ANALYSIS

Given the system model in II, the received vector  $\mathbf{y} \in C^{M \times 1}$  at the  $i$ th BS of (1) can be expressed as

$$\mathbf{y}_i = \sqrt{p^{UE}} \mathbf{x}_p \mathbf{h}_{ii} + \sqrt{p^{UE}} \sum_{l=1, l \neq i}^L \mathbf{x}_p \mathbf{h}_{il} + \mathbf{n}_i \quad (25)$$

where  $\mathbf{h}_{il} (l = 1, 2, \dots, L)$  is the  $M \times L$  MIMO channel matrix between the  $M$  users in the 1st cell and the  $M$ th BS antennas in the  $i$ th cell,  $\mathbf{x}_p$  denote pilot transmitted signals from all users in the  $l$ th cell ( $|\mathbf{x}_p|^2$ , which is equal to the power of the user equipment  $P^{UE}$ ), and  $\mathbf{n}_i$  represents the vector of additive white zero-mean Gaussian noise. The base station processes its received signal vector by multiplying it by the conjugate-transpose of the receiver beamforming filter in [13], as follows

$$\mathbf{r}_i = \mathbf{A}^H \mathbf{y}_i \quad (26)$$

$$\mathbf{r}_i = \sqrt{p^{UE}} \mathbf{a}_n^H \mathbf{x}_p \mathbf{h}_{ii} + \sqrt{p^{UE}} \sum_{l=1, l \neq i}^L \mathbf{a}_n^H \mathbf{x}_p \mathbf{h}_{il} + \mathbf{a}_n^H \mathbf{n}_i \quad (27)$$

The achievable rate can be expressed as [13]

$$R = E\{\log_2(1 + SINR_{Lower}^{UL})\} \quad (28)$$

where the signal to interference and noise ratio can be given as

$$SINR_{Lower} = \frac{|\mathbf{a}_n^H \mathbf{h}_{ii}|^2}{\sum_{l=1}^L |\mathbf{a}_n^H \mathbf{h}_{il}|^2 - |\mathbf{a}_n^H \mathbf{h}_{il}|^2 + \frac{\|\mathbf{a}_n^H\|}{\sqrt{p^{UE}}}} \quad (29)$$

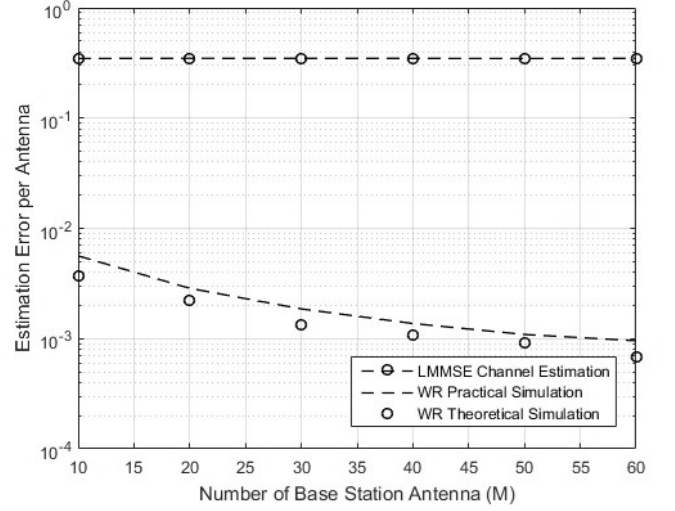
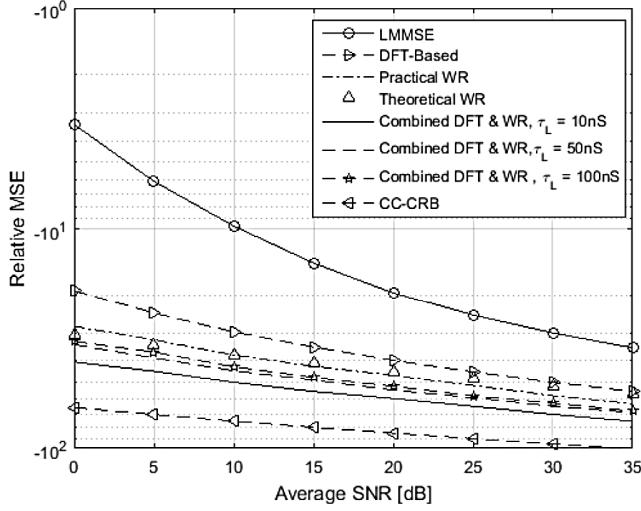


Fig. 4. Computed relative MSE versus the number of antenna at the base station for the conventional LMMSE and practical and theoretical simulation of WR based channel estimation techniques.

