MULTICELL ROBUST DOWNLINK BEAMFORMING

Tshangini, Mati

Awarding institution:
King's College London

DOCTORAL THESIS

MULTICELL ROBUST DOWNLINK BEAMFORMING

Author:

Mati Tshangini

Supervisor:

Dr. Reza Nakhai

A Thesis submitted to King's College London in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the

Centre for Telecommunications Research
Department of Informatics
School of Natural & Mathematical Sciences
KING'S COLLEGE LONDON

January 2016

Dedicated to my beloved Wife Abigail Tawiah Tshangini and blessed Daughters Glorious and Emmanuella Bridger Tawiah Tshangini

Acknowledgements

I would like to gratefully express my appreciation to my supervisor Dr. Mohammad Reza Nakhai for the invaluable time, counsel, and encouragement throughout the years. His flexibility, patience, knowledge of the subjects and cooperation at the time of difficulties were encouraging. Thank you for making every effort to strengthen me throughout my time as your student. I have been extremely blessed to have a supervisor who truly accepted and responded to my family matters and without you it could have been impossible to finish my PhD.

I would also like to thank my colleagues and friends at CTR, including Yaqub, Alexandre, Bright, Shuyu, Saba, Wan and Changtao who have been lovely in a number of ways. In particular, I would like to thank Dr Bi, Dr Adnan and Dr Tuan. We shared a sense of belonging and they were always there for me. Their advice and quality time we shared are unforgettable. I would also like to thank all the members of staff at CTR, the informatics and specially Professor Hamid Aghvami for his advice to all students including me in our annual meeting.

I would like to extend my sincere appreciation, love and gratitude to my b-lessed Wife, Abigail Tawiah Tshangini for her unconditional love, my Daughters, Glorious and Emmanuella Bridger Tawiah Tshangini and my parents Dad, Neil Bridger and Mum, Rosemary Bridger for their support and encouragement during my ups and downs of my PhD. I am amazed by Dad and Mum, who willingly accepted to proof read this thesis. They have shaped me into the person I am now and are my role models.

I would like to thank the UK Governments Engineering and Physical Sciences Research Council (EPSRC) for providing me the funding which allowed me to undertake this research.

Above all, to God Almighty, who has given me strength and courage throughout my PhD degree.

Abstract

The growth in large number of users and data exchange in cellular networks, has led to an urgent improvement of the power efficiency in cellular networks. The capacity and coverage are of main concern due to the fast growing applications and demand in different areas of use. The scarcity of the traditional communications resources like time and spectrum and the safety limits on transmit power from the base stations antennas as well as the mobile terminals demand the use of new additional resources such as spatial dimensions and cooperation for the realisation of the future cellular networks. Different distributively overlaid wireless cellular network systems are being deployed to meet demand for high data rates. However, these distributively overlaid wireless cellular networks can cause higher inter-cell interference if the signals from the source antennas are not combined and coordinated. Therefore, the solution to eliminating interference is considered as the benchmark for reducing the power consumption in the network. This thesis aims to address these concerns by proposing different algorithm techniques based on beamforming for Multi-Cell Processing (MCP) addressed across multiple coordinating multi-antenna base stations.

First, a distributed optimization problem in a standard semidefnite relaxation (SDR) is introduced that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information (CSI). The aim is to ensure that the worst-cases of the signal-to-interference-plus-noise ratio (SINR) at each user remains above the required level. The feasibility solutions are achieved for certain sets of SINRs only due to relaxation of optimal beamforming. To avoid relaxation and achieve higher SINRS, a second-order cone programming (SOCP), is introduced which

is solved efficiently and achieve higher SINRs. Not only for its power efficiency improvement, but, also SOCP algorithm reduces the complexity of the extra signalling overhead.

0.1 Abbreviations

AoD	Angle of departure
BS	Base station
CSI	Channel state information
CBF	Coordinated beamforming
CoMP	Coordinated multi point
CR	Cognitive radio
DAS	Distributed antenna system
DBF	Decentralised beamforming
dB	Decibel
DDA	Distributed-array antenna
DL	Downlink
ECG	Energy Consumption Gain
FDD	Frequency division duplexing
GPS	Global Positioning System
ICI	Inter-cell interference
IEEE	Institute of Electrical and Electronics Engineers
IMT	International mobile telecommunications
JP	Joint processing
KKT	Karush-Kuhn-Tucker
LMI	Linear matrix inequality
LTE	Long term evolution
LTE-A	Long term evolution-Advanced
MAC	Multiple access channel
MS	Mobile station
MCP	Multi-cell processing
MANET	Mobile Ad hoc NETwork
MBR	Maximum bit rate

ME	Mobile equipment
MSC	Mobile switching centre
MU-MIMO	Multiple user MIMO
MBF	Multi-cell beamforming
MIMO	Multiple input multiple output
MISO	Multiple input single output
MMSE	Minimum mean squared error
NC	Network coding
OFDM	Orthogonal frequency division multiplexing
OFDMA	Orthogonal frequency division multiplexing access
OTBF	Orthogonal Transmit BeamForming
PACR	Power Aware Cooperative Routing
PHY	Physical
QoS	Quality of service
RS	Relay station
SINR	Signal-to-interference-plus-noise ratio
SDP	semidefinite programming
SDR	Semidefnite relaxation
SC	Small cell
SC-BS	Small cell base station
SOCP	Second-order cone programming
SU-MIMO	Single user MIMO
SNR	signal-to-noise ratio
UE	User equipment
UHF	Ultra high frequency
UL	Uplink
ULA	Uniform Linear Array

UMTS	Universal Mobile Telecommunications System
UWB	Ultra Wide Band
UPA	user position aware
WAHN	Wireless Ad hoc Network
WiFi	Wireless-Fidelity
WiMax	Worldwide interoperability for Microwave access
WMN	Wireless Mesh Network
WLAN	Wireless Local Area Network
WSN	Wireless Sensor Network
ZF	Zero forcing
ZMCSCG	Zero mean circularly symmetric complex Gaussian

0.2 Symbols

$\mathbf{a}$	vector $\mathbf{a}$
a	scaler a
$\mathbf{A}$	matrix $\mathbf{A}$
$\mathbb{C}$	The set of complex numbers
$\mathbb{C}^n$	n-dimensional complex $\mathbb{C}^n$
$\mathbb{R}$	The set of real numbers
$\mathbb{R}^n$	real vectors $\mathbb{R}^n$
$\mathbb{H}^n$	complex Hermitian matrices $\mathbb{H}^n$
$\mathbf{A} \succeq \mathbf{B}$	$\mathbf{A}$ - $\mathbf{B}$ is a positive semidefinite matrix
$(\mathbf{A})^T$	transpose of $\mathbf{A}$
$(\mathbf{A})^H$	conjugate transpose of $\mathbf{A}$
$\mathrm{Tr}(\mathbf{A})$	trace of matrix of $\mathbf{A}$
$\mathbf{A} \succeq 0$	matrix $\mathbf{A}$ is positive semidefinite
$\mathbb{E}(.)$	the expectation of a random variable
$\mathbf{0}$	Vector with all elements equal to zero
$\mathbf{1}$	Vector with all elements equal to one
dom	Domain of a function
diag(.)	Square diagonal matrix with elements of input vector placed on main diagonal
Pr(.)	Probability operator
exp(.)	Exponential operator
log(.)	Natural logarithm
$\log_2(.)$	Base 2 logarithm
rank(.)	Matrix rank

$\lVert . \rVert$	the Euclidean norm
$\lVert \mathbf{A} \rVert_F$	Frobenius norm of $\mathbf{A}$
$\Re a, \Im a$	real and imaginary parts of a
$[\mathbf{A}]_{i,j}$	(i, j)th entry of $\mathbf{A}$
$\lvert a \rvert$	magnitude of a
$\mathbf{a} \succ 0$	all elements of $\mathbf{a}$ are positive
$\mathbf{a} \succeq 0$	all elements of $\mathbf{a}$ are nonnegative
$\mathbf{a} \succeq \mathbf{b}$	element-wise greater than or equal to
$\mathbf{a} \succ \mathbf{b}$	element-wise greater than
$\mathbf{e}_i$	column unit vector with a suitable size which contains all zeros
$\mathbf{A} * \mathbf{B} = \mathbf{B} * \mathbf{A}$	Scalar (dot) product of two vectors
$\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A})$	Vector (cross) product of two vectors
$\mathbf{I}_{N \times N}$	$N \times N$ identity matrix with a suitable size
$\begin{bmatrix} a \\ \mathbf{a} \end{bmatrix} \succeq_K 0$	$a \geq \lVert \mathbf{a} \rVert$
$\mathbf{A} = [a_1 \quad a_2 \cdots a_n]$	is $m \times n$
$\text{vec}(\mathbf{A}) = [a_1^T \quad a_2^T \cdots a_n^T]^T$	is $mn \times 1$
$\text{vec}(\mathbf{A})$	stacks $\mathbf{A}$ into a vector columnwise[1]
$\mathbf{h}_i$	Channel vector of user i
$\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, ...\hat{\mathbf{h}}_U]$	set of complex channel of locally active users
$\hat{\mathbf{G}} = [\hat{\mathbf{g}}_1, ...\hat{\mathbf{g}}_N]$	set of complex channel of the users in the adjacent cells
$\mathbf{w}_i$	beamforming vector of user i
$\mathbf{W}$	Matrix of all the beamforming vectors

List of Figures

1.1 Small cell base stations deployed in a macrocell. 7

2.1 Convex and non-convex sets. Left. The hexagon which includes its boundary is convex. Middle. The kidney shaped set is not convex. Right. The square contains some boundary points but not others hence is not convex. The graph is reproduced from [1] 16
2.2 The graph is reproduced from [1] 17
2.3 Parabolic and Flat antenna at the focal point. 35
2.4 Conventional Lens 36
2.5 The feed tilted at 9 degree. 37
2.6 Flat antenna at 9 degree. 37
2.7 Schematic of a wavefront impinging across an antenna array. Under the narrowband assumption the antenna outputs are identical except for a complex scalar 39

3.1 An example of user distributions used in Monte-Carlo simulations. 63
3.2 Total transmit power versus targeted SINR for N = 3, M = 6, K = 3. 64
3.3 Total transmit power versus targeted SINR for N = 2, M = 6, K = 4. 65

4.1 Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS for the SOCP Robust Downlink Beamforming with imperfect CSI. 85
4.2 Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS for the centralized robust CBF with instantaneous CSI. 86
4.3 Convergence and transient behavior of transmit power at target SINR 26dB and the error radii of 0.1 and 0.02 with one user per cell and 6 antenna elements per BS 87
4.4 Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS. 92

5.1 Total transmit power versus Userate values in a 3-cell scenario with one user per cell and 6 antenna elements per BS. 107
5.2 Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS. 108
5.3 Total Intercell power versus Userate values in a 3-cell scenario with one user per cell and 6 antenna elements per BS. 109
5.4 Total Intercell power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS. 110

Contents

0.1 Abbreviations i
0.2 Symbols iv

1 Introduction 1

1.1 Motivation 1
1.2 Thesis Overview 8
1.3 Contributions 10
1.4 Publications 12
1.4.1 Publications not Included in the Thesis 12

2 Background study 13

2.1 Introduction 13
2.2 Convex Optimization 13
2.2.1 Convex Set 16
2.2.2 Cone 17
2.2.3 Euclidean Balls and Ellipsoids 18
2.2.4 Norm balls and Norm Cones 18
2.2.5 Positive Semidefinite Cone 19
2.2.6 Convex functions 20
2.2.7 Convex Optimization Problem 21
2.2.8 Semidefinite Programming 22
2.2.9 Schur Complements 23
2.2.10 Semidefinite Relaxation (SDR) 23

2.2.11 Linear Programs 25
2.2.12 Linear Matrix Inequality 26
2.2.13 Cauchy-Schwarz inequality 26
2.3 Second-order Cone Programming (SOCP) 29
2.4 Robust Optimization 31
2.4.1 Uncertainty Models 31
2.4.2 Ellipsoidal Uncertainty 32
2.5 The S-procedure 33
2.6 A fundamental to Multiple Antennas Techniques and Radio Channel 34
2.6.1 Directional antenna techniques 35
2.6.2 Adaptive antenna techniques 38
2.6.3 Linear antenna array 38
2.6.4 Multiuser downlink Beamforming 42
2.6.5 SOCP and SDP algorithms 44
2.6.6 Semidefinite relaxation algorithm 46
2.6.7 Robust Downlink Beamforming 48
2.6.8 Conclusion 52

3 Robust Downlink Beamforming With Imperfect CSI 53
3.1 Introduction 54
3.2 System model and problem formulation 56
3.3 Robust downlink beamforming 57
3.4 Simulation results 62
3.4.1 Simulation setup 62
3.4.2 Performance evaluation 66
3.5 Conclusion 67

4 SOCP Robust Downlink Beamforming 69
4.1 Second-Order Cone Programming For Robust Downlink Beamforming With Imperfect CSI 69

4.2 Introduction 70
4.3 Second-Order Cone Programming For Robust Downlink Beamforming With Imperfect CSI 73
4.4 A Robust Coordinated Beamforming Formulation with Imperfect Instantaneous CSI 80
4.4.1 Convergence algorithm 82
4.5 Simulation results 83
4.5.1 Simulation setup 83
4.5.2 Performance evaluation 85
4.5.3 Convergence analysis 89
4.5.4 Complexity analysis of SDP and SOCP robust beamforming with imperfect CSI 90
4.5.5 Performance evaluation for SDP and SOCP robust beamforming with imperfect CSI 91
4.6 Conclusion 93

5 Intercell Interference Management For Downlink Beamforming With Imperfect CSI 94
5.1 Introduction 95
5.2 System model and problem formulation 97
5.3 Intercell Interference Management For Downlink Beamforming With Imperfect CSI 97
5.4 SIMULATION RESULTS 105
5.4.1 Simulation setup 105
5.4.2 Performance evaluation for Intercell Interference Problem for Downlink Beamforming With Imperfect CSI . . 106
5.5 Conclusion 111

6 Conclusion and future work 112
6.1 Thesis summary 113
6.1.1 Summary of Chapter 1 113

6.1.2 Summary of Chapter 2 113
6.1.3 Summary of Chapter 3 114
6.1.4 Summary of Chapter 4 114
6.1.5 Summary of Chapter 5 115
6.2 Future research directions 115
6.2.1 A green Coordinated beamforming method 115
6.2.2 Cognitive Radio Coordinated beamforming method . . 116
6.2.3 Robust Beamforming With Imperfect CSI using Gauss-Markov autoregressive 116
6.2.4 Iterative MIMO Robust Beamforming With Imperfect CSI 116
6.2.5 5G Heterogeneous networks/small 117

List of Tables

3.1 Simulation parameters 66

4.1 Complexity analysis for SDP and SOCP 90

Chapter 1

Introduction

1.1 Motivation

The rapid population growth and the increased demand for mobile devices has alarmed the field of mobile communication and brought it to the forefront more than ever before. Developing the new network requires careful consideration in terms of power consumption, latency and reliability of the network. One of the key challenges in wireless communication is to cut down the power consumption, whilst at the same time maintaining an acceptable quality of service. The focus is to enhance new techniques to improve the energy efficiency throughout the network, particularly at the base stations. It has been suggested that telecommunication sector contribute about 2% of the global carbon dioxide (CO_2) emissions but, this could rise to 4% by 2020 based on the current growth in mobile communications. With the growth in mobile communications, operators are increasingly concerned regarding the total energy consumption in cellular network [2]. Different technologies are being put in place by many network operators for energy efficient measures. This includes, the installation of a single RAN technology by Vodafone, which allows multiple network radio technologies, 2G, 3G and 4G to be run from a single base station [3], [4]. The IET has reported that a typical 3G base station uses about 500W of input power to produce only about 40W of output RF power [5]. The heat generated by inefficient operation must also be removed, typically by air conditioning, which adds further to the base station's overall power consumption. It has been concluded that a typical average annual energy consumption of a radio technology, 3G base station is approximately 4.5MWh. Hence for a 3G mobile network covering an area such as the entire country of UK, which has around 12,000 base stations, the total energy consumed will be more than 50GWh per year. This causes a large amount of CO2 emission as well as contributing to the network's operating costs. The strategy taken by Vodafone by including activating new energy-saving software features such as transceivers that switch-off automatically in periods of low traffic, has played a major part in reducing the energy consumption at the base station by up to 10%. Whoever, in commercial cellular networks,the basis for measuring the efficiency and effectiveness of a service is whether it is cost effective. Therefore, BSs are densely deployed by the providers of service in order to guarantee a minimum service-level agreement in line with the users needs during peak traffic periods. Also, a great consideration is needed at the planning stage to determine which base stations can be safely turned off, and what adjustments are needed from one or more neighbouring base stations to ensure complete coverage. To completely turn off a group of base stations could lead to the reconfiguration of the entire wireless communications network. Different approaches have been suggested such as the traffic transfer concept between neighbouring cells during the dynamic operation of BSs. When a set of BSs is switched off,the other set of active BSs instead serves traffic demands that were previously associated with the switched-off BSs utilizing coverage compromising techniques, e.g., [6] - [7]. The energy consumption of BSs is further split into two different categories: fixed energy consumption and traffic-proportional energy consumption. The fixed energy consumption, resulting from power amplifier operations and cooling systems, is unavoidable when a BS remains powered on even if the BS is in an idle state handling no traffic [8]. The former category is dependent on the size limit of the cell. This shows that, the proportion of the fixed energy consumption increases as the BS dimensions increase, in which the BS dimension has a limited capacity, which is the maximum combined data rate of all the users in the cell. The limits lead to the existence of the order of cells, femtocells, picocells, microcells, and macro-class BSs. Different operators and telecommunication business leaders are looking at different ways to complement the current macrocellular networks in order to meet the traffic demand and the unstoppable demand for mobile data. This is growing rapidly and the growth could further increase in the future. Different strategies for developments in technology and network topology have been identified to satisfy this demand. These include [9]:

1. **Spectrum sharing**[10] [11]:

It appears almost the entire cellular network is designed for and operate in the dedicated licensed spectrum. The majority of spectrums suitable for wireless communications are occupied and have already been allocated. Neighbouring cells need to have different frequency bands in order to combat the intercell interference for the users of the adjacent cells. At the same time there are other spectrum usage authorization models for wireless communication, such as unlicensed spectrum or, as widely discussed currently but not yet implemented in practice, various forms of licensed shared spectrum. Operators share their licensed spectrum, such as dynamic shared access, when the network capacity varies, enable more efficient re-use of spectrum based on user access where and when it is needed. This is an example of two operators A and B with different users. If one lacks spectral resources and the other has, the insufficient operator can borrow spectrum from the sufficient operator for a short period of time. This allows a better utilization of the spectrum and gives operators the ability to accommodate many users within its network. The main concern to this type of sharing is cooperation between operators.

A lack of cooperation could cause a high intercell interference to each others users. This could lead to a total degradation of performance, especially the users on the edges of the cell and thereby causing a degradation of the Signal-to-Interference-plus-Noise-Ratio (SINR) at the intended receivers.

2. **Support for wider channel bandwidth**[12]:
The use of spectrum aggregation technology becomes most important, where the aggregation of multiple component carriers is important and allows flexible expansion of bandwidth through simultaneous utilization of multiple carriers to fulfill the high data rate requirement in Long Term Evolution(LTE)-Advanced. However, this technology fails to provide significant enhancements as they are reaching the theoretical limits. Such techniques may not be optimal either, especially under low signal-to-interference plus noise ratio (SINR) conditions[13].

3. **Millimeter Wave**[14]:
Mm-wave spectrum has been suggested to be one of the pontential to provide an increased bandwidth, data rate up to multi gigabit-per-second, while the latency needed by the backhaul is greatly reduced. It has also been suggested that Mm-wave frequency could exploit new spatial processing techniques, such as massive MIMO due to its smaller wavelength. With these capabilities, it means that MmWave massive MIMO needs dense BS deployment. One drawback of this, is that power consumption may be high with MmWave. Therefore, the design of low power devices at the BS are needed from the MmWave in order to combat the implications of power consumption.

4. **Advanced Antenna Techniques**[15]:
Recent advanced radio transmission technologies, which includes single-user multi-input multi-output(MIMO), multi-user MIMO and beamforming based on massive MIMO antenna techniques may have allowed the exploitation of

higher frequency bands and achieve both the diversity and multiplexing gain.

5. **Technologies with traffic asymmetry(uplink and donwlink)** [16][17]: Technologies based on Time-division duplex (TDD) can provide efficient and flexible splitting of the common wireless cellular resources between uplink (UL) and downlink (DL) users. By using these techniques, the uplink and downlink capacity can be easily adjusted and match the demand.

6. **Co-ordination between base stations**[18]: Using advanced Coordinated Multipoint (CoMP) Transmission, the cell edge throughput can significantly be improved and improves energy efficiency by joint transmission among cooperative base stations(BSs). Also, in cellular networks coordination among base stations(BSs) has been recognized as an important solution to reduction of inter-cell interference and increase spectral efficiency, but dramatic signaling overhead and high computational complexity are inherent in this advanced co-ordination technology. Types of CoMP:
Centralized coordinated multi-point (CoMP)-BSs share real-time data and CSI within a centralized unit (CU), requires strict synchronisation and intensive backhaul overhead(limits scalability).
Decentralized coordinated multi-point (CoMP)-Each BS transmits data to its local user (no real-time data sharing - relaxed synchronisation and backhaul overhead), relaxed CSI requirement at each BS.

7. **Small cells**[19]:
Small cell consists of a number of low power nodes such as micro/pico/femto base stations (BSs) deployed in a macrocell shown in Figure 1.1, and the integrated architecture is called a heterogeneous network (HetNet)[13]. It has been suggested and known that the densification of macrocellular networks requires a high transmit power for the base stations. Small cell BSs are low-cost low-power BSs, which have similar functionalities as macrocell BSs

but with a much smaller form factor. They are mainly deployed to provide localised coverage and capacity at households or in hot-spot areas such as city centres and transport hubs. These small cells can be deployed in any frequency band, and due to low power, the small cell could take advantage of higher frequency bands. These are:

Picocells[20] - considered as a low-power operator base stations with a uniform backhaul and access features as macrocells. They can be deployed in a centralized form that serves tens of users within a radio range of 300 m or less. Typical transmit power range from 23 to 30 dBm. Picocells are most used for capacity and outdoor or indoor coverage.

Femtocells[20] - These are low-cost low-power user also known as home BSs or home eNBs. They can serve a dozen active users in homes or enterprises. The femtocell radio range is less than 50 m. Typical transmit power less than 23 dBm. They operate in open or restricted access.

Base station densification is a promised method that improves spectral and energy efficiency through enhanced control over coverage. The densification is made possible by dense deployment of small cells. Now, we most know that, in certain cases the base stations are connected to the mobile switch with fibre. In other cases, the base station is connected to another base station using a point-to-point microwave link with a fibre link to the mobile switch. We call the later case backhaul. We see that the densification brings another set of challenges based on the cost of high volume between the radio access and core network needed for connections, which are limited by latency problems. Another main issue with the dense deployment strategy is that, it will bring in interference problems due to the fact that small cells have a relatively small coverage and therefore, there is a limitation in terms of distance between cells. Also the mix of small cell sizes that generate a heterogeneous network will only bring complexity and will also have to operate with full spectrum reuse across all base stations. The sharing of spectrum and the deployment of small cells due to increased demand for transmission capacity

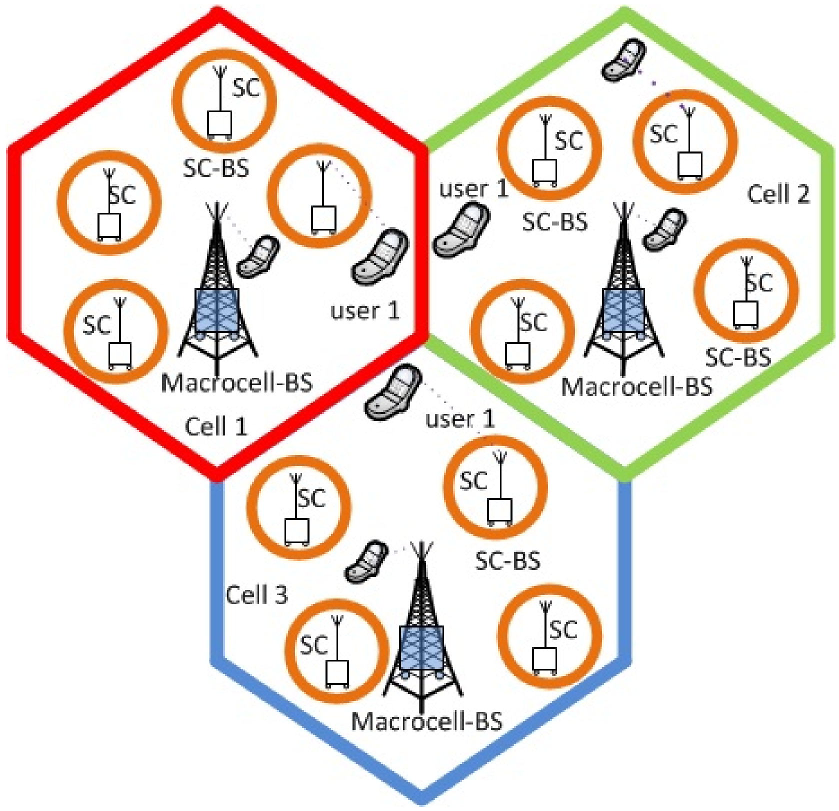

Figure 1.1: Small cell base stations deployed in a macrocell.

will further create a vacuum for increased intercell interference on the users located at the edge region. Therefore, proper mitigation is needed to control the interference in heterogeneous networks with small cells. To mitigate the inter-cell interference in cellular network, transmit beamforming can be used to control the interference, while keeping the desired link quality under control.

1.2 Thesis Overview

We briefly give an outline of the thesis in the following.

Chapter 1 introduces the motivation, overview and the contributions of the thesis.

Chapter 2 provides a background review of the basic properties of convex sets in the multidimensional case: i.e. Unit balls of norms, Ellipsoid. Quadratic optimisation: Semidefinite Relaxation and The S-procedure. The standard conic programs, i.e. second order cone programming (SOCP) and semidefinite programming (SDP). This chapter concludes with antenna techniques aim to build an overview of the spatial multiplexing and the linear antenna array.

Chapter 3 inter-cell alignment is tackled by applying limited cooperation amongst the base stations. The BS independently design their own beamforming using the channel state information at the base station. The base station minimises the combination of its total transmit power and the resulting interference power of the users of the other cell. The formulated semidefinite and relaxation algorithm is compared against the robust CBF with imperfect channel information.

Chapter 4 discusses the complexity analysis and the power efficieny. To reduce the extra signalling, the downlink multicell processing is studied. The objective of this scheme is also to minimise the total combination transmit power across coordinating BSs subject to user SINR constraints. Then a comparison is made between the two proposed methods.

Chapter 5 the time-division duplex (TDD) systems is considered in order to exploit channel reciprocity. The objective is to minimize the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information (CSI), the aim to ensure that the data rate at the cell edge users for any given transmit power remains above the required level.

The thesis concludes with **Chapter** 6 deals with conclusions and the possible future research directions.

1.3 Contributions

The concepts in **Chapter** 2 build the contributions of the thesis and are outlined in following the three different chapters.

Chapter 3 presents a decentralized algorithm to tackle inter-cell interference. The base station within a cell only transmit to its local users. The proposed scheme is referred as robust beamforming with imperfect channel state information. The main objective is to design a set of beamformers for a number of active users such as the combination of the total transmitted power and its aggregate interference induced on the users of the other cells at the base station is minimised, while maintaining a certain level of quality of service within the corresponding cell. The channel state information at the transmitter is assumed to be confined within the ellipsoidal sets. The basic properties of convex sets and the quadratic optimisation reformulate the problem in robust convex semidefinite programing form with linear matrix inequality. The proposed strategy is compared against the conventional and the centralized beamforming. The computer simulations are performed by combining a signal processing tool, i.e., MATLAB and optimization packages.

Chapter 4 a spherical uncertainty set is considered to model the channel state information. To acquire the reduction in signalling overhead, we propose a robust multicell downlink beamforming that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information, whilst also guaranteeing that the worst-cases of the signal-to-interference-plus-noise ratio (SINR) remains above the required level. The problem is reformulated as second order cone programming, and then reformulate the problem into a LMI. For comparison, we reformulated the CBF into a semidefinite relaxation. Then, both schemes are evaluated.

Chapter 5 an optimum multicell downlink beamforming is proposed that minimizes the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information (CSI). The objective is to ensure the data rates of the users at the cell edge, remains above a certain level.

1.4 Publications

The contributions of this thesis have been taken from the following list of publications

1. **Tshangini**, **Mati**.; Nakhai, M.R., "Robust downlink beamforming with imperfect CSI" 2013 IEEE International Conference on in Communications (ICC) , vol., no., pp.4916-4920, 9-13 June 2013

2. **Tshangini**, **Mati**.; Nakhai, M.R., "Second-order cone programming for robust downlink beamforming with imperfect CSI," 2013 IEEE in Global Communications Conference (GLOBECOM), vol., no., pp.3452-3457, 9-13 Dec. 2013

3. **Tshangini**, **Mati**; Nakhai, Mohammad Reza, "Intercell Interference Management For Downlink Beamforming With Imperfect CSI," in European Wireless 2015; 21th European Wireless Conference; Proceedings of , vol., no., pp.1-6, 20-22 May 2015

1.4.1 Publications not Included in the Thesis

The following publication contain work done by the author, but is not included in this thesis.

