DOI:
10.1016/j.jbankfin.2016.11.015

Document Version
Peer reviewed version

Accepted Manuscript

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PII: S0378-4266(16)30218-7
DOI: 10.1016/j.jbankfin.2016.11.015
Reference: JBF 5045

To appear in: *Journal of Banking and Finance*

Received date: 14 July 2015
Revised date: 3 September 2016
Accepted date: 12 November 2016

Please cite this article as: Nicos Koussis , Spiros H. Martzoukos , Lenos Trigeorgis , Corporate liquidity and dividend policy under uncertainty, *Journal of Banking and Finance* (2016), doi: 10.1016/j.jbankfin.2016.11.015

Corporate liquidity and dividend policy under uncertainty

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Acknowledgements:

We are thankful to Elettra Agliardi, Michalis Makrominas, Gloria Gardenal, Elizabeth Whalley, Van Son Lai, two anonymous referees, the editor Bart Lambrecht and conference participants at the 2015 European Financial Management Association, the 2015 Annual International Real Options Conference, the IFORS 2014 conference and the 2013 Multinational Finance Conference.

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Corporate liquidity and dividend policy under uncertainty

Abstract

We examine optimal liquidity (retained earnings) and dividend choice incorporating debt financing with risk of default and bankruptcy costs as well as growth options under revenue uncertainty. We revisit the conditions for dividend policy irrelevancy and the broader role of retained earnings and dividends. Retained earnings have a net positive impact on firm value in the presence of growth options, high external financing costs and low default risk. High levels of retained earnings enhance debt capacity but have a negative effect on equity value due to the likelihood of losing accumulated cash balances in case of default, unless offset by high external financing costs. Opposite directional effects of retained earnings on equity and debt create a U-shaped relation with firm value. The framework is extended to analyze management-shareholder conflicts, demonstrating that managers accumulate higher than optimal cash.

JEL classification: G31, G13

Keywords: dividends; retained earnings; growth options; real options; capital structure; default; debt costs

1. Introduction

Miller and Modigliani (1961) established that dividend policy is irrelevant in frictionless markets. In real markets, dividend irrelevancy does not hold in the presence of costly external financing, default risk, bankruptcy costs and growth option opportunities. Developments using contingent claims analysis or real options provide a framework for an integrated analysis of the firm's interrelated investment, capital structure and dividend policies incorporating both default risk and growth option considerations. A key advantage of the real options approach is the explicit incorporation of uncertainty and managerial or shareholder flexibility to make decisions depending on the future realization of stochastic revenues (see Dixit and Pindyck, 1994; Trigeorgis, 1996). This important strand of literature (e.g., Leland, 1994; Mauer and Triantis, 1994; Childs, Mauer and Ott, 2005; Mauer and Sarkar, 2005; Sundaresan and Wang, 2015; Liu and Mauer, 2011) has, however, largely ignored corporate liquidity and dividend policy. Contingent-based models typically assume that any excess cash is distributed in the form of dividends, while in periods of negative cash flow the firm resorts to external financing to finance the shortfall. One notable difficulty in incorporating liquidity choice in a real options framework is dealing with path-dependency arising from the need to keep track of the history of cash balances retained (e.g., Acharya et al., 2000, p.14). In this paper, we develop a theoretical framework to investigate these issues in a real options setting. We present a number of propositions based on a simple analytic setup and then obtain further insights using a more comprehensive numerical model that incorporates revenue uncertainty, path-dependent liquidity (retained earnings) choice, debt financing with risk of default and costly external financing, as well as future growth options. In our model, retained earnings are held in the form of liquid assets that earn a specified per-period interest. This serves as a substitute to reduce future external financing and the risk of costly default and incurrence of bankruptcy costs. Our main contribution is highlighting the negative disincentive effect to save as cumulated cash savings would be lost at default due to limited liability (and absolute priority rule) and quantifying the tradeoff between the default cost of savings and other important offsetting benefits. We obtain a number of related specific results.

First, we confirm that irrelevancy of retained earnings and dividend policy holds in the absence of default risk, provided retained earnings in the form of liquid assets earn the risk-free rate and there are no external financing costs. Default risk can induce a negative impact of retained earnings on unlevered firm (equity) value because accumulated cash holdings may be lost if the firm cannot make the debt payment and goes bankrupt.

Second, the role of retained earnings is more important the higher the expected benefits from growth options and the greater the external financing costs. External financing costs have a lesser impact on firm value when their only role is to finance liquidity shortages to avoid default. Lins et al. (2010), surveying CFOs from 29 countries, show that the main driver of holding liquidity is the financing of future investment growth opportunities. Brown and Petersen (2011) further suggest that cash balances enable a firm to smooth R&D spending. Riddick and Whited (2009) (see also Milne and Robertson, 1996) put forth a precautionary saving motive of holding cash, while Palazzo (2009) argues that cash-rich firms earn superior returns due to precautionary motives. Our analysis highlights the offsetting role of the risk of default, i.e., the risk of lost cash holdings in case of default offsetting the precautionary motive of accumulated cash.

Third, the incentive to maintain high retained earnings (low dividends) is moderated by firm profitability and available cash balances. For firms with low profitability and low initial cash balances, retaining earnings may not be sufficient to avoid default; in this case, it may be better for shareholders to reduce cash balances and pay higher dividends, resorting to external financing in the future if needed. Conversely, for firms with high profitability and high initial cash balances the negative role of retained earnings is mitigated as the risk of default decreases; in this case, retaining earnings may play a positive role for shareholders because the accumulation of cash balances can be used to finance growth options and reduce costly external financing. This non-linear role of profitability and initial cash balances highlights important differences regarding the behaviour of early-stage firms having low profitability and cash balances vs. more-mature firms with higher profitability and accumulated cash.

Fourth, we show that higher revenue uncertainty increases default risk, thereby reducing the role and importance of retained earnings because equityholders may lose any accumulated cash balances in case of firm default. In the presence of growth options and debt, however, higher retained earnings may be beneficial when volatility is high (despite the higher default risk) because they enhance the value of the growth options, reduce external financing needs for funding valuable growth options while maintaining a high level of debt capacity.

Fifth, higher retained earnings enhance debt capacity as the risk of default is reduced. Unless the firm has valuable growth options or high external financing costs, higher levels of retained earnings have a negative impact on equity value. The opposite directional effects of retained earnings on equity and debt values may thus have a U-shaped effect on firm value. Firm value maximization in this case favours high retained earnings and plowback when the risk of default is low (for firms with high profitability and low volatility) and when growth options and external financing costs are high. A low plowback instead is preferred under high default risk and when the value of growth options and external financing costs are low.

Sixth, in terms of investment timing, when there is low profitability (and low accumulated cash holdings from earlier periods), the firm would delay exercising its growth option to avoid incurring high external financing costs. When the firm faces high profitability (and high accumulated holdings), early investment exercise is more appealing as it can enhance revenues early on without the need to incur high external financing costs.

Finally, we address managerial-shareholder conflicts by conducting an analysis of optimal payout policy based on managerial instead of shareholder optimal retained earnings/dividend choices when there are costs of collective action by shareholders. Such managerial-based policies result in sub-optimally high cumulated cash balances, sub-optimal early exercise of growth options and delayed default. This results in significant agency costs which are more severe in the presence of growth options.

We proceed as follows. Section 2 presents a literature review. Section 3 shows the theoretical framework based on an analytic solution with comparative statics. Section 4 presents numerical sensitivity results based on an extended numerical model. Appendix A proves a key result used in the derivation of analytic comparative statics while Appendix B describes in detail the extended numerical model.

2. Review of literature

Early dividend policy theories have focused on dividend irrelevancy in frictionless markets (Miller and Modigliani, 1961), the effect of corporate and personal taxes (Brennan, 1970; Miller and Scholes, 1978) and the use of dividends as a signalling device for future growth prospects (e.g., Miller and Rock, 1985). As an alternative to an optimal capital structure, pecking order theory (Myers, 1984) motivated by asymmetric information posits that retained earnings should be the first source of financing, followed by external debt issuance, and in last resort by equity. In our model we examine a complementary relation between retained earnings and debt capacity mostly ignored by the pecking order and previous theories on dividend policy. We abstract from asymmetric information (which would increase the costs of external financing in our context) and signalling considerations and set aside the effects of personal taxes.

Agency-based theories have thus far provided prominent explanations of firm dividend decisions. Easterbrook (1984) suggests that paying higher dividends provides a disciplinary mechanism reducing manager-shareholder conflicts; the firm should thus resort to external markets for financing any future investment opportunities. Analogously, Jensen's (1986) "free-cash flow" theory posits that larger dividends reduce the incentives of managers to expropriate value via large accumulated cash balances. Lambrecht and Myers (2008) analyse capital investment policies for a firm facing management-shareholder conflicts and show how bankruptcy costs can distort investment and disinvestment decisions. Our findings provide complementary evidence that managers have strong incentives for maintaining high levels of cash balances to ensure their own benefits. We also show that managers may choose cash-holding policies sub-optimally investing in growth options and leading to high agency costs.[1] Similarly to Lambrecht and Myers (2008), managers may also sub-optimally delay default. Dittmar et al. (2003) find that firms in countries with poor shareholder protection hold more cash since shareholders cannot force managers to disgorge excessive cash balances. Dittmar and Maht-Smith (2007) further show that poor governance mechanisms lead to suboptimal use of cash balances.

Agency conflicts among equity and debt holders over dividend policy are analysed in Hirth and Uhrig-Homburg (2010), who focus on the positive role of liquidity in mitigating the agency costs between debt and equity holders. Our analysis leads to a supportive result showing that cash balances may increase debt capacity. Coculescu (2011) analyses the optimal capital structure and dividend policies for a non-renewable investment (i.e., without a growth option), showing that higher volatility favours higher dividends. She further shows that agency costs over dividend policy are higher for firms with high leverage ratios, bankruptcy costs and distance to default. Other recent evidence favours similar agency explanations of dividend policy and provides less support for signalling theories (Leary and Michaely, 2011).

Gamba and Triantis (2008) analyse a firm's dynamic financing, investment and cash retention policies. Our work shares the premise that accumulated cash holdings can be used to finance growth and avoid costly equity issuance. Unlike them, we show that the presence of debt financing and bankruptcy costs results in a U-shaped relationship with retained earnings. We additionally examine in a simpler

[1] In our model, agency conflicts of this sort can be implicitly captured by reducing the return earned on accumulated cash balances (see also Asvanunt et al., 2010), reflecting increasing agency costs of maintaining high cash balances. A recent article in the *Economist*, titled "The Rise of Distorporation", published on Oct. 26, 2013, highlights that many firms in the US adopt structures such as Master Limited Liability (MLP) that keep no retained earnings to reduce the payment of corporate taxes and enforce market discipline on managers. Allen et al. (2000) suggest another positive side effect of paying high dividends involving an increased monitoring role by institutional investors who represent a clientele for firms paying higher dividends.

setting the conditions violating the irrelevancy of retained earnings and dividends and the role of profitability and revenue uncertainty.