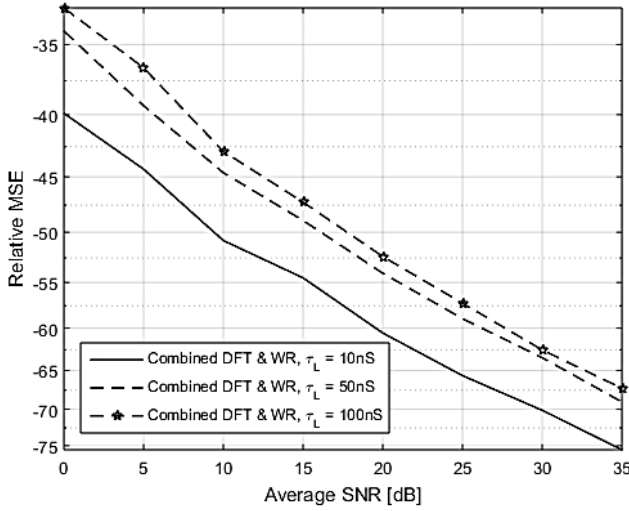


Fig. 3. Relative MSE versus SNR of the conventional LMMSE, theoretical and practical WR, DFT, the combined DFT and WR based channel estimation semi-blind for  $\tau_L = 10, 50$  and  $100$  nS and CC-CRB bound channel estimation for massive MIMO system

## VII. NUMERICAL ILLUSTRATION

In this section, we simulated multi-user multi-cell massive MIMO-OFDM uplink system with  $M = 100$ ,  $N_t = 10$  the N-point DFT is 512, the correlation factor is 0.7 and the number of the path  $L$  is 3.

The performance of the channel estimation is usually measured in-terms of the relative MSE that is normalized with channel covariance matrix and can be given as

$$MSE_{Relative} = \frac{E\{\|\hat{\mathbf{h}} - \mathbf{h}\|_F^2\}}{tr(\mathbf{R})}, \quad (30)$$

where  $\mathbf{R}$  is the channel covariance matrix  $\mathbf{R}$ . For simplicity, the exponential correlation model is adopted to be used in this

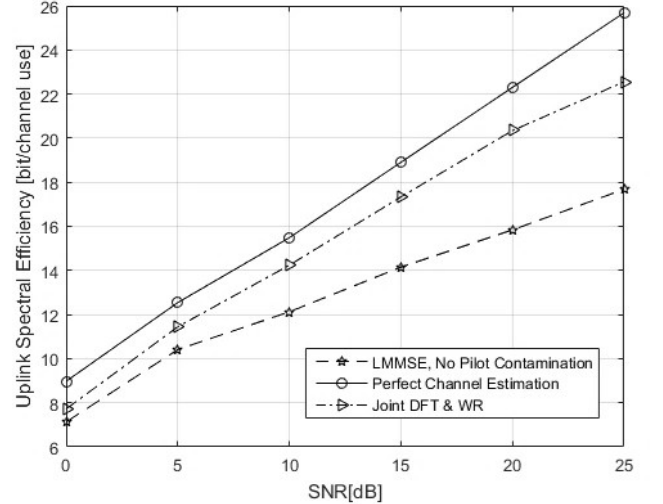


Fig. 5. Achievable Uplink Rate versus SNR for the conventional LMMSE, the Joint DFT and WR based channel estimation techniques with Pilot Contamination and the Perfect Channel Estimation.

paper, the exponential correlation matrix is a special Toeplitz matrix and it is usually used to model the uniform linear array (ULA) [13],[21].

First, we start by comparing the performance of the conventional LMMSE technique with WR semi-blind channel estimation. From the results in Fig.3, the MSE has been plotted versus SNR in the range of 0dB to 35dB for the LMMSE, WR, DFT and the combined DFT and WR for ( $\tau_L=10, 50$  and  $100$  nS, respectively). The CC-CRB of the semi-blind scheme is also plotted as a reference. It can be observed that the performance of the WR semi-blind is better than that of the conventional LMMSE based estimator. Also, it can be found

that the combined DFT and WR based channel estimation provides enhanced performance compared to WR, and its performance is closer to the CRB than the conventional WR algorithm. Also, it can be seen that the estimation accuracy of the combined DFT-based and WR approaches can progressively improve toward the CRB with decreasing values of the delay spread.

In Fig. 4, we consider the MSE versus the number of antenna at the base station from 10 up to 60 for LMMSE and WR approaches. The simulation results show that WR performance is enhanced with increasing the number of the base station antenna while the results show no significant differences for the LMMSE. Also, it can be seen that the performance of the conventional WR are performed better than LMMSE in all cases.

Fig.5. shows the uplink achievable rate for LMMSE with no pilot contamination, combined DFT and WR with delay spread of  $\tau_L=300\text{nS}$  under the effect of pilot contamination and the perfect channel estimation scenarios. It can be seen that the performance of the proposed estimator of the combined DFT and WR has closed performance to the perfect channel estimation and better performance than the LMMSE with no pilot contamination which demonstrate a significant improvement in estimation accuracy and addressing the pilot contamination problem.

### VIII. CONCLUSION

A novel WR based semi-blind channel estimation has been proposed for massive MIMO system. Practical and theoretical simulation results show improved performance of the WR compared to the classical LMMSE technique. We also develop and enhance WR estimation by combining WR with DFT. Simulation results demonstrated improved estimation accuracy performance and significant addressing to the pilot contamination problem of the combined technique compared to the classical WR, the LMMSE and the reference CC-CRB. In addition, it can be concluded that the estimation accuracy of the combined techniques can also be improved by decreasing the values of the delay spread.

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