4. Aijaz, A.; **Tshangini**, **Mati**.; Nakhai, M.R.; Xiaoli Chu; Aghvami, A.-H., "Energy-Efficient Uplink Resource Allocation in LTE Networks With M2M/H2H Co-Existence Under Statistical QoS Guarantees," , IEEE Transactions on Communications, vol.62, no.7, pp.2353-2365, July 2014

Chapter 2

Background study

2.1 Introduction

2.2 Convex Optimization

Recently, novel convex optimization techniques have become one of the key solving powerful tool for cutting edge wireless communication applications. Many of the approaches in communication problems can either be cast as or be converted into convex optimization problem [21]. Convex optimization enfold two important parts. The convex objective and convex constraints. By convention, a minimization of a convex objective function subject to convex constraints. The key advantages to casting communication problems into convex optimization are:

- Any local minimum in a convex optimisation problem is also the global minimum. Therefore, algorithms written to solve a convex problem are much more efficient, faster and reliable.

- Numerical methods are sufficient to solve a convex problem even when a closed form does not exist.

• Can have large number of constraints and can be easily added to the problem

Mostly, a more recent approach to optimization under uncertainty, in which uncertainty is considered as deterministic and set-based. The development of fast interior point methods for convex optimization [1], particularly for semidefinite optimization has brought big interest into the field. Some of the central issue addressed in [22] for the optimization under uncertainty that guarantee the feasibility of the optimal solutions for all realizations of the uncertain data is:

1. **Tractability**. In general, some of the robust optimization problem may not itself be tractable. The tractability depend on the structure of the nominal problem as well as the class of uncertainty set. Many well-known classes of optimization problems, including the two standard conic programs such as second order cone programming(SOCP) and semidefinite programming (SDP), have a robust optimization formulation that can be transformed into tractable. A formulation is needed to ensure that tractability is preserved [23], [24].

2. **Conservativeness and probability guarantees**. This approach is used when the parameter uncertainty is not stochastic, or if distributional information is not readily available. But even if there is an underlying distribution, the tractability benefits of the robust optimization approach may make it more attractive than alternative approaches from stochastic optimization [25].

3. **Flexibility**. Here the robust optimization is used as a tool to imbue the solution with desirable properties, like sparsity, stability, or statistical consistency.

The overall aim of this chapter is based on the developments of beamforming schemes and main aspects of robust , with an emphasis on its tractability, which are used to formulate the algorithms used in chapter 3 and 4. In addition to tractability, a central question in the robust optimization design has been probability guarantees on feasibility under particular distributional assumptions for the disturbance vectors. With this question in mind: what is the smallest value that will satisfy the uncertainty?

Ben-Tal and Nemirovski [26], as well as El Ghaoui et al. [27], have considered the spherical and ellipsoidal uncertainty sets. Controlling the size of these spherical and ellipsoidal sets, as in the next section of convex set, has the interpretation of a budget of uncertainty that the decision-maker selects in order to easily trade off robustness and performance

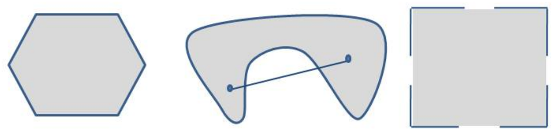

Figure 2.1: Convex and non-convex sets. Left. The hexagon which includes its boundary is convex. Middle. The kidney shaped set is not convex. Right. The square contains some boundary points but not others hence is not convex. The graph is reproduced from [1]

2.2.1 Convex Set

A set C is convex, if and only if a line segment between any two points lie in C. The mathematical representation of the given definition can be written as

$$\theta x_1 + (1-\theta)x_2 \quad \epsilon \quad C \tag{2.1}$$

for any $x_1, x_2 \quad \epsilon \quad C$ and $0 \leq \theta \leq 1$. Examples of convex and non-convex sets are shown in Fig. 2.1. Lines and line segments also form convex sets.

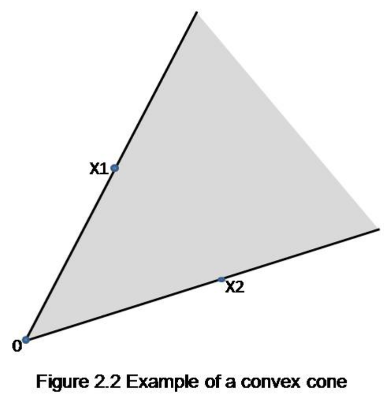

Figure 2.2: The graph is reproduced from [1]

2.2.2 Cone

Definition. A cone C is a subset of the Euclidean space with the property that for all vectors, if $x \quad \epsilon \quad C$ and all the nonnegative scalars $\theta \geq 0 \quad \theta x \quad \epsilon \quad C$. If the set C is convex and a cone then it is called a convex cone [1]. Convex cones satisfy the following condition

$$\theta x_1 + \theta x_2 \quad \epsilon \quad C \tag{2.2}$$

for any $x_1, x_2 \quad \epsilon \quad C$ and $\theta_1, \theta_2 \quad \geq 0$.
For C to be a convex cone, it needs to satisfy the following properties:
1. C is closed
2. C has no empty space
3. if $x \quad \epsilon \quad C$ and $-x \quad \epsilon \quad C$ then $x = 0$.
Figure 2.2 Example of a convex cone with apex 0 and edges passing through $x1$ and $x2$.

2.2.3 Euclidean Balls and Ellipsoids

Euclidean balls forms convex set[1]. For a given $\mathbf{x} \in \mathbb{R}^n$ with a center $\mathbf{x}_c$ and a radius $r > 0$, a closed euclidean ball $B(\mathbf{x}_c, r)$ is the set of point in $\mathbb{R}^n$ and is given as

$$B(\mathbf{x}_c, r) = \{\mathbf{x} \mid \left\|\mathbf{x} - \mathbf{x}_c\right\|_2 \leq r\}, \tag{2.3}$$

Ellipsoids form convex set and has the following form [1]:

$$\varepsilon = \{\mathbf{x}_c + \mathbf{A}\mathbf{u} \mid \left\|\mathbf{u}\right\|_2 \leq 1\}. \tag{2.4}$$

where $\mathbf{x}_c$ is the center of the ellipsoid and $\mathbf{P}$ is a symmetric and positive definite matrix. For an $\mathbf{A}$ square and nonsingular, the ellipsoid can also be represented in another form as

$$\varepsilon = \{\mathbf{x} \mid (\mathbf{x} - \mathbf{x}_c)^T \mathbf{P}^{-1} (\mathbf{x} - \mathbf{x}_c) \leq 1\}, \tag{2.5}$$

2.2.4 Norm balls and Norm Cones

Norm balls and norm cones form convex sets [1]. A norm ball of radius r and center at $\mathbf{x}_c$ is given by

$$B(\mathbf{x}_c, r) = \{\mathbf{x} \mid \left\|\mathbf{x} - \mathbf{x}_c\right\| \leq r\}, \tag{2.6}$$

where $\left\|.\right\|$ is any norm on $\mathbb{R}^n$. The norm cone associated with the norm $\left\|.\right\|$ is the set

$$C = \{(\mathbf{x}, t) \mid \|\mathbf{x}\| \leq t\} \subseteq \mathbb{R}^{n+1}, \tag{2.7}$$

where $t > 0$. The second-order cone is the norm cone for the Euclidean norm. It is also known as the Lorentz cone or ice-cream cone.

2.2.5 Positive Semidefinite Cone

Positive semidefinite matrices form a convex cone, hence a convex set [1]. This can be proven using 2.1 and the definition of positive semidefiniteness as follows. for any $\theta_1, \theta_2 \geq 0$ $\mathbf{A}, \mathbf{B} \in \mathbb{S}^n$ and $\mathbf{x} \in \mathbb{R}^n$, we have

$$\mathbf{x}^T(\theta_1\mathbf{A} + \theta_2\mathbf{B})\mathbf{x} = \theta_1\mathbf{x}^T\mathbf{A}\mathbf{x} + \theta_2\mathbf{x}^T\mathbf{A}\mathbf{x} \geq 0. \tag{2.8}$$

Hence $\theta_1\mathbf{A} + \theta_2\mathbf{B} \in \mathbb{S}^n_+$, which proves that the set of positive semidefinite matrices form a convex cone.

2.2.6 Convex functions

A function [28]-[29]

$$f : \mathbb{R}^n \quad \longrightarrow \quad \mathbb{R} \tag{2.9}$$

is called convex if for any pair of of non-equal x_1, x_2 in the domain of f and any pair of real positive numbers θ_1, θ_2, such that $\theta_1 + \theta_2 = 1$, one has

$$f(\theta_1 x_1 + \theta_2 x_2) \leq \theta_1 f x_1 + \theta_2 f x_2 \tag{2.10}$$

We say f is strictly convex, if and only if, the above inequality is always strict; f is called (strictly) concave iff $-f$ is (*strictly*) convex. Visually, it means that on the graph $(x, y) \quad \epsilon \quad \mathbb{R}^{n+1} : y = f(x)$ of f, for any x lying on the line segment, connecting a pair of chosen points x_1 and x_2 in the domain of f, the point $(x, f(x))$ lies below a chord, connecting the pair of points $(x_1, f(x_1))$ and $(x_2, f(x_2))$, for all the possible choices of the pair x_1, x_2 (the height being measured in terms of the y-coordinate). This makes the majority of the convexity issues essentially one-dimensional: for instance $f(x)$ is convex if and only if for any chosen pair of points x_1 and x_2 in the domain of f, the function
$f(t) = f(tx_1 + (1-t)x_2)$ of one variable $t \quad \epsilon \quad \mathbb{R}$ is convex.

Examples of convex functions

$exp(ax)$ is convex on $\mathbb{R}$

$log(x)$ is concave on $\mathbb{R}$

$xlog(x)$ is convex on $\mathbb{R}_{++}$ or on $\mathbb{R}_{+}$

Every norm on $\mathbb{R}_n$ is convex

2.2.7 Convex Optimization Problem

An optimization problem is a mathematical problem of the form [1], [30]

$$\begin{aligned} \min_{x} \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \leq 0, i = 1 \cdots m \\ & h_i(x) = 0, i = 1 \cdots p \end{aligned} \tag{2.11}$$

$f_0(x)$ is the objective function
$f_i(x) \leq 0$ and $h_i(x) = 0$ are the inequality and equality constraint functions.
x is the optimization variable.
The domain $\mathcal{D}$ of the optimisation problem is the set of points for which the objective and constraint functions are defined. A point $x \in \mathcal{D}$ is called a feasible point if all constraints are satisfied. The optimisation problem is feasible if there is at least one feasible point.

A feasible solution x^* is called a globally optimum solution if $f_0(x^*) \leq f_0(x)$ for all feasible x. A feasible solution $\overline{x}$ is called a locally optimum solution if there exists an $\varepsilon > 0$ such that $f_0(\overline{x}^*) \leq f_0(x)$ for all feasible x that satisfies $\left\|\mathbf{x} - \overline{\mathbf{x}}\right\|_2 \leq \varepsilon$ An optimisation problem is convex if and only if all of the following conditions are satisfied.

$\star$ The objective function is convex.

$\star$ The inequality constraint functions are convex.

$\star$ The equality constraint functions are affine, i.e., have the form $h_i(x) = a_i^T\mathbf{x} + b_i$

$\star$ The domain of the optimisation problem is convex.

2.2.8 Semidefinite Programming

Semidefinite program (SDP) refers to an optimisation problem with linear objective and affine constraints includes equalities, inequalities. Here, we minimize a linear function subject to the constraint that an affine combination of symmetric matrices is positive semidefinite [1], [31], [32] [33].

Semidefinite programming is the exact implementation of conic programming with the positive semidefinite cone, so we can write the general cone program in the form

$$\begin{aligned} \min_{x} \quad & c^T(x) \\ \text{subject to} \quad & F(x) + g \preceq_C 0 \\ & A(x) = b, \end{aligned} \tag{2.12}$$

Where c,g,b are the vector parameters of the problem and F,A are matrices and C is a proper cone. The case, where $C = S_+^n$, then the above equation is called semidefinite program(SDP). The SDP can be represent in the form

$$\begin{aligned} \min_{x} \quad & c^T(x) \\ \text{subject to} \quad & x_0F_0 + x_1F_1 + \cdots x_{n-1}F_{n-1} + G \preceq 0 \\ & A(x) = b, \end{aligned} \tag{2.13}$$

where $G, F_1 \quad \cdots \quad F_n \quad \epsilon \quad \mathbb{S}^k$, and $A \quad \in R^{p\times n}$. The inequality of the above problem is called linear matrix inequality and the semidefinite program truly seen as a generalized of the linear program.

2.2.9 Schur Complements

Suppose $\mathbf{R}$ and $\mathbf{S}$ are Hermitian, it means that if it is equal to its conjugate. That is,
$\mathbf{R} = \mathbf{R}^*$
$\mathbf{S} = \mathbf{S}^*$
or in terms of components
$\mathbf{r}_{ij} = \mathbf{r}^*_{ji}$
$\mathbf{s}_{ij} = \mathbf{s}^*_{ji}$
Then, the following conditions are equivalent:
$\mathbf{R} \prec 0, \quad Q = \mathbf{S} - \mathbf{G}^T\mathbf{R}^{-1}\mathbf{G} \prec 0;$
and

$$\mathbf{P} = \begin{bmatrix} \mathbf{S} & \mathbf{G}^T \\ \mathbf{G}_0^H & \mathbf{R} \end{bmatrix} \prec 0.$$

Schur complement lemmas
$\mathbf{P} \succ 0$ if and only if $\mathbf{S} \succ 0$ and $\mathbf{Q} \succ 0$.

if $\mathbf{S} \succ 0$,then $\mathbf{P} \succeq 0$ if and only if $\mathbf{S} \quad \succeq 0$.

2.2.10 Semidefinite Relaxation (SDR)

Consider a non-convex QCQP of the following form

$$\begin{aligned} \min_{x} \quad & \mathbf{x}^T\mathbf{P}_0 x + \mathbf{q}_0^T x + r_0 \\ \text{subject to} \quad & \mathbf{x}^T\mathbf{P}_i\mathbf{x} + \mathbf{q}_i^T\mathbf{x} + r_i \leq 0, i = 0, 1, \cdots, m \end{aligned} \tag{2.14}$$

where $c, x \in \mathbb{R}^n, \mathbf{P}_i \in \mathbb{S}^n$ $i = 0, 1, \cdots, m$, $\mathbf{q}_i \in \mathbb{R}^n$ and $r_i \in \mathbb{R}$ $\mathbf{A} \in \mathbb{R}^{p\times n}$ and $\mathbf{b} \in \mathbb{R}^p$. Problem 2.14 is non-convex when at least one of the P_i is not positive semidefinite. The SDR makes use of the following properties, $\mathbf{X} = \mathbf{x}\mathbf{x}^T$, in order to linearise problem 2.14. The definition of $\mathbf{X}$

implies that $\text{rank}(\mathbf{X}) = 1$. This definition also implies that $\mathbf{x}^T\mathbf{P}_i\mathbf{x} = \text{tr}(P_i\mathbf{X})$, hence, problem 2.14 can be rewritten as

$$\begin{aligned} \min_{\mathbf{x},\mathbf{X}} \quad & \text{Tr}(\mathbf{P}_0 X) + \mathbf{q}_0^T x + r_0 \\ \text{subject to} \quad & \text{Tr}(\mathbf{P}_i X) + \mathbf{q}_i^T \mathbf{x} + r_i \leq 0 i = 0, 1, \cdots, m \\ & \mathbf{X} = \mathbf{x}\mathbf{x}^T. \end{aligned} \tag{2.15}$$

The constraint $\mathbf{X} = \mathbf{x}\mathbf{x}^T$ is non-convex constraint; however, it can be relaxed by replacing it with the looser positive semidefinite constraint $\mathbf{X} = \mathbf{x}\mathbf{x}^T \succeq 0$. With this relaxation, the relaxed problem can be stated as the following SDP

$$\begin{aligned} \min_{\mathbf{x},\mathbf{X}} \quad & \text{Tr}(\mathbf{P}_0 X) + \mathbf{q}_0^T x + r_0 \\ \text{subject to} \quad & \text{Tr}(\mathbf{P}_i X) + \mathbf{q}_i^T \mathbf{x} + r_i \leq 0 i = 0, 1, \cdots, m \\ & \mathbf{X} = \mathbf{x}\mathbf{x}^T \succeq 0. \end{aligned} \tag{2.16}$$

Utilising the Schur complement in 2.2.9 to represent the last constraint, problem 2.16 can then be rewritten as

$$\begin{aligned} \min_{\mathbf{x},\mathbf{X}} \quad & \text{Tr}(\mathbf{P}_0 X) + \mathbf{q}_0^T x + r_0 \\ \text{subject to} \quad & \text{Tr}(\mathbf{P}_i X) + \mathbf{q}_i^T \mathbf{x} + r_i \leq 0 i = 0, 1, \cdots, m \\ & \begin{bmatrix} \mathbf{X} & \mathbf{x} \\ \mathbf{x}^T & 1 \end{bmatrix} \succeq 0. \end{aligned} \tag{2.17}$$

Problem 2.17 is called the SDP relaxation of the original non-convex problem 2.14. The optimal value of the relaxed problem gives the lower bound on the optimal value of the original non-convex QCQP.

When the objective and constraints of the original problem 2.14 are homogeneous, i.e., there are no linear terms $\mathbf{q}_i^T$ x, then a simpler relaxed problem can be obtained. The definition, $\mathbf{X} = \mathbf{x}\mathbf{x}^T$, is again used to linearise the QCQP. In this case, this definition implies that $\mathbf{X} \succeq 0$ and $\text{Rank}(\mathbf{X}) = 1$. Problem 2.14 can then be rewritten as

$$\begin{aligned}
\min_{\mathbf{X}} \quad & \text{Tr}(\mathbf{P}_0 X) + r_0 \\
\text{subject to} \quad & \text{Tr}(\mathbf{P}_i X) + r_i \leq 0 i = 0, 1, \cdots, m \\
& \mathbf{X} \succeq 0. \\
& \text{Rank}(\mathbf{X}) = 1.
\end{aligned} \tag{2.18}$$

The rank constraint is the only non-convex constraint in problem 2.18 and the problem can be relaxed by dropping this rank constraint. The resulting SDP is stated as

$$\begin{aligned}
\min_{\mathbf{X}} \quad & \text{Tr}(\mathbf{P}_0 X) + r_0 \\
\text{subject to} \quad & \text{Tr}(\mathbf{P}_i X) + r_i \leq 0 i = 0, 1, \cdots, m \\
& \mathbf{X} \succeq 0.
\end{aligned} \tag{2.19}$$

Some of the relaxation techniques can be found [34].

2.2.11 Linear Programs

A linear program (LP) has the following structure

$$\begin{aligned}
\min \quad & \mathbf{c}^T(\text{x}) \\
\text{subject to} \quad & \mathbf{A}\mathbf{x} \preceq \mathbf{b}
\end{aligned} \tag{2.20}$$

where $\mathbf{c}, \mathbf{x} \in \mathbb{R}^n$ and $\mathbf{b} \in \mathbb{R}^m$ and $\mathbf{A} \in \mathbb{R}^{m \times n}$. Now, given the standard form LP, the duality of the (2.20) can be found as

$$\begin{aligned} \max \quad & -\mathbf{b}^T(y) \\ \text{subject to} \quad & \mathbf{A}^T(y) + c = 0 \\ & y \succeq 0 \end{aligned} \tag{2.21}$$

That $y \succeq 0$ means that all the components of the vector y is nonnegative.

2.2.12 Linear Matrix Inequality

A linear matrix inequality(LMI) [35] has the following form

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i > 0, \tag{2.22}$$

where

⋆ $F(x)$ is a positive definite, i.e., $u^T F(x) u > 0$ for all nonzero $u \in \mathbf{R}^n$.
⋆ $x \in \mathbf{R}^m$ is the variable and
⋆ $Fi = F_i^T \in \mathbf{R}^{n \times n}, i = 0, \cdots, m$ are the symmetric matrices.
Convex inequalities are converted to LMI form using Schur complements.

2.2.13 Cauchy-Schwarz inequality

Theorem[36]: Let us consider V to be some vector space that contains a norm and inner product, then for all $\mathbf{a}, \mathbf{b} \in \mathbf{V}$ the following inequality holds:
$| \mathbf{a}.\mathbf{b} | \leq \|\mathbf{a}\| \|\mathbf{b}\|$
The inequality holds if $a = 0$ or $b = 0$.
In the case where $a \neq 0$ or $b \neq 0$, then the expansion of the norm is possible,

i.e. Let consider the following function

$$f(t) = \|\mathbf{a} + t\mathbf{b}\|^2, \tag{2.23}$$

where t is a scalar.Thus, $f(t) \geq 0$ for all t, since $f(t)$ is a sum of squares. Before expanding, it is important to know the following euclidean norms, inner products and the four properties.

The **Euclideam norm** of a vector **a**, denoted $\|\mathbf{a}\|$, is the square root of the sum of the squares if its elements.

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}_1^2 + \mathbf{a}_2^2 + \cdots + \mathbf{a}_n^2} \tag{2.24}$$

The **Euclideam norm** in terms of the **inner product** of a is:

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^T\mathbf{a}} \tag{2.25}$$

The four properties:

Homogeneity. $\|\varepsilon\mathbf{a}\| = \mid \varepsilon \mid \|\mathbf{a}\|$.

Triangle inequality. $\|\mathbf{a} + \mathbf{b}\| \leq \|\mathbf{a}\| + \|\mathbf{b}\|$.

Nonnegativity. $\|\mathbf{a}\| \geq 0$.

Definiteness. $\|\mathbf{a}\| = 0$ only if $x = 0$.

Now, the function (2.23) can now be expanded as

$$
\begin{aligned}
f(t) \quad &= (\mathbf{a} + t\mathbf{b})^T(\mathbf{a} + t\mathbf{b}) \\
&= \mathbf{a}\mathbf{a}^T + t\mathbf{b}^T\mathbf{a} + t\mathbf{a}^T\mathbf{b} + t^2\mathbf{b}^T\mathbf{b} \\
&= \|\mathbf{a}\|^2 + 2t\mathbf{a}^T\mathbf{b} + t^2\|\mathbf{b}\|^2
\end{aligned}
$$

Therefore, the Cauchy-Schwarz inequality

$$
\begin{aligned}
(\mathbf{a}^T\mathbf{b}) \quad &\leq \|\mathbf{a}\|\|\mathbf{b}\| \\
\|\mathbf{a} + \mathbf{b}\|^2 \quad &\leq \|\mathbf{a}\|^2 + 2\|\mathbf{a}\|\|\mathbf{b}\| + \|\mathbf{b}\|^2 \\
\|\mathbf{a} + \mathbf{b}\| \quad &\leq \|\mathbf{a}\| + \|\mathbf{b}\| \qquad (2.26)
\end{aligned}
$$

These is a very important property and it has been applied in the contribution chapters.

2.3 Second-order Cone Programming (SOCP)

A second order cone programming (SOCP) is a generalization of linear and quadratic programming which allows for affine combination of variables to be constrained inside second-order cone. This cone is the set of vectors of $\mathcal{R}^n$ such that the euclidian norm of the $(n-1)$ first components is less than or equal to the $n-th$ components. Linear programs, convex quadratic programs and quadratically constrained convex quadratic programs can all be formulated as SOCP problems.
Let us consider the second-order cone $\mathcal{K}_L$

$$\mathcal{K}_L = \mathbf{x} = (x_0; \overline{\mathbf{x}}) \in \Re^L : x_0 \geq \left\|\overline{\mathbf{x}}\right\| \tag{2.27}$$

Definition 2 The standard form Second-Order Cone Programming (SOCP) problem is

$$\begin{aligned} \min \quad & \mathbf{c_1}^T\mathbf{x_1} + \cdots + \mathbf{c_r}^T\mathbf{x_r} \\ s.t \quad & \mathbf{A_1x_1} + \cdots + \mathbf{A_rx_r} = \mathbf{b} \\ & \mathbf{x_i} \succeq_Q \mathbf{0}, \text{for} i = 1, \cdots, r \end{aligned} \tag{2.28}$$

The second-order cone dual problem is

$$\begin{aligned} \max \quad & \mathbf{b}^T\mathbf{y} \\ s.t \quad & A_i^T\mathbf{y} + \mathbf{z_i} = \mathbf{c_i}\text{for} i = 1, \cdots, r \\ & \mathbf{x_i} \succeq_Q \mathbf{0}, i = 1, \cdots, r \end{aligned} \tag{2.29}$$

The second-order constraint of this problem is usually encountered in the following form

$$\begin{aligned}\min_{\mathbf{x}} \quad & \Re\{\mathbf{f}^H\mathbf{x}\} \\ s.t \quad & \left\|\mathbf{A}_i^H\mathbf{x}+\mathbf{b}_i\right\| \leq \mathbf{c}_i^H\mathbf{x}+d_i, \text{for} i=1,\cdots,r \\ & F(\mathbf{x})=g,\end{aligned} \tag{2.30}$$

where $f \in \Re^n$, $\mathbf{A}_i \in \Re^{n\times n}, b_i \in \Re^{n_i}, c_i \in \Re^n, d_i \in \Re$, with x being the optimizing variable. $F(\mathbf{x})$ is a linear function of $\mathbf{x}$. When $\mathbf{A}_i = 0 \quad \text{for} \quad i = 1,\cdots,r$ SOCP is reduced to a general linear program. When $\mathbf{c}_i = 0 \quad \text{for} \quad i = 1,\cdots,r$ SOCP is equivalent to a convex quadratically constrained quadratic programming (QCQP). Therefore, SOCP falls between linear (LP) and quadratic (QP) programming and SDP. The SOCP constraints can be expressed in the LMI form, then recast the problem as a semidefinite programming.

The second order cone of the first constraint in (2.30),i.e.,
$\left\|\mathbf{A}_i^H\mathbf{x}+\mathbf{b}_i\right\| \leq \mathbf{c_i}^H\mathbf{x}+d_i$
is equivalent to the following LMI constraint:

$$\begin{bmatrix} \mathbf{c}_i^H\mathbf{x}+d_i & \mathbf{x}^H\mathbf{A}_i+\mathbf{b}_i^H \\ \mathbf{A}_i^H\mathbf{x}+\mathbf{b}_i & (\mathbf{c}_i^H\mathbf{x}+d_i)\mathbf{I}\end{bmatrix} \succeq 0.$$

We can cast problem (2.30) into a standard SDP form as:

$$\begin{aligned}\min_{\mathbf{x}} \quad & \Re\{\mathbf{f}^H\mathbf{x}\} \\ s.t \quad & \begin{bmatrix} \mathbf{c}_i^H\mathbf{x}+d_i & \mathbf{x}^H\mathbf{A}_i+\mathbf{b}_i^H \\ \mathbf{A}_i^H\mathbf{x}+\mathbf{b}_i & (\mathbf{c}_i^H\mathbf{x}+d_i)\mathbf{I}\end{bmatrix} \succeq 0.\end{aligned} \tag{2.31}$$

There are several software packages available that can handle SOCP problem. The recasted SDP can be solved using the numerical optimization packages, e.g., the SeDumi solver [37].

2.4 Robust Optimization

Most practical wireless systems try to learn the channel conditions but this raises issues of imperfection. Different optimization techniques are being modelled mathematically in order to formalise real world problems. This includes: assumptions and reduction of complexity. A small variation in the input can cause explosive effect in the output. Therefore, error handling methodology is required to deal with uncertainty. Ben-Tal et al. [38],[39] has suggested three errors for most reasons for data uncertainty. These are: Prediction, measurement and implementation. The most common error corresponds to the contribution of this thesis is the prediction. Prediction: The system has inaccurate knowledge about the parameters, therefore forecasts, often based on historical data, are used instead.

2.4.1 Uncertainty Models

Given a convex mathematical program in the following general formulation [40]:

$$\begin{aligned} \min \quad & f_0(x) \\ \text{subject to} \quad & f_i(x, u_i) \leq 0, i = 1, \cdots, n, \\ & x \in \mathcal{D} \end{aligned} \tag{2.32}$$

Where $f_0, f_1, \cdots, f_n$ are convex functions, $\mathcal{D} \subseteq \mathbb{R}^n$ is a convex set in Euclidean space, and $\mathbf{u} = (u_1, \cdots u_n)$ is a fixed parameter vector. The general bounded uncertainty based robust counterpart of this formulation is given by

$$\begin{aligned} \min \quad & f_0(x) \\ \text{subject to} \quad & f_i(x, u_i) \leq 0, \forall u_i \in \mathcal{U}, i = 1, \cdots, n, \\ & x \in \mathcal{D} \end{aligned} \tag{2.33}$$

where the parameter vector $\mathbf{u}$ is constrained to be in a set $\mathcal{U} = \mathcal{U} \times \cdots \times \mathcal{U} = \mathcal{U}^n$ called the uncertainty set. The main aim of (2.33) is to find $\mathbf{u}^*$ that minimises f_0 for all realisations of u_i within $\mathcal{U}$. The geometrical representation of $\mathcal{U}$ leads to the best tractability solutions and these include: ellipsoidal uncertainty mode set.