Acharya et al. (2000) analyse optimal dividend policy when the firm maintains cash balances to meet future debt obligations under costly equity issuance. Due to path-dependency complications, they focus on the simpler zero-coupon debt case. We examine the more general coupon-paying debt case, while additionally allowing for future growth options. Our setting also differs from Asvanunt et al. (2011) in that we allow for cash balances to be accumulated over longer horizons, capturing longer-term effects of risk faced by shareholders in losing cumulated cash balances as uncertainty unfolds. Copeland and Lyasoff (2005) analyse retention policy for an unlevered firm in the presence of growth options and costly external financing. Our model goes beyond theirs to also allow for default risk and the effects of bankruptcy costs. Kisser (2013) also recognizes that cash holdings have real option value as they can be used to avoid or reduce issuance costs, but he does not explicitly address the role of default risk, debt issuance and bankruptcy costs. His model assumes quadratic agency costs of free-cash flow, amplifying agency costs when cash balances are high. We also confirm that high volatility reduces the value of holding cash, but we attribute this to default risk rather than agency costs. Furthermore, we show that in the presence of growth options the role of retained earnings may be more significant at high volatility levels because they reduce external financing costs of valuable growth options and maintain debt capacity.

Boyle and Guthrie (2003) study the effect of cash balances on the optimal timing of investment in the presence of liquidity constraints. They obtain a U-shaped investment trigger as a function of cash balances: for low cash balances the investment trigger is reduced (i.e., the firm invests earlier), while for high cash balances the trigger is raised (the firm delays investment). Hirth and Viswanatha (2011) attribute the U-shape in the investment trigger to the firm’s balancing the trade-off between present and future financing costs. We show that in the absence of liquidity constraints, the opposite result may obtain: firms with low cash balances may delay investment to avoid external financing costs, while firms with high profitability and high initial cash balances may invest early to expedite receiving the growth option benefits as external financing costs are reduced with higher available cash.

Empirical evidence on dividend policy relevance generally finds that dividend payouts are positively related to firm profitability (measured by ROA) and size (measured by assets), and negatively related to growth opportunities (measured by R&D/assets) (see Fama and French, 2001). Fama and French (2002) relate these findings to predictions of the trade-off and pecking order theories of capital structure. They interpret the empirical finding that firms with higher profitability and lower growth opportunities pay more dividends as consistent with the trade-off theory. To our knowledge, no formal model makes these predictions explicitly. Our model provides an explicit link of dividend payout with growth opportunities, confirming that dividend payout is negatively associated with growth opportunities. However, we show that the effect of firm profitability on dividend payout is much subtler. Equityholders in less-profitable levered firms may have an incentive to pay out more dividends to avoid losing accumulated cash holdings in the future if there is a high risk of default. When the firm has sufficiently large profits, it may comfortably plowback earnings to avoid defaulting or incurring high external financing costs, as the risk of default is lower. Thus, we posit that the observed positive empirical relation between profitability and dividend payout may be due to additional factors not adequately explained by trade-off theory. Bates et al. (2009) document an increase in cash balances over time and that cash holdings are positively associated with volatility, market-to-book and R&D, and negatively associated with leverage. The positive effect of volatility on cash holdings is justified if higher volatility drives higher growth option value, consistent with the hypothesis that firms accumulate cash to finance growth options or in the presence of managerial incentives to reduce employment risk. The positive effect of market-to-book and R&D is also in line with our model's predictions of using accumulated cash balances to finance growth.

A number of empirical studies on corporate cash holdings (e.g., Opler, 1999; Harford, 1999) find supportive evidence in line with our finding that firms with large growth opportunities and limited access to capital markets tend to hold higher cash balances (see also additional evidence on the value-enhancing role of cash for constrained firms by Denis and Sibilkov, 2009). Holt (2003) also finds that due to the presence of irreversibility and financial constraints, younger firms with growth opportunities will pay dividends only after they finance their investment opportunities. Denis (2011) provides an overview of the literature on corporate liquidity, including the role of cumulated cash in alleviating financing constraints. Pinkowitz and Williamson (2007) provide further evidence that the value of cash holdings is more important in high-growth industries, such as computer software and telecommunications. Faulkender and Wang (2006) examine the impact of cash on excess stock returns, providing evidence that the value of cash holdings is more important for firms with limited access to capital markets. Furthermore, in line with our result on the positive role of cash balances on debt (capacity) levels, they find a positive relationship between the level of cash balances and leverage. Anderson and Carverhill (2012), assuming a mean-reverting process for earnings, also find a positive role of cash balances on long-term debt levels. Conversely, they do not find a significant impact of growth options on cash holdings unless firms grow at an unusually high rate.

The theoretical and empirical literature on liquidity has also examined the hedging role of cash balances. Acharya et al. (2007) show that higher cash holdings along with risky debt provide a hedge for constrained firms and should not be considered as negative debt. Froot et al. (1993) discuss the role of cross-state cash flow transfers for hedging purposes. Harford et al. (2014) argue that firms hold larger cash balances to reduce refinancing risk associated with short-term debt. Our model recognizes the positive (hedging) role of cash balances relative to costly external financing but also highlights the important role of default risk which adversely affects the impact of cumulating cash balances. Our main contribution is to bring out this negative effect of the disincentive to save as cumulated cash balances would be lost in case of default due to limited liability. Our numerical analysis provides an investigation and illustration of general conditions in which the negative incentive to save due to limited liability is offset by positive incentives offered by debt capacity, growth options etc. and quantifies this important tradeoff.

3. Theoretical framework

3.1. A simple analytic benchmark and main propositions

In this section, we study a simple model set-up analytically and derive a number of key propositions. We later on develop further insights illustrating other cases based on an extended numerical framework (presented in detail in Appendix B). This framework allows for more realistic features, such as yearly operating costs, debt issuance, taxes, bankruptcy costs, external financing costs and extension to more than two periods. Let *P* denote the firm's (present value of) stochastic revenues, assumed to follow a geometric Brownian motion (GBM) of the form:

$$\frac{dP}{P} = \mu dt + \sigma dZ \tag{1}$$

where μ denotes a constant growth rate, σ the constant volatility of revenues, and dZ a standard Weiner process. Under risk-neutrality, the constant drift μ is replaced with the risk-free rate *r*. We generally assume the firm's revenues actually grow at a rate $r - \delta$, where δ is an opportunity cost or dividend-yield adjustment (e.g., from competitive erosion) reducing the growth rate of revenues.

In the simple set up, we assume away any operating costs, depreciation expense and taxes such that the firm cash flow at $t = 0$ is P_0. The firm operates for two periods, period 0 where the firm chooses its savings and period *T* where the firm decides whether to exercise a growth option. In period 0, it chooses the level of savings or retained earnings, $R_0 \in [0, P_0]$, which in turn determines the level of distributed dividends, $D_0 = P_0 - R_0$. In the second period (at *T*), the firm has the option to pay an investment cost *X* and obtain the revenues during the period (P_T), any cumulated cash balances since period 0 with continuously compounded interest r_x ($R_0 \exp(r_X T)$), and any additional revenues arising from the growth option, equal to an enhanced multiplier factor e of revenue levels at T (eP_T). Non-investment or default in this simple set-up is triggered by the firm not paying the investment cost X at T. This occurs when the annual revenues, the growth investment generated cash flows and any cumulated cash or savings from the earlier period are not sufficient to cover the investment cost X.[2] Following standard contingent claim arguments, the Black-Scholes type equation describing the value of equity (unlevered firm) is:

$$E_0 = P_0 - R_0 + (1+e)P_0 \exp(-\delta T) N(d_1) - \exp(-rT)(X - R_0 \exp(r_X T)) N(d_2) \quad (2)$$

with

$$d_1 = \frac{\ln(((1+e)P_0)/(X - R_0 \exp(r_X T))) + (r - \delta + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}};$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Note that to avoid negative values in the logarithmic terms in parameters d_1 and d_2, when $X - R_0 \exp(r_X T) < 0$, $d_i \to \infty, i = 1,2$.[3] In this case, default risk is eliminated making all cumulative normal terms in Eq. (2) equal to 1. The first two terms in Eq. (2) capture the value of dividends in the initial period, $D_0 = P_0 - R_0$, which depends on the choice of retained earnings by the firm. The third and fourth terms represent the value of the growth option to receive revenues at T (enhanced by a factor e). The exercise price of the growth option X is reduced by cumulated savings of the previous period (earning interest r_X continuously compounded till T). As usual, the term $N(d_2)$ captures the risk-neutral

[2] The investment cost is here for simpliicity treated as non-discretionary in order to allow for default risk in a simple setting simplifying the analytic derivations. In the more general numerical model presented in Appendix B, we separate out the discretionary costs of financing the growth option from non-discretionary operational costs. The main insights relating to default risk remain unchanged.
[3] This is the intuitive condition that the options' exercise price must be non-negative.

probability that equity holders will exercise the growth option at T by paying investment cost X net of the savings.

The sensitivity of equity (unlevered firm) value to retained earnings R_0 is thus given by:

$$\frac{\partial E_0}{\partial R_0} = -1 + \exp((r_X - r)T)\,N(d_2) + \frac{\partial d_1}{\partial R_0}\left[(1+e)P_0\exp(-\delta T)n(d_1) - \exp(-rT)(X - R_0\exp(r_X T))n(d_2)\right]$$

(3)

where $n(d) = \frac{1}{\sqrt{2\pi}}\exp(-\frac{d^2}{2})$ and $\frac{\partial d_1}{\partial R_0} = \frac{\partial d_2}{\partial R_0} = \frac{\exp(r_X T)}{\left[X - R_0\exp(r_X T)\right]\sigma\sqrt{T}}$.

This result is greatly simplified by noting that the term in brackets [.] to the right of Eq. (3) is 0 (see Appendix A for a proof). Thus, the impact of a change in retained earnings on equity (unlevered firm) value is as follows:

$$\frac{\partial E_0}{\partial R_0} = -1 + \exp((r_X - r)T)N(d_2) \quad (4)$$

We thus observe that dividend or retained earnings irrelevancy holds when two conditions are met.

Proposition 1: Dividend/retained earnings irrelevancy (unlevered firm). The retained earnings/dividends choice is irrelevant when:

a) retained earnings earn the risk-free interest rate ($r_x = r$), and

b) there is no default risk.

Proof:

It is essential for irrelevance that $r_x = r$; otherwise, a marginal change in retained earnings may enhance value when $r_x > r$ or it may be value reducing when $r_x <$ r. Absence of default risk further ensures that

$N(d_2) = 1$ and so $\frac{\partial E_0}{\partial R_0} = 0$. This establishes our base result that the level of retained earnings/dividends has no effect on unlevered firm (or equity) value.

Corollary 1. With $r_x = r$, the presence of default risk results in a negative (positive) relation between retained earnings (dividends) and unlevered firm value.

Proof: With positive default risk $N(d_2) < 1$ and $r_x = r$ it readily obtains that $\frac{\partial E_0}{\partial R_0} = -1 + N(d_2) < 0$.

