Robust Optimization for Multi-Antenna Downlink Transmission in Cellular Networks

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Robust Optimization for Multi-Antenna Downlink Transmission in Cellular Networks

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Submitted to the King's College London
for the degree of Doctor of Philosophy

January 2015

This thesis is dedicated to my parents for their love and endless support.

Acknowledgements

Fist and foremost, I like to give an honor to my Lord and savior who have been my strength and my salvation through it all.

The four year long voyage through the rough seas of research has come to an end. Unforgettable moments of agony and ecstasy, anguish and elation. Everything molded into a sense of success and accomplishment. Although a PhD dissertation is a very personal piece of work, it is not less true that the following pages wouldn't have been the same without all the people who directly or indirectly, have influenced me during these last years. It is a pleasant aspect that I have now the opportunity to express my gratitude for all who made this journey smooth and enjoyable.

I would like to express my sincere gratitude to Dr Mohammad Reza Nakhai, for his support, patience, and encouragement throughout my PhD studies. It has been a pleasure to make this long journey under his supervision not only in the good moments, but also in the moments where my enthusiasm was not at its highest. His technical and editorial advice was essential to the completion of this dissertation and has taught me innumerable lessons and insights on the workings of academic research in general. I look forward to have more opportunities to learn from and collaborate with him in the future.

I am also very grateful to Prof. Hamid Aghvami for always providing a stimulating environment at the department and his motivation, knowledge and long lasting experience have been invaluable to me.

I am at dearth of words to express my gratitude to my parents and family members, who have been patient, caring and supportive in every way possible. Their love provided my inspiration and was my driving force. I owe them everything and wish I could show them just how much I appreciate them.

I would like to thank my former colleague, Dr Tuan Anh Le for his assistance and support. Last but not least, to those who indirectly contributed in this

research, your kindness means a lot to me. Thank you very much.

Saba Nasseri

January 2015

Abstract

In multi-cell networks where resources are aggressively reused and the cell sizes are shrinking to accommodate more users, eliminating interference is the key factor to reduce the system energy consumption. This growth in the demand of wireless services has urged the researchers to find new and efficient ways of increasing coverage and reliability, i.e., coordinated signal processing across base stations. The optimum exploitation of the benefits provided by coordinated signal processing can be achieved when a perfect channel state information at transmitter (CSIT) is available. The assumption of having perfect knowledge of the channel is, however, often unrealistic in practice. Noise-prone channel estimation, quantization effects, fast varying environment combined with delay requirements, and hardware limitations are some of the most important factors that cause errors. Providing robustness to imperfect channel state information (CSI) is, therefore, a task of significant practical interest.

Current robust designs address the channel imperfections with the worst-case and stochastic approaches. In worst-case analysis, the channel uncertainties are considered as deterministic and norm-bounded, and the resulting design is a conservative optimization that guarantees a certain quality of service (QoS) for every allowable perturbation. The latter approach focuses on the average performance under the assumption of channel statistics, such as mean and covariance. The system performance could break down when persistent extreme errors occur. Thus, an outage probability-based approach is developed by keeping a low probability that channel condition falls below an acceptable level. Compared to the worst-case methods, this approach can optimize the average performance as well as consider the extreme scenarios proportionally.

In existing literature, robust precoder designs for single-cell downlink transmissions have been extensively investigated, where inter-cell interference was treated as background noise. However, robust multi-cell signal processing has not been adequately explored.

In this thesis, we focus on robust design of downlink beamforming vectors for multiple antenna base stations (BSs) in a multi-cell interference network. We formulate a robust distributed beamforming (DBF) to independently design beamformers for the local users of each BS. In DBF, the combination of each BS's total transmit power and its resulting interference power toward other BSs' users is minimized while the required signal-to-interference-plus-noise-ratios (SINRs) for its local users are maintained.

In our first approach of solving the proposed robust downlink beamforming problem for multiple-input-single-output (MISO) system, we assume only imperfect knowledge of channel covariance is available at the base stations. The uncertainties in the channel covariance matrices are assumed to be confined in an ellipsoids of given sizes and shapes. We obtain exact reformulations of the worst-case quality of service (QoS) and inter-cell interference constraints based on Lagrange duality, avoiding the coarse approximations used by previous solutions. The final problem formulations are converted to convex forms using semidefinite relaxation (SDR). Through simulation results, we investigate the achievable performance and the impact of parameters uncertainty on the overall system performance.

In the second approach, in contrast to the 'average case' and 'worst-case' estimation error scenarios in the literature, to provide the robustness against channel imperfections, the outage probability-based approach is proposed for the aforementioned optimization problem. The outages are due to the uncertainties that naturally emerge in the estimation of channel covariance matrices between a BS and its intra-cell local users as well as the other users of the other cells. We model these uncertainties using random matrices, analyze their statistical behavior and formulate a tractable probabilistic approach to the design of optimal robust downlink beamforming vectors by transforming the probabilistic constraints into a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. The performance and power efficiency of the proposed probabilistic algorithm compare to the worst-case approach are assessed and demonstrated through simulation results.

Finally, we shift to the case where imperfect channel state information is available both at transmitter and receiver sides; hence we adopt a bounded deterministic model for the error in instantaneous CSI and design the downlink beamformers. The robustness criterion is to minimize the transmitted

power while guaranteeing a certain quality of service per user for every possible realization of the channel that is compatible with the available channel state information. To derive closed form solutions for the original non-convex problem we transform the worst-case constraints into a SDP with LMI constraints using the standard rank relaxation and the S-procedure. Superiority of the proposed model is confirmed through simulation results.

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Abbreviations

AoD	angle of departure
BS	base station
CBF	coordinated beamforming
CSI	channel state information
CSIT	channel state information at transmitter
DAS	distributed antenna system
DBF	decentralised beamforming
DDA	distributed-array antenna
FDD	frequency division duplexing
ICI	intercell interference
IMT	international mobile telecommunications
KKT	Karush-Kuhn-Tucker
LMI	linear matrix inequality
LTE	long term evolution
MAC	multiple access channel
MS	mobile station
MBF	multicell beamforming
MCP	multicell proccessing
MIMO	multiple input multiple output
MISO	multiple input single output
MMSE	minimum mean squared error
QoS	quality of service
RS	relay station
SDP	semidefinite programming
SDR	semidefinite relaxation
SINR	signal-to-interference-plus-noise ratio
SNR	signal-to-noise ratio
SOC	second order cone

SOCP	second order cone programming
TDD	time division duplexing
UPA	user position aware
UHF	ultra high frequency
ZF	zero forcing
ZMCSCG	zero mean circularly symmetric complex Gaussian

Symbols

a	scaler a
$\mathbf{a}$	vector $\mathbf{a}$
$\mathbf{A}$	matrix $\mathbf{A}$
$\lvert a \rvert$	magnitude of a
$\lVert \mathbf{a} \rVert$	Euclidean norm of $\mathbf{a}$
$\lVert \mathbf{A} \rVert_F$	Frobenius norm of $\mathbf{A}$
$\mathbf{A}^*$	complex conjugate of $\mathbf{A}$
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$\mathbf{A}^H$	complex conjugate transpose of $\mathbf{A}$
$[\mathbf{A}]_{i,j}$	(i,j)th entry of $\mathbf{A}$
$\mathrm{Tr}\,(\mathbf{A})$	trace of $\mathbf{A}$
$\mathrm{diag}\{\mathbf{A}\}$	diagonal matrix with the diagonal element built from $\mathbf{A}$
$\mathrm{vec}\,(\mathbf{A})$	stacks $\mathbf{A}$ into a vector columnwise[1]
$\mathbb{E}\,(\cdot)$	expectation operator
$\Re\,\{\cdot\},\Im\,\{\cdot\}$	real and imaginary parts of the argument, respectively
$\mathbf{A} \succeq 0$	$\mathbf{A}$ is a positive semidefinite matrix
$\mathbf{A} \succeq \mathbf{B}$	$\mathbf{A} - \mathbf{B}$ is a positive semidefinite matrix
$\mathbf{a} \succ 0$	all elements of $\mathbf{a}$ are positive
$\mathbf{a} \succeq 0$	all elements of $\mathbf{a}$ are nonnegative
$\mathbf{a} \succ \mathbf{b}$	element-wise greater than
$\mathbf{a} \succeq \mathbf{b}$	element-wise greater than or equal to
$\mathbf{I}$	identity matrix with a suitable size
$\approx$	approximately equal to
$\oplus$	bitwise XOR
$\odot$	Hadamard product
$\mathbb{C}^{n \times m}$	The set of n-by-m complex matrices

---

[1] If $\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix}$ is $m \times n$, then $\mathrm{vec}\,(\mathbf{A}) = \begin{bmatrix} \mathbf{a}_1^T & \mathbf{a}_2^T & \cdots & \mathbf{a}_n^T \end{bmatrix}^T$ is $mn \times 1$.

$\mathbb{H}^{n \times m}$	The set of n-by-m complex Hermitian matrices
$\mathbb{R}^{n \times m}$	The set of n-by-m dimensional real matrices
$\mathcal{CN}(\mu, \sigma^2)$	complex scalar Gaussian random distribution with mean μ and variance σ^2
C	convex set (convex cone)
$\mathbf{S}_+^n$	set of positive semidefinite matrices
$\Pr\{A\}$	probability of an event A
$\text{ICDF}(\cdot)$	inverse cumulative distribution function of the Chi-square random variable with m degree of freedom
$x \to a$	x approaches to a
$\text{rank}(\cdot)$	rank of a matrix
$\sum$	summation

Chapter 1

Introduction

1.1 Motivation[1]

Increasing fuel prices and predicted long-term resource scarcity have brought the field of green communications to the forefront in recent times. Rigorous efforts are being made to cut down power consumption, particularly in wireless communications, whilst at the same time maintaining an acceptable quality of service. It is believed that more than 75% of total energy consumption in cellular networks are dissipated on radio parts, i.e. base stations (BSs) [6]. In particular, cooling systems alone consume 40% to 60% of the base station's (BS) energy consumption[2]. Recent analysis by network operators and manufacturers has indicated that current wireless networks are not very energy efficient, particularly the BSs by which user terminals access service from the network [7]. Reducing transmit power at BSs will lead to substantial energy savings for the entire network.

Applications of mobile internet in different areas such as education, health care, smart grids and security have been growing very fast. As a result of increasing dependency on these applications in our day to day activities, demand for a significant increase in user data rate per area and the spectral efficiency are inevitable over the next 10 years. On the other hand, delivering higher data rate per area requires more transmission power which is constrained not only by the safety limits but also by the importance of global warming issues and the need for greener communications. Therefore, high speed transmission would mean diminishing coverage range, as otherwise, an enormous increase of transmission power is required by both mobile terminals and base stations to maintain the current cell size and achieve the ambitious targets of the

---

[1]Source: [5]
[2]Source: Vodafone Group R&D, 2009.

beyond IMT-advanced technologies.

Cell splitting, i.e., dividing large cells into a number of smaller cells, is a promising method that can significantly increase both capacity and coverage of the future cellular networks. But for this approach the network providers need a lot of new base stations. The required infrastructure (electrical energy, mast, fiber or wireless link to the next base station controller, etc.) and the base station itself are very expensive. Moreover the large number of smaller cells increases the number of inter-cell handovers and the signaling load.

Co-channel interference has been identified as one of the major impairments that degrades the performance of wireless systems [8–10]. Co-channel interference is caused by simultaneous transmission of data to proximal users assigned the same frequency-time resources. The presence of interference forces BSs to increase their transmit power if certain quality of service for their user terminals is to be maintained. Therefore, mitigating co-channel interference is a key factor leading to the reduction of BSs' transmit power.

Recently, the idea of multi-cell processing (MCP) in cellular networks has been recognized as an effective technique to overcome inter-cell interference and substantially improve the capacity [11–15]. In MCP, a coordinated virtual architecture is mapped over a cellular infrastructure such that the individual mobile user is collaboratively served by its surrounding base stations rather than only by its designated base station. In this architecture, base stations are equipped with multiple antennas but user terminals can have either single or multiple antennas. Using coordinated scheduling alone or incorporation with beamforming amongst a number of local base stations enables the network to constructively overlay the desired signals at an intended user and eliminate or sufficiently mitigate them at the other unintended users. Ideally, in this way, each user within a cell feels free of inter-cell-interference and, hence, can potentially achieve the highest capacity with the lowest energy consumption under the reuse one regime, i.e., while all the available spectrum is fully reused within the adjacent cells.

Transmit beamforming is a technique using at least two antennas to transmit a radio frequency signal. The phases of the transmissions across these antennas are controlled such that useful signals are constructively added up at a given desired receiver while interfering signals are eliminated at unintended user terminals. Given a fixed transmit power at each antenna element, an ideal transmit beamforming with M antenna elements yields a M^2-fold gain in received power compared to a single-antenna transmission [16]. Therefore in a power-limited regime, transmit beamforming with M antenna elements results in a M-fold increase in rate, a M-fold increase in free space

propagation range or a M-fold decrease in the net transmitted power.

Given the channel state information of a set of active user terminals, the task of a beamforming designer is to calculate beamforming vectors, known as beamformers, for the user terminals under a certain system requirement. It must be noted that channel state information is assumed to be available to the beamforming designer. Due to the limited channel training and/or feedback resources, the downlink channel state information (CSI), in terms of either the downlink channel vectors, i.e., instantaneous CSI in slow-fading scenarios, or the downlink channel covariance matrices, i.e., statistical CSI in fast-fading scenarios, may not be perfectly known at the BS, e.g., in frequency-division duplex (FDD) systems or in time-division duplex (TDD) where, the main problem in CSI acquisition is the channel estimation at the transmitter and the delay before the transmitter resources can be adapted based on the computed channel estimate. To accommodate the scenarios that only estimated and/or erroneous CSI is available at the BS, various robust downlink beamforming schemes have been proposed, see, e.g., [17–28]. The existing contributions on robust beamforming can generally be categorized into two classes, namely the deterministic (worst-case) design, see, e.g., [17–22], and the probabilistic design (also known as chance-constrained approach and outage-constrained approach), see, e.g., [23, 24, 26–28]. Unlike multi-user downlink beamforming with perfect CSI [12, 29–35], the robust multi-user downlink beamforming problem cannot be efficiently solved to optimality, and convex approximation methods are widely applied (see, e.g., [21, 23]). The system requirement in transmit beamforming usually defines an optimization problem.

1.2   Contributions

This thesis contributes towards robustness against uncertainties in channel parameters, in cellular networks. We formulate robust optimization problems that minimize a linear combination of total transmitted power at each BS and the induced aggregate interference power on the users of the other cells. The aim is to solve optimization problems with respect to different channel uncertainty models, to design robust beamforming vectors that guarantee the quality of service (QoS) at mobile users by ensuring that a set of signal-to-interference-plus-noise ratio (SINR) targets are met, despite the presence of erroneous CSI.

The significant contribution of this thesis is to develop approaches that mathematically reformulate physical problems and efficiently solve them with the aid of convex optimization tools. A key aspect of the effort is to recast non-convex optimization

problems into convex optimization ones. In the proposed problems formulations and their solutions by considering more realistic models for defining the channel uncertainties and by avoiding the conservative steps involved in the reformulations of the corresponding beamforming problems we avoid the drawbacks of previous methods. The principal contributions are divided in three Chapters (3, 4 and 5), with respect to assumptions on system setup and channel state information model.

In Chapter 3, an optimization problem is developed under a limited cooperation amongst BSs to both tackle inter-cell interference problem and improve energy efficiency in cellular wireless networks. A significant utilization of backhaul is required to overcome inter-cell interference in multi-cell processing networks where multiple BSs simultaneously transmit to their intended local users with aggressive frequency reuse. Chapter 3 proposes a downlink transmission strategy that enables each BS to design locally its own beamforming vectors without relying on data or downlink CSI of links from other BSs to the users. With only imperfect knowledge of second-order statistical CSI available at each BS, the objective of the proposed scheme is to design a set of beamforming vectors for a number of simultaneously active users, such that a combination of total transmit power and the worst-case of the resulting total interference on the other vulnerable users of the adjacent cells at each BSs is minimized, while the QoS satisfies in the worst-case scenario. In our approach of solving the proposed robust downlink beamforming problem a spherical uncertainty set to model the uncertainty in the channel covariance matrices is assumed. We obtain exact reformulations of the worst-case QoS and interference constraints based on Lagrange duality, avoiding the coarse approximations used by previous solutions [17]. The final problem formulations are converted to a more tractable convex forms using semidefinite relaxation (SDR). These particular contributions have been published in [36].

In Chapter 4 we present an alternative approach to the optimization problem proposed in Chapter 3. In Chapter 3 the CSI errors are adversarially chosen from some bounded set which results in worst-case robust beamformers design; in this approach one does not utilize any distributional properties of the errors. In contrast to the conservative approach of Chapter 3, the analyzed uncertainty model is fundamentally changed in this chapter. We propose a chance-constrained optimization problem, where errors follow a certain fully specified distribution, whose properties are then exploited to yield outage-constrained robust beamformers design. The objectives are to minimize the total transmit power and imposing an upper limit on the interference at the outer-cell users, subject to targeting a lower bound on the received SINR for all (cell-edge) users so that the QoS targets are satisfied with the specified probabilities. The

probability (chance)-constrained problems are known to be difficult to solve because the probabilistic constraints in general do not have closed-form expression and are not convex. Hence by applying Schur complement the original optimization problem with probabilistic constrains can be rewritten with equivalent linear matrix inequality (LMI) constraints in a convex SDR form. Furthermore, the proposed scheme is in contrast to the methods that approximate the probabilistic constraints with their convex upper-bounds and, hence, effectively find a feasible worst-case solution without any optimality guarantee. The analytical details of our novel approach that directly accounts for the probabilistic constraints without approximating them by convex upper-bounds are described in this chapter.

We also establish a novel connection between the proposed probability-constrained stochastic optimization problem in this chapter and the worst-case optimization problem in Chapter 3. In practical scenarios CSI imperfections are unbounded random variables, hence bounded uncertainty region in the worst-case approach would naturally imply that with a certain probability, the uncertain CSI may fall outside of the considered uncertainty region. Thus, in this chapter we provide a relationship between the probability and the uncertainty parameter in the worst-case approach to provide a practical rule for choosing the uncertainty parameter based on the QoS requirements. These particular contributions have been published in [37, 38].