2.4.2 Ellipsoidal Uncertainty

By considering the LP problem in (2.20) and rewrite as follow:

$$\begin{aligned} \min \quad & c^T(x) \\ \text{subject to} \quad & a_i^T x \preceq b, \forall a_i \in \mathcal{U}a_i, \forall b_i \in \mathcal{U}b_i, i = 1, \cdots, m, \end{aligned} \tag{2.34}$$

where $\mathcal{U}b_i \subseteq \mathbb{R}$. The uncertainty $\mathcal{U}$ is the direct product of the partial uncertainty sets $\mathcal{U}a_i$ and this can be expressed as

$$\mathcal{U}a_i = \overline{a}_i + P_i u | \|u\| \leq 1, i = 1, \cdots, m, \tag{2.35}$$

where $P_i \in \mathbb{R}^{n\times n}$ and $\overline{a}_i \in \mathbb{R}^n, i = 1, \cdots, m$ are given. The model has the following advantages:

$\star$ The ellipsoid is a simple mathematical representation as shown in (2.35)

$\star$ If $P_i = \mathbf{I}$, then the uncertainty sets are exactly spheres.

$\star$ If $P_i = 0$, then $\overline{a}_i$ is fixed, and there is no uncertainty.

$\star$ In some of the cases where the uncertain data has an underlying stochastic model, the stochastic uncertainty can be replaced by a deterministic ellipsoidal uncertainty where the ellipsoid is represented using the mean and covariance matrix of the uncertain data [38]

2.5 The S-procedure

Different quadratic functions have different forms. Some quadratic functions can be negative whenever some other quadratic functions are all negative. In some cases, this constraint can be expressed as an LMI in the data defining the quadratic functions or forms; in other cases, we can form an LMI that is a conservative but often useful approximation of the constraint.
Let $F_0, \cdots, F_p$ be quadratic functions of the variables $\zeta \in \mathbf{R}^n$:

Lemma 1 (S-procedure [1], [41]): Let $F_i(\zeta)=\zeta^H \mathbf{T}_i \zeta + 2u_i^T\zeta + v_i \geq 0$, for $i = 0, 1$, where $\mathbf{T}_i \in \mathbb{H}^{M\times M}$ be complex Hermitian matrices, $\mathbf{b}_i \in \mathbb{C}^M$ and $c_i \in \mathbb{R}$.
Suppose that there exist an $\overline{\zeta} \in \mathbb{C}^M$ such that $F_i(\overline{\zeta}) < 0$.
Then the following two conditions are equivalent:

1. $F_0(\zeta) \geq 0$ for all ζ that satisfy $F_1(\zeta) \leq 0$;

Obviously if
there exist $\tau_1 \geq 0, \cdots \tau_p \geq 0$ such that
for all ζ, $F_0(\zeta) + \sum_{i=1}^{p} \tau_i F_i(\zeta) \geq 0$,
we then write,

$$\begin{bmatrix} \mathbf{T}_0 & \mathbf{u}_0 \\ \mathbf{u}_0^H & v_0 \end{bmatrix} + \tau \begin{bmatrix} \mathbf{T}_1 & \mathbf{u}_1 \\ \mathbf{u}_1^H & v_1 \end{bmatrix} \succeq \mathbf{0}.$$

The S-procedure is mostly used in system theory to obtain stability and performance results for nonlinear and under an appropriate uncertain systems. The S-procedure could also be applied in cases when it is not exact. The reason is that it can be used to obtain sufficient conditions in terms of Linear Matrix Inequalities (LMI) for a large number of nonconvex problems in systems analysis [42].

2.6 A fundamental to Multiple Antennas Techniques and Radio Channel

The implementation of multiple antennas in the last few decades have received extensive attention due to its capabilities to improve the overall wireless communication systems performance. Multiple antennas techniques (smart antennas) are able to reduce the interference in both downlink and uplink transmission modes.
Today, the base station can use multiple antennas techniques for radio transmission and reception. The implementation of these techniques at transmitter can provide diversity processing, which increases the received signal power and reduces the amount of fading. By implementing these techniques at both transmitter and receiver can provide multiplexing gain, so as to increase the data rate.
For beamforming, these techniques are aimed to increase the coverage of the cell. The thesis focus most on beamforming, and therefore, smart antenna techniques needs to be addressed in this contest.

The smart antenna techniques in MISO system have two fundamental principles of beamforming in a static network
1. Main beam pointing towards the desired user.
2. Null beam pointing towards the interfering user.

In a multi-users intra-system, where the users are randomly generated or multiple mobile users in a cellular network, these can cause challenges of directing the beam to a specific users. This becomes tedious and challenging. Different antennas techniques are most used to overcome these challenges. These include the following:
1. Directional antenna techniques
2. Adaptive antenna techniques

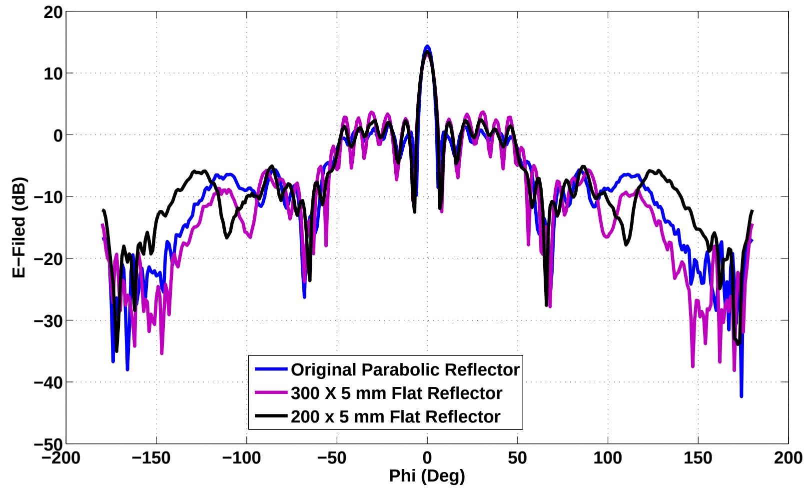

Figure 2.3: Parabolic and Flat antenna at the focal point.

2.6.1 Directional antenna techniques

Switched-beam antennas This is a simplified form of antenna techniques. Different directive antennas are used to generate a limited number of beams that point to a desired directions. This type of antenna is easy to be implemented, but it gives a limited improvement.Tracking at beam switching rate with a disadvantage of a very low gain between beams, limited interference suppression, false locking with shadowing, interference and wide angular spread [43]. Figure 2.3 show a simplified form of a mechanical switched antenna. **Steer-beam antennas(or dynamically phased arrays)** These antennas are arranged with a predefined patterns and the beam can be steered in any direction. The phase can be shifted using different set of angles i.e. The discrete transformation techniques, to allow the conversion from curved devices into flat devices as shown in **Figure** 2.4, then the transformed curve is used to steer the directional beam of the antenna at a desired angle and this is shown in Figure **Figure** 2.5 and 2.6.

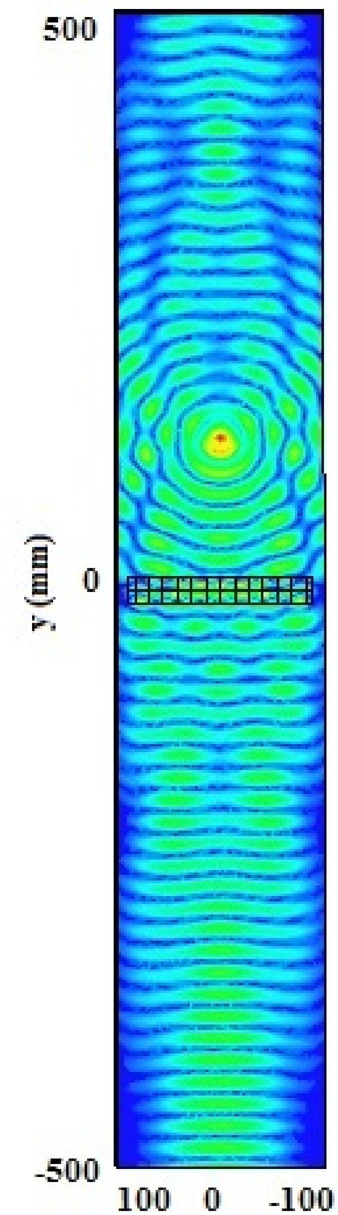

Figure 2.4: Conventional Lens

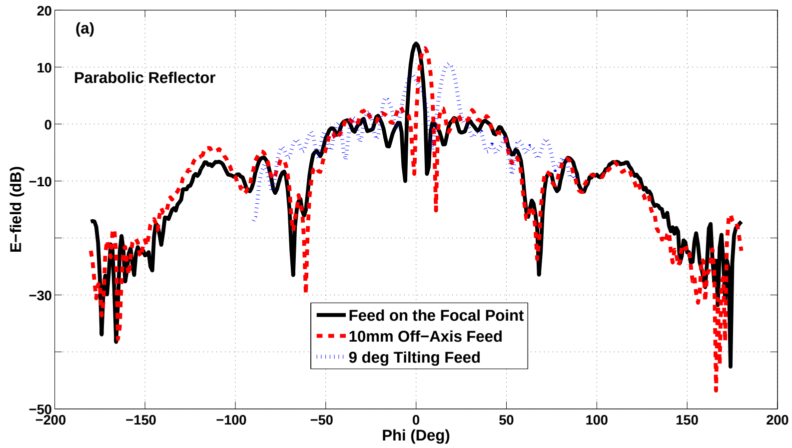

Figure 2.5: The feed tilted at 9 degree.

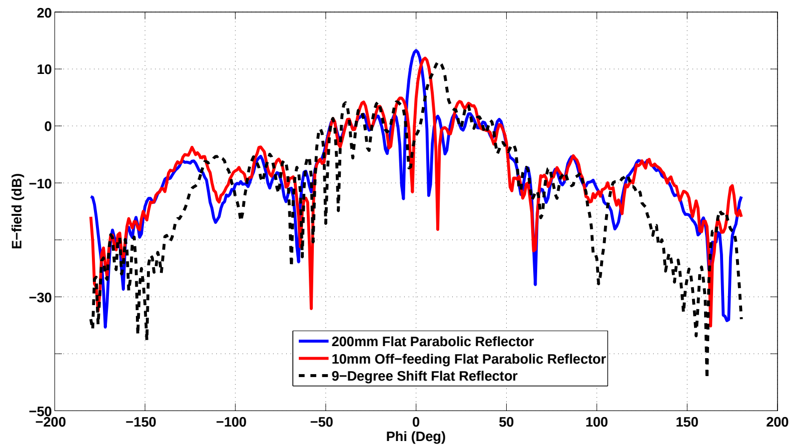

Figure 2.6: Flat antenna at 9 degree.

2.6.2 Adaptive antenna techniques

Adaptive antenna arrays, or smart antenna is a composition of N element arrays of equally spaced that produce desirable antenna patterns of the same amplitude in order to achieve improved performance over that of a single antenna. Such patterns have high gain in the direction of desired signals and nulls in the direction of interferencing signals. The other importance to the structure of the antenna patterns, provides degrees of freedom in the spatial domain compared to a single antenna. The most familiar concept in the 21st century is the advanced in signal processing, which different algorithm-s are being developed and exploited for a very wide range of use, such as signal enhancement, interference suppression. The interference suppression is a very important topic if the wireless community wish to meet the stringent demands of the next-generation wireless systems and beyond. A wide range of applications such as satellite navigation, biomedical engineering and wireless communications are becoming more concerned of the significant increase in mobile data traffic and the effect of energy efficiency. Therefore, beamforming and direction of arrival estimation based on antenna arrays will play an important role in fulfilling the increased demands of various mobile communication services.

2.6.3 Linear antenna array

Smart antennas are the structure of two or more antennas placed along a s-traight line to work in harmony and create a unique radiation pattern, which depends on certain factors, such as geometrical configuration of the array and this can be linear, circular, spherical, and conformal arrays. Also, depends on the distance, phase, amplitude and the relative pattern between the individual elements. The antenna elements are allowed to work in harmony by means of the array element phasing, which is performed with hardware or is carried out digitally [44]. In this section, a concept of a linear antenna array

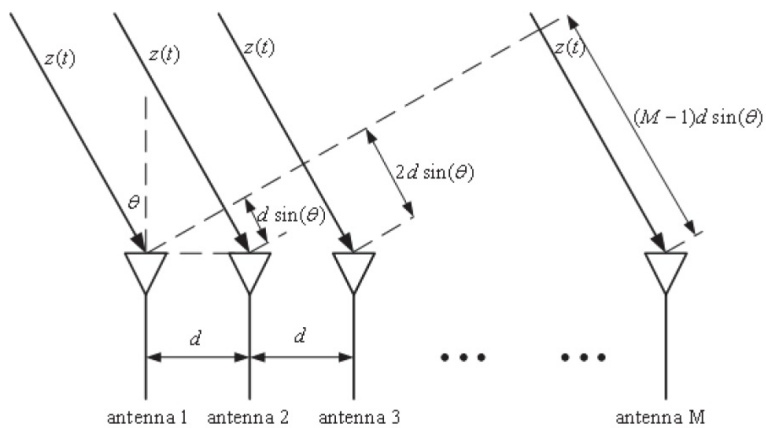

Figure 2.7: Schematic of a wavefront impinging across an antenna array. Under the narrowband assumption the antenna outputs are identical except for a complex scalar

in [45] is reviewed. Thorough treatments for all arrays of antennas can be found in [46] and [47].
Consider a signal wavefront, $z(t)$, impinging on an antenna array comprising M antennas spaced d apart each other at angle θ, shown in Figure. 2.7. It is assumed that the wavefront has a bandwidth B and is expressed as:

$$z(t) = \beta(t)\xi_i e^{j2\Pi v_c t} \tag{2.36}$$

where $\beta(t)$ is the complex envelope representation of the signal and v_c is the carrier frequency. ξ_i captures both effects of channel fading, i.e. fast and slow fading, and pathloss. Let T_z be the traveling time of the wavefront across any two adjacent antennas. It is clear that

$$T_z = \frac{d\sin\theta}{c} \tag{2.37}$$

where c is the speed of light.
The maximum time of the wavefront traveling along one array is assumed to be much smaller than the reciprocal of the bandwidth of all transmitted signals, i.e.,

$$B \ll \frac{1}{(M-1)T_z} \tag{2.38}$$

Assuming that antenna element patterns are identical. Provided the received signal at the first antenna is

$$z(t) = \beta(t)\xi_i e^{j2\Pi v_c t} \tag{2.39}$$

If the signal of the first antenna if $z(t)$. Then the signal of the second antenna will be transmitted after $z(t-T_z)$. Then, the signal at the second antenna is

$$z(t-T_z) = \beta(t-T_z)\xi_i e^{j2\Pi v_c (t-T_z)} \tag{2.40}$$

Under the narrowband assumption in 2.38, $B \ll 1/T_z$. It can be stated that

$$\beta(t-T_z) \approx \beta(t). \tag{2.41}$$

Using the relationship, $v_c/c = 1/\lambda_c$. It is clear that 2.37 can be rewritten as

$$v_c T_z = \sin\theta \frac{d}{\lambda_c} \tag{2.42}$$

where λ_c is the wavelength of the signal information. The equation 2.40 can be rewritten as

$$z(t - T_z) = \beta(t)e^{j2\pi v_c t}\xi_i e^{-j2\Pi \sin\theta \frac{d}{\lambda_c}} \tag{2.43}$$

$$z(t - T_z) = z(t)\xi_i e^{-j2\Pi \sin\theta \frac{d}{\lambda_c}} \tag{2.44}$$

Similarly, the signal at the kth antenna, i.e., $k = 1, 2, \cdots, M$ is given as

$$z_k(t - T_z) = z_k(t)\xi_i e^{-j2\Pi \sin\theta \frac{d}{\lambda_c}(k-1)} \tag{2.45}$$

From 2.39,2.43 and 2.45 , it can be seen that the signals at any two array elements are identical except for a phase shift which depends on the angle of arrival and the array geometry.
Consider a free field environment, i.e., neither scatterers and nor multipath. A planar continuous-wave wavefront of frequency v_c arriving from an angle θ will introduce a spatial signature across the antenna array. This spatial signature is a function of angle of arrival, antenna element patterns and antenna array geometry. The complex

$M \times 1$ vector, $\mathbf{a}(\theta) = [a_1(\theta) \quad a_2(\theta) \cdots \quad a_M(\theta)]^T$ is called the array response vector. For the linear antenna array with identical element patterns, the array response vector is given as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{-j2\Pi \sin\theta \frac{d}{\lambda_c}} \\ \\ \cdot \\ \cdot \\ \cdot \\ e^{-j2\Pi(M-1)\sin\theta \frac{d}{\lambda_c}} \end{bmatrix}$$

Similarly, it is possible to write the array response vector for a transmit linear antenna array with identical element patterns as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{-j2\Pi \sin\theta \frac{d}{\lambda_c}} & \cdots & e^{-j2\Pi(M-1)\sin\theta \frac{d}{\lambda_c}} \end{bmatrix} \tag{2.46}$$

Hence, the MISO channel between the antenna array and a user i can be written as

$$\mathbf{h}_i = \xi_i \mathbf{a}(\theta_i) \tag{2.47}$$

where θ_i is the angle of departure, with respect to the broadside of the antenna array, of the user i.

Using antenna arrays opens up a spatial dimension to improve capacities of wireless communication systems. This improvement is due to the fact that smart beam patterns can be shaped by controlling the phases of individual antennas of the array. Hence power-efficient beams can be steered towards intended users while minimum/non interference are imposed on unintended users. Smart beam patterns are performed via algorithms based on certain criteria. These algorithms can be implemented using hardware. However, it is more easily performed using software, i.e., using digital signal processing [44]. These criteria could be either minimising transmit power with constraints on users SINRs or maximising users sum rate with constraints on transmit power to name a few. In the following section, the first strategy, i.e., minimising transmit power under constraint of users SINR, is reviewed.

2.6.4 Multiuser downlink Beamforming

Let us consider a BS equipped with M antennas, and K single antenna users. The BS communicates with its K associated users using power-efficient beams to deliver their desired levels of SINR. Without loss of generality, the set of its locally active users are denoted $S_l = \{1, \cdots, U\}$ in cell q. Let $\mathbf{h}_i \in \mathbb{C}^{M \times 1}$ contains the channel coefficients between the BS q and the users $i \in S_l$. The received signal at user i is given by

$$y_i = \mathbf{h}_i^H \mathbf{w}_i s_i + \sum_{j=1, j \neq i}^{U} \mathbf{h}_i^H \mathbf{w}_j s_j + n_i, \tag{2.48}$$

where $\mathbf{w}_i \in \mathbb{C}^{M \times 1}$ and s_i are, respectively, the beamforming vector and the data symbol associated to user i, $n_i \sim \mathbb{CN}(0, \sigma^2)$ is zero mean circularly symmetric complex Gaussian noise at user i. Letting the average energy in transmitting the ith symbol s_i be normalized to unity, i.e., $\mathbb{E}_{s_i}(| s_i |^2) = 1$, one can express the SINR at any local user $i \in S_l$ as

$$\text{SINR}_i = \frac{| \mathbf{h}_i^H \mathbf{w}_i |^2}{\sum_{j \neq i} | \mathbf{h}_i^H \mathbf{w}_j |^2 + \sigma^2}, \tag{2.49}$$

A common class of optimal transmit downlink beamforming for multiple users is to calculate or find the optimum downlink beamforming vectors $\mathbf{w}_i$ that minimises the total transmit power while guaranteing all users SINR requirements and this is given for BS q, as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{i \in s_l} \mathbf{w}_i^H \mathbf{w}_i \\ s.t \quad & \text{SINR}_i \geq \gamma_i, \forall_i \in S_l, \end{aligned} \tag{2.50}$$

where γ_i is the SINR level and in (2.50) it is assumed to be feasible. It can be verified that the SINR constraints in (2.50) are non-convex. In the next section, a technique to reformulate (2.50) in SOCP and SDP forms is presented.

2.6.5 SOCP and SDP algorithms

In this section, the two standard conic programs of SOCP and SDP developed in [48] are used to cast (2.50) in convex form. These are reviewed. Let

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{h}_U \end{bmatrix} \text{ and } \quad \mathbf{W} = [\mathbf{w}_1 \quad \mathbf{w}_2 \cdots \mathbf{w}_U]$$

In introducing a real slack variable P_0, (2.50) can be rewritten as [48]:

$$\begin{aligned} \min_{\mathbf{W},P_0} \quad & P_o \\ \text{s.t} \quad & \frac{\mid [\mathbf{HW}]_{i,i} \mid^2}{\sum_{j=1 j\neq i}^{U} \mid [\mathbf{HW}]_{i,j} \mid^2 + \sigma^2} \geq \gamma_i, \qquad \forall i \in S_l \\ & Tr\left(\mathbf{WW}^H\right) \leq P_0 \end{aligned} \tag{2.51}$$

where $[\mathbf{X}]_{i,j}$ represents the (i,j)-th entry of matrix $\mathbf{X}$. Taking this into account, the i-th SINR constraints can now be recast in standard form. Rearranging the constraints and using matrix notations, the constraints yield

$$\frac{1}{\gamma_i} \mid [\mathbf{HW}]_{i,i} \mid^2 \geq \sum_{j=1 j\neq i}^{U} \mid [\mathbf{HW}]_{i,j} \mid^2 + \sigma^2, \qquad \forall i \in S_l \tag{2.52}$$

Adding $\mid [\mathbf{HW}]_{i,i} \mid^2$ to both sides results in

$$\left(1+\frac{1}{\gamma_i}\right) \mid [\mathbf{HW}]_{i,i} \mid^2 \geq \sum_{j=1}^{U} \mid [\mathbf{HW}]_{i,j} \mid^2 + \sigma^2, \qquad \forall i \in S_l \tag{2.53}$$

Equivalently,

$$\left(1+\frac{1}{\gamma_i}\right) \mid [\mathbf{HW}]_{i,i} \mid^2 \geq \quad \left\| \begin{bmatrix} \mathbf{H}^H \mathbf{W}^H \mathbf{e}_i \\ \sigma^2 \end{bmatrix} \right\|^2 \tag{2.54}$$

One can verify the fact that an arbitrary phase rotation can be added to the beamformers without affecting the SINR constraints and objective of 2.51. In other words, if $\mathbf{W}$ is optimal solution to 2.51 then $\mathbf{W}\mathrm{Diag}e^{\{j\phi\}}$ where ϕ for $i = 1, \cdots, \mathbf{U}$ are arbitrary phases, is also an optimal solution. Therefore $\mathbf{W}$ can be selected in such a manner that $[\mathbf{HW}]_{i,i} > 0$, i.e., $[\mathbf{HW}]_{i,i}$ can be chosen to be real, for all i without the loss of generality. Since $[\mathbf{HW}]_{i,i} > 0$, $\forall i$, taking the square root of the equation 2.54 leads to

$$\sqrt{\left(1+\frac{1}{\gamma_i}\right)} \mid [\mathbf{HW}]_{i,i} \mid \geq \quad \left\| \begin{bmatrix} \mathbf{H}^H \mathbf{W}^H \mathbf{e}_i \\ \sigma^2 \end{bmatrix} \right\| \tag{2.55}$$

Using vec(.) operator, one can cast the power constraint of 2.51 as

$$p \geq \left\| \mathrm{Vec}\left(\mathbf{W}\right) \right\| \tag{2.56}$$

where $p = \sqrt{P_0}$. Therefore, problem 2.51 can be reformulated in a SOCP form as

$$\min_{\mathbf{W},p} \quad p$$
$$\text{s.t} \quad \left\| \begin{bmatrix} \mathbf{H}^H\mathbf{W}^H\mathbf{e}_i \\ \sigma^2 \end{bmatrix} \right\| \leq \sqrt{\left(1+\frac{1}{\gamma_i}\right)} \left[\mathbf{HW}\right]_{i,i}, \forall 1 \leq i \leq U \quad (2.57)$$
$$\left\|\text{Vec}\left(\mathbf{W}\right)\right\| \leq p. \quad (2.58)$$

By applying the concept in 2.31, the SOCP in 2.58 can be written in a SDP form as

$$\min_{\mathbf{W},p} \quad p$$
$$\text{s.t} \quad \begin{bmatrix} \sqrt{\left(1+\frac{1}{\gamma_i}\right)} \left[\mathbf{HW}\right]_{i,i} & \begin{bmatrix} \mathbf{e}_i^T\mathbf{HW} & \sigma^2 \end{bmatrix} \\ \begin{bmatrix} \mathbf{H}^H\mathbf{W}^H\mathbf{e}_i \\ \sigma^2 \end{bmatrix} & \sqrt{\left(1+\frac{1}{\gamma_i}\right)} \left[\mathbf{HW}\right]_{i,i} \mathbf{I} \end{bmatrix} \succeq 0 \quad (2.59)$$
$$\begin{bmatrix} p & \text{vec}^H(\mathbf{W}) \\ \text{vec}(\mathbf{W}) & p\mathbf{I} \end{bmatrix} \succeq 0. \quad (2.60)$$

Solving 2.58 or 2.59 provides the optimal beamforming matrix $\mathbf{W}$ and the optimal downlink power as p^2. Beamformer for user i can be obtained as the ith column of $\mathbf{W}$.

2.6.6 Semidefinite relaxation algorithm

Since its introduction in the early years of 2000, the semidefinite relaxation has become an effective approximation technique that relaxes some of the constraints of the optimization problem such that the relaxed problem is easier to solve than the original problem and many practical experiences have suggested and indicated the accuracy of the approximation [49], [50], [51] and references therein. This section outlines a method to cast 2.50 in a convex form using SDR technique. Let $\mathbf{R}_i = \mathbf{h}_i^H\mathbf{h}_i$ and $\mathbf{F}_i = \mathbf{w}_i\mathbf{w}_i^H$. It is clear that $F_i, \forall 1 \leq i \leq U$ is a positive semidefinite and Hermitian matrix. Further more the rank of the matrix is one. The multiuser downlink beamforming problem in 2.50 can be expressed as

$$\begin{aligned}
\min_{\mathbf{w}_i} \quad & \sum_{i \in s_l}^{U} \mathbf{w}_i^H \mathbf{w}_i \\
\text{s.t} \quad & \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{j=1, j \neq i}^{U} \mathbf{w}_j^H \mathbf{R}_i \mathbf{w}_j + \sigma^2}, \forall 1 \leq i \leq U.
\end{aligned} \tag{2.61}$$

Recall the following equality

$$\mathbf{x}^H \mathbf{A} \mathbf{x} = \mathrm{Tr}(\mathbf{A} \mathbf{x} \mathbf{x}^H) \tag{2.62}$$

If $\mathbf{A} = \mathbf{I}$ then

$$\mathbf{x}^H \mathbf{x} = \mathrm{Tr}(\mathbf{x} \mathbf{x}^H). \tag{2.63}$$

Rearrange the *ith* SINR constraints of 2.61, one can arrive at

$$\left(1 + \frac{1}{\gamma_i}\right) \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_i) - \sum_{j=1, j \neq i} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_j) - \sigma^2 \geq 0. \tag{2.64}$$

The problem 2.61 can be posed as

$$\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=l}^{U} \mathrm{Tr} \mathbf{F}_i \\
\text{s.t} \quad & \frac{1}{\gamma_i} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_i) - \sum_{j=1, j \neq i} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_j) - \sigma^2 \geq 0. \\
& \mathbf{F}_i = \mathbf{F}_i^H \succeq 0 \\
& \mathrm{Rank}(\mathbf{F}_i) = 1, \forall 1 \leq i \leq U
\end{aligned} \tag{2.65}$$

The second constraints in 2.66 is to guarantee that $\mathbf{F}_i$, $\forall 1 \leq i \leq U$, is a positive semidefinite and Hermitian matrix. Dropping the last constraints in 2.66, i.e., $\text{rank}(\mathbf{F}_i) = 1$, results in a SDP form, i.e.,

$$\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=l}^{U} \text{Tr}\mathbf{F}_i \\
\text{s.t} \quad & \frac{1}{\gamma_i}\text{Tr}(\mathbf{R}_i\mathbf{F}_i) - \sum_{j=1, j\neq i} \text{Tr}(\mathbf{R}_i\mathbf{F}_j) - \sigma^2 \geq 0. \\
& \mathbf{F}_i = \mathbf{F}_i^H \succeq 0, \forall 1 \leq i \leq U
\end{aligned}$$

Dropping these rank one constraints not only enlarges the feasible set of the problem 2.66 but also leads to a relaxed SDP problem.This relaxation is referred to as semidefinite relaxation technique. For general nonconvex quadratic problems, solving a SDR problem usually gives an optimal solution with rank of larger than one. In such cases, SDR can only provide a lower bound on the optimal objective function and possibly attain an approximate solution to the original problem [52].When using SDR results in $\mathbf{F}_i$ solutions with ranks higher than one, a randomization procedure, e.g., [53], [54], [55], [56], [57] and references therein, can be used to find approximate rank-one solutions.