The presence of a costly investment growth opportunity in this simple setup introduces default risk since, if not exercised, the firm does not invest and defaults. Our sensitivity results based on the extended numerical model of Appendix B (presented in the next section) confirm this key role of default risk that in a more general context may be caused by fixed operating costs or debt coupon payments due. Irrespective of the precise form or source, default risk should be absent for dividend/retained earnings irrelevancy to hold. This basic idea is also found in other papers (e.g., Gertner and Scharfstein (1991)), albeit convoluted with other aspects of investment and financing decisions.

Corollary 1 highlights the significant role of default risk in the dividend/retained earnings choice. In a setting with positive default risk where $r_X = r$, cumulating cash balances actually reduce value for shareholders. Corollary 1 shows that 1 dollar of dividends foregone today and retained as savings for future use has a present worth of $N(d_2) < 1$. This is so because there is a risk that this dollar saved will be foregone in the future in case of default as a result of limited liability and absolute priority rule. The above holds in the presence of future growth options. An enhanced value of the growth option (e.g., manifested via higher values of the growth expansion factor *e*) has a positive effect on retained earnings by positively affecting $N(d_2)$. Proposition 2 below summarizes this result.

Proposition 2: Effect of growth options. A higher level of growth options (*e*) has a positive effect on the role of retained earnings on equity (unlevered firm) value.

Proof: Taking the derivative of Eq. (4) with respect to growth expansion factor *e* results in

$$\frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial e} = \exp((r_X - r)T)n(d_2)\frac{\partial d_2}{\partial e}\text{, where } n(d_2) > 0 \text{ and } \frac{\partial d_2}{\partial e} > 0\text{. Thus, } \frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial e} > 0\text{.}$$

Despite the positive effect of growth options shown in proposition 2, since $N(d_2)$ is bounded above by 1, and for a reasonable case where r_X is close to *r*, a very high value of the growth option (*e*) cannot reverse the sign of $\frac{\partial E_0}{\partial R_0}$ from negative to positive. As we show in the next section, the presence of growth options combined with positive external financing costs can result in a value-enhancing role of cumulating cash balances.

Similarly to the case of growth options in the absence of external financing costs, high profitability alone also cannot reverse the negative impact of retained earnings on equity (unlevered firm) value. This can be readily seen from Eq. (4), where the negative effect of retained earnings can at best become zero as firm profitability gets very large ($P_0 \to \infty$). However, increasing firm profitability will reduce default risk and thereby improve the effect of retained earnings on equity (unlevered firm) value. This is summarized in proposition 3 below.

Proposition 3: Effect of profitability. High current firm profitability (P_0) will have a positive effect on the role of retained earnings on equity (unlevered firm) value.

Proof: Taking the derivative of Eq. (4) with respect to revenues (P_0) results in

$$\frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial P_0} = \exp((r_X - r)T)\, n(d_2)\frac{\partial d_2}{\partial P_0} \text{ where } n(d_2) > 0 \text{ and } \frac{\partial d_2}{\partial P_0} = \frac{1}{P_0 \sigma \sqrt{T}} > 0\text{. Thus, } \frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial P_0} > 0\text{.}$$

Consistent with other real options models, higher firm volatility enhances firm value in the presence of growth options ($\frac{\partial E_0}{\partial \sigma} > 0$). However, when one examines the effect of volatility on the role of retained earnings, default risk again plays a central role. This leads to a negative impact of volatility concerning the role of retained earnings on equity (unlevered firm) value. This result is summarized in proposition 4 below.

Proposition 4: Effect of revenue volatility. Higher revenue volatility (σ) will have a negative effect on the role of retained earnings on equity (unlevered firm) value.

Proof: Taking the derivative of Eq. (4) with respect to σ, we obtain $\frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial \sigma} = \exp((r_X - r)T)n(d_2)\frac{\partial d_2}{\partial \sigma}$.

Note that $n(d_2) > 0$ and $\frac{\partial d_2}{\partial \sigma} = -\frac{\left(\ln(((1+e)P_0)/(X - R_0 \exp(r_X T))) + (r - \delta + \frac{1}{2}\sigma^2)T\right)}{\sigma^2 \sqrt{T}} < 0$. Thus, $\frac{\partial(\frac{\partial E_0}{\partial R_0})}{\partial \sigma} < 0.$

An important issue in these analyses is whether optimal retention/payout choices are always characterized by a corner (0 or 100%) solution. Our analysis in this section shows that optimal retention choice will result in a corner solution of zero plowback in the absence of any external financing costs and debt as long as $r_X = r$ (see proposition 1 and corollary 1). When r_X is sufficiently larger than r, a corner solution at the other end (i.e., 100% plowback) may obtain. No-corner solutions are possible in this setup only in the presence of external financing costs (which generally drive higher retention values) combined with $r_x < r$ (a driving force for lower plowback).

4. Further insights from extended numerical model

We next extend the above simple 2-period set up to one where the firm now operates in 3 periods: $t = 0$ (current period), period $t = 1$ (at T_1) and period $t = 2$ (at T_2). This numerical extension enables gaining additional insights into the role of external financing costs, growth options and leverage. Our extended numerical model is described in more detail in Appendix B. Here, we sketch the key features of the model and the main additional insights it is intended to capture. Under this extended setup, the (potentially levered) firm can invest at intermediate date t = 1 and save both at the initial date t = 0 and at t = 1. Unlike the simple 2-period analytic framework presented earlier, here non-investment in the growth option does not necessarily result in default as the firm may stay in the existing mode of operation(s). Investment in the growth option is discretionary and merely enhances the scale of revenues in period t = 2 via a multiplicative expansion factor e . The firm now faces an overhang of fixed costs denoted by C (either operating C_e or financial C_R), and faces additional costs of external financing which are a proportion v_E of the financial deficit. Corporate profits are taxed with a rate τ. When debt is considered, it is issued at t = 0 and repaid in installment payments (C_R) at the intermediate (t = 1) and last date (t = 2). Equity value obtained by the firm's equity holders at t = 0 (E_0) reflects the risk of default and associated bankruptcy costs which are a proportion b of the value of unlevered assets at default. [4] This extended setup involves key trade-offs facing the firm's shareholders in their choice of retained earnings and dividend policy, which are summarized below.

First, higher retained earnings in the form of cash balances reduce the need and cost to externally finance a future shortfall or growth option. In the presence of debt, internal cash holdings can also be used to avoid intermediate involuntary default and associated bankruptcy costs. Cash holdings can further enhance the debt capacity of the firm and the tax benefits of debt.[5] On the negative side, accumulated cash balances may be lost in case of firm default. As a result of this specific risk of foregone retained earnings in the event of default, debtholders get to benefit from higher levels of retained earnings and cash balances because they are part of the recoverable assets.[6] For the same reason, higher cumulated retained earnings enhance debt capacity.

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[4] We do not consider agency conflicts and risk-shifting between existing and new shareholders or between shareholders and debt holders.

[5] These beneficial roles of retained earnings are more evident in Appendix B (see Eq. (A.1) - Eq.(A.3)) where retained earnings are used to finance a financing shortfall *(F*) and the growth option and thus reduce external financing costs (see Eq. (A.5) and Eq.(A.6)). Additionally, in Eq. (A.7) accumulated retained earnings reduce the risk of default in the last period. The negative effect of cumulating cash balances is seen in Eq. (A.7) and Eq. (A.9) where equity value can become zero and accumulated retained earnings lost.

[6] See Eq. (A.8) and Eq.(A.10) in Appendix B.

Second, our extended setup provides additional insights into the role of retained earnings in the presence of growth options. Because the enhancement of revenues from the growth investment accrues in a future period whereas the investment cost of expansion (*X*) is incurred early on (at T_1), retaining earnings from earlier periods help in reducing external financing costs. The role of retained earnings will be greater the higher the accrued growth investment benefits (*e*) and the more the external financing costs.[7] This allows the firm to pursue the exercise of the expansion option with substantial future dividends accruing to equity holders (due to higher expansion *e*) without the need to incur high external financing costs.

Thirdly, retained earnings here are held as liquid assets earning a return r_x per period until they are used to finance growth or liquidity shortages, are distributed as dividends or end up in the hands of debtholders in case of default. When $r_X > r$, the role of cash balances is enhanced. Boyle and Guthrie (2003) use a separate process to describe the evolution of cash balances allowing a role for the volatility of cash balances. They show that higher cash holdings volatility results in the firm investing earlier to avoid foregoing future cash flows. However, this approach does not allow for the analysis of retention policies. Assuming a fixed return on retained earnings here suggests that retained earnings can be used to reduce volatility and alleviate default in bad states. This has a beneficial role and it enhances debt capacity. The main drawback is that retained earnings may be lost if default occurs.

In what follows subsection 4.1 revisits the basic conditions under which dividend irrelevancy holds using our extended numerical model described in Appendix B analysing equity value without financial leverage (unlevered firm) but in the presence of external financing costs. We confirm the negative role of default risk and present additional insights for an enhanced role of retained earnings in the presence of a growth option and external financing costs. Subsection 4.2 revisits the factors that affect the retained earnings/dividend choice for an unlevered firm in this extended setup focusing on the impact of profitability, revenue volatility and investment growth option. In subsection 4.3 we examine the effect

[7] This result holds because a higher level of financing from cumulated retained earnings (*F*) (chosen in Eq.(A.3)) reduces the variable component of financing costs proportional to the deficit (Eq. (A.6)).

of retained earnings on debt and levered firm value in the presence of bankruptcy costs. Subsection 4.4 considers the issue of investment timing. Subsection 4.5 examines managerial-shareholder conflicts.

4.1. Dividend/retained earnings irrelevancy for an unlevered firm with external financing costs

In this subsection, we discuss numerical results for the case where our model predicts dividend policy and retained earnings irrelevancy for equity (an unlevered firm) while allowing the possibility of external financing costs and discretional exercise of the growth option. Figure 1A presents numerical results in the presence and in absence of a growth option without default risk. Figure 1B considers the case involving positive fixed operating costs driving default risk.

When focusing on the case with no default risk, we assume that operating costs and debt coupon payments are zero (no default risk, $C_e = C_R = 0$). Our parameter choices for the risk-free rate ($r = 0.036$), volatility of revenues ($\sigma = 0.2$), and the corporate tax rate ($\tau = 0.3$) are in line with prior studies (e.g., see Leland, 1994; Mauer and Sarkar, 2005; Goldstein et al., 2001; Hennessy and Whited., 2007). The risk-free rate of 3.6% is based on the historical average of the 3-month US Treasury Bill rate for the period with available data (1934–2015). A 5% ($\delta = 0.05$) "dividend yield" or competitive erosion is used. A 5-year intermediate horizon ($T_1 = 5$) is chosen as typical for firm product development (e.g., see Pennings and Lint, 1997).[8] A 10-year business planning and debt horizon ($T_2 = 10$) is also common. We discuss the implications of varying these and other parameters in subsequent sections. We assume in the base case that the return earned on cash balances or retained earnings equals the risk-free rate (i.e., $r_x = r = 0.036$). For the case "without growth" option in the left panel, we set $e = 1$; for the "growth" investment case in the right panel, we assume an investment (exercise) cost $X = 100$ with $e = 3$. We also consider cases both without ($v_E = 0$) and with ($v_E = 0.10$) variable external financing costs. Hennessy and Whited (2007) estimate external financing costs to be within the range 5%-10.7% depending on firm size.