In Chapter 5 we assume a bounded deterministic model for the error in CSI and adopt a new SINR criterion considering imperfect channel knowledge at both transceivers sides; something that has been lacking in previous studies. It is well-known that imperfect CSI can significantly degrade the system performance [39–41]. In other words, if one derives algorithms for transceiver design based on erroneous channel coefficients as if they were perfect, some promised QoS targets in the system might often be violated. In fact, beamforming designs without estimation-error-proof measures (e.g. robust design) may result in fragile system performance. Hence, it is of great importance to take imperfect CSI as an influential factor in beamforming design. The robust constraints guarantee the quality of service (QoS) at mobile users (MUs) by ensuring that a set of SINR targets are met, despite the presence of erroneous CSI at both the BS and the MU sides. The imperfect CSI affected QoS constraints are then converted into finite number of linear matrix inequalities (LMIs) by utilizing S-procedure and the original intractable non-convex problem is approximated through SDR. Our approach differs from the existing ones in inclusion of a robust ICI controlling cost term in the objective function of the proposed optimization problem. This transmission strategy can increase the scalability of multi-cell networks with highly efficient re-usability of

spectral resources across the adjacent cells. These particular contributions have been published in [42]

1.3 Thesis Outline

The outline of the thesis can be summarized as follows: Chapter 1 includes the motivation and states the contributions of the thesis. Chapter 2 is divided into two main parts: the first part is devoted to the state of the art and the literature reviews of multi-antenna systems, coordinated and robust beamforming, and the second part focuses on a general and introductory presentation of mathematical preliminaries. The presented concepts are used to develop robust beamforming schemes for cellulatr networks discussed in Chapters 3, 4 and 5. This chapter is included to have this thesis self contained, but it is not intended to be comprehensive. For more details of the reviewed content, classical texts are cited as well. In Chapter 3, robust precoders are designed with imperfect knowledge of second-order statistics of channel satisfying worst-case constraints. In Chapter 4, a novel method to design robust beamformers with probabilistic constraints is derived. In Chapter 5, with imperfect CSI at both transceiver sides, a novel technique to compute robust beamforming vectors is developed. The closing Chapter 6 deals with conclusions and future work.

1.3.1 Publications

A collection of contributions for this thesis has been complied from the following list of publications:

1. T. A. Le, S. Nasseri, A. Zarrebin-Esfahani, A. Mills, and M. R. Nakhai, "Power-efficient downlink transmission in multicell networks with limited wireless backhaul," *IEEE Wireless Communications Magazine, Special Issue on Technologies for Green Radio Communication Networks*, vol. 18, no. 5, pp. 82–88, Oct. 2011.

2. S. Nasseri, T. A. Le, and M. R. Nakhai, "Robust and power efficient interference management in downlink multi-cell networks," in *Proc IEEE 24th International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, Dec. 2013, pp. 1097–1101.

3. S. Nasseri and M. R. Nakhai, "Robust Interference Management via Outage-Constrained Downlink Beamforming in Multicell Networks" in *Proc IEEE Global Communications Conference (GLOBECOM 2013)*, Dec. 2013, pp. 3492–3497.

4. S. Nasseri and M. R. Nakhai, "Min-Max Robust Transmit Beamforming for Power Efficient Quality of Service Guarantee," in *Proc IEEE Global Communications Conference (GLOBECOM 2014)*, Dec. 2014, pp 3448–3453.

Chapter 2

Background study

This chapter gives a basic introduction to the topic of this thesis. The framework is based on the concept of design of reliable and efficient communication systems. The practical performance of multi-cell systems is limited by a variety of nonidealities, such as insufficient channel knowledge, high computational complexity, heterogeneous user conditions, limited backhaul capacity, transceiver impairments, and the constrained level of coordination between base stations. A major complication in cellular networks is inter-cell interference that arises and limits the performance when multiple users are served in parallel. To mitigate the inter-cell interference in downlink transmissions many researchers have studied coordinated signal processing across base stations. To improve the throughput, user satisfaction, and revenue of multi-cell systems, we take advantage of multiple antenna which provides diversity gain in spatial domain without extra bandwidth expansion or transmit power. In order to further exploit the benefits of multiple antenna system, transmit beamforming (precoding) is widely implemented for enhancing the performance and increasing the system throughput. A major drawback of most existing transmit beamforming techniques is that they require nearly perfect knowledge of the channel at the transmitter, which is typically not available in practice. The channel imperfections could lead to severe performance degradation. Hence, robust transmit beamforming design is required to provide robustness against the imperfect channel

In this chapter, we give a general overview of topics such as multi-antenna systems, multi-cell coordination, and robust beamforming and its use in wireless communications. The purpose of this chapter is to provide sufficient background to be able to understand the basic research problems that are considered herein and how this thesis contributes to these areas. Next, in Section 2.7, mathematical concepts that are frequently used in the subsequent chapters are summarized.

2.1 Radio Resource Management in a Cellular Concept
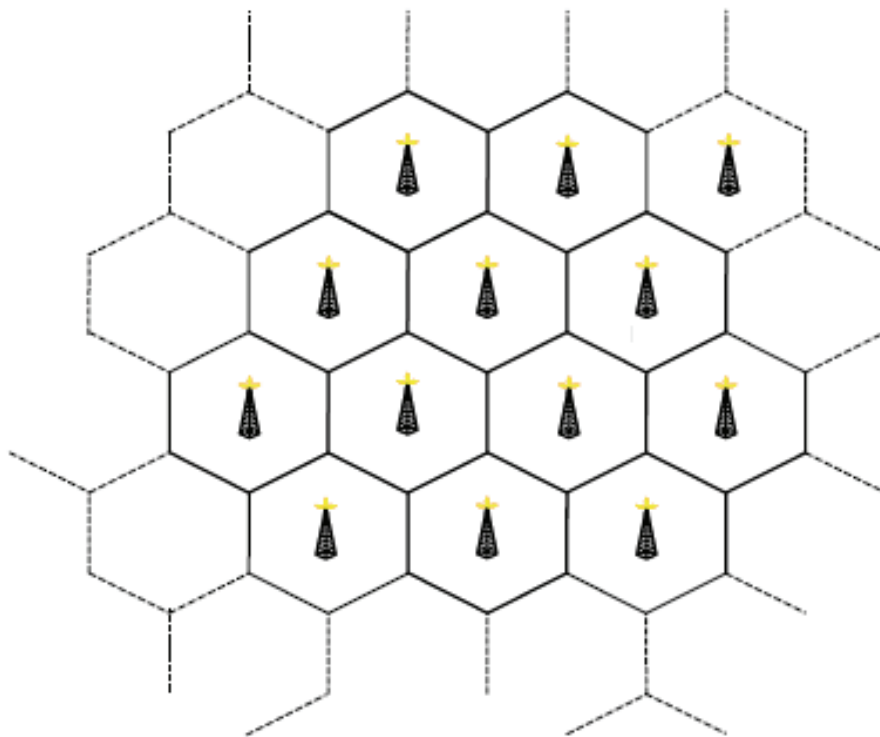
Figure 2.1: Illustration of a cellular system, where each cell is served by one BS in the cell-center.

In order to offer sophisticated mobile communications over a large area, wireless cellular networks divide the covered area into cells, as shown in Fig. 2.1. All communications within each cell are served by one base station (BS) located in the cell-center. The same frequency resource is repeatedly available (reused) for other cells. Hence, the main advantage of using cellular systems is that through reusing radio channels in cells, the network coverage can be provided to areas of any size.

However, how to determine the size and the shape of a cell, as well as how to allocate resources among cells are very important in radio network planning, as they may largely influence the system performance. The size and the shape of each cell depend on signal quality received within the covered cell-area, which is related to many factors, such as the surrounding terrain, buildings, the height of transmission antennas, the transmission power of the BS, the expected traffic demands and density, as well as the atmospheric conditions, etc. Cells are generally represented as idealized
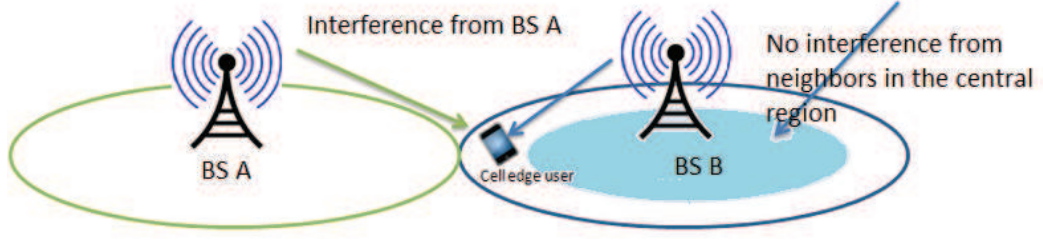
Figure 2.2: A typical two-cell layout, where the cell-edge user is interfered by BS A from the neighboring cell [1]

regular hexagons, but because of topographical and environmental conditions, this is only an approximation of what actually occurs [43]. Naturally, in a real world scenario, the cell shapes are very irregular and overlap with each other by approximately 10 to 15%. This enables users operating near the boundary of a cell to choose which BS they are associated to.

Enhancing the cell coverage by allowing as many users to communicate reliably irrespective of their location and mobility appears to be a primary concern of network service providers. This task is typically fulfilled by doing aggressive spectrum reuse which on one side enhances the spectral efficiency, whereas on the other side it causes severe inter-cell interference (ICI) among the users of same spectrum, particularly cell-edge users located close to the cells boundaries as shown in Fig. 2.2

Radio resource management has been evolved as an efficient tool to coordinate, mitigate and manage ICI while enhancing the network performance in a cellular networks. The incurred ICI in cellular networks with universal frequency reuse is severe and random due to its dependence on the channel statistics and on the dynamics of the multi-user scheduling decisions. Therefore, it is important for the system designers to accurately characterize the behavior of the ICI in order to quantify various network performance metrics and to develop efficient resource allocation and interference mitigation schemes. More specifically, efficient spectrum allocation and power control management solutions are needed to leverage the potential of cellular networks.

This thesis considers cellular networks and studies how the transmissions within a cell should be designed to optimize the performance and how to coordinate the operation of multiple cells. The main focus is on transmission from a BS to multiple user devices, which is commonly viewed as more difficult than transmission in the opposite direction.
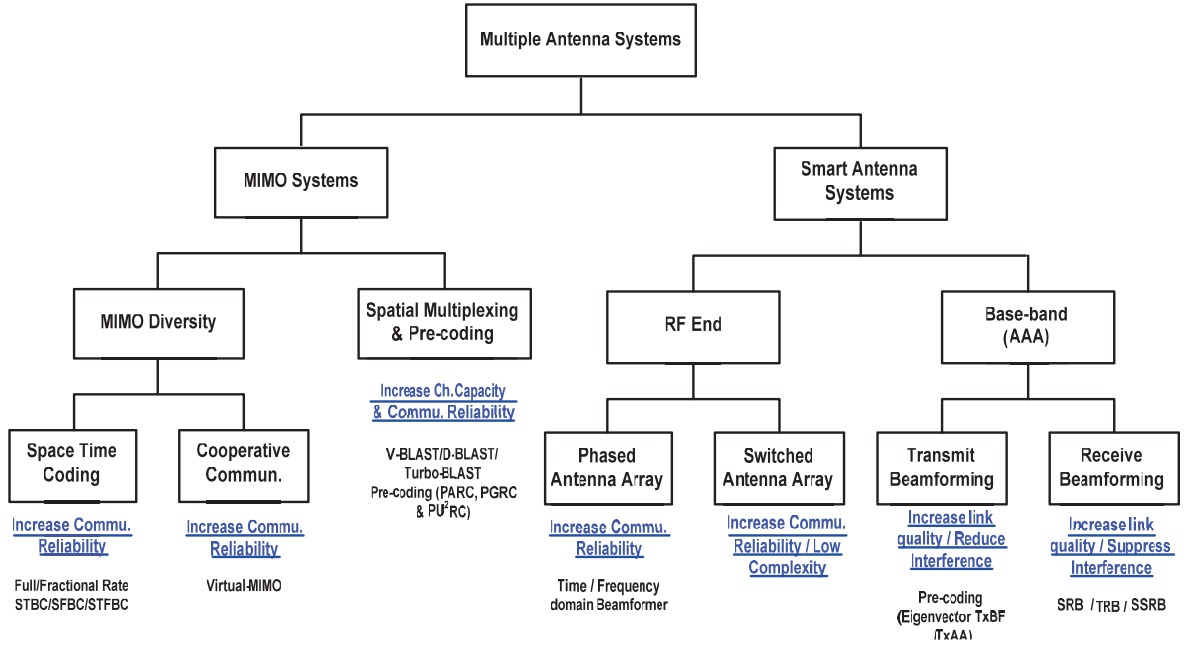
Figure 2.3: Multiple antenna technologies.

2.2 Introduction to Multiple Antenna Systems

Multiple antennas technologies proposed for cellular systems have gained much attention in the last few years because of the huge gain they can introduce in the communication reliability and the channel capacity levels. Furthermore, multiple antenna systems can have a big contribution to reduce the interference both in the uplink and the downlink by employing smart antenna technology. To increase the reliability of the communication systems, multiple antennas can be installed at the transmitter or/and at the receiver. Such systems are called multiple-input multiple-output (MIMO). Each transmit antenna can be viewed as a mouth and each receive antenna as an ear. The extra mouths and ears can be used for diversity or multiplexing. Fig. 2.3 summarizes the different multiple antenna technologies and gives some examples of these technologies. In Fig. 2.3 multiple antenna technologies are categorized into two main groups. In the first group the techniques related to spatial diversity and spatial multiplexing, and in the second group the smart antenna techniques are introduced. The advantages of multiple antenna in gaining transmission efficiency can be boosted by combining the two groups in Fig. 2.3. For example in [44], a transmission scheme that effectively combines conventional transmit beamforming with orthogonal spacetime block coding is proposed, where numerical results demonstrates significant gains over a system using

conventional beamforming systems.

2.2.0.1   Spatial (antenna) Diversity
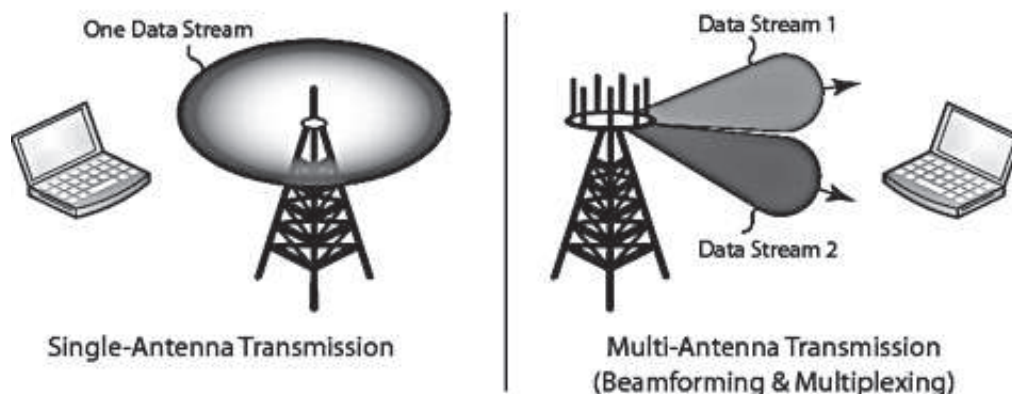
Figure 2.4: transmission comparison of single-antenna and multi-antenna. With a single antenna, The signal propagates in all directions (and most directions will not lead to the user). With multiple antennas, the signal can be directed towards the users (called beamforming). Multiple signals in parallel can be sent using different beamforming (called multiplexing).

The multi-path propagation between each pair of transmit and receive antennas will be different. This creates a diversity of routes that the transmitted signal can travel to the destination. The strongest signal will be carried in one of these routes and should be used for transmission. Certainly, better performance will be achieved by selecting the best route out of many possibilities, compared with the single antenna case that we are stuck with only one possibility. The result can be viewed as speaking with many mouths in such a way that the voice is directed towards the user and using the ears to listen carefully in this direction. Note that the best route is usually not to select one antenna/mouth at the transmitter and one antenna/ear at the receiver, but to combine all of them in a smart way to achieve one strong voice that is easy to hear. This directing is called *beamforming* since it forms a directed signal beam towards the receiver, instead of sending in all directions as with a single antenna; see Fig. 2.4.

2.3   Spatial Multiplexing

Instead of using only the best route as in the diversity case, MIMO techniques can be used to send multiple data signals in parallel. If each transmit antenna can be

viewed as a mouth and each receive antenna as an ear, then the idea can be viewed as listening to different voices with each ear, this is called multiplexing. It can be achieved by directing the signals toward different ears using the beamforming idea in Fig. 2.4. To multiplex four data signals, both the transmit antenna and the receive antenna need to have four antennas, it is the minimum of the number of transmitters and the number of receivers that decides how many signals that can be multiplexed.

It is not obvious whether the antennas should be used to achieve diversity or to perform multiplexing, or a little bit of both. Diversity reduces the risk of errors in the transmission (since all ears are focused on the same signal), while multiplexing increases the total data rate (since ears are listening to different signals). Beamforming requires knowledge of how the channel behaves; otherwise the desirable beam direction will remain unknown. Therefore, multiplexing is preferred if the channel knowledge is accurate, while diversity can protect against inaccuracies. How to perform reliable transmission, robust to imperfection in channel knowledge is the main topic of this thesis.

The advantages of multi-antenna transmission all depend on whether the channels from each transmit antenna to each receive antenna experience different multi-path propagation (i.e., the signals travel different routes). This is not necessarily the case: if the transmitter and receiver are located in a tunnel that acts like a waveguide, there is basically just one route between them irrespectively of how many antennas we employ. Fortunately, such closed environments are rare in practice. Instead, the important thing is that the antennas are sufficiently separated to be able to observe different signal routes. The wavelength decides what is a good separation, and it is short when the frequency is high and vice versa. For frequencies in the range of 0.7-5 GHz, a good separation is one or a few decimeters. Thus, we can expect the next generation of communication systems to employ, for example, two antennas in hand-held devices, up to four antennas in laptops, and perhaps even more at the base stations (which are less size-constrained). Of course, there will always be some similarities between the antennas; this is called *spatial correlation*. Geometrically, it means that transmissions in some spatial directions are more probable to arrive at the receiver and that the receiver is more probable to hear strong signals from certain directions. This behavior is natural; if the base station is placed on a roof top, it is probably better to use beamforming to send signals along a street leading towards the receiver than to send it in a completely different direction.