2.6.7 Robust Downlink Beamforming

The imperfection is estimated of the covariance matrice $\mathbf{R}_i$ and the uncertainty matrice $\mathbf{\Delta}_i$ represents its associated estimation error. Letting the average energy in transmitting the ith symbol s_i be normalized to unity, i.e., $\mathbb{E}_{s_i}(\mid s_i \mid^2) = 1$, one can express the SINR at any local user $i \in S_l$ in 2.61 as

$$\text{SINR}_i = \frac{\mathbf{w}_i^H(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{w}_i}{\sum_{j\neq i} \mathbf{w}_j^H(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{w}_j + \sigma_i^2}, \tag{2.66}$$

where the uncertainty in the estimation of channel covariance matrices is confined in a spherical set defined as

$$\left\|\mathbf{\Delta}_i\right\|_F \leq \varepsilon_i, \forall_i \in S_l, \tag{2.67}$$

where ε_i is the non-negative value indicating the radii of the spherical uncertainty region. The optimization problem that ensures that the data rate of the cell edge users, remains above the required level is introduced as

$$\begin{aligned} \max_{\mathbf{w}_i} \quad & \min_{\|\mathbf{\Delta}_i\| \leq \varepsilon_i} \log_2(1 + \mathrm{SINR}_i) \\ \text{s.t.} \quad & \sum_{i \in s_l} \mathbf{w}_i^H \mathbf{w}_i \leq P_i. \end{aligned} \tag{2.68}$$

The constraint in (2.68) is the total power constraint of P at BS q. By introducing a slack variable k, one can rewrite problem (2.68) as

$$\begin{aligned} \min_{\mathbf{w}_i, k} \quad & k \\ \text{s.t.} \quad & -(\min_{\|\mathbf{\Delta}_i\|_F \leq \varepsilon_i} \log_2(1 + \mathrm{SINR}_i)) \leq k \\ & \sum_{i \in s_l} \mathbf{w}_i^H \mathbf{w}_i \leq P_i, \end{aligned} \tag{2.69}$$

Using the equality in (2.62),then (2.66) can be rewritten as

$$\mathrm{SINR}_i = \frac{\mathrm{Tr}[(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{F}_i]}{\sum_{j \neq i} \mathrm{Tr}[(\mathbf{R}_i + \mathbf{\Delta}_i)\mathbf{F}_j] + \sigma_i^2}. \tag{2.70}$$

Since, the diagonal entries of $\mathbf{F}_i$ are non-negative and the Frobenious norm is sub-multiplicative, i.e., $\rho(\mathbf{\Delta}_i) \leq \|\mathbf{\Delta}_i\|_F \leq \varepsilon_i$, where $\rho(\mathbf{\Delta}_i)$ is the spectral radius of $\mathbf{\Delta}_i$, we can write

$$\mathrm{Tr}(\mathbf{\Delta}_i \mathbf{F}_i) \leq \varepsilon_i \mathrm{Tr}(\mathbf{F}_i). \tag{2.71}$$

Hence, the first constraint in problem (2.69) can be rewritten as

$$\frac{\mathrm{Tr}[(\mathbf{R}_i - \varepsilon_i \mathbf{I})\mathbf{F}_i]}{\sum_{j \neq i} \mathrm{Tr}[(\mathbf{R}_i + \varepsilon_i \mathbf{I})\mathbf{F}_j] + \sigma_i^2} \geq (2^{-k} - 1), \tag{2.72}$$

$$\frac{\mathrm{Tr}[(\mathbf{R}_i - \varepsilon_i \mathbf{I})\mathbf{F}_i]}{(2^{-k} - 1)} \geq \sum_{j \neq i} \mathrm{Tr}[(\mathbf{R}_i + \varepsilon_i \mathbf{I})\mathbf{F}_j] + \sigma_i^2, \tag{2.73}$$

which can be expanded as

$$\begin{aligned}
&\frac{\mathrm{Tr}[(\mathbf{R}_i - \varepsilon_i \mathbf{I})\mathbf{F}_i]}{(2^{-k} - 1)} + \mathrm{Tr}[(\mathbf{R}_i + \varepsilon_i \mathbf{I})\mathbf{F}_i] \geq \\
&\sum_{j \in i} \mathrm{Tr}[(\mathbf{R}_i + \varepsilon_i \mathbf{I})\mathbf{F}_j] + \sigma_i^2, && (2.74)\\
&(1 + \frac{1}{2^{-k} - 1})\mathrm{Tr}(\mathbf{R}_i \mathbf{F}_i) + (1 - \frac{1}{2^{-k} - 1})\mathrm{Tr}(\varepsilon_i \mathbf{I} \mathbf{F}_i) \geq \\
&\sum_{j \in i} \mathrm{Tr}[(\mathbf{R}_i + \varepsilon_i \mathbf{I})\mathbf{F}_j] + \sigma_i^2.. && (2.75)
\end{aligned}$$

The optimization problem in (2.68) can now be formed as

$$\begin{aligned}
\min_{\mathbf{F}_i, k} \quad & k && (2.76)\\
s.t \quad & 2^{-k}\mathrm{Tr}(\mathbf{R}_i \mathbf{F}_i) \geq \sum_{j \in s_l} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_j)(2^{-k} - 1) + q, \\
& \sum_{i \in s_l} \mathrm{Tr}(\mathbf{F}_i) \leq P_i, \\
& \mathbf{F}_i = \mathbf{F}_i^H \succeq 0, \\
& \mathrm{Rank}(\mathbf{F}_i) = 1,
\end{aligned}$$

where

$$\begin{aligned} q = \quad & (\sum_{j \in s_l} \mathrm{Tr}(\varepsilon_i \mathbf{I} \mathbf{F}_j) + \sigma_i^2)(2^{-k} - 1) + \\ & \mathrm{Tr}((\varepsilon_i \mathbf{I} - \varepsilon_i \mathbf{I}(2^{-k} - 1))\mathbf{F}_i). \end{aligned}$$

The relaxed SDP problem is obtained by dropping the rank one constraint and enlarges the feasible set of the problem in (2.76).

$$\begin{aligned} \min_{\mathbf{F}_i, k} \quad & k \\ s.t \quad & 2^{-k} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_i) \geq \sum_{j \in s_l} \mathrm{Tr}(\mathbf{R}_i \mathbf{F}_j)(2^{-k} - 1) + q, \\ & \sum_{i \in s_l} \mathrm{Tr}(\mathbf{F}_i) \leq P_i, \\ & \mathbf{F}_i = \mathbf{F}_i^H \succeq 0, \end{aligned} \tag{2.77}$$

where

$$\begin{aligned} q = \quad & (\sum_{j \in s_l} \mathrm{Tr}(\varepsilon_i \mathbf{I} \mathbf{F}_j) + \sigma_i^2)(2^{-k} - 1) + \\ & \mathrm{Tr}((\varepsilon_i \mathbf{I} - \varepsilon_i \mathbf{I}(2^{-k} - 1))\mathbf{F}_i). \end{aligned}$$

Therefore, (2.77) is exactly equivalent to the original problem (2.68). This fact has been confirmed in [58]. The authors of [58] noticed that the solution to (2.77) always admits rank-one matrices $\mathbf{F}_i$, $\forall_i$, which directly yields the solution to (2.68) using $\mathbf{F}_i = \mathbf{w}_i \mathbf{w}_i^H$.

2.6.8 Conclusion

Convex optimisation theory and techniques used in the majority of this thesis has been presented. The concepts of convex set, cone, convex functions, semidefinite programming, linear programs, linear matrix inequality, schur complements, cauchy-schwarz inequality, second-order cone programming, robust optimization and the S-procedure have been introduced. An optimisation problem to calculate transmit beamformers for multiple active users in a single-cell scenario is sketched and principles of beamforming via linear antenna array along with concepts of second order cone programming and semidefinite programming are oultlined. A powerful technique known as convex relaxation, which allows non-convex problems to be relaxed into convex problems was reviewed. Finally, a robust downlink beamforming in the presence of uncertainty has been demonstrated and discussed.

Chapter 3

Robust Downlink Beamforming With Imperfect CSI

The cooperation of base stations at beamforming level lead to the best efficient use of the resources and improve the performance of the wireless cellular network. This chapter covers an optimum multicell downlink beamforming that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information (CSI). The aim is to ensure that the worst-cases of the signal-to-interference-plus-noise ratio (SINR) at each user remains above the required level. The imperfection in CSI is modeled to be bound within an ellipsoidal set, which is used to model the uncertainty between the true and the estimated channel coefficients. The original non-convex optimization problem is formulated using the S-procedure, then cast the original non-convex problem into a tractable formulation with convex constraints in linear matrix inequality (LMI) form and solve it using the standard semidefinite relaxation (SDR). The results confirm the effectiveness of the proposed robust scheme, in terms of power efficiency at BSs, compared with the conventional method in the presence of imperfect CSI.

3.1 Introduction

Coordinated multi-point (CoMP) communications has been envisaged to be used in the LTE-advanced as an effective mean of confronting the intercell interference [59]. It is aimed to deliver uniformly very high data rates with significantly improved spectral efficiency to the user terminals irrespective of their locations within the cellular network. In a perfect CoMP, the base stations (BSs) share their data and the global channel state information (C-SI) in a central unit to transmit jointly to the users [60]. However, such a centralized processing requires an additional resources of an ideal backhaul, which is prohibited in practical systems as it limits the scalability of the network. Recently, the research interests have been shifted towards as much as possible decentralization of the CoMP systems to relax the backhaul overhead [61].
Furthermore, using multiple antenna at BSs for spatial multiplexing in CoM-P demands the provision of CSI at the transmitter, i.e., CSIT, for effective downlink beamforming towards the user terminals. However, the estimation of CSIT, i.e., CSI at a BS as the transmitter in the downlink, is imperfect due to several reasons, such as estimation error, delay and the quantization error, e.g., as a result of limited feedback from a user terminal. Hence, in addition to decentralization, robust design methodologies against channel uncertainties is another key factor in achieving the promised gains of CoM-P in practical scenarios. In robust formulation of a cellular beamforming problem, the true CSIT is considered to be confined within an uncertainty region and the beamforming vectors are designed such that they remain feasible despite the imperfections in the estimated CSIT [62]. Such problems typically lead to optimization problems with infinite number of constraints and reformulating them to tractable equivalent forms is a challenging task.
Some robust design techniques for cellular networks have been developed in [63], [64], [65], [62] to cite a few. Authors in [63] minimize the transmitted power subject to the worst-case quality-of-service (QoS) constraints per

user. The worst-case solution is achieved using lagrange duality, which explicitly takes into account the positive-semidefinite property of the downlink covariance matrices. It has been assumed that the transmitter has erroneous covariance based channel state information. In [64], a robust beamforming design problem for a MISO downlink channel is proposed. In that scheme, the aim is to minimize the weighted sum power of the BSs under imperfect channel state information subject to SINR constraints in a downlink multiuser MISO system. A semidefinite programming (SDP) relaxation method [66] is considered to be tight under norm-constrained CSIT errors when base station is equipped with two antennas provided that the size of channel errors is sufficiently small. Moreover, the approach in [63] and [64] ignore the overall interference power on the outer-cell users due to transmissions to the locally active users within a cell.

The chapter aims to address the issues of distributiveness and robustness in cellular networks. A distributed optimization problem is formulated that minimizes a combination of total transmitted power at a BS, used to maintain a certain level of quality of service within the corresponding cell, and its aggregate interference induced on the users of the other cells. The algorithmic solution to this problem can be concurrently used at individual BSs of a multicell network over a shared bandwidth to significantly improve the network spectral efficiency. The proposed formulation can be considered as decentralized in a sense that each BS within a cell only transmits data towards the users within the same cell. However, in order to control the induced intercell interference, the BS obtains the CSI between itself and the other vulnerable users of the other cells through feedback channels. As such a feedback may require message passing between the BSs, the proposed scheme may be considered as a partially decenteralised solution. Further, the proposed formulation involves robustness against CSIT uncertainties which are assumed to be confined within the ellipsoidal sets. As the original robust counterpart is intractable, use the S-Procedure and the standard semidefinite relaxation to reformulate the problem in robust convex semidefinite programing form with linear matrix inequality (LMI) constraints [1]. The rest of this chapter is organised as follows: Section 3.2 introduces system model and problem formulation. Section 3.3 presents robust downlink beamforming. Simulation results are presented in section 3.4. Finally, section 3.5 concludes the chapter.

3.2 System model and problem formulation

Consider a cellular scenario of N_c cells. It is assumed that each BS of each cell is equipped with M antennas, and K single antenna users. Within the network, it is assumed that the same carrier frequency is used by the adjacent base stations to support their intracell users. Each BS communicates with its K associated users using power-efficient beams to deliver their desired levels of SINR and restricts the resulting interference on the other users of the adjacent cells. Without loss of generality, denote the set of locally active users as $S_l = \{1, \cdots, U\}$ in cell q and the set of the other users, who are present in the adjacent cells and might be subjected to inter-cell interference because of the transmission by the q-th BS, as $S_o = \{1, \cdots, N\}$. Let $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ contains the channel coefficients between the BS q and the users $i \in S_l$. Let $\mathbf{g}_t \in \mathbb{C}^{M\times 1}$ contains the channel coefficients between the BS q and user $t \in S_o$. The received signal at user $i \in S_l$ is given by

$$y_i = \mathbf{h}_i^H \mathbf{w}_i s_i + \sum_{j=1, j\neq i}^{U} \mathbf{h}_i^H \mathbf{w}_j s_j + v_i + n_i, \tag{3.1}$$

where $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and s_i are, respectively, the beamforming vector and the data symbol associated to user i, $n_i \sim \mathbb{CN}(0, \sigma^2)$ is zero mean circularly symmetric complex Gaussian noise at user i and v_i is the resulting intercell interference seen by user i due to the transmissions of the BSs in the adjacent cells i.e., other than cell q. Letting the average energy in transmitting the ith symbol s_i be normalized to unity, i.e., $\mathbb{E}_{s_i}(\mid s_i \mid^2) = 1$, one can express the SINR at any local user $i \in S_l$ as

$$\mathrm{SINR}_i = \frac{|\ \mathbf{h}_i^H \mathbf{w}_i\ |^2}{\sum_{j \neq i} |\ \mathbf{h}_i^H \mathbf{w}_j\ |^2 + \xi_i + \sigma^2}, \tag{3.2}$$

where $\xi_i = E[|v_i|^2]$ is the imposed total intercell interference power on user i. Furthermore, it is assumed that any i-th local user, i.e., $i \in S_l$, can measure the arrived outer-cell interference power ξ_i and report it to its local BS. The resulting optimization problem that calculate the optimum downlink beamforming vectors at any given BS q, can be written as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mathbf{w}_i^H \mathbf{g}_t \mathbf{g}_t^H \mathbf{w}_i + \sum_{i \in s_l} \eta_i \mathbf{w}_i^H \mathbf{w}_i \\ s.t \quad & \mathrm{SINR}_i \geq \gamma_i, \forall_i \in S_l, \end{aligned} \tag{3.3}$$

where γ_i and η_i are, respectively, the SINR level and the (cost) weighting factor required by an active local user $i \in S_l$ and μ_t is the weighting factor considered for the neighboring outer-cell user t by the BS q. The scheduler uses these coefficients to set the priority levels that depends on the quality (cost) of requested services by different users or to proportionally maintain fairness among the users. The first of the objective function in (3.3) indicates the overall interference power on the outer-cell users due to the transmission of BS q to its locally active users, while the second term is the total signal power transmitted by the BS q.

3.3 Robust downlink beamforming

Let $\hat{\mathbf{g}}_t \in \mathbb{C}^{M \times 1}$ and $\hat{\mathbf{h}}_i \in \mathbb{C}^{M \times 1}$be the estimated CSIs at the BSs. Then the true CSI can be expressed as

$$\begin{aligned} \mathbf{g}_t &= \hat{\mathbf{g}}_t + \mathbf{e}_t, \quad \forall t \in S_o, \\ \mathbf{h}_i &= \hat{\mathbf{h}}_i + \mathbf{e}_i, \quad \forall i \in S_l, \end{aligned} \tag{3.4}$$

where $\mathbf{e}_t \in \mathbb{C}^M$ and $\mathbf{e}_i \in \mathbb{C}^M$ represent the CSI error vectors. It is assumed that these error vectors are bound within the ellipsoidal sets defined as

$$\mathbf{e}_t^H \mathbf{Q}_t \mathbf{e}_t \leq 1, \qquad \forall t \in S_o, \tag{3.5}$$

$$\mathbf{e}_i^H \mathbf{Q}_i \mathbf{e}_i \leq 1, \qquad \forall i \in S_l, \tag{3.6}$$

where $\mathbf{Q}_t \in \mathbb{C}^{M\times M}$ and $\mathbf{Q}_i \in \mathbb{C}^{M\times M}$ are a positive definite matrix and characterizes the shape and size of the ellipsoid. Substituting for $\mathbf{h}_t$ from (3.4) in (3.3), we can write

$$\begin{aligned}
\min_{\mathbf{w}_i} \max_{\mathbf{e}_t^H \mathbf{Q}_t \mathbf{e}_t \leq 1} \quad & \sum_{t\in s_o}\sum_{i\in s_l} \mu_t \mathbf{w}_i^H (\hat{\mathbf{g}}_t + \mathbf{e}_t)(\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{w}_i + \\
& \sum_{i\in s_l} \eta_i \mathbf{w}_i^H \mathbf{w}_i \\
\text{s.t} \min_{\mathbf{e}_i^H \mathbf{Q}_i \mathbf{e}_i \leq 1} \quad & \frac{|\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i\,|^2}{\sum_{j\in s_l j\neq i} |\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j\,|^2 + \xi_i + \sigma^2} \\
& \geq \gamma_i, \qquad \forall i \in S_l.
\end{aligned} \tag{3.7}$$

By introducing a slack variable k, one can rewrite (3.7) as

$$\min_{\mathbf{w}_i, k} \quad k + \sum_{i\in s_l} \eta_i \mathbf{w}_i^H \mathbf{w}_i \tag{3.8}$$

$$\begin{aligned}
s.t \quad & \frac{|\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i\,|^2}{\sum_{j\in s_l j\neq i} |\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j\,|^2 + \xi_i + \sigma^2} \geq \gamma_i, \\
& \mathbf{e}_i^H \mathbf{Q}_i \mathbf{e}_i \leq 1, \qquad \forall i \in S_l,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
& \sum_{t\in s_o}\sum_{i\in s_l} \mu_t \mathbf{w}_i^H (\hat{\mathbf{g}}_t + \mathbf{e}_t)(\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{w}_i \leq k, \\
& \mathbf{e}_t^H \mathbf{Q}_t \mathbf{e}_t \leq 1, \qquad \forall t \in S_o.
\end{aligned} \tag{3.10}$$

The first constraint in (3.8) can be expanded according to (3.11), where $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H$ is a positive semidefinite matrix with unit rank.

$$\begin{aligned}
&\frac{(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i\mathbf{w}_i^H(\hat{\mathbf{h}}_i + \mathbf{e}_i)}{\gamma_i} - \sum_{j\in s_l j\neq i}(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j\mathbf{w}_j^H(\hat{\mathbf{h}}_i + \mathbf{e}_i) \geq \xi_i + \sigma^2,\\
&\left(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H\right)\left(\frac{\mathbf{W}_i}{\gamma_i} - \sum_{j\in s_l j\neq i}\mathbf{W}_j\right)\left(\hat{\mathbf{h}}_i + \mathbf{e}_i\right) \geq \xi_i + \sigma^2, \qquad \forall i \in S_l,
\end{aligned} \tag{3.11}$$

Using the expansion in (3.11), the objective function and its constraints in (3.8) can be written as

$$\begin{aligned}
\min_{\mathbf{W}_i,k} \quad & k + \sum_{i\in s_l}\eta_i Tr(\mathbf{W}_i) \qquad\qquad (3.12)\\
s.t \quad & (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{U}_i(\hat{\mathbf{h}}_i + \mathbf{e}_i) \geq \xi_i + \sigma^2,\\
& \mathbf{e}_i^H\mathbf{Q}_i\mathbf{e}_i \leq 1,\\
& -\sum_{t\in s_o}\sum_{i\in s_l}\mu_t(\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{W}_i(\hat{\mathbf{g}}_t + \mathbf{e}_t) + k \geq 0,\\
& \mathbf{e}_t^H\mathbf{Q}_t\mathbf{e}_t \leq 1, \qquad \forall t \in S_o,\\
& \mathrm{Rank}(\mathbf{W}_i) = 1, \qquad \forall i \in S_l,\\
& \mathbf{W}_i \succeq 0, \quad i = 1\ldots,U,
\end{aligned}$$

where

$$\mathbf{U}_i = \left(\frac{\mathbf{W}_i}{\gamma_i} - \sum_{j\in s_l j\neq i}\mathbf{W}_j\right). \tag{3.13}$$

In the sequel, use the S-Procedure, to rewrite the constraints in (3.12) that involve quadratic inequalities in error vectors in the LMI forms. **Lemma 1** (S-procedure [1], [41]): Let $\Phi_i(\mathbf{e}) = \mathbf{e}^H\mathbf{A}_i\mathbf{e} + \mathbf{b}_i^H\mathbf{e} + \mathbf{e}^H\mathbf{b}_i + c_i \geq 0$, for $i =$ 0, 1, where $\mathbf{A}_i \in \mathbb{H}^{M\times M}$ be complex Hermitian matrices, $\mathbf{b}_i \in \mathbb{C}^M$ and $c_i \in \mathbb{R}$.

Suppose that there exist an $\overline{\mathbf{e}} \in \mathbb{C}^M$ such that $\Phi_i(\overline{\mathbf{e}}) < 0$.

Then the following two conditions are equivalent:

1. $\Phi_0(\mathbf{e}) \geq 0$ for all $\mathbf{e}$ that satisfy $\Phi_1(\mathbf{e}) \leq 0$;
2. There exist a $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A}_0 & \mathbf{b}_0 \\ \mathbf{b}_0^H & c_0 \end{bmatrix} + \lambda \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} \succeq \mathbf{0}.$$

By applying Lemma 1, the constraints in (3.12) can be expressed as

$$\begin{aligned} &\mathbf{e}_i^H \mathbf{U}_i \mathbf{e}_i + \hat{\mathbf{h}}_i^H \mathbf{U}_i \mathbf{e}_i + \mathbf{e}_i^H \mathbf{U}_i \hat{\mathbf{h}}_i + \hat{\mathbf{h}}_i^H \mathbf{U}_i \hat{\mathbf{h}}_i - \xi_i - \sigma^2 \\ &\geq 0, \\ &\mathbf{e}_i^H \mathbf{Q}_i \mathbf{e}_i \leq 1, \qquad \forall i \in S_l, \end{aligned} \tag{3.14}$$

$$\begin{aligned} &-\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mathbf{e}_t^H \mathbf{W}_i \hat{\mathbf{e}}_t - \sum_{t \in s_o} \sum_{i \in s_l} \mu_t \hat{\mathbf{g}_t^H} \mathbf{W}_i \mathbf{e}_t \\ &-\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mathbf{e}_t^H \mathbf{W}_i \hat{\mathbf{g}}_t - \sum_{t \in s_o} \sum_{i \in s_l} \mu_t \hat{\mathbf{g}_t^H} \mathbf{W}_i \hat{\mathbf{g}}_t + k \geq 0, \\ &\mathbf{e}_t^H \mathbf{Q}_t \mathbf{e}_t \leq 1, \qquad \forall t \in S_o. \end{aligned} \tag{3.15}$$

Hence, recast (3.14) and (3.15) as a linear matrix inequality (LMI) and this is shown as,

$$\begin{aligned} \mathbf{\Upsilon_i} = \begin{bmatrix} \mathbf{U}_i + \lambda_i \mathbf{Q}_i & \mathbf{U}_i \hat{\mathbf{h}_i} \\ \hat{\mathbf{h}_i^H} \mathbf{U}_i & \hat{\mathbf{h}_i^H} \mathbf{U}_i \hat{\mathbf{h}_i} - \xi_i - \sigma^2 - \lambda_i \end{bmatrix} \succeq \mathbf{0}, \\ \forall i \in S_l, \end{aligned} \tag{3.16}$$

$$\begin{aligned} \mathbf{\Upsilon_t} = &\begin{bmatrix} -\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mathbf{W}_i + \lambda_t \mathbf{Q}_t & -\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mathbf{W}_i \hat{\mathbf{g}}_t \\ -\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \hat{\mathbf{g}_t^H} \mathbf{W}_i & -\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \hat{\mathbf{g}_t^H} \mathbf{W}_i \hat{\mathbf{g}}_t + k - \lambda_t \end{bmatrix} \\ &\succeq \mathbf{0}, \qquad \forall t \in S_o, \end{aligned}$$

where λ_i and λ_t are the slack variable for all $i = 1 \ldots \mathbf{U} \in S_l$, and $t = 1 \ldots N \in S_o$. Replace the constraints in (3.12) with (3.16) and taking into account the slackness conditions with $\lambda_i \geq 0$. Problem (3.12) can then be written as

$$\begin{aligned} \min_{\mathbf{W}_i,k} \quad & k + \sum_{i \in s_l} \eta_i Tr(\mathbf{W}_i) \\ s.t \quad & \boldsymbol{\Upsilon}_{\mathbf{i}} \succeq \mathbf{0}, \qquad \forall i \in S_l, \\ & \boldsymbol{\Upsilon}_{\mathbf{t}} \succeq \mathbf{0}, \qquad \forall t \in S_o, \\ & \lambda_i \geq, \lambda_t \geq \mathbf{0}, \qquad\qquad \forall_{i,t} \\ & \text{Rank}(\mathbf{W}_i) = 1, \\ & \mathbf{W}_i \succeq \mathbf{0}, \qquad i = 1 \ldots, U. \end{aligned} \tag{3.17}$$

The key point to (3.17) is the fourth constraint, which is a non-convex. It is not easy to solve such a constraint. Therefore, if the obtained optimal solution is rank one, as it is in this case and is the solution to (3.17). Otherwise the general rank is relaxed for some $\mathbf{w}_i$ and we apply the same optimal conditions that were proved in [41]. The optimization problem in (3.17) is in the standard semidefinite programming form and it is in fact convex, and then using the numerical optimization packages, e.g., the SeDumi solver [37], the resulting convex problem can be solved efficiently. For comparison purposes, the robust coordinated beamforming (CBF) is computed using the coordinated beamforming developed in [67] for instantaneous channel. The conventional robust beamforming is obtained by setting $\mu_t = 0$, $\forall t \in S_o$, and problem (3.3) is reduced to a general form of the nominal optimization problem but with the error in the estimation of the instantaneous channel.

3.4 Simulation results

3.4.1 Simulation setup

This section presents some graphical examples illustrating the performance of the proposed formulation against the centralized robust coordinated beamforming approach and the conventional robust approach. The users are distributively generated on the verge of the 3 adjacent cells named critical areas of the neighbouring base stations. Fig. 3.1 illustrates an example of one user distribution with 3 users. Monte Carlo simulations are carried out over 50 users distribution with 10000 channel realizations. Generate 10000 sets of CSI errors satisfying the error bound, $\left\|\mathbf{e}_t\right\|^2 \leq \varepsilon^2$ until a fine stable averaged transmitted power is achieved. To simulate the channel, it is recommended to try to separate the distance between the base stations and indeed for the users in order to avoid other user from receiving a null signal. Not only the simulation setup consider the effect of small-scale fading caused by two antennas separated by a fractional of a meter but also, it considers large-scale fading caused by shadowing conditions. Therefore, the following channel model setup in [41] is used and it is given as

$$\mathbf{g}_t = 10^{-(128.1+37.6\log_{10}(l))/20}.\psi_t.\varphi_t.(\hat{\mathbf{g}}_t + \mathbf{e}_t), \tag{3.18}$$

where l is the distance between the t BS and t user, ψ_t is the shadowing, φ_t is the antenna gain, $\hat{\mathbf{h}}_t$ and $\mathbf{e}_t$ denote the estimated CSI channel and the CSI error respectively. A spherical error as in [41] is considered, with $\mathbf{Q}_t = \varepsilon_t^{-2}\mathbf{I}_{mt}$ for all t where the CSI error satisfy bounds $\varepsilon_t > 0$. If not mentioned specifically, all error radii ε_t are the same and equal to ε. The rest of the parameters, which are based on the LTE standard are shown in the table 3.4.1.

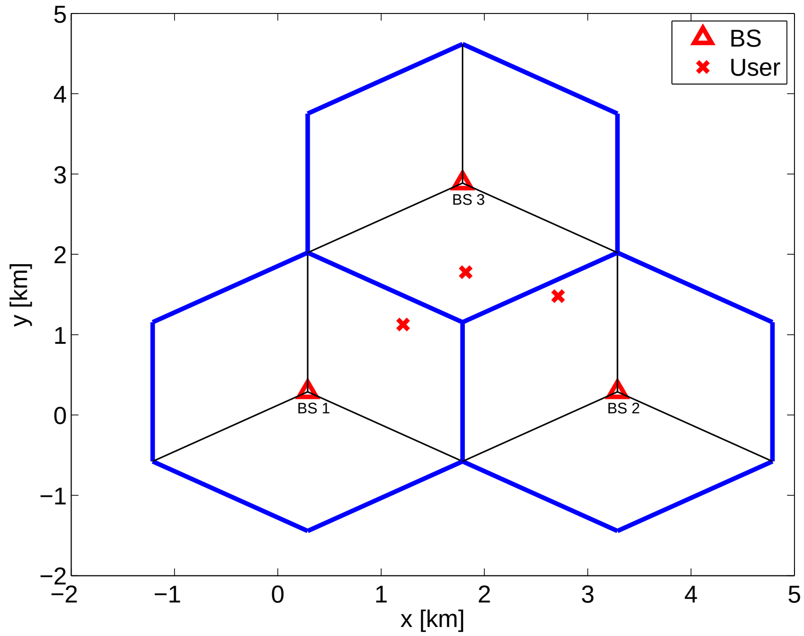

Figure 3.1: An example of user distributions used in Monte-Carlo simulations.