[8] A 5-year horizon may be considered long as it creates a high range of potential revenue outcomes at T_1 and T_2 and reduces the potential for business planning, particularly with respect to retained earnings and future financing needs. We have examined shorter horizons in additional sensitivity results. Our general implications do not change, although we find that shorter horizons allow for better business planning and a somewhat enhanced role for retained earnings.

[Insert Figure 1]

The left panel of Figure 1A confirms that in the absence of debt/default risk and growth options, unlevered firm (equity) value is invariant to the proportion of earnings retained (or the portion paid as dividends). The level of external financing costs here has no effect as there are no anticipated financing needs in the absence of fixed operating costs and debt payments. At intermediate decision time (T_1), a zero retention ratio is optimal irrespective of the level of revenues. A small decline in the return earned on retained earnings, however, results in a violation of dividend/retained earnings irrelevancy. In this case, with $r_x < r$, firm value declines with retained earnings, while when $r_x > r$ firm value increases in retained earnings (latter results not shown for brevity). The right panel of Figure 1A focuses on the case involving a discretionary growth option. Now, given a rising need for external financing, retained earnings acquire more value as they can be used to finance the growth option. In case of no external financing costs, retained earnings remain irrelevant even with a growth option available.

Figure 1B considers the case involving positive fixed operating costs of 80 at T_1 and T_2 (C_e = 80), leading to scenarios with negative profits and the possibility of default. In the "growth" option case of panel B, we again assume X = 100, e = 3. The left panel in Figure 1B shows that even in the absence of external financing costs, dividend/retained earnings policy is relevant when there is a risk of default. An increase in plowback (retained earnings) reduces firm value, with or without external financing costs.[9] The main reason for the negative relation between retained earnings and firm value is that, in the case of default, accumulated cash balances will be foregone for shareholders. With a growth option (right side of panel B), retained earnings play a more beneficial role in the presence of external financing costs.[10]Overall, the results of this extended numerical model are in line with and supportive of the analytic model results of the previous section. They confirm that the retained earnings/dividends choice is

[9] The case with external variable financing costs of 10% shows only minor differences and is thus not reported in the figures.

[10] For this particular set of parameters, a positive role for retained earnings is not that apparent. We discuss in subsequent sections situations where the role of retained earnings is enhanced when external financing costs are substantial or when the firm considers debt financing.

irrelevant when retained earnings earn the risk-free rate ($r_x = r$), when there is no default risk (no fixed debt or operating costs), and when there are no future growth options. Additionally, we show that dividend irrelevancy holds in the absence of external financing costs even when a costly growth option exists. That is because exercise of the growth option is discretional so that no exercise does not result in default and the firm stays in the existing mode of operations.

4.2. Role of profitability and growth options with external financing costs

This subsection re-examines the factors that enhance the role of retained earnings based on our extended numerical model of Appendix B. We focus on the effect of firm profitability, the volatility of revenues, and the impact of growth options on equity without financial leverage (unlevered firm). Figure 2 examines the role of profitability, defined here as the level of revenues relative to operating costs. We consider fixed operating costs ($C_e = 80$) varying relative profitability from low (revenue $P_0 = 50$) to high (revenue $P_0 = 150$). Additionally, for Figure 2 we again assume: $\tau = 0.3$, $r = r_x = 0.036$, $\delta = 0.05$, $\sigma = 0.2$, $T_1 = 5$, T_2=10, $v_E = 0.1$ and that a growth option is available with cost $X = 100$, e = 3. Debt is set to zero ($C_R = 0$, $b = 1$ focusing on an unlevered firm. In Figure 2, higher levels of retained earnings reduce unlevered firm/equity value when profitability is relatively low. However, at higher profitability, retained earnings have a positive impact on firm value. These results confirm the analytic results of proposition 3, however now we see that in the presence of external financing costs a more significant role for retained earnings is acquired.

[Insert Figure 2]

Figure 3 illustrates the effect of revenue volatility. The figure first verifies the well-known real options result that higher volatility increases growth option value and, indirectly, equity value (e.g., Trigeorgis, 1996).[11] However, we show here that at higher revenue volatility, retained earnings have a more negative impact on unlevered firm (equity) value. This is manifested in a steeper negative effect of

[11] With debt financing, high volatility may adversely affect firm value (see Leland, 1994 and our subsequent analysis).

higher retained earnings at higher volatility levels. This result is in line with Kisser (2013). However, in our case, it is driven by operating default risk regardless of whether there is a growth option. The above results regarding volatility are consistent with proposition 4 derived earlier using the simpler analytic setup. In the subsequent section we further analyse the effect of volatility on levered firm value as it illustrates interesting trade-offs between equity and debt financing obtained.

[Insert Figure 3]

Retained earnings can be more value enhancing when there are higher benefits from the growth option exercise. In Figure 4 Panel A, this is illustrated by varying the expansion factor *e* that drives the accrued benefits of the growth investment. The results confirm proposition 2 (derived using the simpler analytic setup) that a higher level of expansion benefits *e* enhances the beneficial role of retained earnings.

[Insert Figure 4]

Figure 4 Panel B assumes higher external financing costs, showing that the role of retained earnings is further enhanced. Compared with Panel A, which assumes financing costs of 10%, Panel B assumes financing costs of 20%, showing that unlevered firm (equity) value can increase in the plowback ratio at lower expansion factors. This extends our earlier result based on the analytic setup, demonstrating that the presence of external financing costs can result in a positive role of cash balances. We summarize this result below as an empirical prediction.

Prediction 1 (external financing costs): A higher level of external financing costs further enhances the positive role of cumulating higher retained earnings.

We note in this regard that with sufficiently high external financing in the presence of a costly growth option, a corner solution can be obtained where 100% plowback is optimal. This assumes $r_X = r$. When r_X is lower than r, internal solutions of optimal retention/payout policy can be obtained.

4.3. The role of debt financing

In the presence of debt, the risk of default is inevitable so dividend irrelevancy does not hold even if there are no operating costs or growth options. Using our extended numerical model of Appendix B we show that if cumulated cash holdings accrue to debtholders in case of default, debt capacity is enhanced and equity value declines with higher retained earnings. This is in line with Morrelec (2001), who shows that liquid assets enhance debt capacity when there are debt covenants protecting debtholders from shareholders disposing liquid assets. We show that under a solution where retention is concurrently chosen with debt financing, the optimal retention ratio is determined by the relative impact of retained earnings on equity and debt values. A higher level of retained earnings enhances debt value while in general results in reduced equity value unless there are significant growth options and external financing costs. Due to this opposite effect of retained earnings on equity and debt values, levered firm value may exhibit a U-shape with respect to retained earnings. Instead of a U-shape, a purely positive (negative) role of retained earnings can be obtained when debt enhancement is more (less) important than the reduction in equity.

Figure 5 illustrates the impact of retained earnings on the levered firm, equity and debt holder values at two different levels of profitability.[12] As noted, a higher level of retained earnings reduces equity but increases debt value. Equity value decreases due to the risk that accumulated cash holdings may be foregone in case of default. The overall effect on total levered firm value depends on the relative impact on equity and debt values, with a U-shaped relationship between firm value and retained earnings possible.

[Insert Figure 5]

[12] In our simulations of Figure 5, because $r_X = r$, corner solutions are obtained. We obtain internal solutions of optimal retained earnings when $r_x < r$. These results are in line with Rochet and Villeneuve (2011) who also obtain internal solutions for cash balance holdings when the return earned on cash holdings is lower than the discount rate.

We next summarize the following empirical prediction regarding the effect of retained earnings on debt, equity, and firm values.

Prediction 2 (debt, equity, and firm value): A higher level of retained earnings can result in opposite directional effects on equity (negative) and debt value (positive), leading to a U-shaped effect on firm value.

Figure 6 shows further insights regarding the impact of volatility on the role of retained earnings in the presence of debt. We use a relatively "normal" level of profitability with revenues P_0 = 120 and a debt coupon of C_R = 100 (operating costs C_e are here assumed zero). We distinguish between two cases, one in the absence of a growth option and one involving a costly growth option. In both cases we assume external financing costs v_E = 10% of external financing needs. First, we observe that retained earnings have a positive impact on firm value both with and without a growth option, at low and high volatility due mainly to their enhancing role on debt capacity. An important insight gained here is that in the presence of a growth option, a high level of volatility combined with high levels of cumulated cash balances result in the highest levered firm value. A high volatility enhances equity value due to the presence of the growth option and higher cumulated cash balances help counterbalance the negative effects on debt capacity and reduce possible external financing needs. By comparison, in the no growth option case a lower volatility results in higher firm value due to the absence of significant upside growth potential for equity holders and significant external financing cost savings arising from financing the growth option with retained earnings. Both in the presence and in the absence of a growth option we verify a negative effect of retained earnings on equity value due to the presence of default risk where cumulated cash balances may be lost. However, we observe that higher volatility creates a more significant (steeper) negative effect of cumulated cash balances on equity value due to the higher default risk. On the other hand, a classic real option result obtains where higher volatility results in higher equity and lower debt values. Finally, we observe a positive role of retained earnings on debt value, which is more substantial for higher volatility as exemplified by the steeper increase in debt value when the volatility of revenues is high.

[Insert Figure 6]

We summarize these results in the following prediction.

Prediction 3 (volatility effect in the presence of debt and growth options): In the presence of growth options and debt, higher volatility enhances the role of retained earnings because it enhances the growth option value obtained by equity holders while the retained earnings can be used to limit external financing costs of financing the growth option. Furthermore, at high volatility a higher level of retained earnings sustains debt capacity.

4.4. Effect of early investment timing (with growth option)

In this section we perform a robustness test investigating a variation of the model that allows for early investment timing at $t = 0$ or at $t = T_1$. The firm here can make the growth investment earlier, which allows for earlier enhancement of growth option revenues. For numerical model implementation see Appendix B. We assume here that with early growth option exercise, revenues are enhanced not just at T_2 (by $e = 2$) but also at T_1 (by $e = 2$). Offsetting this benefit of earlier growth option exercise and intermediate revenue enhancement, the firm may now incur higher external financing costs earlier to finance the growth option if cash holdings available at an earlier stage are not sufficient. Figure 7 shows firm value for the decision to wait (W) or to exercise early (EE) the growth option at various levels of plowback and firm profitability. We contrast a low vs. high level of initial profitability affecting the financing costs of early-exercising the option (high vs. low). Here, the early exercise (EE) strategy is invariant to plowback because the firm in both scenarios does not have cash left after financing the growth option (kept as retained earnings).