2.4   Smart Antenna Systems

Smart antenna was born in the early 1990 when well developed adaptive antenna arrays originate from Radar system. Later, Smart antenna technique is applied in wireless communications system. Recently, Smart antenna technique has been proposed as a promising solution to the future generations of wireless communication systems, such as the Fourth-Generation mobile communication systems, broadband wireless access networks, where a wide variety of services through reliable high-data rate wireless channels are expected. Smart antenna technique can significantly increase the data rate and improve the quality of wireless transmission, which is limited by interference, local scattering and multi-path propagation [45, 46]. Smart antennas offer the following main applications in high data-rate wireless communication systems [47]

- Spatial Diversity

- Co-channel interference reduction

- Angle reuse or space division multiple access (SDMA)

- Spatial multiplexing

Smart antenna system can be categorized into two main groups:

- Switched array systems (switched beamforming) with a finite number of fixed, predefined patterns or combining strategies (sectors)

- Adaptive Array systems (AAS) (adaptive beamforming) with an infinite number of patterns (scenario-based) that are adjusted in real time

Switched beamformers electrically calculate the direction of arrival (DoA) and switch on the fixed beam. The user only has the optimum signal strength along the center of the beam. The adaptive beamformer deals with that problem and adjusts the beam in real time to the moving user equipment (UE). The complexity and the cost of such a system is higher than the first type. To match the characteristics in each radio frequency chain of the transmitter and receiver, on-line calibration is required in smart antenna systems. On-line calibration technique can compensate the errors such as the distortions of radio frequency components due to small environment changes, the nonlinear characteristics of mixer, amplifier and attenuator, I/Q imbalance errors, etc.

2.4.1 Beamforming

Beamforming is a signal processing technique that is used to steer a signal from a set of non-directional antennas into a certain direction. By combining the signals of these antennas a new simulated antenna is created. This simulated antenna can then be electrically pointed in a certain direction, without physically moving the real antennas [48]. Previously a lot of work has been done in the area of beamforming. Initially, not intended for telecommunications, beamforming algorithms found there place in this area as well. Later on, when the theory was further developed for the use of cellular networks, many excellent works were created that describe beamforming for space division purposes in mobile communications[1]. Cox, 1987 [53] was one of the first to develop an adaptive beamforming strategy, similar to ones used in contemporary works. A lot of problems with wireless networks result from environmental or external conditions that interfere with or completely destroy communication paths. Exploring the concept of spatial selectivity opens new opportunities for performance improvements. An ideal situation would be if the base station had perfect knowledge of the exact user locations. An infinite precision in space may lead to the formation of single narrow beams. Such a configuration is impossible in a real situation, with no feasible hardware to make that precision, but still this idea gives some powerful insight of what could be done to benefit the mobile air interface.

Introduction of antenna arrays and an adaptive algorithm, made it possible to calculate and create different radiation patterns. This operation is called beamforming. The variable patterns follow from spatial constructive and destructive interference of the signal wavefront in different directions. Feeding of different antenna elements with a phased signal results in pattern change. Such a procedure generally steers the main lobe in the plane, where the elements are arranged, and eventually steers the nulls as well. Furthermore, a large variety of possible patterns can also be achieved by changing each element's signal amplitude [56]. By dynamically varying both according to some algorithms the array turns into an adaptive one. The adaptive antennas are used for mobile communications nowadays are two dimensional arrays, as shown on Fig. 2.5.

Generally, in the vertical plane elements are ordered in columns and fed with the same amplitude and phase to form a vertical pattern with a narrow beam. Such a beam is needed for each antenna to focus its radiation to the surface of the surrounding area. In the horizontal plane the antenna can be seen as a horizontal array of columns. The signal arriving at, and departing from them is predistored using an adaptive algorithm. The idea is to change the amplitudes and phases of the signals in the different element

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[1]Examples include [12, 27, 49–55] and many others
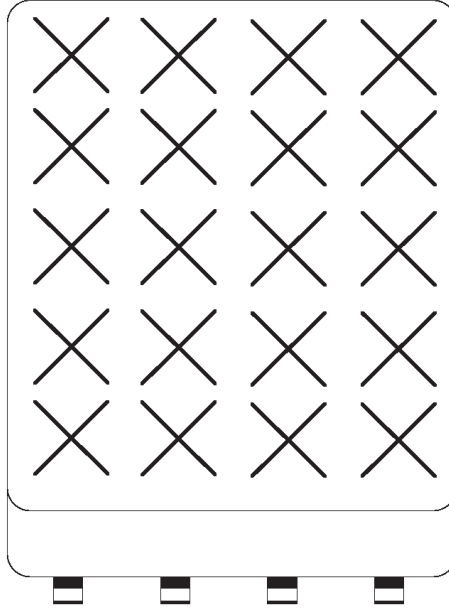
Figure 2.5: Two dimensional antenna array [2]

columns. This can be interpreted as a multiplication with complex coefficients from the base-band signals point of view. This results in steering of the main lobe in azimuth (horizontal plane) so that its peak can target a particular user in the cell. The adaptive algorithm has to calculate the best complex coefficients achieving which is achieved with regard to the user locations. Moreover, in order to not cause interference towards other users, the pattern nulls have to be pointed in their directions. Such a procedure aims to achieve interference rejection and spatial filtering. The adaptive antenna arrays together with the adaptive processing unit are called Smart Antennas or Adaptive Antenna Systems. Conventional directional antenna for mobile communications, a beamforming antenna, and its typical design sketch are shown On Fig. 2.6. It can be seen from 2.6(c) that the beamforming antenna is wider in size, as it contains a number of conventional antennas inside its radome.

The proper formation of the beams requires knowledge of user's locations. Location detection techniques using electronic scanning with antenna arrays are considered a mature technology, and have been heavily developed over the last decades. Mainly for military purposes and satellite communications, and have been recently introduced in cellular networks as well. Many excellent texts, such as [57], give a detailed description of various location techniques. They are usually coined as estimation of direction of (signal) arrival (or angle of arrival). The idea is to find the angle between the user's location and BS and the antenna boresight. The antenna boresight is the plane
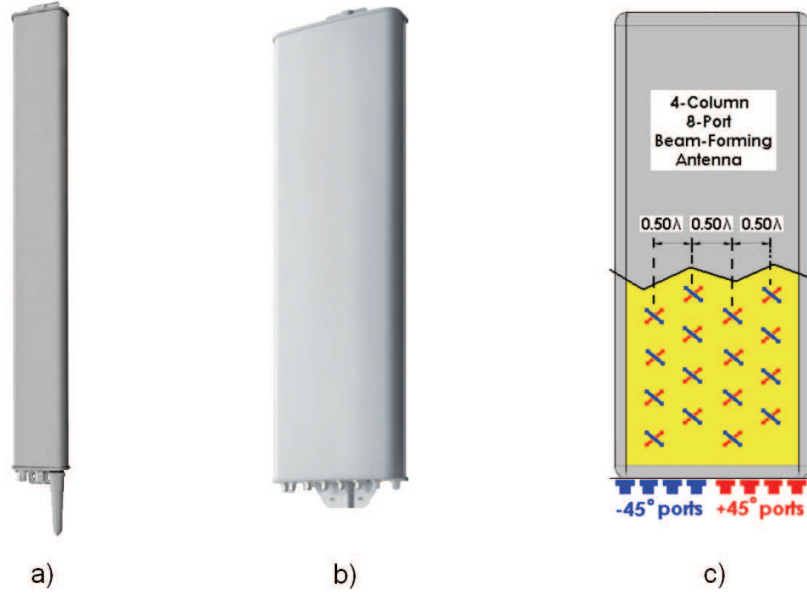
Figure 2.6: a) Conventional antenna for cellular communications b) Beamforming antenna c) Sketch of the structure of a beamforming antenna [2]

perpendicular to the antenna surface. The base station has to respond to the user in the same direction where its signal came from. Therefore in the downlink transmission the angle of departure (AoD) has to be the same as the direction of arrival for the specific user. AoD is a more common term in the standardization documents of 3GPP and their technical report 25.996 assesses issues like AoD and spatial channel models [58]. Fig. 2.7 depicts this angle. The arrays used for BSs are assumed to be equidistant, i.e., distance between each two adjacent elements is equal to d. To find θ, the distance has to be: $d \leq \frac{\lambda}{2}$. Increasing the distance between the antenna elements, increases the sidelobes as well. When $d = \frac{\lambda}{2}$ a critical point is reached. New parasite sidelobes named grating lobes appear as d grows further, and are a result of spatial undersampling of the transmitted/received signal. Grating lobes lead to ambiguities in the directions of the departing/arriving signals, since parasite copies of these signals replicate themselves in space in unwanted directions.

The next key component needed for beamforming is the channel state information (CSI). The base station needs CSI to be aware of the radio environment features-path loss, fading, etc. (Independent of beamforming, CSI has other uses as well, for example choosing the best coding and modulation scheme). Precoding in digital systems is done by applying complex weighting to the signal before radiating it in the air. Without loss of generality it is assumed that in every antenna element branch, the signal has

17
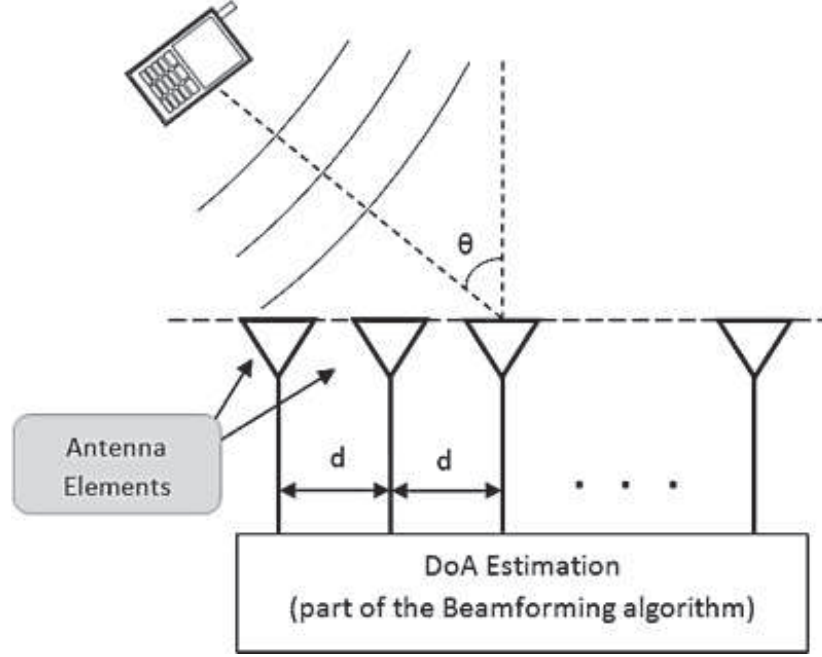
Figure 2.7: Direction of Arrival

two components: in-phase (I) and the shifted by $90\,^\circ$ quadrature one (Q). The signal in each is scaled by a factor. This is equivalent to multiplication of the complex signal samples ($I + jQ$) with a set of complex numbers (beamforming weights). Let us assume that the signal we want to transmit with M antennas is $\mathbf{s}^T \in \mathbb{C}^M$, and the weights are represented with the vector $\mathbf{w} \in \mathbb{C}^M$. Therefore, the resulting output is

$$x = \sum_{i=1}^{M} w_i s_i. \tag{2.1}$$

Many different algorithms with a different degree of precision have been used so far for finding the beamforming vectors $\mathbf{w}$. All of them are based on having the CSI available at the transmitter. Channel estimation techniques can be found in [59–65] and references therein. Finding the optimal CSI is a mathematical optimization problem, which is further discussed in Section 2.7.2.

2.5 Inter-Cell Interference in LTE

A cellular network consists of a large number of cells and each user connects to the closest base station (i.e., the one with the strongest channel). There are invisible edges between each cell where user devices switch between the corresponding base stations.

The activities in one cell will be affected by activities in neighboring cells. For example when two users are next to each other, but at different sides of the cell edge, i.e., belong to different cells, are served in parallel (at the same subchannel), their respective data signals will cause severe interference to each other; see Fig. 2.8. Present LTE systems suffer greatly from this problem. Examples of coverage (downlink received levels), and downlink data rate predictions of a real LTE network are shown in Figs. 2.9 and 2.10 [3]. The base stations are indexed as A-J. It can be seen clearly that in the cell-edge areas, despite of a high received signal power level, the throughput is very low, the deep blue color in Fig. 2.10 shows these areas. Hence, there needs to be some kind of coordination of resource allocation between adjacent cells. Preventing the adjacent cells to use the same subchannels would be the simplest coordination scheme. This will basically remove the interference between cells, but leads to poor exploitation of the scarce frequency resources. Multi-antenna transmission enables more intricate coordination schemes where base stations avoid allocating the same time/frequency resource to adjacent users at the cell edge; see Fig. 2.11. Such schemes require that base stations share decisions with neighboring base stations. In addition, each base station needs to know the channels to all users in adjacent cells that they might cause interference to.
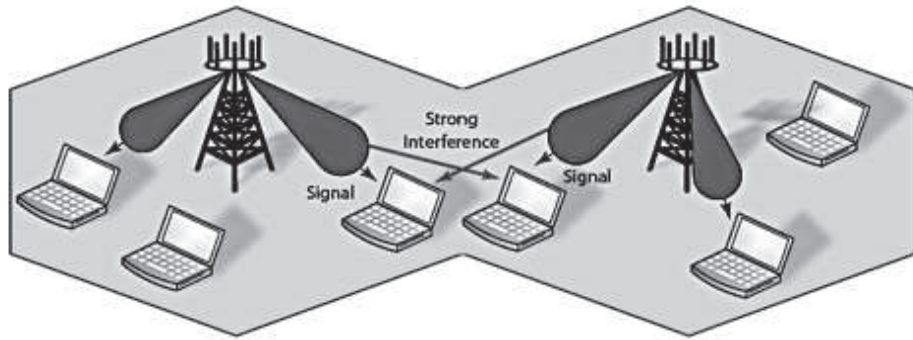
Figure 2.8: Uncoordinated Multi-cell: Strong interference might be caused to cell edge users.

In addition to interference avoidance, multi-cell coordination can also be used to jointly serve certain users through multiple base stations and thereby remove the strict cell edges. Joint transmission to a user is called coordinated multipoint (CoMP) transmission and will ideally make all the cells act as just one cell (with transmit antennas at different locations). This has great potential as it makes the the number of parallel data signals limited by the total number of antennas (mouths) at all base stations. But just as every other advanced transmission scheme, CoMP transmission requires
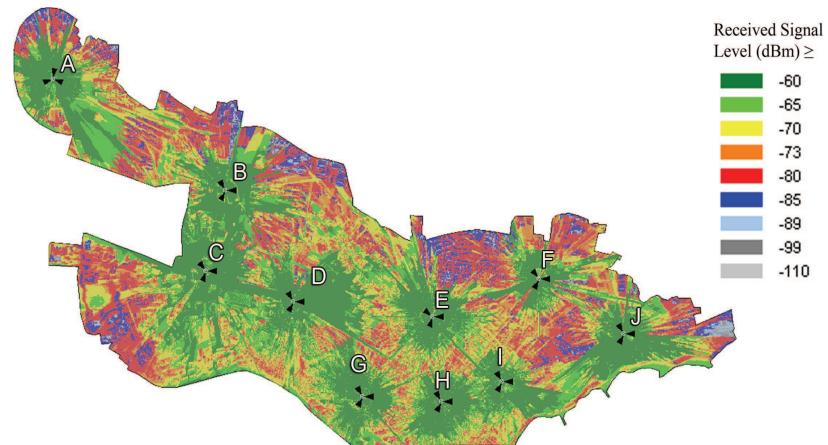
Figure 2.9: Received downlink signal level (dBm) [3]
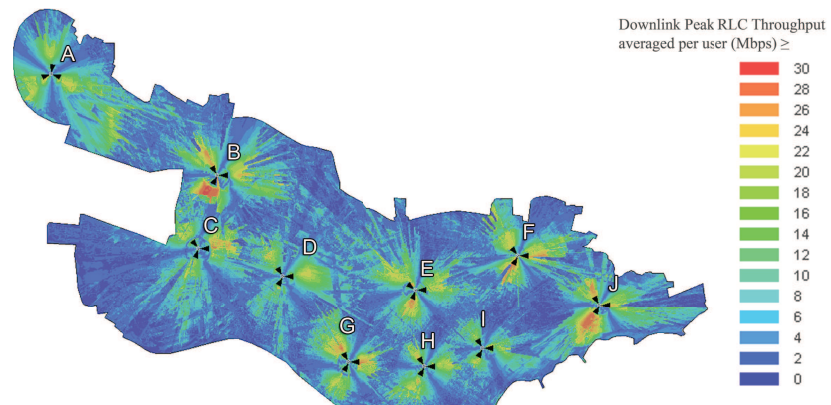
Figure 2.10: Downlink peak throughput per user (Mb/s) [3]
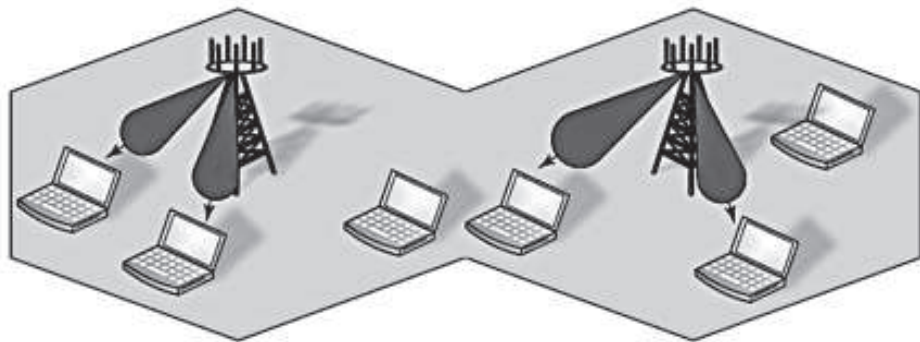
Figure 2.11: Coordinated Interference: Base stations cooperate by only sending parallel transmissions to users in different directions.

very accurate channel knowledge and good backhaul networks between base stations to enable fast coordination.

2.6 Coordinated Multipoint Transmission in LTE-Advanced

CoMP is widely regarded as a technique that will increase data transmission rates and help ensure consistent service quality and throughput on LTE wireless broadband networks as well as on 3G networks. By coordinating and combining signals from multiple antennas, CoMP, will make it possible for mobile users to enjoy consistent performance and quality when they access and share videos, photos and other high-bandwidth services whether they are close to the center of an LTE cell or at its outer edges. This feature can be implemented in both downlink and uplink. A short and precise presentation of CoMP can be found in [66].

CoMP processing schemes are generally divided into *Joint Processing* (JP) and *Coordinated Scheduling/Coordinated Beamforming* (CS/CB).