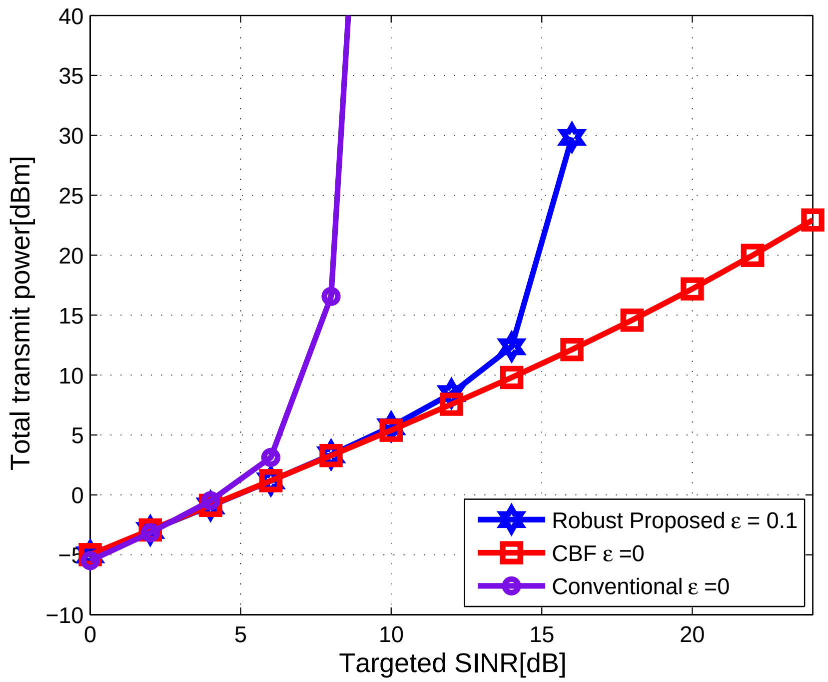

Figure 3.2: Total transmit power versus targeted SINR for N = 3, M = 6, K = 3.

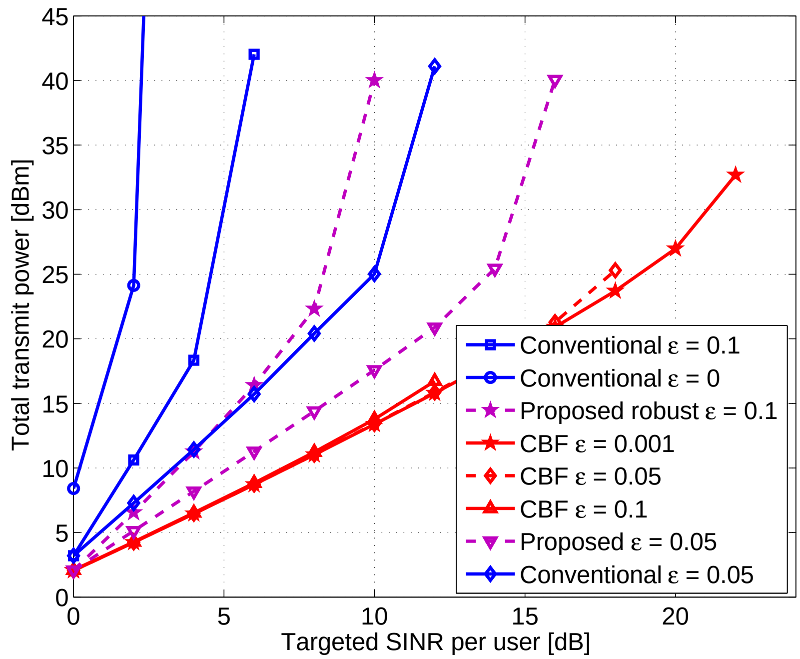

Figure 3.3: Total transmit power versus targeted SINR for N = 2, M = 6, K = 4.

Table 3.1: Simulation parameters

Parameter	**Value**
Number of cells (N)	3
Number of users per cell (K)	1 or 2
Number of antennas per cell (M)	6
Antenna spacing	$\lambda/2$
Array antenna gain	15 dBi
Downlink carrier frequency	2 GHz
Noise power spectral density (all users)	-174 dBm/Hz
Noise figure at user receiver	5 dB
BS-to-BS's distance	3 km
Path loss model (l in km)	$128.1 + 37.6\log_{10}(l)$
Angular offset's standard deviation	$2°$
Log-normal shadowing's standard deviation	8 dB
Number of scatterers per user	5
Subchannel bandwidth's wide	15 kHz

3.4.2 Performance evaluation

In this section, the performance of the robust downlink beamforming with imperfect CSI is analysed and compared with the conventional robust downlink and the robust coordinated beamforming. The problem is formulated into a tractable formulation in a linear matrix inequality (LMI). Then recast the formulated LMI in a semidefinite program [1]. The SeDuMi solver [37] is used to compute the optimal solution. BSs are equipped with multiple antennas and a single antenna per user. For different beamforming techniques in Fig. 3.2 and 3.3, the performance is measured of the change in result of the sum-transmit power as a function of SINR levels at the cell user terminals. Performance evaluation and comparison of Fig. 3.2 with Fig. 3.3 are carried out for the same error bound, $\varepsilon = 0.1$ in achieving the same SINR level. it can be observed that the formulated robust downlink for the 3 users in three cells in Fig. 3.2 is more power efficient than the

4 users in two cells in Fig. 3.3. It can be observed from Fig. 3.2 for the target SINR $\gamma = 10$ dB consumes approximately 6 dBm while in Fig. 3.3, 32 dBm. Now each Figure is evaluated separately. The results obtained in Fig. 3.2 showed that from 0 to 8 dB of the SINR target, the conventional, CBF and robust downlink beamforming with imperfect CSI were almost indistinguishable in terms of transmit power. There is a slight change from 8 to 12 dB, where the robust downlink and robust CBF-centralized continue to have the same performance but there is a 18.85 % increase in terms of transmission power for the conventional beamforming. Therefore, using the robust downlink beamforming with imperfect CSI is recommended instead of the conventional beamforming when CSI error is taken into account in the case of one user per cell. Finally, in Fig. 3.3 it can be observed that the conventional downlink requires higher transmit power than the proposed robust downlink with imperfect CSI and the CBF-centralized in every point of the SINR level. When $\gamma = 10$ dB for the error bound, $\varepsilon = 0.1$, the formulated robust downlink with imperfect CSI design failed beyond the 10 dB SINR level. By decreasing the error bound to 0.05, there was an improvement in terms of transmit power for the proposed robust downlink. As observed, the target SINR of robust downlink beamforming with imperfect CSI degrades rapidly with the channel estimation error. To compare the performance, the formulated robust downlink with imperfect CSI outperform the conventional design. With an average transmit power of 10.45 dBm for the error bound, $\varepsilon = 0.05$, the design can attain 12 dB.

3.5 Conclusion

This chapter presented an overview of robust downlink beamforming with imperfect CSI. The goal was to minimize a combination of the sum power of each BS across the network subject to the worst-case of the SINR level of individual users. The method showed that the proposed robust downlink beamforming with imperfect CSI can be reformulated into a tractable formulation using S-procedure and linear matrix inequality (LMI). It is shown that when accounting for the CSI errors at both the overall intercell interference power in the subject function and the SINR constraints, a feasible solution for certain sets of SINR and the optimal solution were achieved. Simulation results have confirmed that our proposed robust downlink with imperfect CSI outperform the conventional in terms of transmit power even when CSI errors are taken into account. In addition, it is also observed that applying different values of the error bound improved the transmitted power. On the other hand, the increase in error bound reduces the SINR level of the users but at a cost of increased transmit power at the base station.

Chapter 4

SOCP Robust Downlink Beamforming

4.1 Second-Order Cone Programming For Robust Downlink Beamforming With Imperfect CSI

This chapter addresses the problem of downlink multicell processing (MCP) when a channel state information is used to design transmit beamforming. This kind of design entails the complexity that requires extra signalling overhead. However, the channel state information (CSI) may be subjected to estimation and quantization errors. To acquire reduction in signalling overhead, the study proposes a robust multicell downlink beamforming that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information, whilst also guaranteeing that the worst-cases of the signal-to-interference-plus-noise ratio (SINR) remains above the required level. A spherical uncertainty set is used to model the imperfection in CSI between the true and the estimated channel coefficients. The original non-convex problem is formulated as the second-order cone programming (SOCP) and then recast the convex constraints in linear matrix inequality (LMI) form. Also, a coordinated beamforming with imperfect instantaneous CSI is reformulated based on spherical uncertainty set using the standard semidefinite relaxation (SDR). Simulation results confirm the efficiency of the proposed method at BSs, compared with the conventional method in the presence of imperfect CSI to validate the theoretical analysis.

4.2 Introduction

The use of channel state information in the downlink multicell processing has brought a burden on signalling overhead between base stations (BSs) and their user terminals, when the number of cooperative transmissions increases. It has been reported that to avoid the extra overhead outweighing the cooperative gain, a method that independently allows users to have a choice over transmission mode between coherent coordinated multi-point (CoMP) and non-CoMP [68]. It aimed to mitigate the adverse effect of training overhead on the downlink throughput of the CoMP system through the use of the transmission mode selection based on the attained statistical channel information at the user side. In a perfect CoMP, the BSs share their data and the global channel state information (CSI) in a central unit to transmit jointly to the users [60], [69]. However, such a centralized processing requires an additional resource of an ideal backhaul, which is prohibited in practical systems as it limits the scalability of the network. Recently, the research interests have been shifted towards as much as possible decentralization of the CoMP systems to relax the backhaul overhead [61], [70], [71]. Furthermore, in the centralized CoMP all signal processing tasks are carried out by a central unit which requires the exchange of information and control signals among the coordinated base stations. The exchanged information should contain full

CSI and message signals, for effective downlink beamforming towards the user terminals. However, the acquisition of CSI at the transmitter, i.e., at a BS, (CSIT) is imperfect due to a number of factors such as estimation error, delay in channel and the quantization error, e.g., caused by limited feedback from a user terminal to the BS. Hence, in addition to decentralization, robust design against channel uncertainties is another key factor in achieving the promised gains of CoMP in practical scenarios. In robust formulation of a cellular beamforming problem, the true CSIT is considered to be confined within an uncertainty region and the beamforming vectors are designed such that they remain feasible despite the imperfections in the estimated CSIT [62]. Such problems typically lead to optimization problems with infinite number of constraints and reformulating them in tractable equivalent forms is a challenging task. Recently, different approaches have been developed for robust multiple-input single-output (MISO) transmit beamforming. In [72], authors study multi-group multicasting using trace bounds on the covariance mismatches and minimize the total transmitted power subject to the worst-case user quality-of service constraint. In this section, semidefinite relaxation (SDR) is used to approximate the original non-convex worst-case beam-forming problem. A conservative version of robust beamforming problem based on semi-infinite second-order cone program is studied in [73] and [74] using Lorentz-Positive Maps and Quadratic Matrix Inequalities.

In [75] and [76], the authors minimize the transmit power in the presence of imperfect CSI, with confined uncertainty within a Euclidean ball, subject to achieving a lower bound on the received signal-to-interference-plus noise ratio at user terminals. Most of the above studies are based on robust optimization and in [77] have concluded that for a given optimization problem, the design for robust counterpart may significantly suffer from two major difficulties: (i) cast the original non-convex problem into a tractable formulation using approximations, and, (ii) once the tractable formulation is obtained an increase in the complexity of the robust counterpart is seen as

compared to the original problem. Due to the two major difficulties, finding an effective method to design a beamforming to obtain an efficient transmit power can be very challenging due to computational cost and efficiency. From the above challenges, only few authors have proposed different approaches in the study of the comparison between the SDP and SOCP. For i.e., the author in [77] study the signal-to-interference-noise-ratio balancing in multicell multiple-input single-output (MISO) downlink and assumed that there is an imperfection in the CSI. The authors first solved the non-convex problem in semidefinite relaxation method, then proposed SOCP based design method and then offered comparison performance between the two methods.
However, these approaches ignore the overall interference power on the outer-cell users due to transmissions to the locally active users within a cell.
This chapter considers a robust distributed approach where each BS within a cell only sends data to its local users, while controlling the induced interference on the other users of the other cells. This approach can significantly reduce the signaling overhead in comparison with its fully centralized counterpart.
To achieve this goal, a robust multicell downlink beamforming is proposed that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference, induced on the other users of the adjacent cells in the presence of imperfect channel state information, subject to guaranteeing the worst-cases of the signal-to-interference-plus-noise ratio (SINR) remain above a prescribed threshold. It is assumed that the imperfection in CSI between the true and the estimated channel coefficients is confined within a spherical uncertainty set. The original non-convex robust problem is reformulated as a second-order cone programming (SOCP), then, the SOCP convex constraints are transformed into linear matrix inequality (LMI) forms. The proposed scheme is then compared against a centralized scheme. To achieve this, a coordinated beamforming scheme proposed in [67] is refor-

mulated, with imperfect instantaneous CSI using the standard semidefinite relaxation [66].

This chapter is organized as follows: Section 4.3 presents second order cone programming formulation and coordinated beamforming with imperfect instantaneous CSI . Simulation results are presented in section 4.5. Finally, subsection 4.6 concludes the section.

4.3 Second-Order Cone Programming For Robust Downlink Beamforming With Imperfect CSI

In the following, the formulation of the original problem to SOCP is presented. Firstly, the overall interference power on the outer-cell users of problem (3.7) is considered and this can be expressed as follows
Problem in (3.7) can be equivalently rewritten as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & q(\mathbf{w}_i, \mathbf{e}_t) + \sum_{i \in s_l} \eta_i \mathbf{w}_i^H \mathbf{w}_i \\ \text{s.t} \quad & \frac{\mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i \mid^2}{\sum_{j \in s_l j \neq i} \mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j \mid^2 + \xi_i + \sigma^2} \\ & \geq \gamma_i, \qquad \forall i \in S_l. \end{aligned} \tag{4.1}$$

where

$$q(\mathbf{w}_i, \mathbf{e}_t) = \max_{\left\|\mathbf{e}_t\right\| \leq \varepsilon_t} \sum_{t \in s_o} \sum_{i \in s_l} \left| \, (\hat{\mathbf{g}}_t^H \mathbf{w}_i + \mathbf{e}_t^H \mathbf{w}_i) \, \right|^2 \tag{4.2}$$

We assume that $\mathbf{Q}_t$ is given as $\mathbf{Q}_t = \varepsilon_t^{-2}\mathbf{I}_M$. The maximization term, $\mathbf{e}_t^H \mathbf{Q}_t \mathbf{e}_t \leq 1$, can be reexpressed as $\left\|\mathbf{e}_t\right\| \leq \varepsilon_t$, where ε_t represents the size of the sphere. The upper-bound in (4.2) is satisfied when $\mathbf{e}_t$ lies within the maximum bound when $\|\mathbf{e}_t\| = \varepsilon_t$. Therefore, we write the function that maximizes over $\mathbf{e}_t$ as follow

$$q(\mathbf{e}_t) = \max_{\|\mathbf{e}_t\| \leq \varepsilon_t} \sum_{t \in s_o} \sum_{i \in s_l} \mid (\hat{\mathbf{g}}_t^H \mathbf{w}_i + \mathbf{e}_t^H \mathbf{w}_i) \mid^2 \tag{4.3}$$

Therefore, for $\mathbf{Q}_t = \varepsilon_t^{-2} \mathbf{I}_M$ for all t where the CSI error satisfy bounds $\varepsilon_t > 0$, problem (3.5) and (3.6) are reduced to be spherically bounded. We reexpress (3.5) and (3.6) as $\|\mathbf{e}_i\|_2 \leq \varepsilon_i$ and $\|\mathbf{e}_t\|_2 \leq \varepsilon_t$, where ε_i and ε_t are the nonnegative values of the uncertainty region. Using the triangle inequality and the Cauchy-Schwarz inequality, and the fact that $\max_{\|\mathbf{e}_t\|_2 \leq \varepsilon_t} \mid \mathbf{e}_t^H \mathbf{w}_t \mid = \varepsilon_t \|\mathbf{w}_t\|_2$, one can write problem (4.3)

$$\begin{aligned} &\sum_{t \in s_o} \sum_{i \in s_l} \mu_t \mid (\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H) \mathbf{w}_i \mid^2 \\ &\leq \sum_{t \in s_o} \sum_{i \in s_l} \mu_t (\mid \hat{\mathbf{g}}_t^H \mathbf{w}_i \mid + \varepsilon_t \|\mathbf{w}_i\|_2)^2. \end{aligned} \tag{4.4}$$

The right hand side (RHS) of (4.4) can be expanded as

$$
\begin{aligned}
&\sum_{t\in s_o}\sum_{i\in s_l}\mu_t(\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid +\varepsilon_t\|\mathbf{w}_i\|_2)^2\\
&=\sum_{t\in s_o}\sum_{i\in s_l}\mu_t(\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid^2 +\varepsilon_t^2\|\mathbf{w}_i\|_2^2+\\
&2\varepsilon_t\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid \|\mathbf{w}_i\|_2)\\
&\leq\sum_{t\in s_o}\sum_{i\in s_l}\mu_t(\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid^2 +\varepsilon_t(\varepsilon_t+2\|\hat{\mathbf{g}}_t\|_2)\|\mathbf{w}_i\|_2^2)\\
&=\sum_{t\in s_o}\sum_{i\in s_l}\mu_t(\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid^2 +\varepsilon_t(\varepsilon_t+2\sqrt{\hat{\mathbf{g}}_t^H\hat{\mathbf{g}}_t})\mathbf{w}_i^H\mathbf{w}_i)\\
&=\sum_{t\in s_o}\sum_{i\in s_l}\mu_t\mid \hat{\mathbf{g}}_t^H\mathbf{w}_i\mid^2+\sum_{t\in s_o}\sum_{i\in s_l}\mu_t\mathbf{w}_i^H m_t\mathbf{w}_i,
\end{aligned}
\tag{4.5}
$$

where $m_t=\varepsilon_t{}^2+2\varepsilon_t\sqrt{\hat{\mathbf{g}}_t^H\hat{\mathbf{g}}_t}$ and the inequality in (4.5) is due to the application of Cauchy-Schwarz inequality, i.e., $\mid \mathbf{e}_t^H\mathbf{w}_i\mid\leq\|\mathbf{e}_t\|\|\mathbf{w}_i\|\leq\varepsilon_t\|\mathbf{w}_i\|$.

Furthermore, one can write

$$
\begin{aligned}
&\mid (\hat{\mathbf{h}}_i^H+\mathbf{e}_i^H)\mathbf{w}_i\mid^2\\
&\geq(\mid \hat{\mathbf{h}}_i^H\mathbf{w}_i\mid-\mid \mathbf{e}_i^H\mathbf{w}_i\mid)^2\\
&\geq\mid \hat{\mathbf{h}}_i^H\mathbf{w}_i\mid^2+\mathbf{w}_i^H(\varepsilon_i^2-2\varepsilon_i\sqrt{\hat{\mathbf{h}}_i^H\hat{\mathbf{h}}_i})\mathbf{w}_i
\end{aligned}
\tag{4.6}
$$

and

$$
\begin{aligned}
&\mid (\hat{\mathbf{h}}_i^H+\mathbf{e}_i^H)\mathbf{w}_j\mid^2\\
&\leq(\mid \hat{\mathbf{h}}_i^H\mathbf{w}_j\mid+\mid \mathbf{e}_i^H\mathbf{w}_j\mid)^2\\
&\leq\mid \hat{\mathbf{h}}_i^H\mathbf{w}_j\mid^2+\mathbf{w}_j^H(\varepsilon_i^2+2\varepsilon_i\sqrt{\hat{\mathbf{h}}_i^H\hat{\mathbf{h}}_i})\mathbf{w}_j.
\end{aligned}
\tag{4.7}
$$

Apply the lower bound and the upper bound derived in (4.6) and (4.7), respectively, in the constraints of the optimization problem introduced in (4.1) to account for the worst case of SINR constraints in terms of uncertainties in CSI. Using the upper bound derived in (4.5) in the objective function, one can rewrite the problem in (4.1) as

$$
\begin{aligned}
\min_{\mathbf{w}_i} \quad & \sum_{t\in s_o}\sum_{i\in s_l}\mu_t \left|\, \hat{\mathbf{g}}_t^H \mathbf{w}_i \,\right|^2 + \sum_{t\in s_o}\sum_{i\in s_l}\mu_t \mathbf{w}_i^H m_t \mathbf{w}_i + \\
& \sum_{i\in s_l}\eta_i \mathbf{w}_i^H \mathbf{w}_i \qquad\qquad (4.8)\\
\text{s.t} \quad & \frac{\mid \hat{\mathbf{h}}_i^H \mathbf{w}_i \mid^2 + \mathbf{w}_i^H m_i \mathbf{w}_i}{\sum_{j\neq i}\mid \hat{\mathbf{h}}_i^H \mathbf{w}_j \mid^2 + \sum_{j\neq i}\mathbf{w}_j^H m_{ii}\mathbf{w}_j + \xi_i + \sigma^2} \\
& \geq \gamma_i, \qquad \forall i \in S_l,
\end{aligned}
$$

where

$$
m_i = {\varepsilon_i}^2 - 2\varepsilon_i\sqrt{\hat{\mathbf{h}}_i^H\hat{\mathbf{h}}_i},
$$

$$
m_{ii} = {\varepsilon_i}^2 + 2\varepsilon_i\sqrt{\hat{\mathbf{h}}_i^H\hat{\mathbf{h}}_i}.
$$

Furthermore, one can expand the constraint in (4.8) as

$$
\left(1+\frac{1}{\gamma}\right)\left|\, \hat{\mathbf{h}}_i^H \mathbf{w}_i \,\right|^2 \geq \sum_{j\in s_l}\left|\, \hat{\mathbf{h}}_i^H \mathbf{w}_j \,\right|^2 + q_i, \tag{4.9}
$$

where

$$
q_i = \sum_{j\in s_l}\mathbf{w}_j^H m_{ii}\mathbf{w}_j - \mathbf{w}_i^H\left(m_{ii} + \frac{m_i}{\gamma}\right)\mathbf{w}_i + \xi_i + \sigma^2.
$$

Let us define $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_U]$, $\mathbf{W}_s = \mathbf{W}\mathrm{Diag}[\sqrt{\eta_1}, \cdots, \sqrt{\eta_U}]$, the indicator vectors $\mathbf{x}_i$ as a $U\times 1$ vector with unity at the ith dimension and zeros elsewhere and $\mathbf{x}_t$ as a $N\times 1$ vector with unity at the tth dimension and zeros elsewhere, where $N = (N_c - 1)U$. Then, $\sum_{i\in s_l}\mathbf{w}_i^H\mathbf{w}_i = \mathrm{Tr}(\mathbf{W}^H\mathbf{W}) = \left\|\mathbf{W}\right\|_F^2$ and $\sum_{i\in s_l}\eta_i\mathbf{w}_i^H\mathbf{w}_i = \left\|\mathbf{W}_s\right\|_F^2$. Hence, using (4.9), one can restate the problem in (4.8) as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{t\in s_o}\sum_{i\in s_l}\mu_t \left| \mathbf{x}_t^T\hat{\mathbf{G}}^H\mathbf{W}\mathbf{x}_i \right|^2 + \\ & \sum_{t\in s_o}\mu_t m_t \left\|\mathbf{W}\right\|_F^2 + \left\|\mathbf{W}_s\right\|_F^2 \\ \text{s.t} \quad & \left(1+\frac{1}{\gamma}\right) \left| \mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i \right|^2 \geq \sum_{j\in s_l} \left| \mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_j \right|^2 + \\ & q_i, \end{aligned} \tag{4.10}$$

where

$$\begin{aligned} m_t = & \quad {\varepsilon_t}^2 + 2\varepsilon_t\sqrt{\mathbf{x}_t^T\hat{\mathbf{G}}^H\hat{\mathbf{G}}\mathbf{x}_t}, \\ m_i = & \quad {\varepsilon_i}^2 - 2\varepsilon_i\sqrt{\mathbf{x}_i^T\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{x}_i}, \\ m_{ii} = & \quad {\varepsilon_i}^2 + 2\varepsilon_i\sqrt{\mathbf{x}_i^T\hat{\mathbf{H}}^H\hat{\mathbf{H}}\mathbf{x}_i}, \\ q_i = & \quad -\left(m_{ii}+\frac{m_i}{\gamma}\right)\mathbf{x}_i^T\mathbf{W}^H\mathbf{W}\mathbf{x}_i + \\ & \quad m_{ii}\left\|\mathbf{W}\right\|_F^2 + \xi_i + \sigma^2, \end{aligned}$$

where $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, ...\hat{\mathbf{h}}_U]$ and $\hat{\mathbf{G}} = [\hat{\mathbf{g}}_1, ...\hat{\mathbf{g}}_N]$ denote the set of complex channel of locally active users and a set of complex channel of the users in the adjacent cells. It is observed that for an optimal $\mathbf{W}$ satisfying problem (4.10), $\mathbf{W}diag[x^{j\psi_1}, ...x^{j\psi_U}]$, where ψ_i is an arbitrary phase, is also an optimal solution. Therefore, one can design the beamforming matrix $\mathbf{W}$ up to an arbitrary phase scaling and the scalar $\mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i$ is non-negative and real. By introducing a slack nonnegative variable P_o. Problem (4.10) can be recast as

$$\begin{aligned}
\min_{\mathbf{w}_i, P_o} \quad & P_o \\
\text{s.t} \quad & \left(1+\frac{1}{\gamma}\right) \left| \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_i \right|^2 \geq \sum_{j \in s_l} \left| \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_j \right|^2 + \\
& q_i, \\
& \sum_{t \in s_o} \sum_{i \in s_l} \mu_t \left| \mathbf{x}_t^T \hat{\mathbf{G}}^H \mathbf{W} \mathbf{x}_i \right|^2 + \\
& \sum_{t \in s_o} \mu_t m_t \left\| \mathbf{W} \right\|_F^2 + \left\| \mathbf{W}_s \right\|_F^2 \leq P_o.
\end{aligned} \tag{4.11}$$

The constraints in (4.11) can be expressed as

$$\left(1+\frac{1}{\gamma}\right) \left| \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_i \right|^2 \geq \left\| \left(\mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \right) \right\|^2 + q_i \tag{4.12}$$

$$\left\| \mathrm{Vec} \left(\mathbf{P} \hat{\mathbf{G}}^H \mathbf{W} \right) \right\|^2 + \sum_{t \in s_o} \mu_t m_t \left\| \mathbf{W} \right\|_F^2 + \left\| \mathbf{W}_s \right\|_F^2 \leq P_o \tag{4.13}$$

where $\mathbf{P} = \mathrm{Diag}[\sqrt{\mu_1}, \cdots, \sqrt{\mu_N}]$ is a $N \times N$ diagonal weighting matrix. It can be easily verified that (4.12) and (4.13) can be equivalently rewritten in the second order cone forms or SOCP constraints as

$$\left\| \begin{matrix} (\hat{\mathbf{H}}^H \mathbf{W})^T \mathbf{x}_i \\ \sqrt{q_i} \end{matrix} \right\|^2 \leq \left(1+\frac{1}{\gamma}\right) \left| \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_i \right|^2, \tag{4.14}$$

$$\left\| \begin{matrix} \mathrm{vec} \left(\mathbf{P} \hat{\mathbf{G}}^H \mathbf{W} \right) \\ \sqrt{\sum_{t \in s_o} \mu_t m_t \left\| \mathbf{W} \right\|_F^2 + \left\| \mathbf{W}_s \right\|_F^2} \end{matrix} \right\|^2 \leq P_0 \tag{4.15}$$

Finally according to Schur complement, (4.14) and (4.15) are equivalent to the following linear matrix inequalities, i.e.,

$$S = \begin{bmatrix} \sqrt{1+\frac{1}{\gamma}}\mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i & \begin{bmatrix}\mathbf{x}_i{}^T\left(\hat{\mathbf{H}}^H\mathbf{W}\right) & \sqrt{q_i}\end{bmatrix} \\ \begin{bmatrix}(\hat{\mathbf{H}}^H\mathbf{W})^T\mathbf{x}_i \\ \sqrt{q_i}\end{bmatrix} & \sqrt{1+\frac{1}{\gamma}}\mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i\mathbf{I} \end{bmatrix}, \tag{4.16}$$

and (4.17), respectively.

$$L = \begin{bmatrix} \sqrt{P_o} & \begin{bmatrix}\mathrm{vec}^T\left(\mathbf{P}\hat{\mathbf{G}}^H\mathbf{W}\right) & \sqrt{\sum_{t\in s_o}\mu_t m_t\left\|\mathbf{W}\right\|_F^2+\left\|\mathbf{W}_s\right\|_F^2}\end{bmatrix} \\ \begin{bmatrix}\mathrm{vec}\left(\mathbf{P}\hat{\mathbf{G}}^H\mathbf{W}\right) \\ \sqrt{\sum_{t\in s_o}\mu_t m_t\left\|\mathbf{W}\right\|_F^2+\left\|\mathbf{W}_s\right\|_F^2}\end{bmatrix} & \sqrt{P_o}\mathbf{I} \end{bmatrix}. \tag{4.17}$$

Hence, the optimization problem in (4.11) can be equivalently restated as

$$\begin{aligned} \min_{\mathbf{W},P_o} \quad & P_o \\ \text{s.t} \quad & S \succeq 0, \\ & L \succeq 0. \end{aligned} \tag{4.18}$$

The convex problem in (4.18) can be solved using the numerical optimization packages, e.g., the SeDumi solver [37].