[Insert Figure 7]

When initial profitability is low (and hence cash balances are insufficient to finance the growth investment), the firm may delay exercise of the investment option. Even though firm value is decreasing in plowback when the firm waits, firm values for the wait strategy (W) are higher than when the firm exercises early (EE). Firm value under early exercise (EE) is lower as the firm may lack sufficient cash balances and incurs high external financing costs early on to finance the growth option. Waiting (W) allows the firm to attain lower external financing costs as it may enter a more favourable state of revenues next period. In the second panel, the situation is reversed. When initial profitability is high and the funding gap small, external financing costs of exercising the growth option early (EE) are small, while the firm can benefit from the earlier enhancement of revenues. Our results regarding investment timing are summarized next.

Prediction 4 (effect of retained earnings on optimal investment timing): when firm profitability or accumulated cash holdings from earlier periods are low, early investing is less appealing due to high external financing costs needed for exercising the growth option. With high firm profitability or high accumulated earnings, early investing in the growth option is more beneficial as the firm need not incur high external financing costs.

4.5. Management-shareholder conflicts

Our analysis so far has been silent concerning the conflicts of interest between management and the firm's shareholders. The management-shareholder conflict can have important shareholder value implications. For example, Gormley and Matsa (2016) show that managers may undertake value-destroying decisions such as increasing cash holdings and lowering volatility. We next examine the impact of managerial-driven optimizing decisions --as opposed to firm (shareholder) value maximization assumed earlier -- to analyse the role of conflicts of interest between the management and shareholders.

To more readily incorporate this type of analysis into our base numerical model setup, we assume that firm operating costs C_e reflect payments related to managerial compensation and focus on an unlevered firm (zero debt). We further assume that shareholders face costs of collective action (see Lambrecht and Myers, 2008) so that if they intervene they obtain a fraction α times the value of the firm at t = 0, i.e., αV_0, where $0 \leq \alpha \leq 1$ and V_0 is the optimal firm value (see Eq. (A.11) in Appendix B). The costs of collective action also hold at T_1, such that if shareholders intervene they obtain α times the firm value under a growth or no growth policy depending on which is optimal (see Eq. (A.9) in Appendix B). When the costs of collective action are zero or negligible (α = 1), our solutions reduce to those obtained under shareholder value maximization.

As the costs of shareholders' collective action increase (i.e., for values of α lower than 1), managerial-based optimization results in excessive cash holdings at t = 0. Intuitively, management chooses a high plowback upfront in order to reduce downside risk and ensure firm continuity and the collection of future managerial compensation. Confirming this insight, we also find that the firm defaults much later (at a lower revenue trigger) compared to the shareholder-value based optimal solution. Managerial-based optimization also results in earlier exercise of the growth option as it is partly financed with retained earnings. Earlier exercise of the growth option by management ensures enhanced revenues in the future period and thus reduced risk of loss of managerial compensation. We also find that at intermediate time T_1, management may choose a lower plowback (compared to that used at t = 0) providing shareholders with a sufficient incentive to continue with operations (see also Lambrecht and Myers, 2012). We further find that for sufficiently high revenue levels, managers may choose to wait instead of exercise the growth option, i.e., the decision regions become {default, wait, exercise the growth option, wait} starting from low to high revenue values.[13] Deviations from optimal firm valuation are more severe in the presence of growth options.

[13] Because the costs of collective action are proportional to firm value, enforcing exercise of the growth option at high revenue states is very costly for shareholders. From the management perspective, at these very high revenue states, the risk of default and the loss of future managerial compensation is at a minimum even if the firm waits (does not enhance future revenues by exercising the growth option). The result is that managers choose to wait instead of exercising the growth option at these high revenue states.

To illustrate our findings, consider a case with managerial compensation costs of 80 (C_e = 80) given the earlier base-case parameters (P_0 =100, τ = 0.3, r = r_x = 0.036, δ = 0.05, σ = 0.2, T_1 = 5, T_2=10, v_E = 0.1) with the growth option (X = 100, e = 3). Debt is zero (C_R = 0, b = 1). Firm value optimization in the absence of shareholders' costs of collective action (α = 1) results in a value of 107.43. With costs of collective action at 10% of firm value (i.e., α = 0.9), firm value reduces to 103.52, involving agency costs of 3.78% and optimal managerial-driven plowback of 100% at t = 0. In an extreme case where shareholder intervention is not possible (α = 0), firm value declines to 88.55, resulting in agency costs of over 21%. These differences are due to a sub-optimally chosen high level of cumulated cash holdings at t = 0 and to managerial-driven investment and default policies resulting in earlier investment in the growth option, non-exercise of the growth option at very high revenue levels and in delayed default. To illustrate the differences in decision regions, we compare the decision region for the case with no costs of collective action (α = 1) with the case where costs of collective action are at the highest level (α = 0). In the case of α = 1, the firm defaults at T_1 when revenues $P \in [0, 76.46)$. By comparison, when α = 0 the firm delays default to much lower levels, when $P \in [0, 19.99)$. When α = 1 the firm waits when $P \in [76.46, 109.35)$ and then exercises the growth option at revenue levels higher than 109.35. By contrast, when α = 0 the firm waits when $P \in [19.99, 58.47)$ and exercises the growth option much earlier (at lower revenue levels), in the region $P \in [58.47, 2737)$. Moreover, for revenue levels that are very high, the firm waits. When α = 0 firm plowback at t = 1 ranges between 55% and 15% for revenue levels $P \in [19.99, 119.5)$ and zero plowback for revenues higher than 119.59. On the other hand, under a shareholder-based optimal solution plowback remains zero for all revenue levels.

In the absence of the growth option in the above situation, management will choose 100% plowback at t = 0 when α = 0 to minimize default risk, whereas in the absence of costs of collective action (α = 1) plowback at t = 0 is zero. Firm value in this case with no shareholder intervention drops to 81.84 (compared to a firm optimal of 94.51 when α = 1). The maximum level of agency costs due to managerial-shareholder conflicts in the no-growth case is less than 16% (compared to 21% in the case with a growth option). We offer the following empirical prediction.

Prediction 5 (manager-shareholder conflicts): In the presence of costs of collective action by shareholders, payout policies chosen by managers to maximize own managerial compensation can result in excessive levels of cash balances and delayed default. Resulting firm value reduction (compared to shareholder-based value optimization) is more severe in the presence of growth options as managerially-driven payout solutions result in sub-optimal exercising of the growth options.

It is worthy here to relate our findings to recent studies justifying dividend smoothing. In general our setup does not result in dividend smoothing behaviour. When costs of collective action are low (high α) we find that managerial-based policies are aligned with shareholders' preferences to distribute excess cash. This implies erratic changes in dividends from period to period that depend on the variation in profitability. When costs of collective action are high (α is low), managers tend to cumulate large cash balances initially but may pay substantially higher dividends later on to keep shareholders happy to continue operations.[14] This also implies some erratic (rather than smooth) dividends behavior across periods. Lambrecht and Myers (2012) obtain dividend smoothing when shareholder costs of collective action are combined with managerial risk aversion and habit formation. In another context, Acharya and Lambrecht (2015) justify smoothing in the presence of information differences between managers and shareholders as managers smooth dividends to meet shareholders' expectations. Our model does not incorporate these features, however it captures default, limited liability and growth options. It would be an interesting avenue for further research to extend our setup to multiple stages of growth in the presence of

[14] When future revenues are high, managers pay high dividends even when the costs of collective action are high (we typically obtain that plowback is zero at high revenue levels). Thus, managers align dividend policy with the interests of shareholders because they face little risk of losing future managerial compensation. When revenues are at medium levels, managers use part of retained earnings to finance the growth option. In this case it is unlikely that dividends will be paid out to shareholders, however shareholders will tend to accept this since they avoid the costs of collective action and external financing costs while they capture high future revenue growth. For low level of revenues, exercise of the growth option may no longer be possible, so managers choose to remain in the existing mode of operation and to partly finance any shortfall with cumulated cash balances. Because in this case the growth option investment cost is avoided, managers may pay some dividends to shareholders allowing them enough benefits to continue instead of defaulting.

default risk examining whether there are conditions where dividend smoothing might be obtained within our current setup.[15]

5. Conclusions

We have developed a real options model of retained earnings (dividend) choice in the presence of growth options, debt financing with risk of default and bankruptcy costs under revenue uncertainty. Our simpler analytic setup helped clarify the conditions for retained earnings or dividend irrelevancy uncovering the main drivers affecting the retained earnings/dividend choice. Our model confirms that dividend policy irrelevancy depends critically on the absence of default risk. In the presence of growth options, dividend irrelevancy holds if there are no costs of external financing. Lower firm profitability and higher revenue volatility favor higher dividend payout due to the risk that shareholders will lose any accumulated cash balances in case of default. A significant role of retained earnings under high volatility is obtained in the presence of significant growth options and debt financing. We highlight a complementary role of retained earnings on debt capacity especially when there is high risk of default such as in case of low profitability and high revenue volatility. With regard to investment timing, we show that when current profitability and cumulated cash balances are high the firm will favor early exercise of the growth option avoiding high costs of external financing. We also discuss management-shareholder conflicts over payout policy when shareholders face significant costs of collective action. We find that deviations from optimal firm value are more pronounced when there is suboptimal exercise of growth options and that managers choose excessive cash holdings to avoid default and the loss of managerial compensation. Several testable implications and predictions stand out, providing a platform for further empirical work in this important area at the intersection of investment, financing and dividend/retention policies.

[15] We have also investigated the impact of superimposing a partial adjustment (dividend smoothing) model of Michaely and Roberts (2012) in our numerical model framework. The presence of a partial adjustment equation for dividends acts as an additional constraint resulting in firm value reductions that may reach 16%-21% depending on whether there are growth options or not. Of course, such estimates are indicative on the true impact on firms since our model is not fully calibrated with the data (and furthermore it is not a multi-period model).

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Appendix A: Proof of equation (4) deriving the sensitivity of equity value with respect to retained earnings

In order to prove equation (4) we prove that:

$$(1+e)P_0 \exp(-\delta T)n(d_1) - \exp(-rT)(X - R_0 \exp(r_x T))n(d_2) = 0$$

Define $P^{'} = (1+e)P_0$, $X^{'} = (X - R_0 \exp(r_X T))$.

We need to show that $\exp(-rT)X' n(d_2) = P' \exp(-\delta T)n(d_1)$ which is verified below:

$$\begin{aligned}
\exp(-rT)X' n(d_2) &= \exp(-rT)X' n(d_1 - \sigma\sqrt{T}) \\
&= \exp(-rT)X' \frac{1}{\sqrt{2\pi}} \exp(-\frac{(d_1 - \sigma\sqrt{T})^2}{2}) \\
&= \exp(-rT)X' \frac{1}{\sqrt{2\pi}} \exp(-\frac{d_1^2}{2}) \exp(-\frac{(-2d_1\sigma\sqrt{T} + \sigma^2 T)}{2}) \\
&= \exp(-rT)X' n(d_1) \exp(d_1\sigma\sqrt{T}) \exp(\frac{-\sigma^2 T}{2}) \\
&= X' n(d_1) \exp(-rT + \ln(P'/X') + (r - \delta + \frac{1}{2}\sigma^2)T - \frac{\sigma^2}{2}T) \\
&= X' n(d_1) \exp((P'/X') - \delta T) \\
&= P' \exp(-\delta T)n(d_1).
\end{aligned}$$

Thus, $(1+e)P_0 \exp(-\delta T)n(d_1) - \exp(-rT)(X - R_0 e^{r_x T})n(d_2) = 0$ and after this simplification we can thus obtain equation (4): $\frac{\partial E_0}{\partial R_0} = -1 + \exp((r_X - r)T)N(d_2)$.