In JP a number of BSs transmit towards a single mobile user (MS) and the MS data has to be available at each BS. A major drawback of JP is the backhaul between the coordinated transmission points because every station has to receive the user information and also has to exchange its CSI with all of the neighboring stations, in due time. A more backhaul friendly type of CoMP, which is the focus of this thesis is coordinated beamforming (CB), because transmission takes place at one base station only, so there is no need to exchange the user data. The only requirement is to exchange user locations and their channel parameters. Example scenarios of coordinated beamforming and joint transmission are shown in Fig. 2.12. Generally successful beamforming is
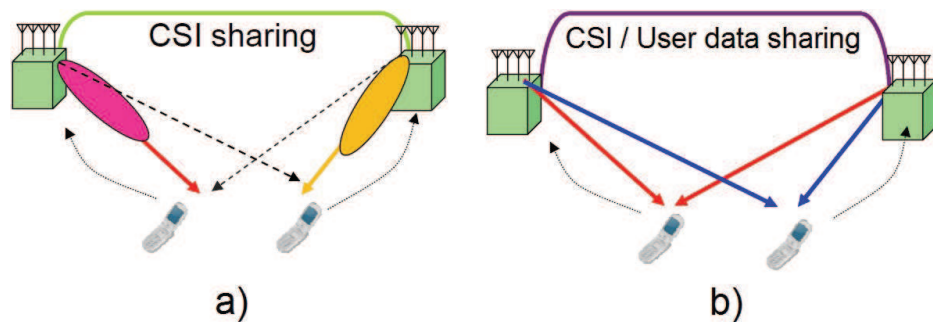
Figure 2.12: a) Coordinated beamforming b) Joint transmission

not always possible, because it heavily depends on user locations. If users are too close

to one another or are located at the cell edges or two or more of them are aligned towards a particular BS, the required transmit power to serve them would go very high. In these situations, whenever one station is increasing its level to serve its own user it will automatically cause interference to the other users due to their proximity of alignment. This results in a demand for more power, radiated towards the second user, which would increase the interference imposed on the first one. Whenever this happens, scheduling [1] is needed to enable both users to operate properly. This is why CB is having a scheduling counterpart as well and the whole strategy is called CS/CB. In the rest of this project, we use the term CB instead of CS/CB for convenience and that the focus is only on the beamforming aspects of transmission.

2.6.1 Centralized and Decentralized in CoMP

Depending on where the coordination procedure takes place, the solution may be centralized or distributed. In centralized CoMP systems, cooperative BSs should be connected with a central processing unit (CU) by low latency backhaul links [68, 69]. Under such a framework, each user feeds back its CSI to its local BS, and then the CU collects CSI from all BSs through backhaul links. With CSI of all users at CU, the centralized CoMP systems enable globally optimal cooperation among BSs, which however pays the penalty of increasing infrastructural costs, unaffordable feedback overhead and difficulty of network upgrading. In currently deployed cellular systems and emerging mobile standards, the backhaul latency is in an order of 10 to 20 milliseconds [70]. This leads to severe performance deterioration of multi-user multiple-antenna CoMP systems. Hence, recently considerable research has been directed towards the decentralization of CoMP in order to cope with the drawbacks of the centralized CoMP systems [71–74]. Furthermore, CoMP requires the provision of full CSI at the transmitter, i.e., CSIT, in order to effectively design downlink beamforming towards the end user terminals. Yet, the CSI at the transmitter might not be perfect, due to various reasons such as uncertainties from channel estimation, feedback delay and quantization errors between a user and the BS. The CoMP designs based on the assumption of perfect CSIT may yield unpredictable results in practical scenarios where the captured CSIT may be imperfect. Hence, robustness against CSI errors in conjunction with distributiveness is another key consideration in achieving the promised gains of CoMP.

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[1]As previously mentioned the frequency reuse factor of the LTE-advanced systems is one, which means that all BSs use the whole available spectrum. A scheduler is used to avoid ICI by allocating, based on some parameters, orthogonal time or frequency resources to different MSs. Since scheduling is out of the scope of this project, the reader is recommended to read [67] for more information

2.6.2 Robust Signal Processing under Model Uncertainties
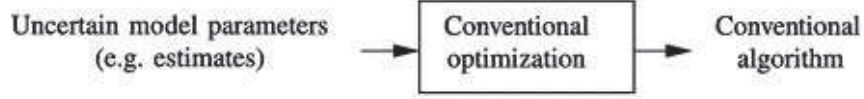
Figure 2.13: Conventional approach to deal with model uncertainties: Treat the estimated parameters and the model as if they were true and perfectly known. The parameters for the signal processing algorithm are optimized under these idealized conditions [4].
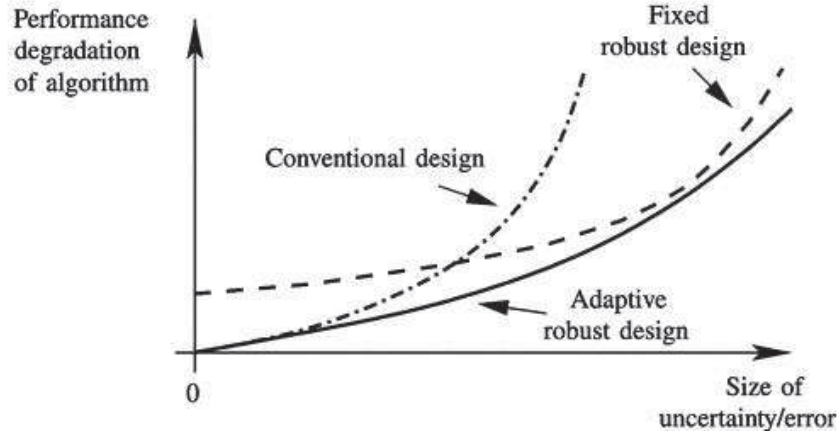
Figure 2.14: Different design paradigms and their sensitivity to the size of the model uncertainty or the parameter error [4].

The design of adaptive signal processing relies on a model of the underlying physical technical system. The choice of a suitable model follows the traditional principle: it should be as accurate as necessary and as simple as possible. Typically, with the model complexity, the complexity of signal processing algorithms increases. On the other hand the performance degrades in case of model-inaccuracies. For example, the following practical constraints lead to an imperfect characterization of the real system:

- To obtain an acceptable complexity of the model, some properties are not modeled explicitly.

- The model parameters, which may be time-variant, have to be estimated. Thus, they are not known perfectly.
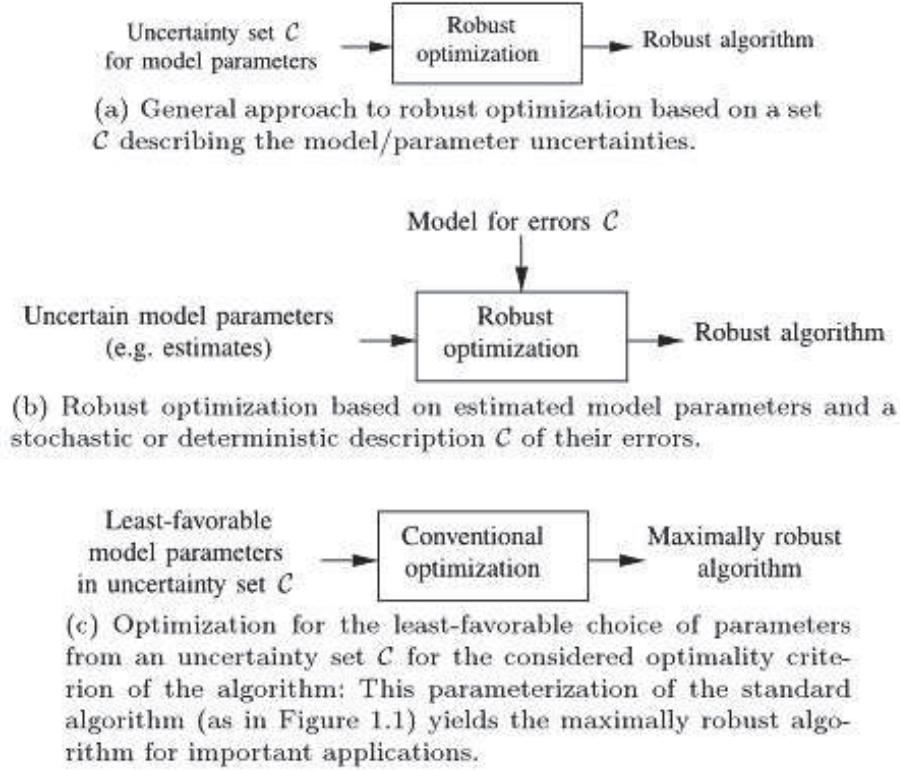
Figure 2.15: General approach to robust optimization of signal processing algorithms under model uncertainties and two important special cases [4].

Often a pragmatic design approach is pursued (Fig 2.13) which is characterized by two design steps:

- An algorithm is designed assuming the model is correct and its parameters are known perfectly.

- The model uncertainties are ignored and the estimated parameters are applied as if they were error-free.

It yields satisfactory results as long as the model errors are "small". *A robust algorithm design* aims at minimizing the performance degradation due to model errors or uncertainties. Certainly, the first step towards a robust performance is an accurate parameter estimation which exploits all available information about the system. But in a second step, we would like to find algorithms which are robust, i.e., less sensitive, to the remaining model uncertainties.

Sometime suboptimum algorithms turn out to be less sensitive although they do not model the uncertainties explicitly; they give a *fixed robust design* which cannot adapt to the size of uncertainties (Fig 2.14).

An *adaptive robust design* of signal processing yields the optimum performance for a perfect model match (no model uncertainties) and an improved or in some sense optimum performance for increasing errors (Fig 2.14). Conceptually, this can be achieved by

- defining a mathematical model of the considered certainties and

- constructing an optimization problem which includes these uncertainties.

Practically, this corresponds to an enhanced interface between system identification and signal processing (Fig 2.15(a)). Now, both tasks are not optimized independently from each other but jointly.

In this thesis, we focus on two important types of uncertainties in the context of wireless communications:

- Parameter errors with stochastic error model,

- Parameter errors with a deterministic error model.

The two underlying design paradigms are depicted in Fig 2.15(b) and 2.15(c), which are special case of the general approach in Fig 2.15(a); the first clearly shows the enhanced interference compared to Fig 2.13 and is suitable for treating parameter errors (2.15(b)). The second version guarantees a worst-case performance for the uncertainty set C employing a maxmin or minimax criterion; in a first step, it chooses the least-favorable model or parameters in C w.r.t the conventional optimization criterion of the considered signal processing task. Thus, optimization of the algorithm is identical to Fig 2.13, but based on the worst-case model. In important practical cases, this is identical to the problem of designing a maximally robust algorithm.

Finally, let us emphasize that the systematic approach to robust design has a long history and many applications; since the early 1960s robust optimization has been treated systematically in mathematics, engineering, and other sciences.

2.6.3 Imperfect Knowledge of Wireless Channels and Related Works

The benefits promised by the earlier mentioned beamforming scheme depend upon the quality of the channel state information. The fast varying wireless environment, in combination with often very stringent delay constraints, makes the provision of perfect CSI extremely difficult. Mostly it is assumed that the receive side may obtain this

knowledge through pilot transmission process in which a set of certainly known data is transmitted towards the receiver [75], [76]. This process is inherently erroneous. At the transmitters, some additional obstacles appear. In time-division-duplex (TDD) systems, the transmitter might be able to exploit the channel reciprocity. This means using estimated channels from the phase where it operated as a receiver. Other option is the supply of CSI from the receiver using feedback channels. In both scenarios, the rapidly changing wireless environment can result in outdated estimates. Furthermore, the feedback links are typically of limited capacity [77], which increases the uncertainty in the CSI at the transmitter.

It is well-known that imperfect CSI can significantly degrade the system performance [40, 41, 53, 75, 78, 79]. In other words, designing beamforming vectors based on perfect CSI can violate the promised quality-of-service (QoS) at users end. Therefore, designing beamforming vectors which are robust to imperfect CSI is a task of great practical interest. To cope with imperfect CSI, the paradigm of robust optimization has been employed. There are various ways to introduce robustness to CSI errors in wireless systems. For example, the limited feedback channels have attracted a lot of attention recently [80]. In this approach, the receiver uses estimates of the channel to inform the transmitter by quantizing either the channel coefficients themselves or the required properties of the transmitted signal [77, 81, 82]. The accent in the work on limited feedback is put either on vector quantization of the channel or on construction of precoding codebooks.

Downlink beamforming techniques have been implemented in many modern wireless communications standards such as the WiMAX, LTE, and LTE-A [83, 84]. As a result multi-user downlink beamforming has become an active area of research in recent years (see for example [78]] and the references therein). There are a number of beamforming techniques that have been developed assuming the availability of perfect CSI. In the schemes of [30, 31, 85–87] the availability of perfect instantaneous CSI at the transmitter is considered. On the other hand, in this thesis, contrary to the related work in the literature which is mainly based on the assumption of perfect channel state information at both transmitter and receiver, we will focus on robust transmission. Furthermore, when imperfections in CSI occur due to channel estimation errors, it will be assumed that the CSI errors have certain properties, either in terms of shapes of uncertainty regions, or statistics. The model is, therefore, quite general. The main goal is the provision of some QoS targets despite the channel uncertainty. For this problem formulation, optimization based on worst-case or based on chance-constraints are two methodologies of particular interest. One of the oldest, well-established approaches in

robust designs is the worst-case philosophy [88]. In worst-case methods, one usually assumes that the errors in channel knowledge are bounded. Typically, no statistical assumption on the mismatch is needed, which indeed might not exist. The aim is to optimize the system to guarantee certain performance for all channels from the uncertainty regions. Recently, there has been a number of applications of this concept in wireless communications. The modern treatment of the robust beamforming started with the seminal works of Bengtson and Ottersten [17, 49]. In these works, the authors used the worst-case design approach to guarantee the performance of the beamformer even when the least favorite channel realizations are occurring. The authors recast the original formulation to be in a semidefinite optimization form and then relaxed it to be convex. It is noteworthy that this way of treatment had a great impact for upcoming research in this area, as the semidefinite relaxation is an important tool, and is adopted in many beamforming research papers afterwards. Robust worst-case approach, applying semidefinite optimization for point-to-point MIMO channels, have been developed in [89, 90]. Robust designs with SINR as a QoS measure were developed in [91–96]. Worst-case approaches present a convenient framework for modeling quantization errors, since these errors are normally bounded. They are also appropriate for handling slow fading channels, where no sufficient statistics for the averaging is available [97]. However, the requirement to satisfy some goals for all uncertain channels from a specified region might sometimes lead to designs which are overly conservative. Furthermore, the CSI errors are often unbounded. The most prominent example is the channel estimation process. The estimation errors are typically modeled as random variables with the Gaussian distribution. In the unbounded case, it is usually not possible to guarantee any QoS targets with an absolute certainty. The idea to exploit statistical properties of the CSI error, if they exist, and require the fulfillment of some performance targets with certain probabilities, leads to the concept of probabilistically constrained (also coined as chance constrained, or outage based) signal processing [98, 99]. The knowledge about the error might be more precise, as in the case when the exact distribution is available. Sometimes, however, only some parameters, like the error covariance matrix, could be provided. The first applications of these ideas in the contexts of estimation theory and wireless communications have been proposed recently in [24, 100–103]. Most of the problems in this young area remain open though. There exists also a related group of stochastic (or Bayesian) robust designs which exploit the statistical information about the error and improve the average performance of the system. Relied on the knowledge of statistical distribution of uncertainty in channel covariance, and under the constraint that the outage probability does not exceed, the

total downlink transmmit power is minimized in [24]. Later, a sub-optimal method based on SDP that guarantees the probabilistic SINR requirements was proposed in [104].

In [52, 105], for a MISO system, the problem of minimizing the sum transmit power in the network subject to QoS constraints was examined. A novel S-procedure method was presented to solve the robust multi-cell downlink problem [52].

2.7 Mathematical Preliminaries

2.7.1 Introduction

Convex optimization, a subfield of optimization, which includes least-squares and linear programming problems, studies the problem of minimizing convex function over convex set. It is well known that least-squares and linear programming problems have a fairly complete theory, arise in a variety of applications, and can be solved numerically very efficiently.

Several related recent developments have stimulated new interest in convex optimization. These new methods allow us to solve certain new classes of convex optimization problems, such as semidefinite programs and second-order cone programs, almost as easily as linear programs, i.e., a program with linear objective function and linear/affine constraints.

There are great advantages to recognizing or formulating a problem as a convex optimization problem. The most basic advantage is that the problem can then be solved, very reliably and efficiently, using interior-point methods or other special methods for convex optimization. These solution methods are reliable enough to be embedded in a computer-aided design or analysis tool, or even a real-time reactive or automatic control system.

This Section concisely reviews concepts of convex optimization. Linear programming, and semidefinite programming (SDP) methods, together with its application for solving the problem of multi-user beamforming in a single-cell scenario are discussed. Finally a description of Lagrange duality in convex optimization is given. The concepts presented in this chapter are beneficial to the developments of beamforming schemes introduced in Chapters 3, 4 and 5. Readers interested in convex optimization and applications of convex optimization in communications are referred to [106], [107] and [108] for more details.

2.7.2 Convex Optimization

A convex program is an optimization problem where we seek the minimum of a convex function over a convex set. Its objective function as well as the constraints are convex. Convex optimization problems often occur in signal processing, communications, structural analysis and many other fields. Convex problems can be solved numerically with great efficiency and global optimums can be obtained. Efficient interior-point methods are available for the solution of convex optimization problems. However, the difficulty is often to recognize convexity; convexity is harder to recognize than say, linearity. One important feature of convexity is that it is possible to address difficult, nonconvex problems (such as combinatorial optimization problems) using convex approximations that are more efficient than classical linear ones. Convex optimization is especially relevant when the data of the problem at hand is uncertain, and robust solutions are sought.

2.7.2.1 Convex Set

A convex set C is the region such that, for every pair of points within the region C, every point on the straight line segment that joins the pair of points is also within the C, i.e., if for any $x_1, x_2 \in C$ and any θ with $0 \leq \theta \leq 1$, we have

$$\theta x_1 + (1 - \theta)\, x_2 \in C. \tag{2.2}$$
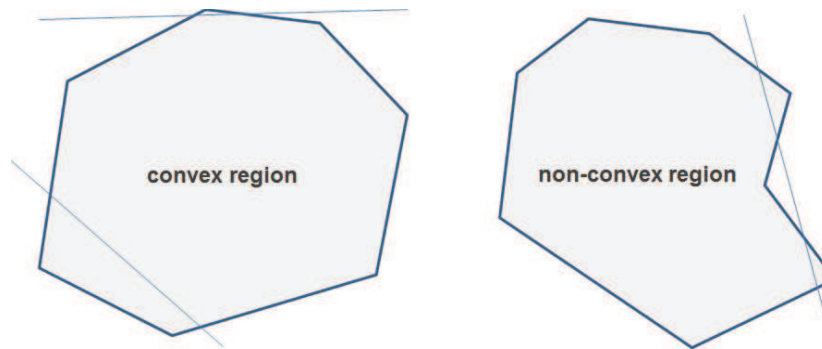
Figure 2.16: Some simple convex and non-convex sets

Some simple convex and non-convex sets is shown in Fig. 2.16. The hexagon including its boundary is a convex set whereas the kidney shaped set is not convex, since the line segment between the two points is partly not contained in the set.