4.4 A Robust Coordinated Beamforming Formulation with Imperfect Instantaneous CSI

In [67], a centralized coordinated beamforming (CBF) problem has been proposed and formulated with imperfect second order statistical CSI. In the sequel and for the purpose of comparison, one can reformulate this problem with imperfect instantaneous CSI. Although, this reformulation is straightforward, it requires some considerations in terms of characterization of uncertainty region for fair comparison with the proposed method in (4.18). The centralized CBF can be rewritten as

$$\begin{aligned} \min_{\mathbf{w}_{i(q)}} \quad & \sum_{q=1}^{N_c} \sum_{i=1}^{U} \mathbf{w}_{i(q)}^H \mathbf{w}_{i(q)} \\ \text{s.t} \quad & \mathrm{SINR}_{i(q)} \geq \gamma_{i(q)}, \forall i \in \mathcal{S}_l, 1 \leq q \leq N_c \end{aligned} \tag{4.19}$$

where the notation $i(q)$ indicates user i in cell q. Notice that for our distributed formulation in (4.18), The cell index q has been dropped for the simplicity of notations, but, hereafter, the centralized CBF formulation is considered. The intercell interference on user $i(q)$, i.e., $\xi_{i(q)}$, is given by

$$\xi_{i(q)} = \sum_{t=1, t\neq q}^{N} \sum_{m=1}^{U} \mid (\hat{\mathbf{g}}_{i(q),t}^H + \mathbf{e}_{i(q),t}^H)\mathbf{w}_{m(t)} \mid^2, \tag{4.20}$$

where $\hat{\mathbf{g}}_{i(q),t}$ is the channel vector between user $i(q)$ and any BS t, $t \neq q$, and $\mathbf{e}_{i(q),t}$ is the corresponding uncertainty.

$$\mid (\hat{\mathbf{h}}_{i(q)} + \mathbf{e}_{i(q)})^H \mathbf{w}_{i(q)} \mid^2 = \quad \mathbf{w}_{i(q)}^H \hat{\mathbf{H}}_{i(q)} \mathbf{w}_{i(q)}, \tag{4.21}$$

$$\mid (\hat{\mathbf{g}}_{i(q),t} + \mathbf{e}_{i(q),t})^H \mathbf{w}_{m(t)} \mid^2 = \quad \mathbf{w}_{m(t)}^H \hat{\mathbf{G}}_{i(q),t} \mathbf{w}_{m(t)}, \tag{4.22}$$

where $\hat{\mathbf{h}}_{i(q)}$ is the direct channel between user $i(q)$ and its local BS in cell q and $\mathbf{e}_{i(q)}$ indicates the corresponding uncertainty. We can write

$$\hat{\mathbf{H}}_{i(q)} = \hat{\mathbf{h}}_{i(q)}\hat{\mathbf{h}}_{i(q)}^H + \mathbf{\Delta}_{i(q)},$$
$$\hat{\mathbf{G}}_{i(q),t} = \hat{\mathbf{g}}_{i(q),t}\hat{\mathbf{g}}_{i(q),t}^H + \mathbf{\Delta}_{i(q),t},$$

where

$$\mathbf{\Delta}_{i(q)} = \hat{\mathbf{h}}_{i(q)}\hat{\mathbf{e}}_{i(q)}^H + \hat{\mathbf{e}}_{i(q)}\hat{\mathbf{h}}_{i(q)}^H + \hat{\mathbf{e}}_{i(q)}\hat{\mathbf{e}}_{i(q)}^H, \tag{4.23}$$
$$\mathbf{\Delta}_{i(q),t} = \hat{\mathbf{g}}_{i(q),t}\hat{\mathbf{e}}_{i(q),t}^H + \hat{\mathbf{e}}_{i(q),t}\hat{\mathbf{g}}_{i(q),t}^H + \hat{\mathbf{e}}_{i(q),t}\hat{\mathbf{e}}_{i(q),t}^H, \tag{4.24}$$

such that, [76],

$$\delta_{i(q)} = \|\mathbf{\Delta}_{i(q)}\|_F \leqslant \varepsilon_{i(q)}^2 + 2\varepsilon_{i(q)}\|\hat{\mathbf{h}}_{i(q)}\|, \tag{4.25}$$
$$\delta_{i(q),t} = \|\mathbf{\Delta}_{i(q),t}\|_F \leqslant \varepsilon_{i(q),t}^2 + 2\varepsilon_{i(q),t}\|\hat{\mathbf{g}}_{i(q),t}\|. \tag{4.26}$$

Note that $\delta_{i(q)}$, $\delta_{i(q),t}$ indicate the error bounds on the direct channel vector of user $i(q)$ and the indirect channel vector of user $i(q)$ as seen by BS t, respectively. Following similar steps in [67], The centralized robust CBF with instantaneous CSI can be formulated as

$$\begin{aligned}
\min_{\mathbf{W}_{i(q)},K} \quad & K \\
\text{s.t} \quad & T_{i(q)} \geq 0, \\
& K - \sum_{q=1}^{N_c}\sum_{i=1}^{U} \mathrm{Tr}(\mathbf{W}_{i(q)}) \geq 0, \\
& \mathbf{W}_{i(q)} = \mathbf{W}_{i(q)}^H \succeq \mathbf{0},\ \forall i \in \mathcal{S}_l, 1 \leq q \leq N_c \\
& \mathrm{Rank}(\mathbf{W}_{i(q)}) = 1,
\end{aligned} \tag{4.27}$$

where $\mathbf{W}_{i(q)}$=$\mathbf{w}_{i(q)}\mathbf{w}_{i(q)}^H$ is a positive semidefinite matrix with unit rank and $T_{i(q)} = \left(1 + \frac{1}{\gamma_{i(q)}}\right) \mathrm{Tr}(\hat{\mathbf{h}}_{i(q)}\hat{\mathbf{h}}_{i(q)}^H\mathbf{W}_{i(q)}) - \sum_{t=1}^{N_c}\sum_{m=1}^{U} \mathrm{Tr}(\hat{\mathbf{g}}_{i(q),t}\hat{\mathbf{g}}_{i(q),t}^H\mathbf{W}_{m(t)}) -$

$\sigma^2_{i(q)} - \sum_{t=1}^{N_c}\sum_{m=1}^{U}\delta_{i(q),t}\text{Tr}(\mathbf{W}_{m(t)}) + \delta_{i(q)}\left(1-\frac{1}{\gamma_{i(q)}}\right)\text{Tr}(\mathbf{W}_{i(q)}) \geqslant 0$. In this formulation, the approximation of $\boldsymbol{\Delta}_i(q) = \delta_i(q)\mathbf{I}_M$ and $\boldsymbol{\Delta}_{i(q),t} = \delta_{i(q),t}\mathbf{I}_M$ have been considered. In the above derivation, the following notation has been considered that $\hat{\mathbf{g}}_{i(q),q} = \hat{\mathbf{h}}_{i(q)}$. The problem in (4.27) can be solved using the convex optimization packages by first relaxing the rank one constraint, which is the only non-convex constraint and then selecting the rank one solutions as the feasible ones [67]. Note also that in simulations, it has been assumed that $\delta = \delta_{i(q)} = \delta_{i(q),t}$.

4.4.1 Convergence algorithm

To arrive at a tractable solution, let consider the channel vector and its corresponding uncertainty in (4.21) and (4.22), respectively.
Then, the problem in (4.1) can be rewritten as

$$\begin{aligned} \min_{\mathbf{w}_i,\mathbf{r}_i} \quad & q(\mathbf{w}_i,\mathbf{e}_t) + \sum_{i\in s_l}\eta_i \mathbf{w}_i^H\mathbf{w}_i \\ \text{s.t} \quad & \frac{|\ \mathbf{r}_i^H\hat{\mathbf{H}}_i\mathbf{w}_i\ |^2}{\sum_{j\in s_l j\neq i}|\ \mathbf{r}_i^H\hat{\mathbf{H}}_i\mathbf{w}_j\ |^2 + \left\|\mathbf{r}_i\right\|^2\xi_i + \left\|\mathbf{r}_i\right\|^2\sigma^2} \\ & \geq \gamma_i, \qquad \forall i\in S_l. \end{aligned} \tag{4.28}$$

where

$$q(\mathbf{w}_i,\mathbf{e}_t) = \max_{\left\|\mathbf{e}_t\right\|\leq\varepsilon_t}\sum_{t\in s_o}\sum_{i\in s_l}\left|\ \mathbf{r}_i^H\hat{\mathbf{G}}_t\mathbf{w}_i)\ \right|^2, \tag{4.29}$$

where $\mathbf{r}_i \in \mathbb{C}^{M\times 1}$ denotes the received antenna vector that is used to retrieve the data symbol s_j.
It is observed that for an optimal $\mathbf{w}_i$ to be a solution, then $\mathbf{w}_i e^{j\psi_1}$, where ψ_i is an arbitrary phase, is also an optimal solution. The phase of $\mathbf{w}_i$, such that

$\mathbf{r}_i^H \hat{\mathbf{H}}_i \mathbf{w}_i$ is non-negative and real. Problem (4.28) can be rewritten

$$\begin{aligned} \min_{\mathbf{w}_i, \mathbf{r}_i, P_o} \quad & P_o \\ \text{s.t} \quad & \mid \mathbf{r}_i^H \hat{\mathbf{H}}_i \mathbf{w}_i \mid^2 \geq \sum_{j \in s_l j \neq i} \mid \gamma_i \mathbf{r}_i^H \hat{\mathbf{H}}_i \mathbf{w}_j \mid^2 + \gamma_i \left\| \mathbf{r}_i \right\|^2 \xi_i + \gamma_i \left\| \mathbf{r}_i \right\|^2 \sigma^2 \\ & \max_{\left\| \mathbf{e}_t \right\| \leq \varepsilon_t} \sum_{t \in s_o} \sum_{i \in s_l} \mid \mathbf{r}_i^H \hat{\mathbf{G}}_t \mathbf{w}_i) \mid^2 + \sum_{i \in s_l} \eta_i \mathbf{w}_i^H \mathbf{w}_i \leq P_o \qquad \forall i \in S_l. \end{aligned} \tag{4.30}$$

For a given receive vector $\mathbf{r}_i$, problem (4.30) can be reformulated as a SOCP. This can be done using the same steps taken in section (4.3) of this thesis. The minimum-mean-square-error (MMSE) receiver $\mathbf{r}_i$, and its proof are given in [78] and the references therein. This is expressed as

$$\mathbf{r}_i = \left[\hat{\mathbf{H}}_i (\mathbf{w}_i \mathbf{w}_i^H) \hat{\mathbf{H}}_i^H + \xi_i \mathbf{I} + \sigma^2 \mathbf{I} \right]^{-1} \hat{\mathbf{H}}_i \mathbf{w}_i \tag{4.31}$$

The steps of the proposed Convergence algorithm is summarized in Algorithm (1). Regarding the estimation of the intercell interference at step 7 of Algorithm (1), interested readers are referred to [79], where the details of MMSE intercell interference estimation approaches are described.

4.5 Simulation results

4.5.1 Simulation setup

In this section, the performance of the proposed approach is compared against the conventional approach within 3 adjacent sectors of 3 neighboring cells, as shown in Fig (3.1). In particular, 1 user per sector is randomly distributed such that, the user of each cell is located within the 3 adjacent sectors of 3 neighbouring cells. Monte Carlo simulations are carried out over 50 users distribution and generate 10000 sets of uncertainty channel realizations per user satisfying $\left\| \mathbf{e}_i \right\|^2 \leq {\varepsilon_i}^2$. The channel model used in [41] is applied to this

Algorithm 1 Convergence algorithm implemented at the BS of each cell l

1: Define a set of locally active users S_l with their corresponding SINR requirements and a stopping point Γ of the algorithm
2: **Inputs:** Γ, $\hat{\mathbf{h}}$,$\hat{\mathbf{g}}$,b, ε, σ^2;
3: **Initialize:** receiver vectors $(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_M)$, $n = 1$, $\xi_i(\mathrm{n}) \geq 0$, $\forall i \in S_l$;
4: **Repeat**
5: Solve the SOCP of (4.30) to find $\mathbf{W}(n)$ and extract $\mathbf{w}_i(n)$, $\forall i \in S_l$. This gives a feasible solutions as $\{\mathbf{w}_1(n), \mathbf{w}_2(n) \cdots, \mathbf{w}_M(n), \mathbf{r}_1(n), \mathbf{r}_2(n) \cdots \mathbf{r}_M(n)\}$, at the nth iteration.
6: Transmit with $\mathbf{w}_i(n)$, $\forall i \in S_l$;
7: Update the user $\mathbf{r}_1$ with the MMSE solution in (4.31) and each user i estimates its received intercell interference as $\xi_i(n+1)$, e.g., using the technique in [79], and reports the updated $\xi_i(n) = \xi_i(n+1)$ back to its local BS. For the next iteration, we denote as $\{\mathbf{r}_1(n+1), \mathbf{r}_2(n+1) \cdots \mathbf{r}_M(n+1), \xi_i(n+1)\}$
8: $n = n + 1$;
9: Repeat lines 5 and 8 **Until** $\sum_{i \in S_l} \left\|\mathbf{w}_i(n)\right\|^2 \geq \sum_{i \in S_l} \left\|\mathbf{w}_i(n+1)\right\|^2 \leq \Gamma$; and SINR of (4.28) are satisfied, then it is said to have converged.
10: **If** the SINR of (4.28) is satisfied for all i, **then**
11: go to line 15
12: **Else if** condition in 9 is not satisfied for a user i **then**
13: go to line 4
14: **End if**
15: **Output**: $\mathbf{w}_i(n)$, $\forall i \in S_l$.

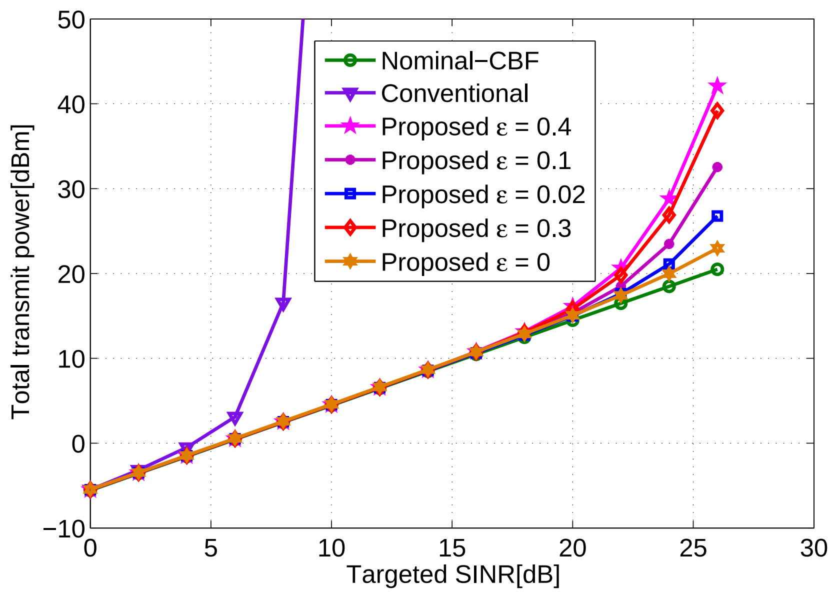

Figure 4.1: Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS for the SOCP Robust Downlink Beamforming with imperfect CSI.

proposed method and is expressed as

$$\mathbf{h}_i = 10^{-(128.1+37.6\log_{10}(l))/20}.\psi_i.\varphi_i.(\hat{\mathbf{h}}_i + \mathbf{e}_i), \tag{4.32}$$

where l is the distance between the i BS and i user, ψ_i is the shadowing, φ_i is the antenna gain, $\hat{\mathbf{h}}_i$ and $\mathbf{e}_i$ denote the estimated CSI channel and the CSI error respectively. The rest of the parameters as shown in the table of the section (3.4.1).

4.5.2 Performance evaluation

Fig. 4.1 shows a plot of total transmit power in a 3-cell cellular scenario versus the SINR target per user for the conventional, the nominal central-

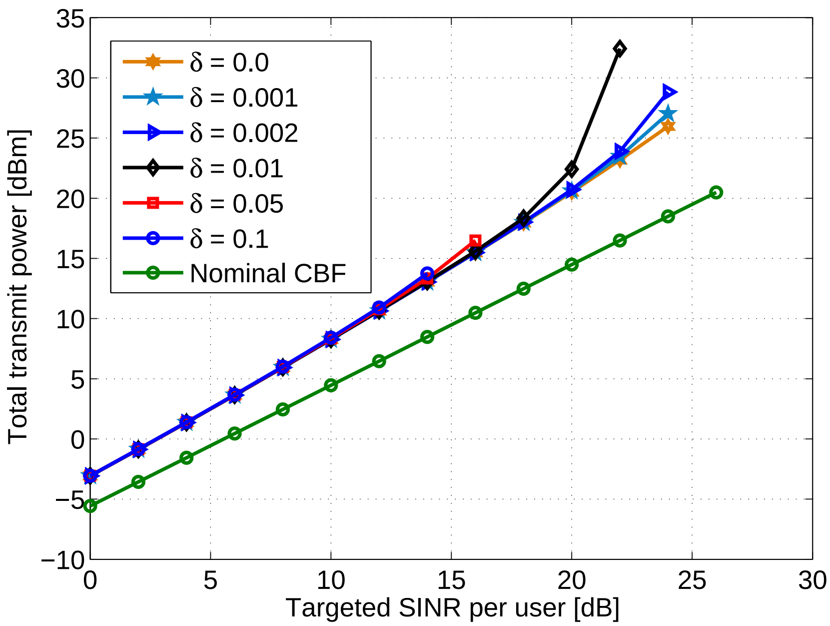

Figure 4.2: Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS for the centralized robust CBF with instantaneous CSI.

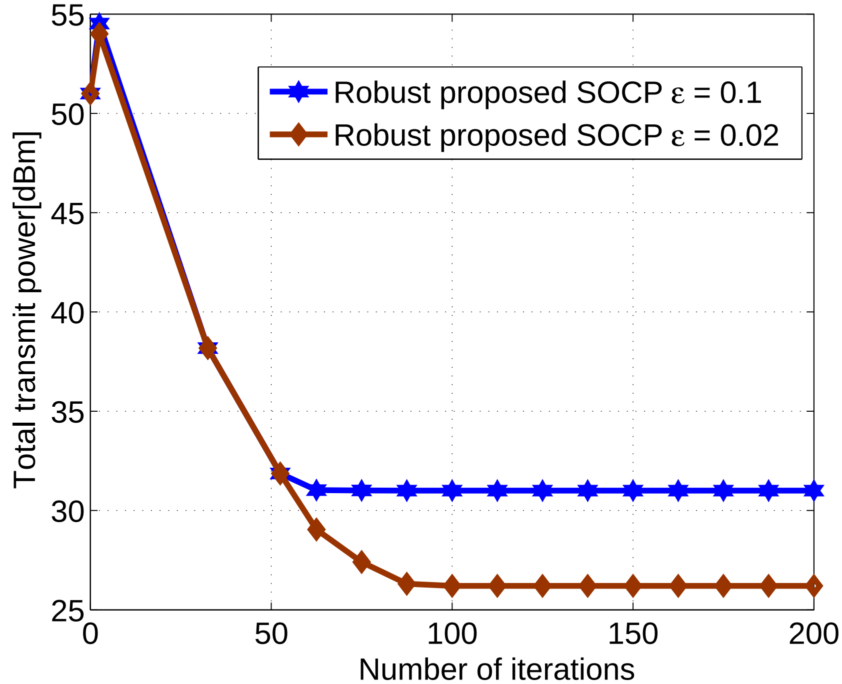

Figure 4.3: Convergence and transient behavior of transmit power at target SINR 26dB and the error radii of 0.1 and 0.02 with one user per cell and 6 antenna elements per BS

ized coordinated beamforming (CBF) [67] with instantaneous CSI and the proposed robust and distributed approach in problem (4.1) for different values of the error radii, i.e. $\varepsilon = 0.0$, 0.02, 0.1, 0.3 and 0.4. The conventional approach is obtained by inserting $\mu_t = 0, \forall t \in S_o$ in problem (4.1) to remove the intercell interference controlling term from the objective function of the proposed optimization problem. A comparison of the performance differences between the conventional approach and the proposed scheme with different levels of uncertainties in Fig. 4.1 shows the effectiveness of the proposed robust formulations in (4.8) and finally in (4.18) in terms of energy efficiency. Furthermore, the performance results of the proposed scheme with different uncertainty levels in Fig. 4.1 show a nearly perfect match with the nominal CBF at low and average target SINR levels. A very intimate match between the no error, i.e., $\varepsilon = 0.0$, performance and the nominal CBF in Fig. 4.1 reveals a negligible cost of robustness in terms of energy consumption at low and the average SINR values of up to 20 dB. It should be noted that the nominal CBF is formulated using the SOCP approach and without semidefinite relaxation (i.e., the similar approach used to formulate the proposed scheme) in a centralized way. By the centralized design, it means that the beamforming vectors of all BSs are jointly designed in a centralized unit and, then, sent to the individual BSs to serve their corresponding local (intracell) users, only. The results in Fig. 4.2 for the robust CBF, as reformulated in section (4.4) using semidefinite relaxation, shows a consistently increasing power gap between the nominal CBF and the robust CBF formulation with $\delta = 0.0$ as the SINR targets increasing. This observation is as a result of using two different formulation techniques of SOCP and the semidefinite relaxation. Comparing the results in Figs. 4.1 and 4.2 with the corresponding no-error performances, it can be observed that a high sensitivity of the robust formulation in section (4.4) to different levels of errors, i.e., the values of δ parameters in Fig. 4.2. For instance, for the uncertainty level of $\delta = 0.1$, at SINR=14 dB in Fig. 4.2, the required transmitted power is 13.76 dBm and the design fails to function efficiently beyond 14 dB of SINR, as it requires a very high power consumption. Whereas the proposed SOCP robust formulation in section (4.3) shows a closer match with the no error performance over the low and average SINR levels as illustrated in Fig. (4.1).

4.5.3 Convergence analysis

Fig.4.3 illustrates the convergence behaviour summarized in algorithm 1 for different values of the error radii for the proposed SOCP robust beamforming with imperfect CSI. These results were achieved by solving (4.30) using algorithm 1 over 1000 channel realizations, for a target SINR of 26dB and error radii of 0.1 and 0.02. One can see that the algorithm converges to an optimal point for the given conditions in step 9 of the algorithm (1). For the error radii, $\varepsilon = 0.1$, the convergence is faster compared to the $\varepsilon = 0.02$. The reason is that, for every feasibility solution $\mathbf{w}_i(n)$ also produces another feasibility solution of $\mathbf{w}_i(n+1)$. As a result, the transmit power monotonically decreasing as the iterations increases.

Table 4.1: Complexity analysis for SDP and SOCP

Methods	**Number of Variables**
SDP	N=12
Methods	Complexity
SDP	$\mathcal{O}(12^{3.5})$
Methods	Number of Variables
SOCP	N=9
Methods	Complexity
SOCP	$\mathcal{O}(9^{3.5})$

4.5.4 Complexity analysis of SDP and SOCP robust beamforming with imperfect CSI

Table 4.1, shows the comparison of both schemes. The complexity analysis is defined in terms of the number of optimization variables, equality and inequality constraints. This gives an estimate of the computational time. This thesis follows the worst case computational complexity for SDP and SOCP developed in [80] and this is approximated as $\mathcal{O}(N^{3.5})$, where N is the number of variables. For the SDP, the variable $\mathbf{W}_i$ defined in (3.8) consist of $A \times A, U$ matrices, where $A = 6$ and $U = 1$ are the number of antennas and user respectively. Giving the total of N= 12, this includes other variables such as slack variables in the simulations declaration of the optimization. For the SOCP, the number of variables, N=8. This showed that, the SOCP yield lower complexity than the SDP, and this can be seen from the number of constraints in chapter (3) compared to chapter (4) from their simulations problem (3.17) and (4.18) respectively.

4.5.5 Performance evaluation for SDP and SOCP robust beamforming with imperfect CSI

This section evaluates the performance between chapter (3) and chapter (4). In chapter (3), the original non-convex problem (3.3) is formulated as a standard semidefinite relaxation (SDR). Then in chapter (4), the same original non-convex problem (3.3) is reformulated as a second-order cone programming (SOCP).Then, the fidelity of both schemes is compared against the coordinated centralized beamforming. The simulation results are provided, in order to compare the effectiveness and efficiency of the SDR in the presence of imperfect CSI with respective of the proposed SOCP robust schemes, in terms of transmitting power at BSs.

In Fig. 4.4 shows a plot of total transmit power in a 3-cell cellular scenario versus the SINR target per user for the conventional, the nominal centralized coordinated beamforming and the proposed robust and distributed approach in problem (3.3) for the sdp and socp in chapter (3) and (4) respectively. For the same values of the error radii, i.e. $\varepsilon = 0.02$ and 0.1. It can be seen that, the sdp and socp their performance at low SINR values up to a target of 10 dB are similar and coincide with the performance of the nominal CBF, but the sdp approach in terms of power efficient support of higher SINR targets, improves as a result of a decreased error radius. The socp continue similar performance with the nominal CBF up to a target of 22 dB, which is almost twice as that of the sdp. This is clearly shows that the socp is more power efficient. Not only for its power efficiency is compared against the standard semidefinite relaxation, also the socp formulation is less complex due to its low number of variables. Therefore, performance loss in the SDR can also be caused by the second order method SeDuMi [37] used, because the method is unable to handle problem with large dimension as the computational complexity is limited.

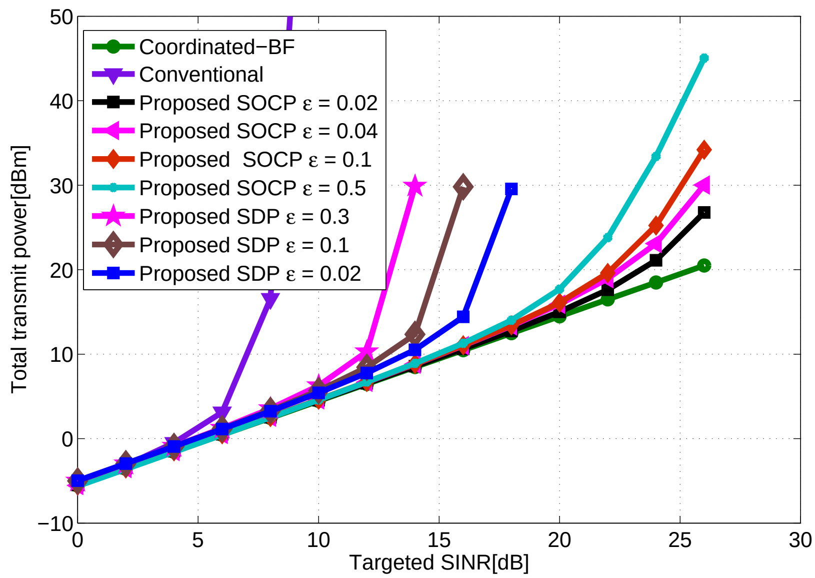

Figure 4.4: Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS.

4.6 Conclusion

This chapter provides a solution to the problem of downlink multicell processing with imperfect channel state information and distributed beamforming. The goal is to minimize a combination of the sum power of each BS across the network subject to the worst-case of the SINR level of individual users. The optimization problem has been formulated as the second-order cone programming (SOCP) and then recast the convex constraints in linear matrix inequality (LMI) form in the presence of imperfect channel state information. The reformulated centralized coordinated beamforming was used to compare and evaluate the efficiency of the proposed formulated scheme. The simulation results showed that the proposed SOCP for robust downlink beamforming with imperfect CSI formulation outperforms the conventional method and shows a close fidelity to a nominal centralized coordinated beamforming design. The second-order cone programming (SOCP)outperformed the SDP in terms of performance and complexity.

Chapter 5

Intercell Interference Management For Downlink Beamforming With Imperfect CSI

This chapter considers an optimum multicell downlink beamforming that minimizes the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect channel state information (CSI). Users on the cell edge mostly suffer from severe interference, which result in low data rate. The aim is to ensure that the data rate at the cell edge users for any given transmit power remains above the required level. A spherical uncertainty set is considered to model the imperfection in CSI between the true and the estimated channel coefficients. The original non-convex problem is formulated as the second-order cone programming (SOCP) and then recast the convex constraints into a tractable formulation in linear matrix inequality (LMI) form.