Appendix B: An extended numerical model

In this appendix we present a numerical model that extends the simple analytic setup in 3 periods, t = 0 , t = 1 (at T_1) and t = 2 (at T_2). P denotes firm revenues following a GBM shown in Eq. (1) of the main text. In this more general setting we allow for operating costs per period C_e for periods t = 0, t = 1 and t = 2 . Debt ($\tilde{B}_0$) issued at t = 0 and maturing at T_2 requires financial costs C_R in periods t = 1 and t = 2. Total costs (sum of operating plus financial) in periods 1 and 2 is denoted by C. The firm has a growth option at T_1 by incurring a cost X that if paid results in an enhancement in revenues in the last period by e . We define two mode of operations: mode S where the firm does not exercise the growth option (stays in existing mode) and mode G where the firm exercises the growth option. We suppress investment timing flexibility.[16]

A constant corporate tax rate τ applies, so that after-tax profits per period are $\Pi_0 = (P_0 - C_e)(1-\tau)$ at t = 0 and $\Pi_1 = (P_1 - C)(1-\tau)$ at t = 1. After-tax profits at t = 2 are $\Pi_2^i = ((1+e^i)P_2 - C)(1-\tau)$, $i = S, G$ and depend on whether the firm's equityholders have not exercised the growth option (mode S) at T_1 in which case $e^S = 0$ or have exercised the growth option at T_1 (mode G) in which case $e^G = e$. Unlevered after-tax profits are $\Pi_{tU} = (P_t - C_e)(1-\tau)$ at t = 0,1 and $\Pi_{2U}^i = ((1+e^i)P_2 - C_e)(1-\tau)$, i= S, G, at t = 2 with $e^S = 0$ and $e^G = e$. The expected present value of future unlevered cash flows is denoted by $\tilde{V}_{tU}^i$, $i = S, G.$ The value of unlevered assets includes an option to abandon so it cannot become negative.

The debt amount $\tilde{B}_0$ raised at t = 0 reflects the expected present value of financial (coupon) payments C_R in periods t = 1 and t = 2 or the value of unlevered assets after incurring proportional bankruptcy costs b in the event of default. Debt values in period t = 1 (at T_1) and period t = 2 (at T_2) are denoted by $B_t^i, t = 1,2,\ i = S, G$ since they depend on whether the growth option is exercised or not.

[16] The version of the model with investment timing flexibility is available uppon request.

The firm's control variable x_t defines the proportion of the period revenues P_t and any cumulated cash balances retained by the firm in each period *t*, $t = 0,1$. In period 0 retained earnings are $R_0 = x_0 \Pi_0$. We assume liquid assets earn r_X annually (continuously compounded).[17] Retained earnings in period *t* = 1 (at T_1) depend on whether the firm exercises or not the growth option and is denoted by $R_1^i, i = S, G$. They depend on the choice of proportion $x_t^i, i = S, G$, on cumulated cash balances from earlier periods and whether the firm is profitable in period *t* = 1, thus:

$$R_1^i = x_1^i (\Pi_1 + R_0 \exp(r_X T_1)) \quad \text{if } \Pi_1 \geq 0$$

(A.1)

$$R_1^i = x_1^i (R_0 \exp(r_X T_1)) \quad \text{if} \quad \Pi_1 < 0 \qquad i = S, G$$

(A.2)

Eq. (A.1) shows that when profits are positive the firm may choose to retain earnings from current period profits and cumulated cash balances from earlier periods. In Eq. (A.2) we allow for partial or total transfer of any cumulated earnings from earlier periods to the next period even when the profitability of period 1 is negative.[18] In the last period (T_2) retained earnings accumulated, i.e, $R_2^i = R_1^i \exp((r_X (T_2 - T_1)) \; i = S, G$ are paid out to shareholders, unless the firm defaults.

When profits are negative, the choice of x_1^i in Eq. (A.2) determines the level of internal financing F_1^i used by the firm:

$$F_1^i = (1 - x_1^i) R_0 \exp(r_X T_1)) \text{ if } \Pi_1 < 0 \quad i = S, G.$$

(A.3)

[17] We implictly assume that there is no working capital and thus net income is equivalent to cash flows. These cash flows if not distributed are assumed to be kept in the form of liquid assets earning r_x. A tax-disadvatage of cumulating cash (compared to debt financing) may be incorporated by reducing the return earned on cash balances.
[18] This may hold even when external financing is costly when the return earned on liquid balances r_X is sufficiently high compared to the discount rate in which case the firm may not use all cumulated cash to finance shortfalls but instead partially or fully finance the deficit using external financing.

The choice of retained earnings in each period determines dividends. Thus, dividends at t = 0 are $D_0 = (1 - x_0)\Pi_0$. In period t = 1 (at T_1) dividends depend on the profits of the period, any cumulated cash balances from earlier periods and whether or not there is an exercise of the growth option, thus:

$$D_1^i = (1 - x_1^i)(\Pi_1 - X^i + R_0 \exp(r_X T_1)) \quad \text{if} \quad \Pi_1 > 0$$

(A.4)

$$D_1^i = \Pi_1 - X^i + F_1^i \quad \text{if} \quad \Pi_1 < 0 \quad i = S, G.$$

In Eq. (A.4) $X^S = 0, X^G = X$ and F_1^i is defined in Eq. (A.3).

We define an indicator function I_E that takes the value of 1 if external financing is needed and 0 otherwise as follows:

$$I_E = 1 \qquad \text{if} \qquad \Pi_1 - X^i + F_1^i < 0$$

(A.5)

$$I_E = 0 \quad \text{otherwise.}$$

In Eq. (A.5) $X^S = 0, X^G = X$ and F_1^i for $i = S, G$ are defined in Eq. (A.3).

If the firm resorts to external financing this involves a cost X_E which is proportion v_E to the level of external financing needed:

$$\begin{aligned} X_E &= v_E(\Pi_1 - X^i + F_1^i) \quad &&\text{if } I_E = 1 \\ X_E &= 0 &&\text{if } I_E = 0 \quad i = S, G. \end{aligned}$$

(A.6)

The optimal retained earnings is selected over a discrete set of choices $x_0 \in [0,1]$ in period t = 0 and $x_1^i \in [0,1], i = S, G$ in period t = 1 (at T_1) while all cumulated cash flows are distributed as dividends to the equity holders in the last period t = 2 (at T_2) unless the firm defaults. Define the equity value function at time t as $E_t^i(P_t, R_t^i)$ which depends on the state of revenues P_t and the control variable R_t^i (controlled by equity holders through x_t^i) for modes i = S,G. The forward-backward dynamic programming optimization works as follows. Given the choice of retained earnings in earlier periods, moving forward in the last period t = 2 (at T_2) we have the following optimality condition for equity holders:

$$E_2^i(P_2, R_2^i) = \max(\Pi_2^i + R_2^i, 0) \quad i = S, G \tag{A.7}$$

$$\text{where} \quad R_2^i = \exp(r_X(T_2 - T_1))R_1^i.$$

The resulting debt value based on coupon level C_R in period t = 2 (at T_2) is:

$$B_2^i = C_R \quad i = S, G \quad \text{if} \quad E_2^i(P_2, R_2) > 0 \quad \text{or} \tag{A.8}$$

$$B_2^i = (1-b)(\max(\Pi_{2U}^i, 0) + R_2^i) \quad \text{if} \quad E_2^i(P_2, R_2) = 0$$

Then, moving backwards, in period t = 1 (at T_1) the following optimality condition holds for equity holders:

$$E_1(P_1, R_1^*) = \left[\max_{x_1^S}(E_1^S(P_1, R_1^S), \max_{x_1^G} E_1^G(P_1, R_1^G) \right] \tag{A.9}$$

where

$$E_1^S(P_1, R_1^S) = \max(D_1^S - X_E \cdot I_E + \tilde{E}_1^S, 0),$$

$$E_1^G(P_1, R_1^S) = \max(D_1^G - X_E \cdot I_E + \tilde{E}_1^G, 0).$$

The values for $D_1^i, i = S, G$, X_E and I_E are defined in Eq. (A.4-A.6). Note that $\tilde{E}_1^i$ (and $\tilde{B}_1^i$ that follows in the next equation) denote the expected continuation value of equity and debt calculated using the binomial probabilities as of T_1.

Debt value in period t = 1 (at T_1) is calculated as follows:

$$B_1^i = C_R + \tilde{B}_1^i \quad \text{if} \quad E_1^i(P_1, R_1) > 0 \tag{A.10}$$

$$B_1^i = (1-b)(\max(\Pi_{1U} + \tilde{V}_{1U}^i, 0) + R_1^i) \text{ if } E_1^i(P_1, R_1) = 0 \quad i = S, G.$$

At t = 0 the following optimality condition holds in order to obtain total levered firm value (obtained by equity holders):

$$V_0(P_0, R_0^*) = \max_{x_0}(D_0 + \tilde{E}_0 + \tilde{B}_0) \tag{A.11}$$

The optimization is based on a first-best optimal firm value taking into consideration both $\tilde{E}_0$ which is the expected continuation value of equity and the level of debt financing $\tilde{B}_0$ raised by equity holders at t = 0. When debt is zero (effected by setting C_R =0 and b = 1) then V_0 in Eq.(A.11) is the unlevered firm (equity value). We solve the problem numerically by building a forward-backward algorithm of exhaustive search over all retained earnings choices using binomial trees (see Cox, Ross and Rubinstein, 1979).[19] We use N_1 steps for the first period T_1. For *each* end-state value of the stochastic revenue (P_t) at T_1, a new lattice for the remaining period ($T_2 - T_1$) is built using N_2 steps, where $N_2 = (T_2 - T_1)/dt$ with dt as above. Thus, several binomial lattices emanate during T_2 from each ending node at T_1. Our algorithm applies an exhaustive search among a grid of plowback choices at t = 0 (ranging from 0 to 100% with small increments), then goes forward and for *each* end-state at T_1 evaluates the optimal plowback using a new grid search for plowback choices, goes backwards to find firm value at t = 0 and then repeats the process using new plowback choices at t = 0 until the level that maximizes firm value is obtained. Further details on construction of binomial trees using a forward-backward approach are provided in Koussis et al. (2013).