2.7.2.2 Convex Functions

A function $f : \mathbb{R}^n \to \mathbb{R}$ is convex if the domain of f is a convex set and if for all x, y that belong to the domain of f and for any $0 \leq \theta \leq 1$, we have

$$f\left(\theta x + (1 - \theta)\, y\right) \leq \theta f\left(x\right) + (1 - \theta)\, f\left(y\right). \tag{2.3}$$

Geometrically, the inequality 2.3 can be interpreted as a line segment between $(x, f(x))$
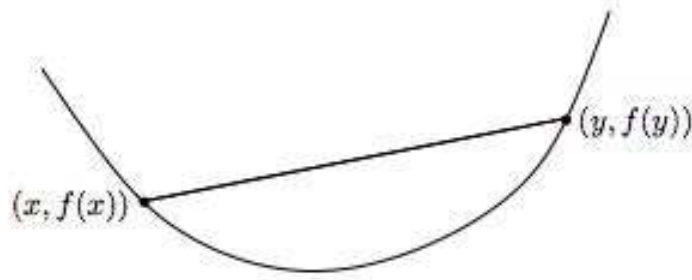
Figure 2.17: Graph of a convex function

and $(y, f(y))$ that lies above the graph of f, Fig 2.17. A function f is strictly convex if strict inequality holds in 2.3 whenever $x \neq y$ and $0 < \theta < 1$. f is said to be concave if $-f$ is convex, and strictly concave if $-f$ is strictly convex. Examples of convex functions are

- The exponential function $f(x) = e^{ax}$ is convex on $\mathbb{R}$, for any $a \in \mathbb{R}$.

- Function $f(x) = |x|^p$, for $p \geq 1$, is convex on $\mathbb{R}$.

- Function $f(x) = x^a$, for $a \geq 1$ or $a \leq 0$, is convex on $\mathbb{R}_{++}$.

2.7.2.3 Convex Optimization Problem

The generic standard form of an optimization problem [109] is as follows

$$\begin{aligned}
\min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\
\text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, 2, .., m, \\
& h_i(\mathbf{x}) = 0, \quad i = 1, 2, .., p,
\end{aligned} \tag{2.4}$$

where $x \in \mathbb{R}^n$ is the optimization variable, $f_0 : \mathbb{R}^n \to \mathbb{R}$ is the objective function, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, 2, .., m$, are the inequality constraint functions, and $h_i : \mathbb{R}^n \to \mathbb{R}$,

$i = 1, 2, .., p$ are the equality constraint functions. This notation is to describe a problem which tries to find the minimum value of the objective function $f_0(x)$ subject to m and p inequality and equality constraints, respectively.

In its standard form, a convex optimization problem can be expressed as

$$
\begin{aligned}
\min_{(\mathbf{x})} \quad & f_0(\mathbf{x}) \\
\text{subject to} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, 2, .., m, \\
& \mathbf{a}_i^T(\mathbf{x}) = b_i, \quad i = 1, 2, .., p,
\end{aligned}
\tag{2.5}
$$

where $f_0, f_1, ..., f_m$ are convex and a_i and $b_i, i = 1, 2, .., p$, are fixed parameters.

The convexity is often considered as a criterion that separates efficiently solvable from difficult optimization problems. Almost all convex problems can be solved, either in a closed form or using iterative algorithms. Some classes of them have very efficient numerical solutions.

2.7.3 Linear Programming

Linear programming (LP) is a considerable field of optimization. Many practical problems in operations research can be expressed as linear programming. An optimization of a linear objective function, subject to linear equality and linear inequality constraints can be expressed in canonical form as:

$$
\begin{aligned}
\min \quad & \mathbf{f}^T \mathbf{x} \\
\text{subject to} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\
\text{and} \quad & \mathbf{x} \geq \mathbf{0}
\end{aligned}
\tag{2.6}
$$

where the vector $\mathbf{x}$ is a variable to be determined, $\mathbf{c}$ and $\mathbf{b}$ are vector of coefficients, $\mathbf{A}$ is a matrix of coefficients. Solving linear programs are reliable and computation time proportional to $n^2 p$ if $p \geq n$.

More special cases of convex optimization problems that are mostly used in science and technology, namely, Second Order Cone Programming (SOCP) problems and Semidefnite Programming (SDP) problems. In this thesis we mostly focus on SDPs, as the objective and constraints of beamforming design problems are stated using this problem.

2.7.4   Semidefinite Programming

In a SDP we minimize a linear function of a variable $\mathbf{x}\mathbb{R}^\backslash$ subject to a matrix inequality. The standard form of a SDP is defined as [34, 106]:

$$\min_{\mathbf{x}} \quad \mathbf{f}^T\mathbf{x}$$
$$\text{subject to} \quad \mathbf{A}(\mathbf{x}) \succeq 0 \tag{2.7}$$

where

$$\mathbf{A} = \mathbf{A}_0 + \sum_{i=1}^{n} x_i \mathbf{A}_i \tag{2.8}$$

is a Hermitian matrix that depends affinely on $\mathbf{x}$ and the $n \times n$ Hermitian matrix $\mathbf{A}_i$, $\forall 1 \leq i \leq n$, is deterministic data.

SDP unifies several standard problems (e.g., linear and quadratic programming) and finds many applications in engineering and combinatorial optimization. Although SDP problems are much more general than LP problems, they are not much harder to solve. Most interior-point methods for LP have been generalized to SDP problems. As in LP, these methods have polynomial worst-case complexity, and perform very well in practice.

The dual problem associated with the SDP (2.7) is [110]

$$\max_{\mathbf{Z}} \quad -\operatorname{Tr}(\mathbf{A}_0\mathbf{Z})$$
$$\text{subject to} \quad \operatorname{Tr}(\mathbf{A}_i\mathbf{Z}) = f_i, \quad i = 1, 2, \cdots, n \tag{2.9}$$
$$\mathbf{Z} = \mathbf{Z}^H \succeq 0.$$

The dual problem (2.9) is also a SDP, i.e., it can be cast in the same form as the primal problem (2.7). For simplicity, we assume that the matrices $\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_n$ are linearly independent. Then the affine set $\operatorname{Tr}(\mathbf{A}_i\mathbf{Z}) = f_i, \forall i$ can be expressed in the form:

$$\{\mathbf{G}(\mathbf{y}) = \mathbf{G}_0 + y_1\mathbf{G}_1 + \cdots + y_p\mathbf{G}_p\} \tag{2.10}$$

where $p = m(m+1)/2 - n$ and $\mathbf{G}_i$ are appropriate matrices. Defining

$$\mathbf{d} = \left[\operatorname{Tr}(\mathbf{F}_0\mathbf{G}_1) \quad \operatorname{Tr}(\mathbf{F}_0\mathbf{G}_2) \quad \cdots \quad \operatorname{Tr}(\mathbf{F}_0\mathbf{G}_p)\right]^T.$$

Hence,

$$\mathbf{d}^T\mathbf{y} = \operatorname{Tr}(\mathbf{F}_0[\mathbf{G}(\mathbf{y}) - \mathbf{G}_0]).$$

Therefore, the dual problem (2.9) becomes

$$\min_{\mathbf{y}} \quad \mathbf{d}^T \mathbf{y}$$
$$\text{subject to} \quad \mathbf{G}(\mathbf{y}) \succeq 0 \tag{2.11}$$

which is a standard SDP form defined in (2.7). This concludes that the problem (2.9) is also a SDP.

Many convex optimization problems, e.g., LP and (convex) quadratically constrained quadratic programming, can be cast as SDP problems, so SDP offers a unified way to study the properties and derive algorithms for a wide variety of convex optimization problems. Most importantly SDP problems can be solved more efficiently, both in theory and in practice.

The SeDuMi solver [111] is a common optimization packet that can be used to solve SOCP and SDP. An elegant Matlab-based modeling system for convex optimization, i.e., CVX which supports the SeDuMi solver, has been developed by Michael Grant and Stephen Boyd [112].

The complexity of SeDuMi solver is given in [113] as follows

- **Asymptotic computational complexity**. Let n be the number of decision variables and m the number of rows of the LMIs. The computational complexity of SeDuMi (including main and inner iterations) is in $O\left(n^2 m^2.5 + m^3.5\right)$ while the algorithm in [114] has a complexity $O\left(n^3 m\right)$. The former algorithm is more efficient for problems with a large number of variables. This is of major interest when solving large scale problems or when implementing LMI-based iterative algorithms as in [115–118].

For more details on convex optimization, or interior-point methods and their complexity analysis interested readers are referred to [106]

2.7.5 Multi-user Downlink Beamforming Algorithm

Consider a base station (BS) equipped with an array of M antenna elements tra'nsmitting to U single-antenna users. The signal received by an user i, i.e., $y_i, i \in \{1, \cdots, U\}$, is given by

$$y_i = \mathbf{h}_i \mathbf{w}_i s_i + \sum_{j=1, j \neq i}^{U} \mathbf{h}_i \mathbf{w}_j s_j + n_i \tag{2.12}$$

where $\mathbf{h}_i \in \mathcal{C}^{1 \times M}$ is the MISO vector channel between user i and the BS, $\mathbf{w}_i \in \mathcal{C}^{M \times 1}$ represents the beamforming vector for user i, s_i is the intended symbol for user i

and finally n_i is the zero mean circularly symmetric complex Gaussian (ZMCSCG) random variable, i.e., $n_i \sim \mathcal{N}(0, \sigma^2)$, modeling the additive white Gaussian noise at the receiving point of user i. Without loss of generality, assuming that $\mathbb{E}\left(|s_i|^2\right) = 1, \forall i$. The signal-to-interference-plus-noise ratio for any user i is expressed as

$$\mathrm{SINR}_i = \frac{|\mathbf{h}_i \mathbf{w}_i|^2}{\sum_{j=1, j \neq i}^{U} |\mathbf{h}_i \mathbf{w}_j|^2 + \sigma^2}. \tag{2.13}$$

A common class of optimal transmit downlink beamforming for multiple users is to find a set of $\mathbf{w}_i$ that minimizes the total transmit power while guaranteeing all users' SINR requirements $\gamma_i, \forall i$:

$$\begin{aligned}
\min_{\mathbf{w}_i} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \\
\text{subject to} \quad & \frac{|\mathbf{h}_i \mathbf{w}_i|^2}{\sum_{j=1, j \neq i}^{U} |\mathbf{h}_i \mathbf{w}_j|^2 + \sigma^2} \geq \gamma_i, \forall 1 \leq i \leq U.
\end{aligned} \tag{2.14}$$

For simplicity, it is assumed that the set of γ_i in (2.14) is feasible. It can be verified that the SINR constraints in (2.14) are non-convex. In the next section, a technique to reformulate (2.14) in Semidefinite relaxation (SDR) form is presented.

2.7.6 Semidefinite Relaxation Algorithm

The introduction of the SDR technique in early 2000s has provided a capability of obtaining accurate, and sometime near optimal, approximation convex forms from non-convex problems, see [119], [120] and references therein. This section illustrates a method to cast (2.14) in a convex form using the SDR technique.

Let $\mathbf{R}_i = \mathbf{h}_i^H \mathbf{h}_i$ and $\mathbf{F}_i = \mathbf{w}_i \mathbf{w}_i^H$. It is clear that $\mathbf{F}_i$, $\forall 1 \leq i \leq U$, is a positive semidefinite and Hermitian matrix. Further more the rank of the matrix is one. The multiuser downlink beamforming problem in (2.14) can be expressed as

$$\begin{aligned}
\min_{\mathbf{w}_i} \quad & \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i \\
\text{subject to} \quad & \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum_{j=1, j \neq i}^{U} \mathbf{w}_j^H \mathbf{R}_i \mathbf{w}_j + \sigma^2} \geq \gamma_i, \forall 1 \leq i \leq U.
\end{aligned} \tag{2.15}$$

Using the following equality:

$$\mathbf{x}^H \mathbf{A} \mathbf{x} = \mathrm{Tr}\left(\mathbf{A} \mathbf{x} \mathbf{x}^H\right). \tag{2.16}$$

If $\mathbf{A} = \mathbf{I}$ then

$$\mathbf{x}^H \mathbf{x} = \text{Tr} \left(\mathbf{x}\mathbf{x}^H \right). \tag{2.17}$$

Rearranging the ith SINR constraints of (2.15), we arrive at

$$\left(1 + \frac{1}{\gamma_i} \right) \text{Tr} \left(\mathbf{R}_i \mathbf{F}_i \right) - \sum_{j=1, j \neq i} \text{Tr} \left(\mathbf{R}_i \mathbf{F}_j \right) - \sigma^2 \geq 0. \tag{2.18}$$

The problem (2.15) can be rewritten as

$$
\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=1}^{U} \text{Tr} \left(\mathbf{F}_i \right) \\
\text{subject to} \quad & \frac{1}{\gamma_i} \text{Tr} \left(\mathbf{R}_i \mathbf{F}_i \right) - \sum_{j=1, j \neq i} \text{Tr} \left(\mathbf{R}_i \mathbf{F}_j \right) - \sigma^2 \geq 0, \\
& \mathbf{F}_i = \mathbf{F}_i^H \succeq 0, \\
& \text{rank} \left(\mathbf{F}_i \right) = 1, \forall 1 \leq i \leq U.
\end{aligned}
\tag{2.19}
$$

The second constraints in (2.19) is to guarantee that $\mathbf{F}_i$, $\forall 1 \leq i \leq U$, is a positive semidefinite and Hermitian matrix. Dropping the last constraints in (2.19), i.e., $\text{rank} \left(\mathbf{F}_i \right) = 1$, results in a SDP form, i.e.,

$$
\begin{aligned}
\min_{\mathbf{F}_i} \quad & \sum_{i=1}^{U} \text{Tr} \left(\mathbf{F}_i \right) \\
\text{subject to} \quad & \frac{1}{\gamma_i} \text{Tr} \left(\mathbf{R}_i \mathbf{F}_i \right) - \sum_{j=1, j \neq i} \text{Tr} \left(\mathbf{R}_i \mathbf{F}_j \right) - \sigma^2 \geq 0, \\
& \mathbf{F}_i = \mathbf{F}_i^H \succeq 0, \forall 1 \leq i \leq U.
\end{aligned}
\tag{2.20}
$$

Dropping these rank one constraints not only enlarges the feasible set of the problem (2.19) but also leads to a relaxed SDP problem. This relaxation is referred to as semidefinite relaxation technique. For general nonconvex quadratic problems, solving a SDR problem usually gives an optimal solution with rank of larger than one. In such cases, SDR can only provide a lower bound on the optimal objective function and possibly attain an approximate solution to the original problem [120]. When using SDR results in $\mathbf{F}_i$ solutions with ranks higher than one, a randomization procedure, e.g., see [119], [121] and [86], can be used to find approximate rank-one solutions.

Since (2.14) has a specific structure that it can be turned into a convex form, strong duality holds for (2.14). Furthermore, it can be shown that the SDR of (2.19), i.e.,

(2.20), is the Lagrangian dual of (2.14) [120]. Therefore, (2.20) is exactly equivalent to the original problem (2.14). This fact has been confirmed in [49]. The authors of [49] noticed that the solution to (2.20) always admits rank-one matrices $\mathbf{F}_i, \forall i$, which directly yields the solution to (2.14) using $\mathbf{F}_i = \mathbf{w}_i \mathbf{w}_i^H$.

2.7.7 The Lagrange dual function

This section reviews main steps for Lagrangian duality in convex optimization for single-cell-multi-user beamforming.

We define the Lagrangian associated with the problem (2.14) as

$$
\begin{aligned}
L(\mathbf{w}_i, \lambda_i) &= \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i + \sum_{i=1}^{U} \lambda_i \left[\sum_{j=1, j \neq i}^{U} |\mathbf{h}_i \mathbf{w}_j|^2 + \sigma^2 - \frac{1}{\gamma_i} |\mathbf{h}_i \mathbf{w}_i|^2 \right] \\
&= \sum_{i=1}^{U} \mathbf{w}_i^H \mathbf{w}_i + \sum_{i=1}^{U} \lambda_i \left[\sum_{j=1, j \neq i}^{U} \mathbf{w}_j^H \mathbf{h}_i^H \mathbf{h}_i \mathbf{w}_j + \sigma^2 - \frac{1}{\gamma_i} \mathbf{w}_i^H \mathbf{h}_i^H \mathbf{h}_i \mathbf{w}_i \right].
\end{aligned}
$$

We refer to λ_i as the ith Lagrange multiplier associated with the ith constraint. Equivalently,

$$
L(\mathbf{w}_i, \lambda_i) = \sum_{i=1}^{U} \lambda_i \sigma^2 + \sum_{i=1}^{U} \mathbf{w}_i^H \left[\mathbf{I} + \sum_{j=1, j \neq i}^{U} \lambda_j \mathbf{h}_i^H \mathbf{h}_i - \frac{\lambda_i}{\gamma_i} \mathbf{h}_i^H \mathbf{h}_i \right] \mathbf{w}_i. \tag{2.21}
$$

The dual function is

$$
g(\lambda_i) = \inf_{\mathbf{w}_i} L(\mathbf{w}_i, \lambda_i). \tag{2.22}
$$

The dual function takes on the values

$$
g(\lambda_i) = \begin{cases} \sum_{i=1}^{U} \lambda_i \sigma^2, & \text{if } \left(\mathbf{I} + \sum_{j=1, j \neq i}^{U} \lambda_j \mathbf{h}_i^H \mathbf{h}_i - \frac{\lambda_i}{\gamma_i} \mathbf{h}_i^H \mathbf{h}_i \right) \succeq 0 \\ -\infty, & \text{otherwise .} \end{cases} \tag{2.23}
$$

Therefore, the Lagrangian dual problem can be stated as

$$
\begin{aligned}
\max_{\lambda_i} \quad & \sum_{i=1}^{U} \lambda_i \sigma^2 \\
\text{subject to} \quad & \left(\mathbf{I} + \sum_{j=1, j \neq i}^{U} \lambda_j \mathbf{h}_i^H \mathbf{h}_i - \frac{\lambda_i}{\gamma_i} \mathbf{h}_i^H \mathbf{h}_i \right) \succeq 0, \forall 1 \leq i \leq U.
\end{aligned} \tag{2.24}
$$

2.8 Conclusion

This chapter provides the background study on the main topic of this thesis, robust optimization for multi-antenna downlink transmission in cellular networks, and reviews basic concepts on multi-antenna transmission, cooperative beamforming, robust optimization. In addition we give the mathematical preliminaries that are used in the subsequent chapters and review topics such as convex optimization, linear and semidefinite programming, followed by SDP application in finding optimal solutions to the multi-user beamforming problem. Finally, a brief review on Lagrangian duality technique is given.