5.1 Introduction

In current cellular networks, inter-cell interference presents severe limitations to achieving high data rates. This is due to the aggressive frequency reuse among neighbouring cells. Most users located at the cell edge suffer from multiple channel conditions effects and severe inter-cell interference problems which result in very low data rates. As a result, to improve their performance, different techniques have been studied to mitigate inter-cell interference in recent literatures, [81], [82], [83]. However, it has been reported that, they require high computational complexity and extensive backhaul signaling and present no fairness guarantees [84]. Coordinated multi-point (CoMP) communications is a solution to the problem. A potential drawback is that the CoMP requires the provision of full CSI at the transmitter, i.e., CSIT, in order to effectively design downlink beamforming towards the end user terminals. Yet, the CSI at the transmitter might not be perfect, and to establish an ideal backhaul link is not affordable due to the limitations in scalability of communications resources. Recently, achieving the promised gains of CoMP in practical scenarios has motivated the use of robust design methodologies against channel uncertainties. In order to tackle the challenges of such real practical scenarios in downlink beamforming, many authors have suggested different design techniques to the above problem, e.g., [85], [41], [86]. In [85] a robust multi-cell coordinated beamforming (MCBF) is considered, the authors use semidefinite relaxation to handle the worst case, that minimizes the weighted sum transmission power of the base stations (BSs) subject to some worst-case SINR constraints on MSs. In [41], the authors adopt an elliptical uncertainty set to model the channel parameters to devise a distributed optimization problem that minimizes the weighted sum power of BSs subject to worst-case signal-to-interference-plus-noise ratio (SINR) constraints on the MSs. A balancing approach is introduced in [86] that maximizes the sum rate of intra-cell users and minimizes the resulting interference on users of the other cells, with an assumption of the availability of perfect CSI. However, none of the above considered minimizing the interference power on the outer-cell users due to transmissions to the locally active users within a cell. In this chapter, the strategic aim is to minimize the aggregated interference power induced on the users of the other cells by any individual base station. The time-division duplex (TDD) systems is taken into consideration in order to exploit channel reciprocity, which can then be exploited to reduce feedback overhead and the possibility of CSI reporting accuracy [87]. The proposed scheme is considered as decentralized in a manner that each base station designs its beamforming independently. In real scenario, no CSI is considered to be perfect and is limited by the capacity of the transmission, and therefore, uncertainty modelling is of interest. In order to solve the problem, one can expect to minimize the worst-case of the resulting overall interference induced on the other users of the adjacent cells in the presence of imperfect CSI. The aim is to ensure that the data rate at the cell edge users, for any given transmit power remains above the required level. The proposed formulation involves robustness against CSIT uncertainties which are assumed to be confined within the spherical uncertainty sets. The original non-convex problem is formulated as the second-order cone programming (SOCP) and then recast to a final optimization problem with the constraints in linear matrix inequality (LMI) form [1]. The rest of the chapter is organized as follows: Subsection II introduces system model and problem formulation. Subsection III presents Intercell Interference Problem robust downlink beamforming. Simulation results are presented in subsection IV. Finally, subsection V concludes the section.

5.2 System model and problem formulation

An optimization problem that calculate the optimum downlink beamforming vectors at any given BS q, can be written as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \sum_{t\in S_o}\sum_{i\in S_q} \mathbf{w}_i^H \mathbf{g}_t \mathbf{g}_t^H \mathbf{w}_i \\ s.t \quad & \log(1+\mathrm{SINR}_i) \geq b, \\ & \sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i \leq P, \forall_i \in S_q. \end{aligned} \tag{5.1}$$

The objective function in (5.1) indicates the overall interference power on the outer-cell users due to the transmission of BS q to its locally active users. The first and the second constraints in (5.1) are to ensure that the required number of b, b/sec/Hz, is delivered to an active user under a given total power constraint of P at BS q and the SINR of problem (3.2) is considered.

5.3 Intercell Interference Management For Downlink Beamforming With Imperfect CSI

Let consider the true CSI defined in (3.4). It is assumed that these error vectors are bound within a spherical uncertainty set defined in (4.2). Substituting for $\mathbf{h}_i$ and $\mathbf{g}_t$ from (3.4) in (5.1), it can be written as

$$\begin{aligned} \min_{\mathbf{w}_i} \quad & \max_{\|\mathbf{e}_t\|_2\leq\varepsilon_t} \sum_{t\in S_o}\sum_{i\in S_q} \mid (\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{w}_i \mid^2 \qquad (5.2) \\ \text{s.t.} \quad & \min_{\|\mathbf{e}_t\|_2\leq\varepsilon_t} \log\Bigg(1 + \frac{\mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i \mid^2}{\sum_{\substack{j\in S_q \\ j\neq i}} \mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j \mid^2 + \xi_i + \sigma^2}\Bigg) \\ & \geq b, \\ & \sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i \leq P, \qquad \forall i \in S_q. \end{aligned}$$

By introducing a slack variable k, one can rewrite (5.2) as

$$\begin{aligned}
\min_{\mathbf{w}_i, k} \quad & k && (5.3)\\
\text{s.t.} \quad & \min_{\|\mathbf{e}_t\|_2 \le \varepsilon_t} \log\Big(1 + \frac{|\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i\,|^2}{\sum_{\substack{j\in S_q\\ j\ne i}} |\,(\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j\,|^2 + \xi_i + \sigma^2}\Big) \\
& \ge b, \qquad \forall i \in S_q \\
& \max_{\|\mathbf{e}_t\|_2 \le \varepsilon_t} \sum_{t\in S_o}\sum_{i\in S_q} |\,(\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{w}_i\,|^2 \le k \\
& \sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i \le P.
\end{aligned}$$

Using the Cauchy-Schwarz inequality, i.e., $|\,\mathbf{e}_t^H\mathbf{w}_i\,| \le \|\mathbf{e}_t\|\|\mathbf{w}_i\| \le \varepsilon_t\|\mathbf{w}_i\|$, and applying the triangle inequality, one can rewrite the second constraint in problem (5.3) as

$$\begin{aligned}
&\max_{\|\mathbf{e}_t\|_2 \le \varepsilon_t} \sum_{t\in S_o}\sum_{i\in S_q} |\,(\hat{\mathbf{g}}_t^H + \mathbf{e}_t^H)\mathbf{w}_i\,|^2 \\
&\le \sum_{t\in S_o}\sum_{i\in S_q}(|\,\hat{\mathbf{g}}_t^H\mathbf{w}_i\,| + \varepsilon_t\|\mathbf{w}_i\|_2)^2. \qquad (5.4)
\end{aligned}$$

The right-hand-side (RHS) of (5.4) can be written as

$$\begin{aligned}
&\sum_{t\in S_o}\sum_{i\in S_q}\Big(|\,\hat{\mathbf{g}}_t^H\mathbf{w}_i\,|^2 + \varepsilon_t(\varepsilon_t + 2\|\hat{\mathbf{g}}_t\|_2)\|\mathbf{w}_i\|_2^2\Big) \\
&= \sum_{t\in S_o}\sum_{i\in S_q}|\,\hat{\mathbf{g}}_t^H\mathbf{w}_i\,|^2 + \sum_{t\in S_o}\sum_{i\in S_q}\mathbf{w}_i^H m_t \mathbf{w}_i, \qquad (5.5)
\end{aligned}$$

where $m_t = {\varepsilon_t}^2 + 2\varepsilon_t\sqrt{\hat{\mathbf{g}}_t^H\hat{\mathbf{g}}_t}$. Similarly, by applying the Cauchy-Schwarz inequality and the triangular inequality, the numerator of the SINR can be shown as

$$\begin{aligned}
&\mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_i \mid^2 \\
&\geq \Big(\mid \hat{\mathbf{h}}_i^H \mathbf{w}_i \mid^2 + \mathbf{w}_i^H (\varepsilon_i^2 - 2\varepsilon_i \sqrt{\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i}) \mathbf{w}_i \Big)
\end{aligned} \tag{5.6}$$

and

$$\begin{aligned}
&\mid (\hat{\mathbf{h}}_i^H + \mathbf{e}_i^H)\mathbf{w}_j \mid^2 \\
&\leq \sum_{j \in S_q j \neq i} \mid \hat{\mathbf{h}}_i^H \mathbf{w}_j \mid^2 + \sum_{j \in S_q j \neq i} \mathbf{w}_j^H \Big(\varepsilon_i^2 + 2\varepsilon_i \sqrt{\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i} \Big) \mathbf{w}_j.
\end{aligned} \tag{5.7}$$

The left-hand-side (LHS) of the first constraint in (5.2), i.e., the rate constraint in the worst case, is calculated by substituting the lower and the upper bounds in (5.6) and (5.7) in the SINR expression in (5.1). Using the upper bound expression in (5.5) for the LHS of the second constraint in (5.3), one can express the problem in (5.3) as

$$\begin{aligned}
\min_{\mathbf{w}_i, k} \quad & k \\
\text{s.t} \quad & \log(1 + \text{SINR}_i) \geq b, \forall_i \in S_q, \\
& \sum_{t \in S_o} \sum_{i \in S_q} \mid \hat{\mathbf{g}}_t^H \mathbf{w}_i \mid^2 + \sum_{t \in S_o} \sum_{i \in S_q} \mathbf{w}_i^H m_t \mathbf{w}_i \leq k \\
& \sum_{i \in S_q} \mathbf{w}_i^H \mathbf{w}_i \leq P,
\end{aligned} \tag{5.8}$$

where

$$\begin{aligned}
\text{SINR}_i = & \quad \frac{\mid \hat{\mathbf{h}}_i^H \mathbf{w}_i \mid^2 + \mathbf{w}_i^H m_i \mathbf{w}_i}{\sum_{j \neq i} \mid \hat{\mathbf{h}}_i^H \mathbf{w}_j \mid^2 + \sum_{j \neq i} \mathbf{w}_j^H m_{ii} \mathbf{w}_j + \xi_i + \sigma^2}, \\
m_i = & \quad {\varepsilon_i}^2 - 2\varepsilon_i \sqrt{\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i}, \\
m_{ii} = & \quad {\varepsilon_i}^2 + 2\varepsilon_i \sqrt{\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i}.
\end{aligned}$$

Furthermore, the first constraint in (5.8) can be rewritten as

$$\frac{|\hat{\mathbf{h}}_i^H \mathbf{w}_i|^2 + \mathbf{w}_i^H m_i \mathbf{w}_i}{\sum_{j\neq i} |\hat{\mathbf{h}}_i^H \mathbf{w}_j|^2 + \sum_{j\neq i} \mathbf{w}_j^H m_{ii} \mathbf{w}_j + \xi_i + \sigma^2} \geq (2^b - 1), \tag{5.9}$$

which can be expanded as (5.10).

$$\begin{aligned}
&|\hat{\mathbf{h}}_i^H \mathbf{w}_i|^2 + \mathbf{w}_i^H m_i \mathbf{w}_i \geq (\sum_{j\neq i} |\hat{\mathbf{h}}_i^H \mathbf{w}_j|^2 + \sum_{j\neq i} \mathbf{w}_j^H m_{ii} \mathbf{w}_j + \xi_i + \sigma^2)(2^b - 1) \\
&2^b |\hat{\mathbf{h}}_i^H \mathbf{w}_i|^2 \geq \sum_{j\in S_q} |\hat{\mathbf{h}}_i^H \mathbf{w}_j|^2 (2^b - 1) + (\sum_{j\in S_q} \mathbf{w}_j^H m_{ii} \mathbf{w}_j + \xi_i + \sigma^2)(2^b - 1) \\
&-\mathbf{w}_i^H (m_i + m_{ii}(2^b - 1))\mathbf{w}_i
\end{aligned} \tag{5.10}$$

By plugging (5.10) into the first constraint in (5.8), the optimization problem in (5.8) can be rewritten

$$\begin{aligned}
\min_{\mathbf{w}_i, k} \quad & k \\
\text{s.t} \quad & 2^b |\hat{\mathbf{h}}_i^H \mathbf{w}_i|^2 \geq \sum_{j\in S_q} |\hat{\mathbf{h}}_i^H \mathbf{w}_j|^2 (2^b - 1) + q_i, \\
& \sum_{t\in S_o} \sum_{i\in S_q} |\hat{\mathbf{g}}_t^H \mathbf{w}_i|^2 + \sum_{t\in S_o} \sum_{i\in S_q} \mathbf{w}_i^H m_t \mathbf{w}_i \leq k \\
& \sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i \leq P
\end{aligned} \tag{5.11}$$

where

$$q_i = \Big(\sum_{j\in S_q} \mathbf{w}_j^H m_{ii} \mathbf{w}_j + \xi_i + \sigma^2\Big)(2^b - 1) - \mathbf{w}_i^H L \mathbf{w}_i,$$

and $L = m_i + m_{ii}(2^b - 1)$. Let denote $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_K]$. Then, $\sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i = \mathrm{Tr}(\mathbf{W}^H\mathbf{W}) = \left\|\mathbf{W}\right\|^2$, and its can restate the problem in (5.11) as

$$\begin{aligned}
\min_{\mathbf{w}_i,k} \quad & k \\
\text{s.t} \quad & 2^b \mid \mathbf{x}_i^T \hat{\mathbf{H}}_i^H \mathbf{W}\mathbf{x}_i \mid^2 \geq \sum_{i\in S_q} \mid \mathbf{x}_t^T \hat{\mathbf{H}}_i^H \mathbf{W}\mathbf{x}_i \mid^2 (2^b - 1) \\
& + q_i, \\
& \sum_{t\in S_o}\sum_{i\in S_q} \mid \mathbf{x}_t^T \hat{\mathbf{G}}_t^H \mathbf{W}\mathbf{x}_i \mid^2 + \sum_{t\in S_o} m_t \left\|\mathbf{W}\right\|^2 \leq k \\
& \left\|\mathbf{W}\right\|^2 \leq P
\end{aligned} \tag{5.12}$$

here

$$\begin{aligned}
m_t = & \quad {\varepsilon_t}^2 + 2\varepsilon_t \sqrt{\mathbf{x}_t^T \hat{\mathbf{G}}^H \hat{\mathbf{G}} \mathbf{x}_t}, \\
m_i = & \quad {\varepsilon_i}^2 - 2\varepsilon_i \sqrt{\mathbf{x}_i^T \hat{\mathbf{H}}^H \hat{\mathbf{H}} \mathbf{x}_i}, \\
m_{ii} = & \quad {\varepsilon_i}^2 + 2\varepsilon_i \sqrt{\mathbf{x}_i^T \hat{\mathbf{H}}^H \hat{\mathbf{H}} \mathbf{x}_i}, \\
q_i = & \quad \left(m_{ii}\left\|\mathbf{W}\right\|^2 + \xi_i + \sigma^2\right)(2^b - 1) - L\left\|\mathbf{W}\right\|^2,
\end{aligned}$$

where $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, ...\hat{\mathbf{h}}_K]$ and $\hat{\mathbf{G}} = [\hat{\mathbf{g}}_1, ...\hat{\mathbf{g}}_N]$ denote the set of complex channel of locally active users and a set of complex channel of the users in the adjacent cells. The indicator vectors $\mathbf{x}_i$ as a $K \times 1$ vector with unity at the ith dimension and zeros elsewhere and $\mathbf{x}_t$ as a $N \times 1$ vector with unity at the tth dimension and zeros elsewhere, where $N = (N_c - 1)K$. It is observed that for an optimal $\mathbf{W}$ satisfying problem (5.12), $\mathbf{W}diag[e^{j\psi_1}, ...e^{j\psi_K}]$, where ψ_i is an arbitrary phase, is also an optimal solution. Therefore, one can design the beamforming matrix $\mathbf{W}$ up to an arbitrary phase scaling and the scalar $\mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W}\mathbf{x}_i$ is non-negative and real. The constraints in (5.12) can be expressed as

$$2^b \mid \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W}\mathbf{x}_i \mid^2 \geq \left(2^b - 1\right)\left\|\left(\mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W}\right)\right\|^2 + q_i \tag{5.13}$$

$$\left\|\mathrm{Vec}\left(\hat{\mathbf{G}}^H\mathbf{W}\right)\right\|^2 + \sum_{t\in S_o} m_t\left\|\mathbf{W}\right\|^2 \leq k \tag{5.14}$$

It can be easily verified that (5.13) and (5.14) can be equivalently rewritten in the second order cone forms or SOCP constraints as

$$\left\| \frac{(\hat{\mathbf{H}}^H\mathbf{W})^T\mathbf{x}_i}{\sqrt{\frac{q_i}{\left(2^b-1\right)}}} \right\|^2 \leq \frac{2^b}{\left(2^b-1\right)} \mid \mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i \mid^2, \tag{5.15}$$

$$\left\| \frac{\mathrm{vec}\left(\hat{\mathbf{G}}^H\mathbf{W}\right)}{\sqrt{\sum_{t\in S_o} m_t\left\|\mathbf{W}\right\|^2}} \right\|^2 \leq k \tag{5.16}$$

$$\sqrt{P} \geq \left\| \ \mathrm{vec}\left(\mathbf{W}\right) \right\|_2 \tag{5.17}$$

Finally according to Schur complement, (5.15), (5.16) and (5.17) are equivalent to the following linear matrix inequalities, i.e.,

$$Z = \begin{bmatrix} \sqrt{P} & \mathrm{vec}^T\left(\mathbf{W}\right) \\ \mathrm{vec}\left(\mathbf{W}\right) & \sqrt{P}\mathbf{I} \end{bmatrix} \tag{5.18}$$

(5.19) and (5.20), respectively.

$$S = \begin{bmatrix} \frac{2^b}{\left(2^b-1\right)}\mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i & \left[\mathbf{x}_i{}^T\left(\hat{\mathbf{H}}^H\mathbf{W}\right) \quad \sqrt{\frac{q_i}{\left(2^b-1\right)}}\right] \\ \left[\frac{(\hat{\mathbf{H}}^H\mathbf{W})^T\mathbf{x}_i}{\sqrt{\frac{q_i}{\left(2^b-1\right)}}}\right] & \frac{2^b}{\left(2^b-1\right)}\mathbf{x}_i^T\hat{\mathbf{H}}^H\mathbf{W}\mathbf{x}_i\mathbf{I} \end{bmatrix} \tag{5.19}$$

$$L = \begin{bmatrix} \sqrt{k} & \left[\mathrm{vec}^T\left(\hat{\mathbf{G}}^H\mathbf{W}\right) \quad \sqrt{\sum_{t\in S_o} m_t\left\|\mathbf{W}\right\|^2}\right] \\ \left[\frac{\mathrm{vec}\left(\hat{\mathbf{G}}^H\mathbf{W}\right)}{\sqrt{\sum_{t\in S_o} m_t\left\|\mathbf{W}\right\|^2}}\right] & \sqrt{k}\mathbf{I} \end{bmatrix} \tag{5.20}$$

Hence, the optimization problem in (5.12) can be equivalently restated as

$$\begin{aligned} \min_{\mathbf{W},k} \quad & k \\ \text{s.t} \quad & S \succeq 0, L \succeq 0, Z \succeq 0. \end{aligned} \tag{5.21}$$

The convex problem in (5.21) can be solved using numerical optimization packages, e.g., the SeDumi solver [37]. the non-robust approach is developed by setting $m_t = m_i = m_{ii} = 0$, and problem (5.3) can now be rewritten as

$$\begin{aligned} \min_{\mathbf{w}_i,k} \quad & k \\ \text{s.t.} \quad & \frac{\mid \hat{\mathbf{h}}_i^H \mathbf{w}_i \mid^2}{\sum_{\substack{j\in S_q \\ j\neq i}} \mid \hat{\mathbf{h}}_i^H \mathbf{w}_j \mid^2 + \xi_i + \sigma^2} \geq 2^b - 1, \qquad \forall i \in S_q \\ & \sum_{t\in S_o} \sum_{i\in S_q} \mid \hat{\mathbf{g}}_t^H \mathbf{w}_i \mid^2 \leq k, \quad \sum_{i\in S_q} \mathbf{w}_i^H \mathbf{w}_i \leq P. \end{aligned} \tag{5.22}$$

After some manipulations to problem (5.22), the following linear matrix inequalities are obtained, i.e.,

$$L2 = \begin{bmatrix} \sqrt{k} & \text{vec}^T\left(\hat{\mathbf{G}}^H \mathbf{W}\right) \\ \text{vec}\left(\hat{\mathbf{G}}^H \mathbf{W}\right) & \sqrt{k}\mathbf{I} \end{bmatrix} \tag{5.23}$$

$$Z2 = \begin{bmatrix} \sqrt{P} & \text{vec}^T(\mathbf{W}) \\ \text{vec}(\mathbf{W}) & \sqrt{P}\mathbf{I} \end{bmatrix} \tag{5.24}$$

and (5.25). Its equivalent SOCP can be stated as

$$S2 = \begin{bmatrix} \frac{2^b}{\left(2^b-1\right)} \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_i & \begin{bmatrix} \mathbf{x}_i{}^T \left(\hat{\mathbf{H}}^H \mathbf{W}\right) & \sqrt{\xi_i + \sigma^2} \end{bmatrix} \\ \begin{bmatrix} (\hat{\mathbf{H}}^H \mathbf{W})^T \mathbf{x}_i \\ \sqrt{\xi_i + \sigma^2} \end{bmatrix} & \frac{2^b}{\left(2^b-1\right)} \mathbf{x}_i^T \hat{\mathbf{H}}^H \mathbf{W} \mathbf{x}_i \mathbf{I} \end{bmatrix} \tag{5.25}$$

$$\begin{aligned} \min_{\mathbf{W},k} \quad & k \\ \text{s.t} \quad & S2 \succeq 0, L2 \succeq 0, Z2 \succeq 0. \end{aligned} \tag{5.26}$$

The convex problem in (5.26) can be solved using the same numerical optimization packages stated in (5.21). The steps of the proposed robust algorithm is summarized in Algorithm 2, where Γ and n are, respectively the stopping threshold and the execution iteration number of the algorithm. Regarding the estimation of the intercell interference at step 6 of Algorithm 2, interested readers are referred to [79], where the details of MMSE intercell interference estimation approaches are described.

Algorithm 2 The proposed algorithm implemented at the BS of each cell q

Inputs: Γ, $\hat{\mathbf{h}}$,$\hat{\mathbf{g}}$,b, ε, σ^2;
2: **Initialize:** $n = 1$, $\xi_i(\text{n}) \geq 0$, $\forall i \in S_q$;
Repeat
4: Solve (5.21) to find $\mathbf{W}(n)$ and extract $\mathbf{w}_i(n)$, $\forall i \in S_q$;
Transmit with $\mathbf{w}_i(n)$, $\forall i \in S_q$;
6: Each user i estimates its received intercell interference as $\xi_i(n+1)$, e.g., using the technique in [79], and reports the updated $\xi_i(n) = \xi_i(n+1)$ back to its local BS;
$n = n + 1$;
8: **Until** $\sum_{i \in S_q} \left(\left\| (\mathbf{w}_i(n-2)) \right\|^2 - \left\| \mathbf{w}_i(n-1) \right\|^2 \right) \leq \Gamma$;
Output: $\mathbf{w}_i(n)$, $\forall i \in S_q$.

5.4 SIMULATION RESULTS

5.4.1 Simulation setup

In this section, the simulations are used to study the effectiveness of the proposed method and evaluate the results using different values of the error radii and compared against the conventional and non-robust scheme. Fig. 3.1 illustrates an example of one user distribution with $K = 3$ users that are distributively generated on the verge of the 3 adjacent cells named critical areas of the neighbouring base stations. Monte Carlo simulations are carried out over 50 users distribution with 10000 channel realizations. Generate 10000 sets of CSI errors satisfying the error bound, $\left\|\mathbf{e}_t\right\|^2 \leq \varepsilon^2$ until a fine stable averaged transmitted power is achieved. To simulate the channel,

the distance between the base stations is an important factor. The distribution of users are also taken into account in order to avoid other user from receiving a null signal. Eminent consideration of large-scale fading caused by shadowing conditions. Therefore, the model setup used in [41] can be used and it is given as

$$\mathbf{g}_t = 10^{-(128.1+37.6\log_{10}(l))/20}.\psi_t.\varphi_t.(\hat{\mathbf{g}}_t + \mathbf{e}_t), \tag{5.27}$$

where l is the distance between the t BS and K user, ψ_t is the shadowing gain, φ_t is the antenna gain, $\hat{\mathbf{h}}_t$ and $\mathbf{e}_t$ denote the estimated CSI channel and the CSI error respectively. The rest of the parameters, which are based on the LTE standard as shown in the table of the section 3.4.1.

5.4.2 Performance evaluation for Intercell Interference Problem for Downlink Beamforming With Imperfect CSI

Fig. 5.1 shows a plot of total transmit power (in dBm) of each BS versus user data rate (in b/s/Hz) per user of the conventional, non-robust and the proposed approach for different values of the error radii, i.e., ε = 0.1, 0.2, 0.3, and 0.4. The base station is equipped with 6 antennas. it is noticed that for the data rate below 5 b/s/Hz, the sensitivity to error is very low, and the proposed scheme closely follow the non-robust scheme. For the data rate $\geq$ 5 b/s/Hz, it has been observed that the variation of the transmitted power is proportional to the error radii. Fig 5.2, shows a plot of total transmit power versus the SINR target per user for any given value of b/s/Hz. Fig 5.2 indicates the effectiveness of the proposed intercell interference problem for downlink beamforming with imperfect CSI in (5.8) and substantially in (5.21) in terms of energy efficiency. Fig. 5.3, shows a plot of total intercell power versus user data rate per user. It is noticed that for a given user data rate, i.e., 4 b/s/Hz, the intercell power is considerable, increasing as the error radii increases. For instance, at 5 b/s/Hz with the error radii of $\varepsilon = 0.2$, the intercell power is 11.2 dBm, while for the same b/s/Hz but with different ε = 0.4, the intercell power is 13.22 dBm. This is an increase of 2.02 dBm. Fig. 5.4, shows a plot of total intercell power versus the SINR target per user for any given value of the user data rate in Fig. 5.3. It is observed that as the error radii increases, the intercell power also increases. For both Fig 5.2 and Fig. 5.4, that at a very low error radii, i.e., 0.1, the proposed approach maintain similar performance for i.e., 18 db, the design achieved the total transmit of 18 dBm and the proposed is more resilient to channel estimate for low b/sec/Hz. Finally, the simulations showed that the non-robust scheme outperforms the proposed scheme at higher rate in b/s/Hz and nearly perfect match with the proposed at low and moderate rate in b/s/Hz.

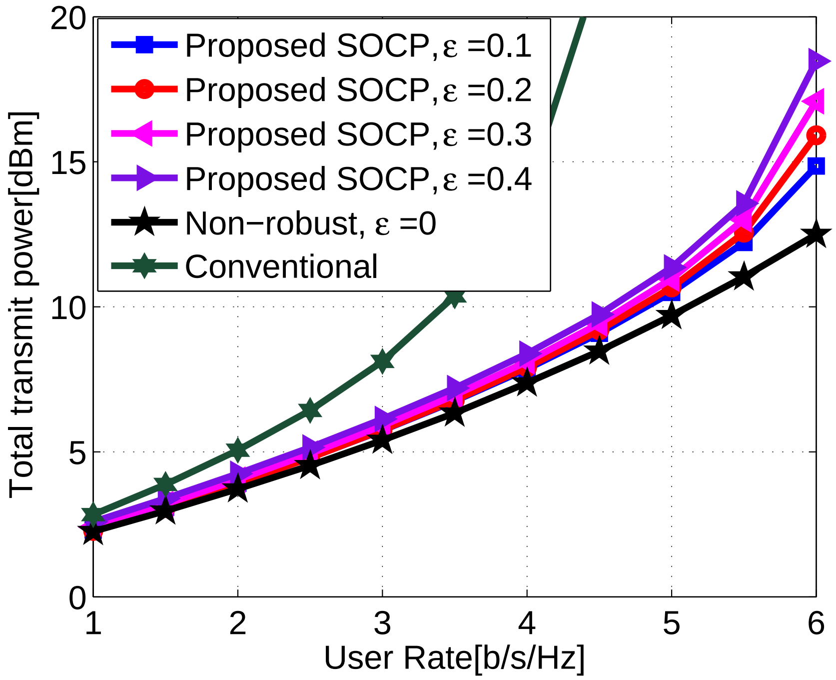

Figure 5.1: Total transmit power versus Userate values in a 3-cell scenario with one user per cell and 6 antenna elements per BS.