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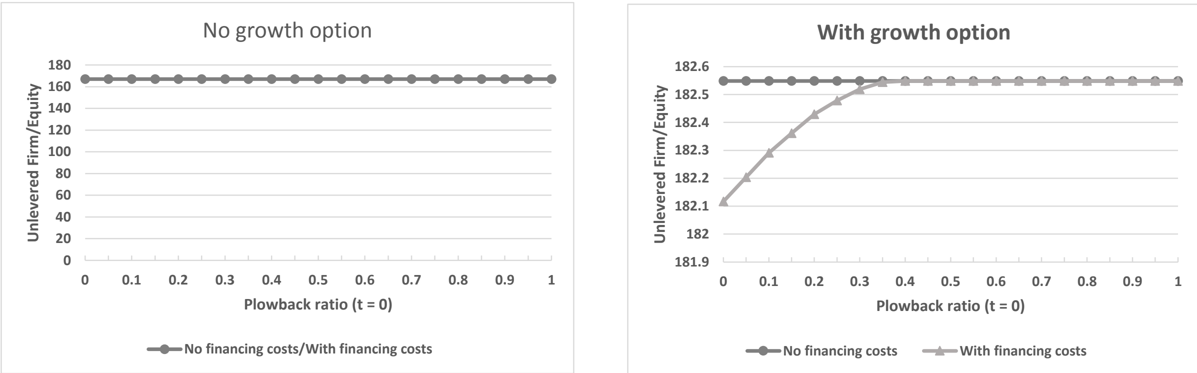
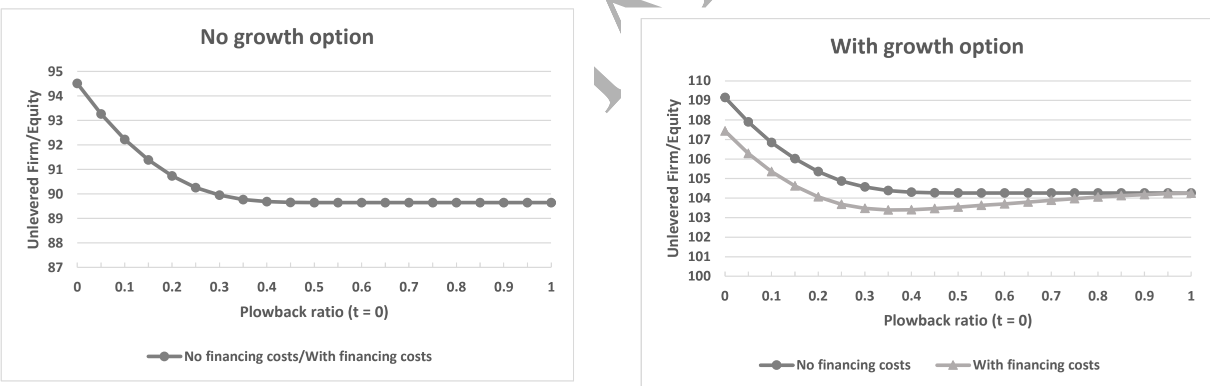
[19] The lattice parameters for the up and down multiplicative jumps and the up and down probabilities are: $u = \exp(\sigma\sqrt{dt})$, $d = 1/u$, $p_u = \frac{\exp((r-\delta)dt) - d}{u-d}$, $p_d = 1 - p_u$, where $dt = T_1 / N_1$.

Figure 1. Optimal retained earnings (plowback ratio) at *t* = 0 for an unlevered firm (equity) in possible presence of external financing costs

A. Without default risk

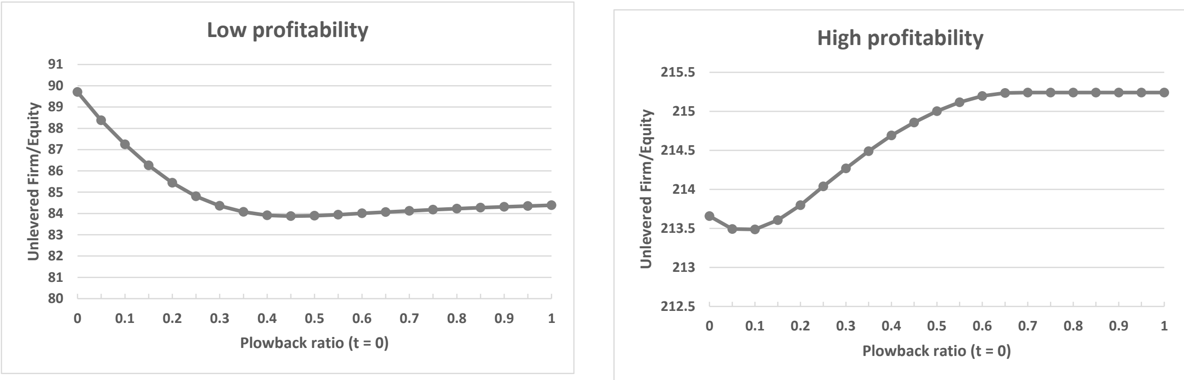


B. With default risk



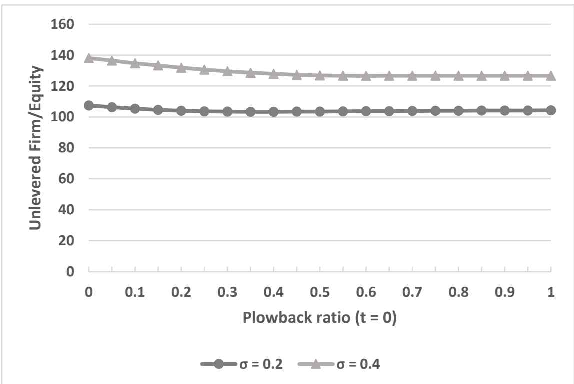
Note: In Figure 1A (without default) we assume P_0 =100, operatng costs C_e = 0, zero debt (C_R = 0, b = 1), τ = 0.3, r = r_x = 0.036, δ = 0.05, σ = 0.2, T_1 = 5, T_2 = 10 years. In "no growth option" case we use e = 1 and in "growth option" case exercise cost is X = 100 and e = 3. We assume either v_E = 0 or variable external financing costs v_E = 0.1. In Figure 1B (with default risk) we set C_e = 80 (adding risk of default) and in "growth option" case exercise cost X = 100 and e = 3. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 2. Impact of firm profitability on retained earnings (plowback) policy for an unlevered firm (equity) in the presence of external financing costs



Note: We assume operating costs C_e = 80, zero debt (C_R = 0, b =1), τ = 0.3, r = r_x= 0.036, δ = 0.05, σ = 0.2, T_1 = 5, T_2 = 10 years, variable external financing costs v_E = 0.1. For the growth option we assume exercise cost is X = 100, e = 3. For low firm profitability we use P_0 = 90 and for high profitability P_0 = 150. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 3. Impact of revenue volatility on retained earnings (plowback) policy at *t* = 0 on unlevered firm/equity in the presence of external financing costs

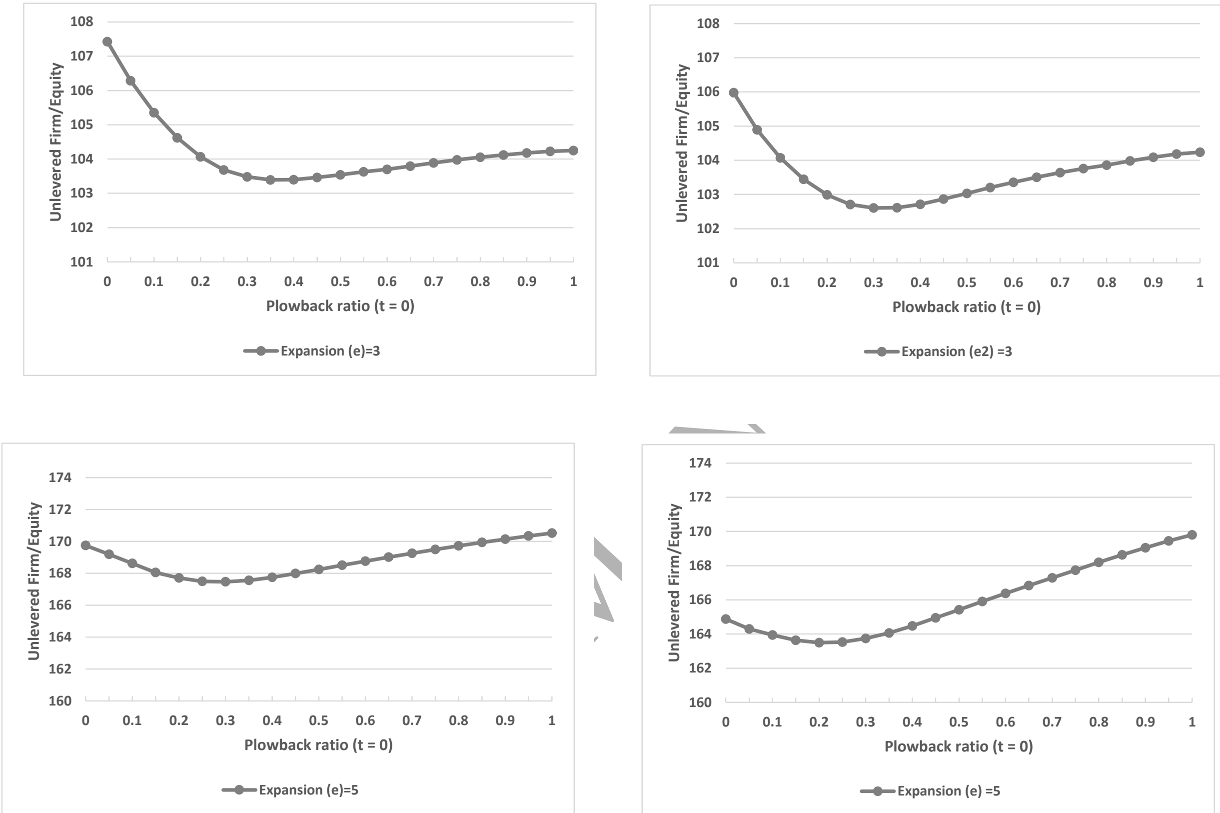


Note: In panel A we assume P_0 =100, operating costs C_e = 80, zero debt (C_R = 0, b =1), τ = 0.3, r = r_x = 0.036, δ = 0.05, σ = 0.2, T_1 =5, T_2 = 10 years and variable external financing costs v_E = 0.1. We also consider a growth option with exercise cost X = 100, e = 3. We vary revenue volatility (σ) between 0.2 and 0.4. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 4. Impact of growth (expansion) option and financing costs on retained earnings (plowback) policy at t = 0 for unlevered firm (equity)