Chapter 3

Worst-Case Robust Optimization of Downlink Multi-Cell Networks

In this chapter, we focus on robust design of downlink beamforming vectors for multiple antenna base stations (BSs) in a multi-cell interference network. We formulate a robust optimization problem where an individual BS within a cell designs its beamforming vectors to minimize a combination of its sum-power, used for assuring a desired quality of service at its local users, and its aggregate induced interference on the users of the other cells, to balance inter-cell interference across the multiple cells. The proposed robust formulation uses spherical uncertainty sets to model imperfections in the second-order statistical channel knowledge between the BS and the users. To maintain tractability of the robust solutions, we derive an equivalent semidefinite programming (SDP) formulation that is convex under standard rank relaxation. The numerical results confirm the effectiveness of the proposed algorithm under various sizes of uncertainty set and the fact that the attained robust solutions always satisfy the rank constraint.

3.1   Introduction

Joint signal processing across the base stations (BSs) with multiple antennas for co-ordinated downlink beamforming has shown promising results in enhancing spectral efficiency and providing a uniform capacity coverage in cellular networks, e.g., [122–125]. However, most of these works make the important assumption that channel state information (CSI) is accurately and globally available to all BSs via an ideal backhaul network. Whereas in practical scenarios, available CSI is inaccurate and establishing ideal backhaul links is not affordable due to the scarcity of communications resources. Thus, there has recently been a growing interest in decentralized and robust design for

cellular networks.

Assuming precisely known CSI and ignoring the influence of parameter uncertainties in designing optimization models for cellular networks lead to generating optimal solutions that may violate critical constraints and show unpredictably poor performance in realistic wireless channel conditions. As explained in section 2.6.3, in order to address such real-world problems in downlink beamforming, a number of authors have recently reported robust modeling and designing techniques, e.g., [17, 126]. In [126], a conservative model that captures uncertainties in channel parameters is used to map a nominal optimization problem for sum-power minimization to its robust counterpart. In [17], error is assumed inside the estimate of channel covariance matrix and a semidefinite programming (SDP) based numerical method, ignoring the positive semidefinite property of the covariance matrix is proposed, which yields a suboptimal solution to the downlink (DL) power optimization problem. However, the robust formulation for multi-cell coordinated beamforming is more challenging since the associated SINR constraints involve CSI errors not only in the desired signal and intra-cell interference terms, but also in the inter-cell interference. Hence the techniques of [17, 126], if applied to multi-cell networks, encounter severe performance degradation. This motivates the design of transmission techniques that can utilize the knowledge of inter-cell interference. Only very recently did there appear a few attempts to develop multi-cell beamforming under imperfect CSI [105]. In [105], bounded channel error is assumed, and suboptimal solutions are obtained by optimizing only the lower bound of the worst-case SINR, which yields some power penalty. To deal with this problem, the authors in [52] approximate the worst-case SINR constraints as linear matrix inequalities (LMIs) by semidefinite relaxation method and can obtain the exact solution. However, these approaches ignore the overall interference power on the outer-cell users due to transmissions to the locally active users within a cell.

In this chapter, we minimize the transmitting power by individual BSs and the resulting inter-cell interference jointly and in a robust manner. In cellular systems where each BS can capture the cross channel information between itself and the users of adjacent cells, e.g., via overhearing and exploiting the channel reciprocity in the time division duplex (TDD) systems [14], or where the cross-channel information of the users of the neighboring cells are attained from their corresponding BSs via backhaul, the scheme can be classified as decentralized; in a sense that each BS designs its beamforming vectors independently. In either case, channel parameters are prone to imperfection and the robustness of the designed downlink beamforming vectors against uncertainties in channel statistics is a critical task from a practical point of view. Under a spheri-

cal uncertainty model for second order statistical CSI, locally measured at individual BSs, we formulate a robust optimization problem that minimizes a combination of two utility functions subject to satisfying certain signal-to-interference-plus-noise ratio (SINR) levels at user terminals. While the first utility function attempts to maintain local users' SINR demands in power efficient way, the second utility function tries to bring into balance and stabilize the multi-cell network at an equilibrium by controlling inter-cell interference.

The main contribution of this chapter is the consideration of channel uncertainties in both of the objective function and the constrains, which makes the optimization problem analytically and numerically intractable. Using Lagrange duality we overcome this problem and present exact reformulations of the part of the objective function and the constraints affected by uncertainties in equivalent and convenient forms. Finally, we transform the original non-convex robust optimization problem to a convex semidefinite programming (SDP) [110] using the standard rank relaxation method.

The rest of this chapter is organized as follows. System model and problem formulation are given in Section 3.2. A robust beamforming scheme is proposed in Section 3.3. Simulation results are presented in Section 3.4. Finally, Section 3.5 concludes the chapter.

3.2   System Model and Problem Formulation
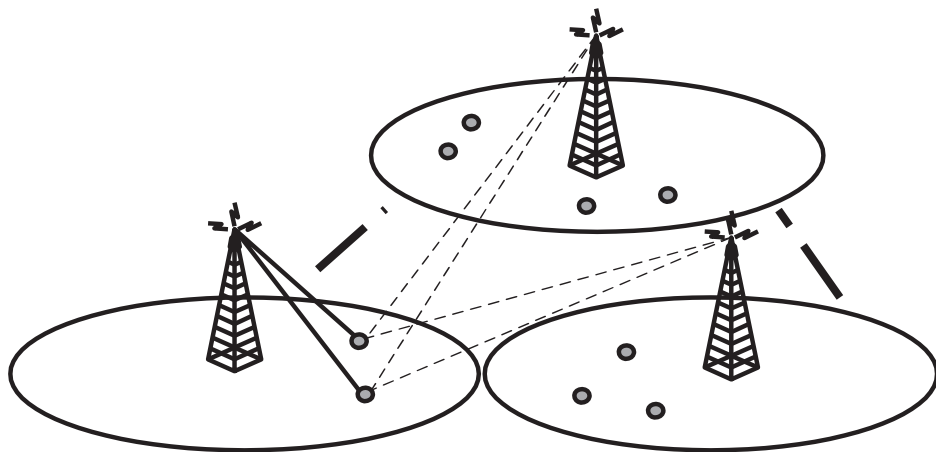
Figure 3.1: Illustration of multiuser multi-cell networks.

Consider a downlink multi-cell network where each cell consists of a single BS with M transmit antennas and U single-antenna users. Let the set of indices of BSs in the network be denoted as $\mathcal{S}_b = \{1, \cdots, N\}$ and the set of active users in each cell

as $\mathcal{S}_l = \{1, \cdots, U\}$, where index $i(q), i \in \mathcal{S}_l$ and $q \in \mathcal{S}_b$, indicates the ith user in cell q. Each BS communicates with its intra-cell users over the same frequency band as the adjacent BSs via the corresponding downlink beamforming vectors. Assume that $\mathbf{w}_{i(q)} \in \mathbb{C}^{M \times 1}$ and $\mathbf{h}_{i(q)}(q) \in \mathbb{C}^{M \times 1}$ are, respectively, the beamforming vector and the vector of channel coefficients of user $i(q)$ as seen by the BS of cell q. Hence, the received signal at user $i(q)$ can be written as

$$y_{i(q)} = \mathbf{h}_{i(q)}^H(q)\mathbf{w}_{i(q)}s_{i(q)} + \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{h}_{i(q)}^H(q)\mathbf{w}_{j(q)}s_{j(q)} + \zeta_{i(q)} + n_{i(q)}, \qquad (3.1)$$

where $s_{i(q)}$ represents data symbol intended for user $i(q)$ and $n_{i(q)} \sim \mathcal{CN}(0, \sigma_n^2)$ is assumed to be zero mean circularly symmetric complex Gaussian (ZMCSCG) noise. In (5.2), $\zeta_{i(q)}$ denotes the induced ICI on user $i(q)$ due to the transmissions of all BSs, other than cell q, in the network. Let $\widetilde{\mathbf{R}}_{i(q)}(q) = \mathbb{E}\left(\mathbf{h}_{i(q)}(q)\mathbf{h}_{i(q)}^H(q)\right)$ and $\widetilde{\mathbf{R}}_{t(k)}(q) = \mathbb{E}\left(\mathbf{h}_{t(k)}(q)\mathbf{h}_{t(k)}(q)^H\right)$ indicate, respectively, the channel covariance matrix of user $i(q)$ and the cross-channel (i.e., the ICI channel) covariance matrix of user t of cell k, as seen by the BS in cell q. We assume that only an imperfect knowledge of $\widetilde{\mathbf{R}}_{i(q)}(q)$ and $\widetilde{\mathbf{R}}_{t(k)}(q)$, i.e., $\mathbf{R}_{i(q)}(q)$ and $\mathbf{R}_{t(k)}(q)$, respectively, are available to the BS q, such that

$$\widetilde{\mathbf{R}}_{i(q)}(q) = \mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)},$$
$$\widetilde{\mathbf{R}}_{t(k)}(q) = \mathbf{R}_{t(k)}(q) + \mathbf{\Delta}_{t(k)},$$

where $\mathbf{\Delta}_{i(q)}$ and $\mathbf{\Delta}_{t(k)}$ are the uncertainty matrices. Letting the average energy in transmitting the ith symbol $s_{i(q)}$ be normalized to unity, i.e., $\mathbb{E}_{s_{i(q)}}\left(\left|s_{i(q)}\right|^2\right) = 1$, SINR can be expressed at any local user $i \in \mathcal{S}_l$ as

$$\text{SINR}_{i(q)} = \frac{\mathbf{w}_{i(q)}^H\left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)}\right)\mathbf{w}_{i(q)}}{\sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{w}_{j(q)}^H\left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)}\right)\mathbf{w}_{j(q)} + \xi_{i(q)} + \sigma_n^2}, \qquad (3.2)$$

where $\xi_{i(q)} = \mathbb{E}\left(\left|\zeta_{i(q)}\right|^2\right)$ is the total inter-cell interference power imposed on user $i(q)$. Furthermore, we have assumed a Gaussian model for the inter-cell interference and that any local user $i(q) \in \mathcal{S}_l$, can measure the arrived outer-cell interference, i.e., using the MMSE approach described in [127], and report it to its local BS. The BSs use the received information, i.e., $\xi_{i(q)}$, to design their beamforming vectors towards their corresponding users. In the sequel, we introduce an optimization problem to calculate

the downlink beamforming vectors at a given BS, as

$$\min_{\mathbf{w}_{i(q)}} \quad \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)} \right) \mathbf{w}_{i(q)} + \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \mathbf{w}_{i(q)} \tag{3.3}$$

$$\text{subject to} \quad \text{SINR}_{i(q)} \geq \gamma_{i(q)}, \quad \forall i \in \mathcal{S}_l, q \in \mathcal{S}_b,$$

where $\gamma_{i(q)}$ is the target SINR level required by an active local user $i \in \mathcal{S}_l$ in cell q. The first term of the objective function in (3.3) indicates overall interference power on the outer-cell users due to the transmissions to the locally active users within a cell, while the second term represents total signal power transmitted to local users. By setting $\boldsymbol{\Delta}_{t(k)} = \boldsymbol{\Delta}_{i(q)} = \mathbf{0}$, where $\mathbf{0}$ indicates a matrix with all zero elements, the problem (3.3) reduces to the non-robust optimization form, where precise channel parameters are assumed.

3.3 Robust Downlink Beamforming

In this section, we map problem (3.3) to an optimization problem that is robust against worst-case imperfection in CSI values, as follows

$$\min_{\mathbf{w}_{i(q)}} \quad \max_{\boldsymbol{\Delta}_{t(k)}} \quad \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} C_{t(k)} + \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \mathbf{w}_{i(q)} \tag{3.4}$$

$$\text{subject to} \quad \min_{\boldsymbol{\Delta}_{i(q)}} \quad \text{SINR}_{i(q)} \geq \gamma_i, \quad \forall i \in \mathcal{S}_l, q \in \mathcal{S}_b,$$

where

$$C_{t(k)} = \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)} \right) \mathbf{w}_{i(q)}. \tag{3.5}$$

We assume that the uncertainty in the estimation of channel covariance matrices is confined in an ellipsoid set defined as

$$\mathcal{E}_{t(k)} = \left\{ \boldsymbol{\Delta}_{t(k)} \in \mathbb{C}^{M \times M} : \text{Tr}(\boldsymbol{\Delta}_{t(k)} \mathbf{T}_{t(k)} \boldsymbol{\Delta}_{t(k)}^H) \leq \delta_t^2 \right\} \tag{3.6}$$

for the outer-cell users' indices $t \in \mathcal{S}_l$ and, similarly, $\mathcal{E}_{i(q)}$ for local users' indices $i(q) \in \mathcal{S}_l$. In (3.6), the weight matrix $\mathbf{T}_{t(k)}$ is positive definite. In this chapter, and without loss of generality, we have used a spherical uncertainty set by setting $\mathbf{T}_{t(k)} = \mathbf{I}$ in (3.6). Hence, $\text{Tr}(\boldsymbol{\Delta}_{t(k)} \mathbf{I} \boldsymbol{\Delta}_{t(k)}^H) = \|\boldsymbol{\Delta}_{t(k)}\|_F^2 \leq \delta_t^2$ and similarly $\|\boldsymbol{\Delta}_{i(q)}\|_F^2 \leq \delta_i^2$. For simplicity and without loss of generality, it is assumed $\delta_t = \delta_i = \delta$. In the sequel, we use the uncertainty set (3.6) that captures the variations of interfering channel covariance matrices and find an expression for the worst case inter-cell interference,

i.e., $\max\limits_{\left\|\boldsymbol{\Delta}_{t(k)}\right\|_F \leq \delta} \sum\limits_{k \in S_b, k \neq q} \sum\limits_{t \in S_l} C_{t(k)}$ in (3.4). In this direction and substituting for $C_{t(k)}$ from (3.5), we solve the following individual problems

$$\max\limits_{\left\|\boldsymbol{\Delta}_{t(k)}\right\|_F \leq \delta} \sum\limits_{i \in S_l} \mathbf{w}_{i(q)}^H \left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right) \mathbf{w}_{i(q)}, \quad \forall t \in S_l. \tag{3.7}$$

Using the property $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \mathrm{Tr}\left(\mathbf{Y}\mathbf{x}\mathbf{x}^H\right)$, problem (3.7) can be rewritten in the following equivalent optimization problem for any $t \in S_o$.

$$\begin{aligned} \min\limits_{\boldsymbol{\Delta}_{t(k)}} \quad & -\left(\mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\mathbf{A}\right) + \mathrm{Tr}\left(\boldsymbol{\Delta}_{t(k)}\mathbf{A}\right)\right) \\ \text{subject to} \quad & \left\|\boldsymbol{\Delta}_{t(k)}\right\|_F \leq \delta \\ & -\mathbf{R}_{t(k)}(q) - \boldsymbol{\Delta}_{t(k)} \preceq 0 \end{aligned} \tag{3.8}$$

where $\mathbf{A} = \sum_{i \in S_l} \mathbf{F}_{i(q)}$ and $\mathbf{F}_{i(q)} = \mathbf{w}_{i(q)}\mathbf{w}_{i(q)}^H$. The second constraint in (3.8) guarantees that $\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}$ is a positive semidefinite matrix. The Lagrangian of (3.8) can be formed as

$$\begin{aligned} L & \left(\lambda_{t(k)}, \boldsymbol{\Delta}_{t(k)}, \mathbf{S}_{t(k)}\right) \\ = & -\mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\mathbf{A}\right) - \mathrm{Tr}\left(\boldsymbol{\Delta}_{t(k)}\mathbf{A}\right) + \lambda_{t(k)}\left(\left\|\boldsymbol{\Delta}_{t(k)}\right\|^2 - \delta^2\right) - \mathrm{Tr}\left(\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right)\mathbf{S}_{t(k)}\right) \\ = & -\mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{A} + \mathbf{S}_{t(k)}\right)\right) - \mathrm{Tr}\left(\boldsymbol{\Delta}_{t(k)}\left(\mathbf{A} + \mathbf{S}_{t(k)}\right)\right) + \lambda_{t(k)}\left(\left\|\boldsymbol{\Delta}_{t(k)}\right\|^2 - \delta^2\right) \end{aligned} \tag{3.9}$$

where $\lambda_t \succeq 0$ and $\mathbf{S}_t \succeq 0$ are Lagrange multipliers for the first and second constraints in (3.8), respectively. To find the infimum in (3.9), we differentiate with respect to $\boldsymbol{\Delta}_{t(k)}$ and equate the resulting expression to zero, i.e., $\nabla_{\boldsymbol{\Delta}_{t(k)}} L\left(\lambda_{t(k)}, \boldsymbol{\Delta}_{t(k)}, \mathbf{S}_{t(k)}\right) = 0$. Using the property $\frac{\partial}{\partial \mathbf{X}} = \|\mathbf{X}\|_F^2 = \frac{\partial}{\partial \mathbf{X}}\mathrm{Tr}\left(\mathbf{X}\mathbf{X}^H\right) = 2\mathbf{X}$ and utilizing the fact that the matrices $\mathbf{A}, \boldsymbol{\Delta}_{t(k)}, \mathbf{S}_{t(k)}$ are all Hermitian, we find the optimal value for $\boldsymbol{\Delta}_{t(k)}$ as

$$\boldsymbol{\Delta}_{t(k)}^{opt} = \frac{1}{2\lambda_{t(k)}}\left(\mathbf{A} + \mathbf{S}_{t(k)}\right). \tag{3.10}$$

Plugging $\boldsymbol{\Delta}_{t(k)}^{opt}$ from (3.10) back in (3.9), we can write

$$\begin{aligned} g\left(\lambda_{t(k)}, \mathbf{S}_{t(k)}\right) & = \inf\limits_{\boldsymbol{\Delta}_{t(k)}} L\left(\lambda_{t(k)}, \boldsymbol{\Delta}_{t(k)}, \mathbf{S}_{t(k)}\right) \\ & = -\frac{\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|^2}{4\lambda_{t(k)}} - \lambda_{t(k)}\delta^2 - \mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{S}_{t(k)} + \mathbf{A}\right)\right). \end{aligned}$$

Hence, the dual problem associated with the primal problem (3.8) can be expressed as

$$
\max_{\lambda_{t(k)}, \mathbf{S}_{t(k)}} \quad -\frac{\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|_F^2}{4\lambda_{t(k)}} - \lambda_{t(k)}\delta^2 - \mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{S}_{t(k)} + \mathbf{A}\right)\right)
$$

$$
\text{subject to} \quad \lambda_{t(k)} \geq 0, \mathbf{S}_{t(k)} \succeq 0. \tag{3.11}
$$

We proceed by solving (3.11) with respect to $\lambda_{t(k)} \geq 0$ for any given $\mathbf{S}_{t(k)}$. Differentiating the objective function in (3.11) with respect to λ_t and equating the resulting expression to zero, one can get the optimal value of λ_t as

$$
\lambda_{t(k)}^{opt} = \frac{\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|_F}{2\delta}. \tag{3.12}
$$

Plugging $\lambda_{t(k)}^{opt}$ from (3.12) back in (3.11), we arrive at the following equivalent dual problem

$$
\max_{\mathbf{S}_{t(k)}} \quad -\delta\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|_F - \mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{S}_{t(k)} + \mathbf{A}\right)\right)
$$

$$
\text{subject to} \quad \mathbf{S}_{t(k)} \succeq 0, \tag{3.13}
$$

which is, in turn, equivalent to

$$
\min_{\mathbf{S}_{t(k)}} \quad \delta\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|_F + \mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{S}_{t(k)} + \mathbf{A}\right)\right)
$$

$$
\text{subject to} \quad \mathbf{S}_{t(k)} \succeq 0. \tag{3.14}
$$

Since (3.8) is a convex optimization problem for any given $\mathbf{A}$, strong duality holds between the primal problem (3.8) and its associated equivalent dual problem (3.14). Hence, (3.8) and (3.14) have the same optimal solutions.