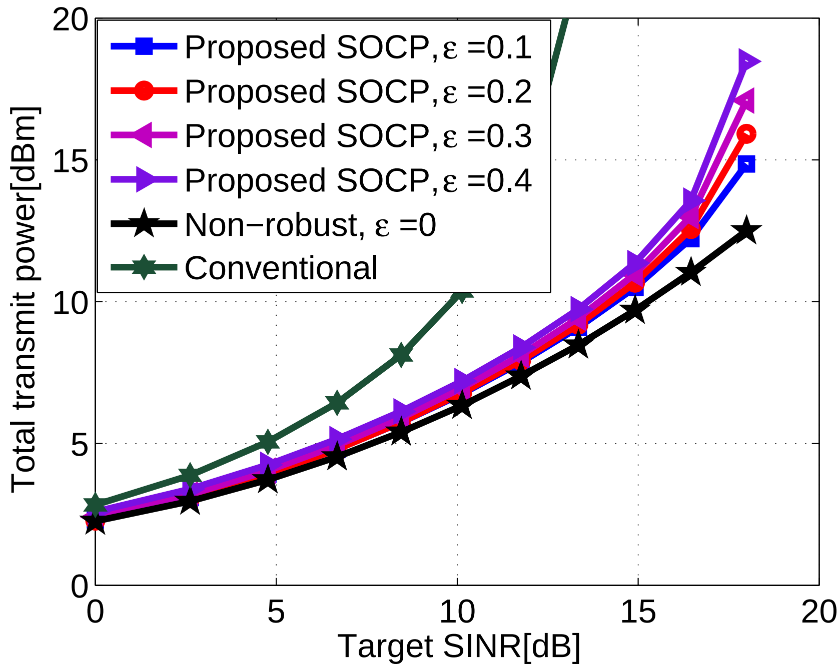

Figure 5.2: Total transmit power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS.

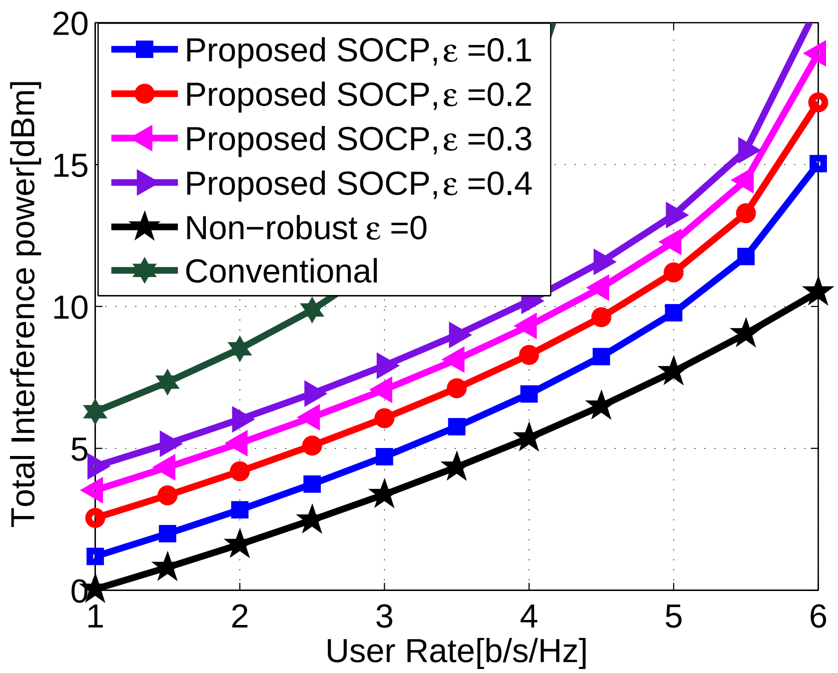

Figure 5.3: Total Intercell power versus Userate values in a 3-cell scenario with one user per cell and 6 antenna elements per BS.

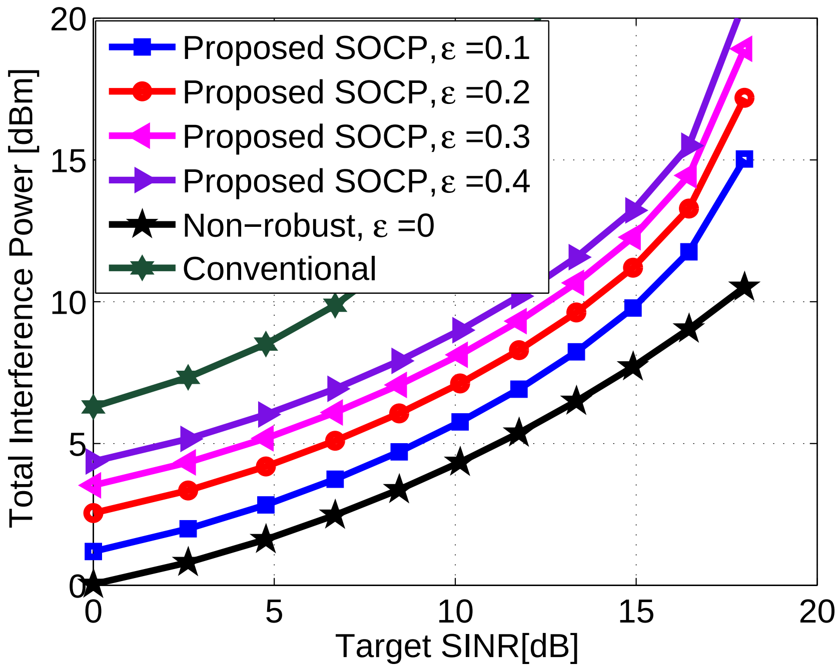

Figure 5.4: Total Intercell power versus targeted SINR values in a 3-cell scenario with one user per cell and 6 antenna elements per BS.

5.5 Conclusion

In this chapter, an interference management scheme in multicell networks has been introduced, where each BS primarily minimizes its total interference on the users of the other cells while delivering certain data rates to its intracell users. The scheme is formulated under a robust optimization problem that accounts for the uncertainties in channel state information at base stations. As the original problem is intractable due to the presence of robust constraints, the first concept is to reformulate the constraints in the deterministic forms by assuming that the uncertainties are bound within a hypersphere. Finally, the problem is restated in the second order cone programming (SOCP) form. The results indicate that the proposed scheme shows more resilience against channel estimation error at moderate and low rates in b/sec/Hz and outperformed the conventional approach, but showed a close fidelity to non-robust at a low and moderate rates in b/sec/Hz.

Chapter 6

Conclusion and future work

Convex optimization has recently been recognized as a powerful tool by the signal processing community. The main advantage of a convex formulation of a problem is that the optimisation problem has only one local optimum, which is also a global optimum and a rigorous optimality condition and duality theory exist to prove the optimal solution. Hence, convex optimisation problems can always be solved, either analytically or numerically, to obtain the optimum solution. This thesis exploited these tools of optimisation to reduce the combination of the total transmitted power and its aggregated interference induced on the users of the other cells of the cellular network while ensuring required levels of signal to-interference-plus-noise ratios for all active user terminals. A decentralised approach was developed at beamforming cooperative level, where the base station can independently perform signal processing tasks using the locally attained CSI between itself and the other vulnerable users of the other cells through feedback channels. The thesis focus has been on the robust formulation of these problems in convex form, with the assumption that the transmitter has erroneous channel state information. In this thesis, it is generally believed, mitigating co-channel interference is the main factor that led to the reduction of the transmit power. The challenges faced in decentralized are the infinite number of constraints and this needed to be reformulated to a tractable form. The provided solution tend to provide robustness against imperfect CSI at the cost of increased power but, with an improved reduction in signaling overhead. Finally, an algorithm is proposed that improved the data rate of the users at the edge of the cells. The outcome proved to show more resilience against channel estimation error at moderate and low level, but a close fidelity match to the non-robust design at low and moderate rates.

6.1 Thesis summary

6.1.1 Summary of Chapter 1

The introductory chapter outlined the motivation of this thesis and highlighted the importance of the proposed research topic in context of interference management in multi-cell networks. Further, the contributions of this thesis were also oultlined.

6.1.2 Summary of Chapter 2

This chapter provided a review of convex optimisation theory and techniques. A fundamental to multiple antennas techniques and radio channel were discussed along the adaptive antenna techniques and the configurations used for this thesis. An optimisation problem was presented to calculate transmit beamformers for multiple active users in a single-cell scenario along with concepts of second order cone programming and semidefinite programming. Finally, a robust downlink beamforming in the presence of uncertainty for multiple active users in a single-cell was briefed step by step.

6.1.3 Summary of Chapter 3

In this chapter, an inter-cell alignment is tackled by applying limited cooperation amongst the base stations. The BS independently design their own beamforming using the channel state information at the base station. The base station minimises the combination of its total transmit power and the resulting interference power of the users of the other cell. The tractable formulation is achieved using the S-procedure and the linear matrix inequality. The formulated semidefinite and relaxation algorithm is compared against the robust CBF with imperfect channel information. It is shown that when accounting for the CSI errors at both the overall intercell interference power in the subject function and the SINR constraints, a feasible solution for certain sets of SINR and the optimal solution were achieved. It was demonstrated that, different values of the error bounds improved the transmitted power.

6.1.4 Summary of Chapter 4

This chapter discusses extra signalling over-head issue. To reduce the extra signalling, the downlink multicell processing is studied. A robust multicell downlink beamforming that minimizes a combination of the sum-power, used by each base station (BS) to transmit data to its local users, and the worst-case of the resulting overall interference, induced on the other users of the adjacent cells in the presence of imperfect channel state information, subject to guaranteeing the worst-cases of the signal-to-interference-plus-noise ratio (SINR) remain above a prescribed threshold. we assume that the imperfection in CSI between the true and the estimated channel coefficients is confined within a spherical uncertainty set. It was shown that this problem can be cast into a convex SOCP and reformulate the problem into a linear matrix inequality form. The simulation results showed that the proposed SOCP for robust downlink beamforming with imperfect CSI formulation outperformed the conventional and the non-robust, and it is the most efficient compared to the SDR.

6.1.5 Summary of Chapter 5

In this chapter, An algorithm is designed at each base station to minimise the worst-case of the resulting overall interference power, then the time-division duplex (TDD) systems is considered in order to exploit channel reciprocity. Then, the data rates of the users at the cell edge are analyzed. The simulation results indicated that the proposed scheme showed more resilience against channel estimation error at moderate and low rates in b/sec/Hz and outperformed the conventional approach, but showed a close fidelity to non-robust at a low and moderate rates in b/sec/Hz.

6.2 Future research directions

6.2.1 A green Coordinated beamforming method

A coordinated beamforming method to minimize the sum of intercell interference and transmit power under minimum SINR constraints are shown in Chapter 3 and 4. Power saving and minimization have the in communication networks while ensuring reliable, secure data transmission across the network. Many of the wireless systems could benefit from the cost saving of the energy. Various algorithms could be run and handle the amount of data in a short period of time. Therefore, the computational complexity to obtain beamforming weight using SDP and SOCP relaxation based methods could also be proposed to see the difference in terms of performance and power saving between the SDP and SOCP method. Also the time its takes for both schemes to converge could be verified and the results would give a clear view of the efficient method that can be used for greener energy optimization performances.

6.2.2 Cognitive Radio Coordinated beamforming method

The power minimization in chapter 3 and 4 could be extended to cellular CR system because interference management has become an important topic in cognitive radio in order to manage and fulfill the regulatory constraints. Here, the CR is used as a spectrum sharing with other licence users to check for availability and only use the spectrum when it is deemed to be free. This is achieved by controlling the transmit and the interference power. Other type of performance metrics such as sum rate, outage probability, required feedback overhead are desirable for this method.

6.2.3 Robust Beamforming With Imperfect CSI using Gauss-Markov autoregressive

A significant contribution could be made by developing a distributed optimization problem that minimizes the sum of intercell interference and transmit power in heterogeneous networks with small cells. The imperfection in CSI in this thesis is modeled to be bound within an ellipsoidal set, which is used to model the uncertainty between the true and the estimated channel coefficients. There are many other models than can accurately model the CSI uncertainty, e.g., Gauss-Markov autoregressive model, which generate a range of errors. This is a very efficient methods to achieve the actual measurement and accurately capture the characteristics of realistic user movements in wireless networks.

6.2.4 Iterative MIMO Robust Beamforming With Imperfect CSI

In chapter 3 and 4, the user terminals are equipped with single antenna. if both the user terminals and base stations have multiple antennas, the design of the global optimum can be efficiently achieved and duality theory could be used to prove the optimal solution. An iteratively optimising transmit and receive beamforming can achieve the best global optimum value. The statistical correlation of the channel matrices to estimate intercell interference and tradeoff between optimality and complexity are open problems for research

6.2.5 5G Heterogeneous networks/small

One of the implications of using MmWave for massive MIMO is the need of dense BS deployment. However, network densification comes with multiple challenges in terms of interference, higher backhaul cost and mobility traffic volumes. It has been suggested that, the generation of 5G radio access network (RAN) could be a true worldwide wireless web (wwww)[88]. However different issues of the 5G RAN such as scalability, spectrum backward flexibility, the extent of interference and battery lifetime especially in machine-type communications (MTCs) and device-to-device (D2D)communications. Therefore, spectrum allocation could be arranged dynamically in conjunction with various levels of cooperation between the network nodes. This is achievable if the total network frequency resource is shared, but with the consequences of increased interference. These alight the need to reduce the interference in the 5G Heterogeneous networks. Therefore, in chapter 3 to obtain the optimal beamforming vectors $\mathbf{w}_i$,$i = 1 \cdots U$, one is mainly interested in $\mathbf{W}_i$ solution of (3.17) that are rank 1. If the optimal $\mathbf{w}_i$ is not rank 1, the solution can be obtained using Gaussian randomization that could find the suboptimal rank 1 solution. Then, extracting the eigenvector associated to the only nonzero eigenvalue, through the eigen decomposition of resulting $\mathbf{w}_i^\star$. Then, the projected stochastic gradient method could be used to solve the problem, where the final approximate solution $\mathbf{w}_i^\star$ is the weighted average.

Bibliography

[1] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[2] [Online]. Available: http://www.smart2020.org/_assets/files/02_smart2020Report.pdf

[3] [Online]. Available: http://www.analysysmason.com/About-Us/News/Newsletter/Single-RAN-LTE-overlay-Oct2013-RDTW0/

[4] [Online]. Available: http://www.huawei.com/uk/solutions/broader-smarter/hw-077174.htm

[5] [Online]. Available: http://www.techradar.com/news/networking/taking-control-of-mobile-network-power-consumption-1217035

[6] K. Davaslioglu and E. Ayanoglu, "Quantifying potential energy efficiency gain in green cellular wireless networks," *IEEE Communications Surveys Tutorials*, vol. 16, no. 4, pp. 2065–2091, Fourthquarter 2014.

[7] Z. Niu, Y. Wu, J. Gong, and Z. Yang, "Cell zooming for cost-efficient green cellular networks," *IEEE Communications Magazine*, vol. 48, no. 11, pp. 74–79, November 2010.

[8] S. H. Lee and I. Sohn, "Affinity propagation for energy-efficient bs operations in green cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 8, pp. 4534–4545, Aug 2015.

[9] [Online]. Available: http://http://www.ofcom.org.uk/

[10] T. Irnich, J. Kronander, Y. Selen, and G. Li, "Spectrum sharing scenarios and resulting technical requirements for 5g systems," in *Personal, Indoor and Mobile Radio Communications (PIMRC Workshops), 2013 IEEE 24th International Symposium on*, 2013, pp. 127–132.

[11] A. Ghasemi and E. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 649–658, Feb 2007.

[12] M. Alotaibi, M. Sirbu, and J. Peha, "Impact of spectrum aggregation technology and frequency on cellular networks performance," in *Dynamic Spectrum Access Networks (DySPAN), 2015 IEEE International Symposium on*, Sept 2015, pp. 326–335.

[13] H. Zhang, S. Chen, X. Li, H. Ji, and X. Du, "Interference management for heterogeneous networks with spectral efficiency improvement," *IEEE Wireless Communications*, vol. 22, no. 2, pp. 101–107, April 2015.

[14] T. S. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez, "Millimeter wave mobile communications for 5g cellular: It will work!" *IEEE Access*, vol. 1, pp. 335–349, 2013.

[15] Y.-H. Lee, "Advanced multi-antenna techniques," in *49th IEEE International Midwest Symposium on Circuits and Systems, 2006. MWSCAS '06.*, vol. 1, Aug 2006, pp. xxxviii–xxxix.

[16] E. Larsson, O. Edfors, F. Tufvesson, and T. Marzetta, "Massive mimo for next generation wireless systems," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 186–195, February 2014.

[17] S. Chen, S. Sun, Y. Wang, G. Xiao, and R. Tamrakar, "A comprehensive survey of tdd-based mobile communication systems from td-scdma 3g to

td-lte(a) 4g and 5g directions," *China Communications,*, vol. 12, no. 2, pp. 40–60, Feb 2015.

[18] A. He, L. Wang, Y. Chen, M. Hansard, and T. Zhangy, "Comparison of comp and miso for energy efficiency in hetnets," in *2015 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)*, March 2015, pp. 352–357.

[19] A. Jafari, D. Lpez-Prez, H. Song, H. Claussen, L. Ho, and J. Zhang, "Small cell backhaul: challenges and prospective solutions," *EURASIP Journal on Wireless Communications and Networking*, vol. 2015, no. 1, 2015. [Online]. Available: http://dx.doi.org/10.1186/s13638-015-0426-y

[20] D. Lopez-Perez, I. Guvenc, G. de la Roche, M. Kountouris, T. Q. S. Quek, and J. Zhang, "Enhanced intercell interference coordination challenges in heterogeneous networks," *IEEE Wireless Communications*, vol. 18, no. 3, pp. 22–30, June 2011.

[21] Z.-Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1426–1438, Aug 2006.

[22] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Review*, vol. 53, no. 3, pp. 464–501, 2011.

[23] H. Sun, S. Yan, and U. Svensson, "Robust minimum sidelobe beamforming for spherical microphone arrays," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 19, no. 4, pp. 1045–1051, May 2011.

[24] O. Tervo, L.-N. Tran, and M. Juntti, "Energy-efficient transmit beamforming for miso downlink via sequential convex approximation," in *2015 IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 2015, pp. 415–419.

[25] Y. Shi, J. Zhang, and K. Letaief, "Optimal stochastic coordinated beamforming for wireless cooperative networks with csi uncertainty," *IEEE Transactions on Signal Processing*, vol. 63, no. 4, pp. 960–973, 2015.

[26] A. Ben-Tal and A. Nemirovski, *Lectures on modern convex optimization: analysis, algorithms, and engineering applications.* Siam, 2001, vol. 2.

[27] L. E. Ghaoui and H. Lebret, "Robust solutions to least-squares problems with uncertain data," *SIAM Journal on Matrix Analysis and Applications*, vol. 18, no. 4, pp. 1035–1064, 1997. [Online]. Available: http://dx.doi.org/10.1137/S0895479896298130

[28] J. Nie, "First order conditions for semidefinite representations of convex sets defined by rational or singular polynomials," *Mathematical Programming.*

[29] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory.* Society for Industrial and Applied Mathematics, 1994. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9781611970777

[30] *Chapter 8: Convex Optimization*, ch. 8, pp. 147–168. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9781611973655.ch8

[31] P. A. Parrilo, *Chapter 2: Semidefinite Optimization*, ch. 2.

[32] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Review*, vol. 53, no. 3, pp. 464–501, 2011. [Online]. Available: http://dx.doi.org/10.1137/080734510

[33] *Semidefinite Programming*, ch. 4, pp. 139–334. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9780898718829.ch4

[34] Z.-Q. Luo and T.-H. Chang, "Sdp relaxation of homogeneous quadratic optimization: approximation bounds and applications," in *Convex Optimization in Signal Processing and Communications*, D. P. Palomar and Y. C. Eldar, Eds.

[35] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics, 1994. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9781611970777

[36] *Inequalities*, ch. 8, pp. 126–143. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/1.9781611971170.ch8

[37] J. F. Sturm, *Using Sedumi 1.02, a MATLAB toolbox for optimization over symmetric cones, Optimization Methods and Software*, 1999, software available at http://sedumi.mcmaster.ca/.

[38] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*, ser. Princeton Series in Applied Mathematics. Princeton University Press, October 2009.

[39] A. Ben-Tal, A. Nemirovski, and C. Roos, "Robust solutions of uncertain quadratic and conic-quadratic problems," *SIAM Journal on Optimization*, vol. 13, no. 2, pp. 535–560.

[40] A. Ben-Tal, E. Hazan, T. Koren, and S. Mannor, "Oracle-based robust optimization via online learning," *Operations Research*, vol. 63, no. 3, pp. 628–638, 2015. [Online]. Available: http://dx.doi.org/10.1287/opre.2015.1374

[41] C. Shen, T.-H. Chang, K.-Y. Wang, Z. Qiu, and C.-Y. Chi, "Distributed robust multicell coordinated beamforming with imperfect csi: An admm approach," *IEEE Transactions on Signal Processing*, vol. 60, no. 6, pp. 2988 –3003, June 2012.

[42] S. Boyd, L. El Ghaoui, E.Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, ser. Studies in Applied Mathematics. Philadelphia, PA: SIAM, jun 1994, vol. 15.

[43] G. Tsoulos, "Smart antennas for mobile communication systems: benefits and challenges," *Electronics Communication Engineering Journal*, vol. 11, no. 2, pp. 84–94, Apr 1999.

[44] F. Gross, *Smart Antennas for Wireless Communications*, McGraw-Hill, 2005.

[45] R. N. A. Paulraj and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2006.

[46] J. D. Kraus and R. Marhefka, *Antennas for All Applications*, McGraw-Hill, 2002.

[47] C. A. Balanis, *Antenna Theory: Analysis and Design*, John Wiley and Sons, 2005.

[48] A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed mimo receivers," *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 161–176, Jan 2006.

[49] W.-K. Ma, T. Davidson, K. M. Wong, Z.-Q. Luo, and P.-C. Ching, "Quasi-maximum-likelihood multiuser detection using semi-definite relaxation with application to synchronous cdma," *IEEE Transactions on Signal Processing*, vol. 50, no. 4, pp. 912–922, April 2002.

[50] M. X. Goemans and D. Williamson, "Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming," *Journal of the ACM*, vol. 42, pp. 1115–1145, 1995.

[51] Z.-Q. Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, May 2010.

[52] A. Gershman, N. Sidiropoulos, S. ShahbazPanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 62–75, May 2010.

[53] S. X. Wu and W.-K. Ma, "Multicast transmit beamforming using a randomize-in-time strategy," in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 3376–3379.

[54] Y. Huang and D. Palomar, "A dual perspective on separable semidefinite programming with applications to optimal downlink beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4254–4271, Aug 2010.

[55] M. Beko, "Efficient beamforming in cognitive radio multicast transmission," *IEEE Transactions on Wireless Communications,*, vol. 11, no. 11, pp. 4108–4117, November 2012.

[56] E. Karipidis, N. Sidiropoulos, and Z.-Q. Luo, "Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups," *IEEE Transactions on Signal Processing*, vol. 56, no. 3, pp. 1268–12 792, March 2008.

[57] N. Bornhorst, P. Davarmanesh, and M. Pesavento, "An extended interior-point method for transmit beamforming in multi-group multicasting," in *2012 Proceedings of the 20th European Signal Processing Conference (EUSIPCO)*, Aug 2012, pp. 6–10.

[58] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2646–2660, June 2007.

[59] C. Khirallah, D. Vukobratovic, and J. Thompson, "On energy efficiency of joint transmission coordinated multi-point in lte-advanced," in *2012 International ITG Workshop on Smart Antennas (WSA)*, Mar 2012, pp. 54 –61.

[60] M. Karakayali, G. Foschini, and R. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Communications Magazine*, vol. 13, no. 4, pp. 56–61, Aug 2006.

[61] S. Han, Q. Zhang, and C. Yang, "Distributed coordinated multi-point downlink transmission with over-the-air communication," in *2010 5th International ICST Conference on Communications and Networking in China (CHINACOM)*, Mar 2010, pp. 1 – 5.

[62] M. Botros and T. N. Davidson, "Convex conic formulations of robust downlink precoder designs with quality of service constraints," *IEEE Journal of Selected Topics in Signal Processing*, vol. 1, no. 4, pp. 714–724, Dec 2007.

[63] I. Wajid, Y. C. Eldar, and A. Gershman, "Robust downlink beamforming using covariance channel state information," in *Acoustics, Speech and Signal Processing, 2009. ICASSP 2009. IEEE International Conference on*, Apr 2009, pp. 2285 –2288.

[64] E. Song, Q. Shi, M. Sanjabi, R. Sun, and Z.-Q. Luo, "Robust sinr-constrained miso downlink beamforming: When is semidefinite programming relaxation tight?" in *2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 3096 –3099.

[65] N. Vucic and H. Boche, "Downlink precoding for multiuser miso systems with imperfect channel knowledge," in *Acoustics, Speech and Signal Processing, 2008. ICASSP 2008. IEEE International Conference on*, 31 2008-april 4 2008, pp. 3121 –3124.

[66] Z. quan Luo, W.-K. Ma, A.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20 – 34, May 2010.

[67] T. A. Le and M. Nakhai, "Coordinated beamforming using semidefinite programming," in *2012 IEEE International Conference on Communications (ICC)*, June 2012, pp. 3790 –3794.

[68] Q. Zhang and C. Yang, "Transmission mode selection for downlink coordinated multipoint systems," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 1, pp. 465 –471, Jan 2013.

[69] A. Yousafzai and M. R. Nakhai, "Block qr decomposition and near-optimal ordering in intercell cooperative multiple-input multiple-output-orthogonal frequency division multiplexing," *IET Communications*, vol. 4, no. 12, pp. 1452–1462, Aug. 2010.

[70] T. A. Le and M. R. Nakhai, "Downlink optimization with interference pricing and statistical csi," *IEEE Transactions on Communications*, vol. 61, no. 6, pp. 2339–2349, Jun. 2013.

[71] T. A. Le and M. Nakhai, "A decentralized downlink beamforming algorithm for multicell processing," in *IEEE Global Communications Conference (GLOBECOM)*, Dec. 2012, pp. 3924–3929.

[72] K. Law, I. Wajid, and M. Pesavento, "Robust downlink beamforming in multi-group multicasting using trace bounds on the covariance mismatches," in *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Mar 2012, pp. 3229 –3232.

[73] Y. Huang, D. Palomar, and S. Zhang, "Lorentz-positive maps and quadratic matrix inequalities with applications to robust miso transmit beamforming," *IEEE Transactions on Signal Processing*, vol. 61, no. 5, pp. 1121 –1130, Mar 2013.

[74] ——, "Lorentz-positive mapswith applications to robust miso downlink beamforming," in *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Mar 2012, pp. 2801 –2804.

[75] E. Gharavol and Y.-C. Liang, "Robust downlink beamforming in multiuser miso cognitive radio networks," in *2009 IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications*, Sept 2009, pp. 808 –812.

[76] E. Gharavol, Y.-C. Liang, and K. Mouthaan, "Robust downlink beamforming in multiuser miso cognitive radio networks with imperfect channel-state information," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 6, pp. 2852 –2860, July 2010.

[77] M. F. Hanif, L. N. Tran, A. Tlli, and M. Juntti, "Computationally efficient robust beamforming for sinr balancing in multicell downlink with applications to large antenna array systems," *IEEE Transactions on Communications*, vol. 62, no. 6, pp. 1908–1920, June 2014.

[78] K.-K. Wong, G. Zheng, and T.-S. Ng, "Convergence analysis of downlink mimo antenna systems using second-order cone programming," in *2005 IEEE 62nd Vehicular Technology Conference VTC-2005-Fall*, vol. 1, Sept 2005, pp. 492–496.

[79] F. Penna, S. Stanczak, Z. Ren, and P. Fertl, "MMSE interference estimation in lte networks," in *2014 IEEE International Conference on Communications (ICC)*, June 2014, pp. 4548–4552.

[80] R. Zhang, Y. C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two-way multi-antenna relay channel with analogue network coding," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 5, pp. 699–712, June 2009.

[81] P. Rost and K. Samdanis, "The two-way interference channel: Robust cooperation to exploit limited backhaul efficiently," in *Proceedings of 2013 9th International ITG Conference on Systems, Communication and Coding (SCC)*, Jan 2013, pp. 1–6.

[82] M. Umair, Y. Dongkai, D. Xiao, and F. Fanny, "Identification of interferers in het-net in lte-a systems based on feicic with cell range expansion," in *International Conference of Information and Communication Technology (ICoICT)*, March 2013, pp. 198–201.

[83] A. Hamza, S. Khalifa, H. Hamza, and K. Elsayed, "A survey on inter-cell interference coordination techniques in ofdma-based cellular networks," in *IEEE Communications Surveys Tutorials*, vol. PP, no. 99, 2013, pp. 1–29.

[84] J. F. Awad, M. A. Imran, and R. Tafazolli, "Reduced complexity interference avoidance scheme based on user grouping in dense cellular networks," in *Proceedings of the 2013 19th European on Wireless Conference (EW)*, April 2013, pp. 1–6.

[85] C. Shen, K.-Y. Wang, T.-H. Chang, Z. Qiu, and C.-Y. Chi, "Worst-case sinr constrained robust coordinated beamforming for multicell wireless systems," in *IEEE International Conference on Communications (ICC)*, June 2011, pp. 1–5.

[86] Z. Ho and D. Gesbert, "Balancing egoism and altruism on interference channel: The mimo case," in *IEEE International Conference on Communications (ICC)*, May 2010, pp. 1–5.

[87] S. Sun, Q. Gao, Y. Peng, Y. Wang, and L. Song, "Interference management through comp in 3gpp lte-advanced networks," *IEEE Wireless Communications*, vol. 20, no. 1, pp. 59–66, March 2013.

[88] T. O. Olwal, K. Djouani, and A. M. Kurien, "A survey of resource management towards 5g radio access networks," *IEEE Communications Surveys Tutorials*, vol. PP, no. 99, pp. 1–1, April 2016.