A. Variable financing costs 10%

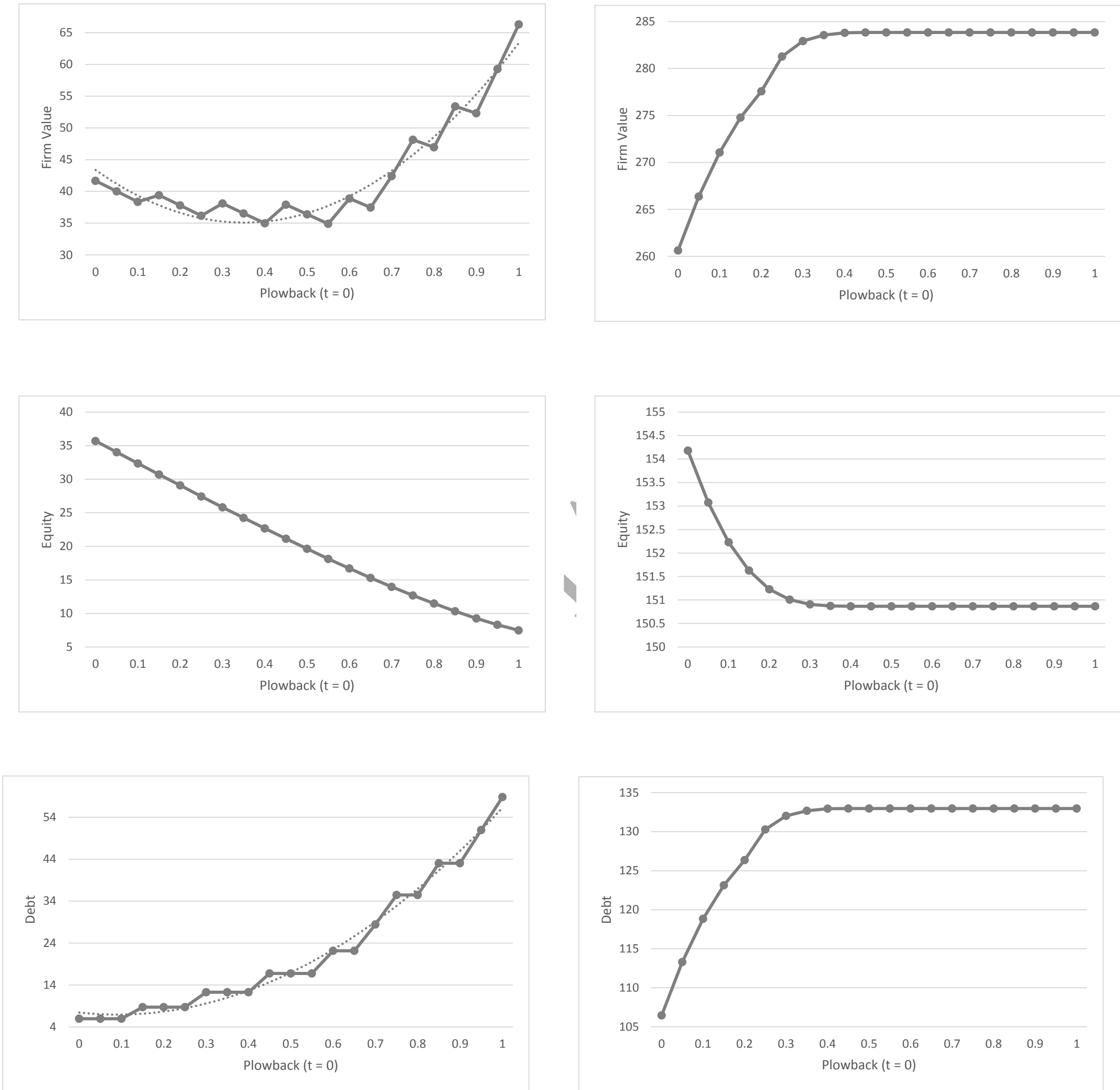
B. Variable financing costs 20%



Note: We assume P_0 =100, operating costs C_e = 80, zero debt (C_R = 0, b = 1), τ = 0.3, r =r_x = 0.036, δ =0.05, σ = 0.2, T_1 =5, T_2 = 10 years, growth option has exercise cost X = 100 with e = 3 or e = 5. In Panel A we use variable external financing costs v_E = 0.1 and in Panel B v_E = 0.2. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 5. Impact of profitability on equity, debt and firm value with debt financing

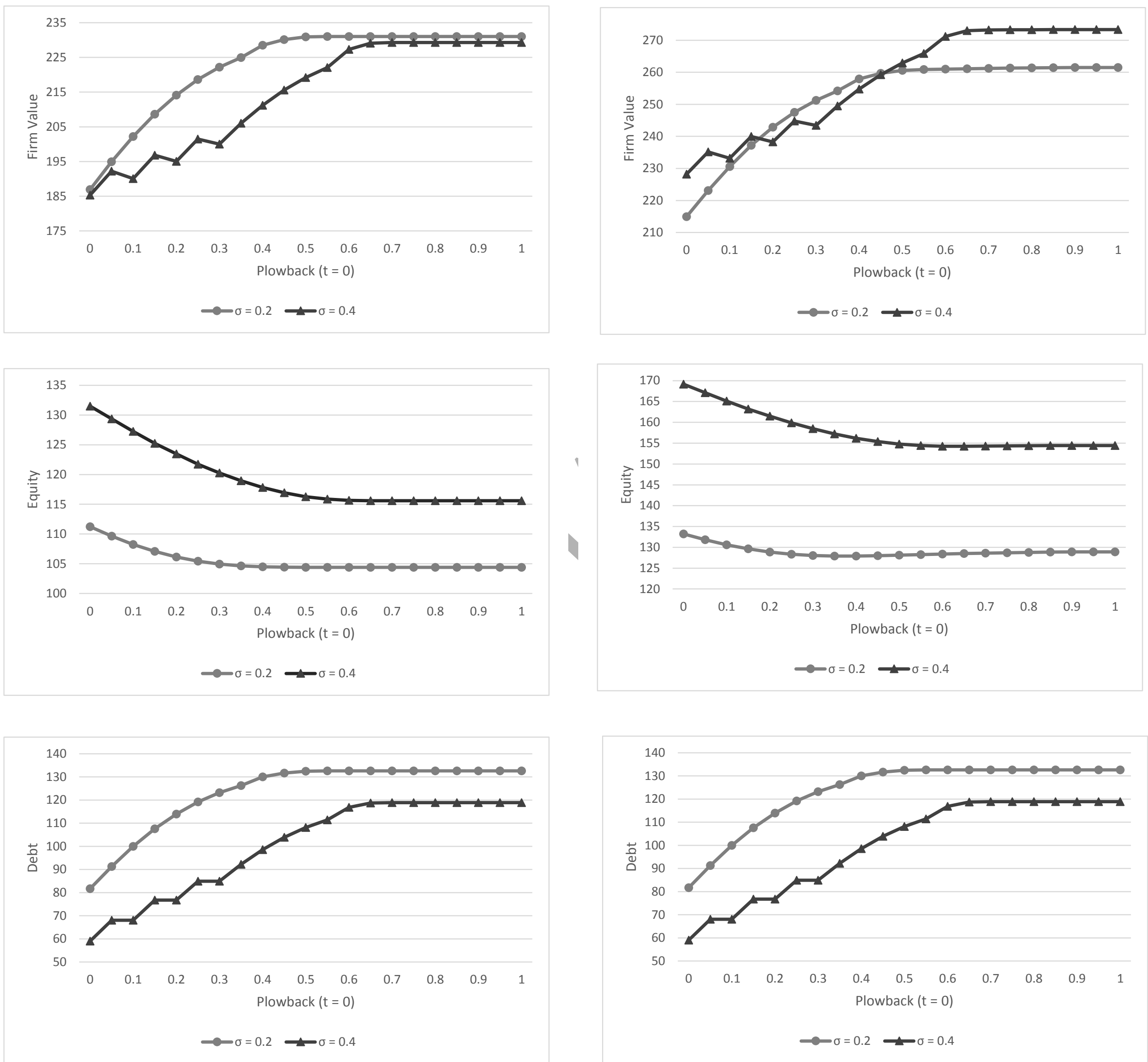
A. Low profitability **B. High profitability**



Note: We assume debt coupon C_R = 100, operating costs C_e =0, τ = 0.3, b = 0.2, r = r_x = 0.036, δ = 0.05, σ = 0.2, T_1 = 5, T_2 = 10 years and e = 1 (no growth option). For high profitability we assume P_0 = 150 and for low profitability we use P_0 = 50. Variable external financing costs v_E = 0.1. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 6. Impact of revenue volatility on retained earnings (plowback ratio) policy with debt financing

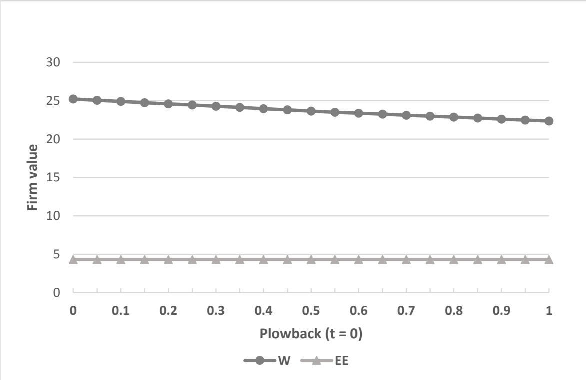
A. No growth option **B. With growth option**



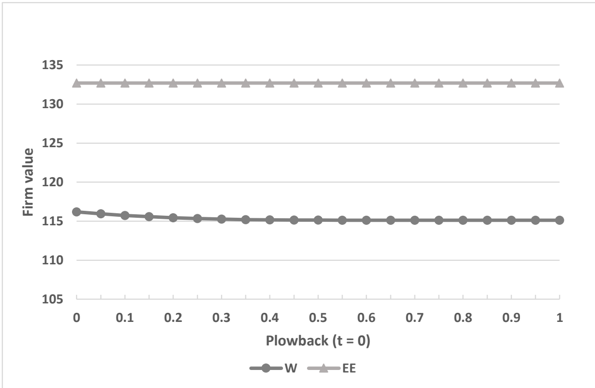
Note: We assume P_0 =120, debt coupon C_R = 100, operating costs C_e =0, τ = 0.3, b = 0.2, r = r_x = 0.036, δ = 0.05, T_1 = 5, T_2 = 10 years, and e = 1 (no growth option). With growth option (Panel B) we assume growth option costs X = 100 with e = 3. For high volatility we assume σ = 0.4 and for low volatility σ = 0.2.Variable external financing costs v_E = 0.1. The numerical solutions are based on the lattice model with N_1 = 100 steps.

Figure 7. Impact of investment timing in growth option exercise on retained earnings (plowback) policy at different levels of profitability

A. Low profitability



B. High profitability



Note: We assume P_0 =150 or 90, operating cost C_e = 80, zero debt (C_R = 0, b =1), τ = 0.3, r = r_x = 0.036, δ = 0.05, σ = 0.2, T_1 = 5, T_2 = 10 years. For the growth option we here assume e = 1.5 with cost X = 50. Variable external financing costs v_E = 0.1. "W" defines the value of the firm when it chooses to wait to exercise the growth option until T_1, and "EE" denotes early exercise of the growth option at t = 0. In the latter case the firm needs to finance the shortage with costly external financing when retained earnings do not suffice. Panel A represents the case of low profitability, assuming P_0 = 90. Panel B represents high profitability (P_0 = 150). The numerical solutions are based on the lattice model with N_1 = 100 steps.