In the sequel, we further simplify the optimization problem (3.4) by deriving an equivalent expression for its constraint,

$$
\min_{\left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta} \quad \mathrm{SINR}_{i(q)} \geq \gamma_{i(q)}. \tag{3.15}
$$

Defining a new variable $\mathbf{B}_{i(q)} = \gamma_i i(q) \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{F}_{j(q)} - \mathbf{F}_{i(q)}$, and using the equality $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \mathrm{Tr}\left(\mathbf{Y} \mathbf{x} \mathbf{x}^H\right)$, we can rewrite (3.15) as

$$
-\left[\mathrm{Tr}\left(\mathbf{R}_{i(q)}(q)\mathbf{B}_{i(q)}\right) + \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)\right] \geq \max_{\left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta} \mathrm{Tr}\left(\boldsymbol{\Delta}_{i(q)}\mathbf{B}_{i(q)}\right), \tag{3.16}
$$

or equivalently,

$$\min_{\left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta} - \left[\operatorname{Tr}\left(\mathbf{R}_{i(q)}(q)\mathbf{B}_{i(q)}\right) + \operatorname{Tr}\left(\boldsymbol{\Delta}_{i(q)}\mathbf{B}_{i(q)}\right) + \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)\right] \geq 0. \quad (3.17)$$

Following the approach used for inter-cell constraint (3.7), we can observe that the minimization on the left side of (3.17) can be substituted by the closed form solution of the following optimization problem:

$$
\begin{aligned}
\min_{\boldsymbol{\Delta}_{i(q)}} \quad & -\left[\operatorname{Tr}\left(\mathbf{R}_{i(q)}(q)\mathbf{B}_{i(q)}\right) + \operatorname{Tr}\left(\boldsymbol{\Delta}_{i(q)}\mathbf{B}_{i(q)}\right) + \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)\right] \\
\text{subject to} \quad & \left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta \\
& -\mathbf{R}_{i(q)}(q) - \boldsymbol{\Delta}_{i(q)} \preceq 0
\end{aligned}
\quad (3.18)
$$

For a given matrix $\mathbf{B}_{i(q)}$, (3.18) is a convex problem in variable $\boldsymbol{\Delta}_{i(q)}$. Using the fact that the Lagrange dual of (3.18) would provide a lower bound to its solution, we replace (3.18) by its dual. Therefore, we write the Lagrange dual function for problem (3.18) as

$$g\left(\lambda_{i(q)}, \mathbf{Z}_{i(q)}\right) = \inf_{\boldsymbol{\Delta}_{i(q)}} L\left(\lambda_{i(q)}, \boldsymbol{\Delta}_{i(q)}, \mathbf{Z}_{i(q)}\right), \quad (3.19)$$

where the Lagrangian function $L\left(\lambda_{i(q)}, \boldsymbol{\Delta}_{i(q)}, \mathbf{Z}_{i(q)}\right)$ is given as

$$
\begin{aligned}
L & \left(\lambda_{i(q)}, \boldsymbol{\Delta}_{i(q)}, \mathbf{Z}_{i(q)}\right) \\
= & -\operatorname{Tr}\left(\mathbf{R}_{i(q)}(q)\mathbf{B}_{i(q)}\right) - \operatorname{Tr}\left(\boldsymbol{\Delta}_{i(q)}\mathbf{B}_{i(q)}\right) - \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right) + \lambda_{i(q)}\left(\left\|\boldsymbol{\Delta}_{i(q)}\right\|^2 - \delta^2\right) \\
& - \operatorname{Tr}\left(\left(\mathbf{R}_{i(q)} + \boldsymbol{\Delta}_{i(q)}\right)\mathbf{Z}_{i(q)}\right) \\
= & -\operatorname{Tr}\left(\mathbf{R}_{i(q)}\left(\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right)\right) - \operatorname{Tr}\left(\boldsymbol{\Delta}_{i(q)}\left(\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right)\right) + \lambda_{i(q)}\left(\left\|\boldsymbol{\Delta}_{i(q)}\right\|^2 - \delta^2\right) \\
& - \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)
\end{aligned}
\quad (3.20)
$$

where $\lambda_{i(q)} \succeq 0$ and $\mathbf{Z}_i \succeq 0$ are Lagrange multipliers for the first and second constraints in (3.18), respectively. Utilizing the fact that the matrices $\mathbf{Z}_{i(q)}, \lambda_{i(q)}, \mathbf{B}_{i(q)}$ and $\mathbf{R}_{i(q)}$ are all Hermitian, we differentiating with respect to $\boldsymbol{\Delta}_{i(q)}$ and equating the resulting expression to zero, i.e., $\nabla_{\boldsymbol{\Delta}_{i(q)}} L\left(\lambda_{i(q)}, \boldsymbol{\Delta}_{i(q)}, \mathbf{Z}_{i(q)}\right) = 0$, hence we find the optimal value for $\boldsymbol{\Delta}_{i(q)}$ as

$$\boldsymbol{\Delta}_{i(q)}^{opt} = \frac{1}{2\lambda_{i(q)}}\left(\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right). \quad (3.21)$$

Substituting $\boldsymbol{\Delta}_{i(q)}^{opt}$ from (3.21) in (3.20), the Lagrange dual function (3.19) becomes

$$g\left(\lambda_{i(q)}, \mathbf{Z}_{i(q)}\right) = -\frac{\left\|\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right\|^2}{4\lambda_{i(q)}} - \mathrm{Tr}\left(\mathbf{R}_{i(q)}(q)\left(\mathbf{Z}_{i(q)} + \mathbf{B}_{i(q)}\right)\right) - \lambda_{i(q)}\delta^2 - \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right).$$
(3.22)

Hence the dual problem corresponding to (3.19) can be written as

$$\max_{\lambda_{i(q)}, \mathbf{Z}_{i(q)}} \quad -\frac{\left\|\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right\|_F^2}{4\lambda_{i(q)}} - \lambda_{i(q)}\delta^2 - \mathrm{Tr}(\mathbf{R}_{i(q)}(q)(\mathbf{Z}_{i(q)} + \mathbf{B}_{i(q)})) - \gamma_{i(q)}(\xi_{i(q)} + \sigma_n^2)$$

$$\text{subject to} \quad \lambda_{i(q)} \geq 0, \mathbf{Z}_{i(q)} \succeq 0.$$
(3.23)

Maximizing the objective function in (3.23) with respect to λ_i leads to the following Lagrange dual to the problem (3.19) :

$$\max_{\mathbf{Z}_{i(q)}} \quad -\delta\left\|\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right\|_F - \mathrm{Tr}\left[\mathbf{R}_{i(q)}\left(\mathbf{Z}_{i(q)} + \mathbf{B}_{i(q)}\right) - \gamma_i\left(\xi_{i(q)} + \sigma_n^2\right)\right],$$

$$\text{subject to} \quad \mathbf{Z}_{i(q)} \succeq 0$$
(3.24)

which is, in turn, equivalent to,

$$-\delta\left\|\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right\|_F - \mathrm{Tr}\left[\mathbf{R}_{i(q)}(q)\left(\mathbf{Z}_{i(q)} + \mathbf{B}_{i(q)}\right)\right] \geq \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right),$$
(3.25)

if there exists some positive semidefinite matrices $\mathbf{Z}_{i(q)}$, i.e., $\mathbf{Z}_{i(q)} \succeq 0$, that satisfy (3.25). Finally, using (3.14) and the fact that the condition in (3.25) together with $\mathbf{Z}_{i(q)} \succeq 0$ are equivalent to the constraint in (3.15), we can rewrite the original optimization problem (3.4) in the following equivalent form

$$\min_{\mathbf{F}_{i(q)}, \mathbf{S}_{t(k)}, \mathbf{Z}_{i(q)}} \quad \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left(\delta\left\|\mathbf{A} + \mathbf{S}_{t(k)}\right\|_F + \mathrm{Tr}\left(\mathbf{R}_{t(k)}(q)\left(\mathbf{S}_{t(k)} + \mathbf{A}\right)\right)\right) + \sum_{i \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{F}_{i(q)}\right)$$

$$\text{subject to} \quad -\delta\left\|\mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)}\right\|_F - \mathrm{Tr}\left(\mathbf{R}_{i(q)}(q)\left(\mathbf{Z}_{i(q)} + \mathbf{B}_{i(q)}\right)\right) - \gamma_i\left(\xi_{i(q)} + \sigma_n^2\right) \geq 0,$$

$$\mathbf{S}_{t(k)} \succeq 0, \quad \mathbf{Z}_{i(q)} \succeq 0, \quad \mathbf{F}_{i(q)} = \mathbf{F}_{i(q)}^H \succeq 0, \quad \mathrm{rank}\left(\mathbf{F}_{i(q)}\right) = 1,$$

$$\forall i \in \mathcal{S}_l, q \in \mathcal{S}_b,$$
(3.26)

The optimization problem in (3.26) is in the standard SDP form if the non-convex rank-one constraint of $\mathrm{rank}\left(\mathbf{F}_{i(q)}\right) = 1$ is relaxed. Then using the CVX [112], the resulting convex problem can be solved efficiently. Interestingly, our numerical solutions confirm that the resulting optimal solutions always admit the dropped rank-one constraint.

In general, if solving the relaxed rank-one problem results in a set of rank-one

matrices $\mathbf{F}_{i(q)}$ for all $i(q)$, then $\mathbf{F}_{i(q)}$ is also the optimal solution to the problem (3.26). Otherwise, the randomization technique in [12] will be used to generate a set of rank-one solutions of $\mathbf{F}_{i(q)}$. Given a rank-one $\mathbf{F}_{i(q)}$ solution, one can obtain the ith beamforming vector $\mathbf{w}_{i(q)}$ as $\mathbf{w}_{i(q)} = \sqrt{\rho_{i(q)}}\mathbf{x}_{i(q)}$, where $\rho_{i(q)}$ and $\mathbf{x}_{i(q)}$ are the eigenvalue and the eigenvector of $\mathbf{F}_{i(q)}$, respectively.

From the robust objective function in (3.26), it can be verified that the term $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \delta \left\| \mathbf{A} + \mathbf{S}_{t(k)} \right\| + \text{Tr} \left(\mathbf{R}_{t(k)}(q)\mathbf{S}_{t(k)} \right)$ provides necessary protection to the nominal objective function. However, this protection comes at the price of a worse optimized objective function value than the original nominal counterpart. Similarly, a comparison between the robust constraint in (3.26) and the nominal constraint indicates that the term $-\delta \left\| \mathbf{B}_{i(q)} + \mathbf{Z}_{i(q)} \right\| - \text{Tr} \left(\mathbf{R}_{i(q)}\mathbf{Z}_{i(q)} \right)$ gives required protection to the ith constraint. In fact, these functions and parameter δ determine and control the trade off between robustness and optimality.

3.4 Simulation results

3.4.1 Simulation setup
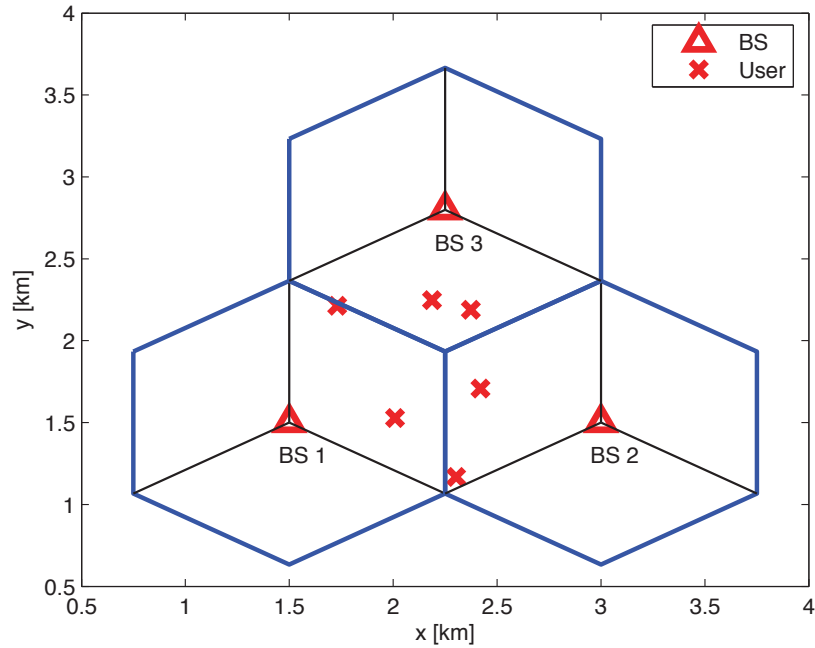
Figure 3.2: An example of random user distribution.

Table 3.1: Simulation parameters

Parameter	Value
Number of cells	3
Number of locally active users per cell	2
Number of antenna elements per sector	6
Antenna spacing	$\lambda/2$
Array antenna gain	15 dBi
Noise power spectral density (all users)	-174 dBm/Hz
BS-to-BS's distance	0.5 km
Path loss model ($l > 35$ in meter)	$34.53 + 38\log_{10}(l)$
Angular offset's standard deviation	2°
Log-normal shadowing's standard deviation	10 dB
Complex Gaussian fading coefficient's variance	1/2 per dimension

In this section, the performance of the proposed approach is observed and compared against the non-robust scheme [128], where perfect statistical CSI is assumed, a centralized robust coordinated beamforming (CBF)[50] and a robust conventional approach within three adjacent sectors of three neighboring BSs. Two users are randomly selected/dropped in each sector. A set of locations of six random users is referred to as one user distribution. Monte-Carlo simulations are carried out over 30 independent user distributions. Fig. 3.2 shows an example of one user distribution. Such a 3-cell scenario is also used in a number of other papers, e.g., [129, 130], for simulation purposes.

Throughout the analytical parts of this paper, we use a similar model as in [49, 51] and model the entries of $[\tilde{\mathbf{R}}_{i(q)}(q)]_{nm}$ and $[\tilde{\mathbf{R}}_{t(k)}(q)]_{nm}$ as

$$e^{\frac{j2\pi\Lambda}{\lambda}\left[(n-m)\sin\theta_{i(p)}\right]}e^{-2\left[\frac{\pi\Lambda\sigma_{as}}{\lambda}\left\{(n-m)\cos\theta_{i(p)}\right\}\right]^2}, \tag{3.27}$$

where λ is the carrier wavelength, $\Lambda = \lambda/2$ is the antenna spacing at BSs, and $\theta_{i(p)}$ is the angle of departure for user $i(p)$ with respect to the broadside of the antenna array. Furthermore it is assumed that the resulting angle spread/offset due to the scatterers is distributed as normal with zero mean and standard deviation of $\sigma_{as} = 2°$. In order to capture the effects of fading, path-loss and shadowing, we have scaled the channel covariance matrices and their corresponding uncertainty matrices by $L_{i(q)}(q)\sigma_F^2 e^{-0.5\frac{(\sigma_s\ln10)^2}{100}}$, where $L_{i(q)}(q)$ is the path loss coefficient between BS q and user $i(p)$ according to $34.53 + 38\log_{10}(\ell)$, i.e., where ℓ is the distance between the BS and the user, $\sigma_F^2 = 1$ is the variance of the complex Gaussian fading coefficient and

$\sigma_s = 10$ is the standard deviation of the log-normal shadow fading coefficient. In our simulation, each and every user estimates the variance of arriving aggregate inter-cell interference as a result of the concurrent transmissions of the BSs over the duration of the current frame and reports it back to its corresponding BS. The BSs use the received information, i.e., $\xi_{i(q)}, i \in \mathcal{S}_l$, to design their beamforming vectors towards their corresponding users. Table 3.1 characterizes the parameters used in our simulations in the following Section.

3.4.2 Simulation results

Let

$$\psi_{i(q)} = \frac{\mathbf{w}_{i(q)}^H \left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)} \right) \mathbf{w}_{i(q)}}{\gamma_{i(q)} \left(\sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{w}_{j(q)}^H \left(\mathbf{R}_{i(q)}(q) + \boldsymbol{\Delta}_{i(q)} \right) \mathbf{w}_{j(q)} + \xi_{i(q)} + \sigma_n^2 \right)} \tag{3.28}$$

denote the normalized SINR constraint value with respect to the target SINR at user $i(q)$. According to this definition, the SINR constraint of user $i(q)$ is satisfied if $\psi_{i(q)} \geq 1$. Fig. 3.3 illustrates the histograms of normalized SINR constraints for $\delta = 0.1$ and a target SINR of $\gamma_{i(q)} = 10$ dB at all users. As the non-robust scheme does not provide any protection against perturbations in CSI, it fails to satisfy about half of the constraints, whereas, the proposed robust scheme guarantees that all of the constraints are satisfied above the target SINR level.
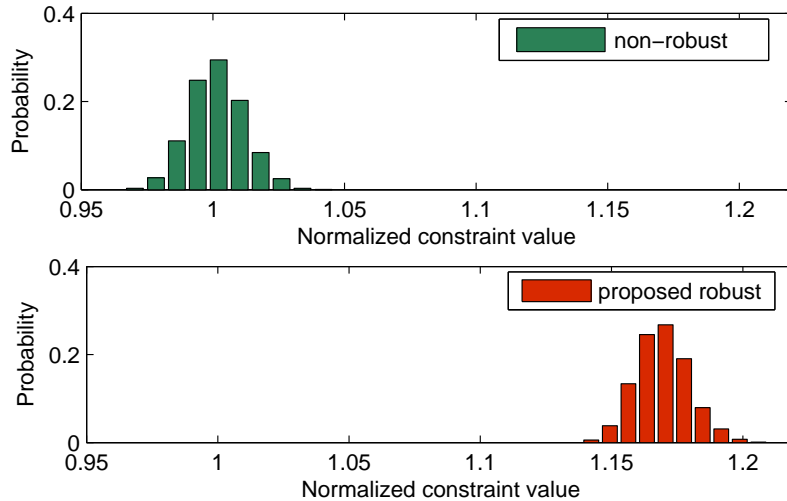
Figure 3.3: Histogram of normalized SINR constraints for $\delta = 0.1$ and target SINR values of 10 dB.

Fig. 3.4 illustrates the effect of various levels of δ on the performance of the proposed scheme. The figure shows that at a given SINR, the total transmit power increases as
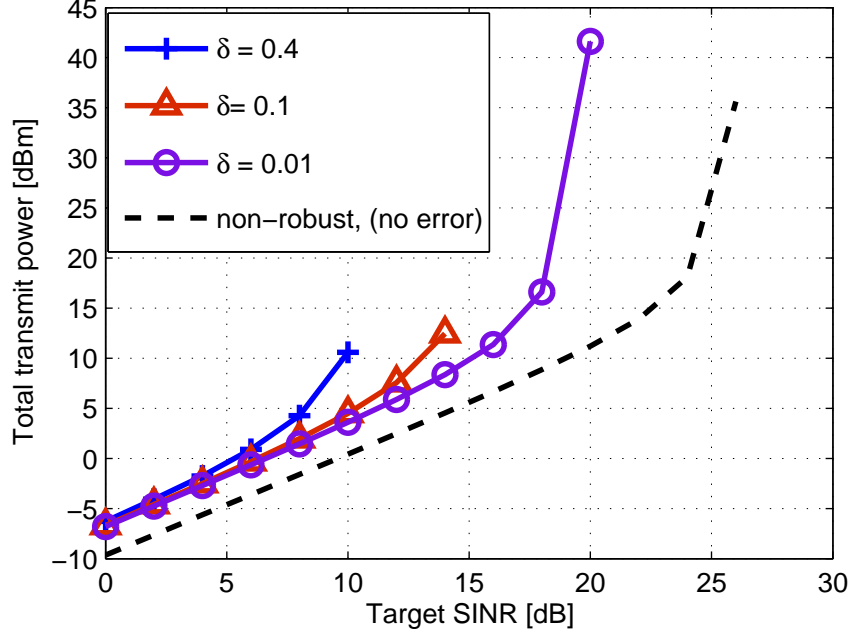
Figure 3.4: Total transmit power versus the target SINR at each user for the proposed scheme with various values of δ.

uncertainty level increases. The results also confirm that achieving robustness at higher uncertainty levels comes at the expense of lower achievable limits of SINR targets at affordable power levels. From Fig. 3.4, it can be observed as a cost of robustness, the robust scheme requires more transmit power at a given SINR target with respect to the non-robust scheme, but it clearly follows from Fig. 3.3, the non robust scheme does not guarantee to satisfy all users' SINR constraints, hence the proposed robust scheme is preferred.

Fig. 3.5 compares the proposed scheme with the conventional method and the coordinated beamforming (CBF) scheme, i.e., [50, section III.B], in terms of total transmit power versus SINR, using perturbed CSI with $\delta = 0.05$. As for the robust conventional method, we used the proposed optimization problem in (3.3) without the first utility function that accounts for the effect of induced total inter-cell interference on the users of the other cells. The CBF scheme jointly designs the beamforming vectors of all BSs in a centralized manner. The results in Fig. 3.5 confirm the effectiveness of the inter-cell interference balancing term in the proposed objective function of (3.3) in reducing the total transmit power at BSs. For instance, with an average transmit power of 6.55 dBm and at $\delta = 0.05$, the proposed scheme can attain 12 dB of target SINR, whereas the robust conventional method requires 18.56 dBm to support 12 dB of SINR target at the same error radius. Furthermore, it also can be seen that for
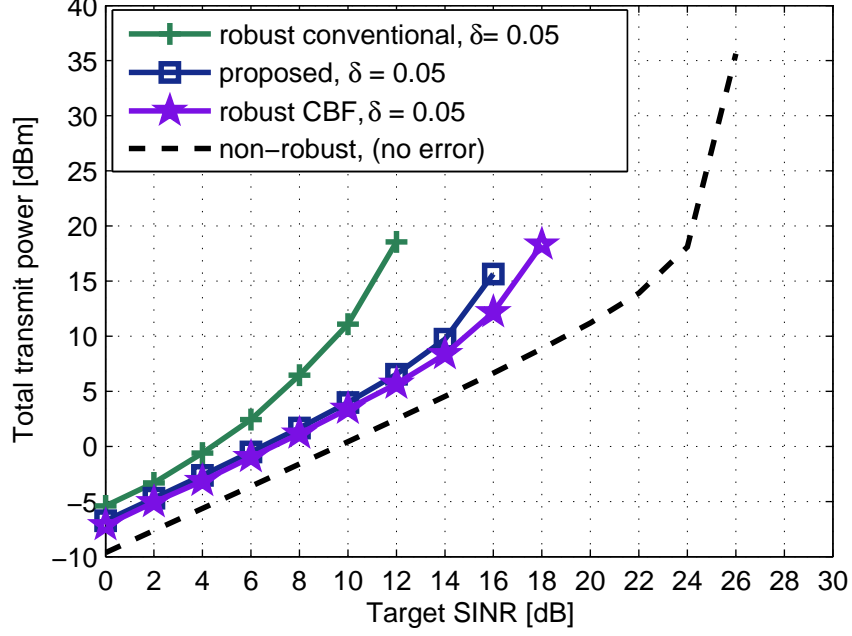
Figure 3.5: Total transmit power of 3 BSs of various schemes against the targeted SINR at each user.

a given level of power budget, the robust conventional method can support shorter range of SINR targets than the proposed robust scheme. For instance the robust conventional downlink beamforming scheme can only achieve the SINR target of up to 12 dB with affordable levels of power consumption, whereas, the proposed robust scheme can support target SINR levels of up to 16 dB. This is due to the so-called ping-pong effect in a multi-cell environment, where each BS keeps increasing its transmit power to maintain its users SINR requirements and, inevitably, keeps increasing its interference on the users of the other cells. Whereas the second term of the objective function of the proposed optimization problem in (3.4) controls the inflicted inter-cell interference by each BS and stabilizes the egoistic dynamic of the conventional network in an equilibrium point, agreed by all BSs. Furthermore a comparison of proposed decentralized scheme against the robust centralized CBF, shows that the proposed scheme closely follows the power efficiency of the centralized CBF up to target SINR of 12 dB and then gradually departs as the target SINR values increase. However, such a centralized processing requires an additional resources of an ideal backhaul, which is prohibited in practical systems as it limits the scalability of the network. Hence, recently the research interests have been shifted towards as much as possible decentralization of the coordinated multi-point (CoMP) communications systems to relax the backhaul overhead [27, 71, 131, 132].

3.5 Conclusion

We formulated an optimum downlink beamforming scheme that accounts for imperfection and the uncertain nature of the estimated channel parameters at base stations in cellular networks. The aim is to provide desired levels of quality of services for the users who are located within the adjacent cells and communicate with their corresponding BSs over a shared bandwidth. We demonstrate the performance of the proposed robust scheme under various channel uncertainties. We show the trade-off between reliably achievable quality of services at user terminals and the cost of robustness in terms of power consumption. The results confirm that as the uncertainty region of the channel parameters grows, power consumption at BSs increases and the range of quality of service in term of SINR targets, achievable at affordable levels of power consumption at BSs, shrinks.

Chapter 4

Chance-Constrained beamforming design for Downlink Multi-cell Networks

We introduce a downlink robust optimization approach that minimizes a combination of total transmit power by a multiple antenna base station (BS) within a cell and the resulting aggregate inter-cell interference (ICI) power on the users of the other cells. This optimization is constrained to assure that a set of signal-to-interference-plus-noise ratio (SINR) targets are met at user terminals with certain outage probabilities. The outages are due to the uncertainties that naturally emerge in the estimation of channel covariance matrices between a BS and its intra-cell local users as well as the other users of the other cells. We model these uncertainties using random matrices, analyze their statistical behavior and formulate a tractable probabilistic approach to the design of optimal robust downlink beamforming vectors. The proposed approach reformulates the original intractable non-convex problem in a semidefinite programming (SDP) form with linear matrix inequality (LMI) constraints. The resulting SDP formulation is convex and numerically tractable under the standard rank relaxation. We compare the performance of the proposed chance-constrained approach with that of the worst-case robustness. The simulation results confirm that the proposed approach can be considerably more power efficient than its conservative worst-case counterpart, specially, when higher SINR targets are desired.

4.1 Introduction

Joint signal processing across the base stations (BSs) with multiple antennas for coordinated downlink beamforming has shown promising results in enhancing spectral efficiency and providing a uniform capacity coverage in cellular networks, e.g., [122–125]. An effective downlink beamforming requires the availability of an accurate channel state information (CSI) at BSs. However, the assumption that the CSI are accurately and globally available to all BSs via an ideal backhaul network is not a realistic one. In many practical scenarios, the available CSI at BSs is imperfect due to several reasons, e.g., estimation error, delay and the quantization error that may arise as a result of limited feedback from a user terminal to a BS. Ignoring the effect of CSI uncertainties in forming optimization models for cellular networks can lead to optimal solutions that may violate critical constraints and results in a poor outcome in realistic channel conditions [41]. These practical considerations have recently motivated a growing interest towards robust design of cellular networks.

Commonly, there are two methods of deterministic and stochastic modeling of imperfect CSI. In the former, the imperfection in the CSI is assumed to be bounded within an uncertainty region and the objective is to provide worst-case guarantees for the performance of the network. More specifically, robust designs based on the deterministic model are conservative, make no assumptions on the distribution of error and optimize the worst-case performance of the system, e.g., see the works in [19, 36, 89, 105, 133, 134] for worst-case CSI modeling examples. Although, the deterministic optimization approaches provide robustness against CSI imperfections, the actual worst-case may occur with a very slim chance in practice. Hence, a deterministic design may lead to an inefficient design, as most system resources could be dedicated to provide guarantees for the worst-case scenarios. In order to provide less conservative solutions in favor of improved resource-efficient design, in the second approach, the perturbations in CSI are modeled to be statistically unbounded according to some known distributions. In the designs based on the stochastic modeling, the beamforming vectors are designed such that the quality-of service (QoS) requirements are met with a high probability, e.g., [135]. The probability (chance)-constrained problems are known to be difficult to solve because the probabilistic signal-to-interference-plus-noise ratio (SINR) constraints in general do not have closed-form expression and are not convex. The majority of available algorithms mainly rely on deriving analytical convex upper bounds for the probabilistic constrains and only find a feasible worst-case solution without any optimality guarantee. In [135], weighted variable-penalty alternating direction method of multipliers is used for a chance-constrained robust multi-cell beamforming problem

to minimize the sum power of all BSs subject to SINR constraints at user terminals in a distributed fashion. In this approach, the probabilistic constraints are upper-bounded by tractable convex approximating functions. Transceiver design with QoS guarantee in the presence of uncertain CSI at the transmitter is studied for a broadcast scenario with a multi-antenna BS and single antenna user terminals in [136]. In this study, the scenario is formulated as an optimization problem and conservative approaches that yield deterministic convex approximation for randomly perturbed second order cone constraints are used to guarantee the satisfaction of the probabilistic constraints. In a similar broadcast scenario, [137] studies power allocation strategies to satisfy QoS targets at user terminals in the presence of channel estimation error with Gaussian distribution. The authors in [137] use Vysochanskii-Petunin inequality in combination with the theory of interference functions to find conservative solutions to the problem. The transmit power minimization subject to probabilistic SINR constraints in a single-cell beamforming scenario is considered in [28] and [23]. The authors used conservative methods based on Bernstein inequality in [28] and relaxation-restriction approach in [23] to approximate the probabilistic constraints.

In this chapter, we introduce a chance-constraint downlink beamforming approach that minimizes a linear combination of total transmit power at individual BSs and the resulting overall interference on the other users of the other cells, subject to satisfying outage-based probabilistic QoS requirements (i.e., in terms of SINR) at user terminals in the presence of channel uncertainties. The outage-based constraints are motivated by the fact that most wireless systems can tolerate occasional outages in the QoS requirements [138–140]. While the proposed objective function maintains the local users' QoS demands in a robust and power efficient way, it balances the inter-cell interference (ICI) in an optimal way across the multiple cells under imperfect CSI and frequency reuse of one. In the following, we summarize the contributions of this chapter.

- We provide a new chance-constraint resource allocation formulation to handle the second-order CSI uncertainty with controlled percentage of outages in MISO multi-cell downlink beamforming channels.

- We derive a new analytical approach to solve the problem by directly characterizing the statistical behavior of the random matrix, modeling the imperfection in estimated second order statistical CSI. Our approach is in contrast to the methods that approximate the probabilistic constraints with their convex upper-bounds and effectively find a feasible worst-case solution without any optimality guarantee.

- We find a relationship between the Frobenius norm of the random matrix, modeling the radius of a hyper-spherical uncertainty region in the worst-case approach, and the outage parameter controlling the probability of the satisfaction of QoS requirement at users in the chance-constraint approach. This relation reveals and quantifies the implicit outage in the worst-case approach, i.e., due to the fact that the uncertainties in practical scenarios are statistically unbounded, and helps to compare it with the chance-constraint approach, fairly.

The rest of this chapter is organized as follows. System model and problem formulation are given in Section 4.2. The proposed beamforming problem is formulated as a probability constrained stochastic optimization problem in Section 4.3. In Section 4.4, we develop a technique based on the outage probability and show its relationship to the worst-case based approach of Chapter 3. Simulation results are presented and discussed in Section 4.5. Finally, Section 4.6 concludes the chapter.

4.2   Problem Formulation

Adopting the same system model as Chapter 3, recall that the estimate of the true channel covariance matrix of user $i(q)$ and the cross-channel (i.e., the ICI channel) covariance matrix of user t of cell k, as seen by the BS in cell q are represented as $\widetilde{\mathbf{R}}_{i(q)}(q)$ and $\widetilde{\mathbf{R}}_{t(k)}(q)$, respectively. We assume that only an imperfect knowledge of $\widetilde{\mathbf{R}}_{i(q)}(q)$ and $\widetilde{\mathbf{R}}_{t(k)}(q)$, i.e., $\mathbf{R}_{i(q)}(q)$ and $\mathbf{R}_{t(k)}(q)$, respectively, are available to the BS q, such that

$$\widetilde{\mathbf{R}}_{i(q)}(q) = \mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)},$$
$$\widetilde{\mathbf{R}}_{t(k)}(q) = \mathbf{R}_{t(k)}(q) + \mathbf{\Delta}_{t(k)},$$

$\mathbf{\Delta}_{i(q)}$ and $\mathbf{\Delta}_{t(k)}$ are random error matrices with respective rd-entries of $[\mathbf{\Delta}_{i(q)}]_{rd}$ and $[\mathbf{\Delta}_{t(k)}]_{rd}$, independently distributed as $[\mathbf{\Delta}_{i(q)}]_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$ and $[\mathbf{\Delta}_{i(q)}]_{rd} \sim \mathcal{CN}(0, \sigma_{rd}^2)$. In Section 3.3, we introduced a robust optimization problem as

$$\min_{\mathbf{w}_{i(q)}} \max_{\mathbf{\Delta}_{t(k)}} \quad \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \mathbf{w}_{i(q)} + \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H \left(\mathbf{R}_{t(k)}(q) + \mathbf{\Delta}_{t(k)} \right) \mathbf{w}_{i(q)}$$

$$\text{subject to} \quad \text{SINR}_{i(q)} = \frac{\mathbf{w}_{i(q)}^H \left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)} \right) \mathbf{w}_{i(q)}}{\sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{w}_{j(q)}^H \left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)} \right) \mathbf{w}_{j(q)} + \xi_{i(q)} + \sigma_n^2} \geq \gamma_{i(q)},$$

$$\forall i \in \mathcal{S}_l, q \in \mathcal{S}_b, \tag{4.1}$$

to design the transmit beamforming vectors and compute the power allocations at BS antennas in the downlink in the presence of channel uncertainties, in the worst-case scenario. In the following sections we provide a new chance-constrained approach to solve the optimization problem (4.1). More specifically, this approach measures the channel uncertainties by using the outage probability, i.e., the probability that the performance degradation caused by the error falls below a certain threshold. This approach is related to the statistical approach, which assumes that the covariance of the mismatched error matrix is known at transmitter.

4.3 Outage based probabilistic optimization

In this section, we reformulate the optimization problem in (4.1) with chance-constrained settings. Defining $\mathbf{A} = \sum_{i \in \mathcal{S}_l} \mathbf{F}_{i(q)}$, where $\mathbf{F}_{i(q)} = \mathbf{w}_{i(q)} \mathbf{w}_{i(q)}^H$ and using $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \text{Tr}\left(\mathbf{Y} \mathbf{x} \mathbf{x}^H\right)$, we can rewrite the problem in (4.1) as

$$
\begin{aligned}
\min_{\mathbf{F}_{i(q)}, \nu} \quad & \sum_{i \in \mathcal{S}_l} \text{Tr}\left(\mathbf{F}_{i(q)}\right) + \nu \\
\text{subject to} \quad & \Pr\left(\text{SINR}_{i(q)} \geq \gamma_{i(q)}\right) \geq 1 - \rho, \\
& \Pr\left(\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \text{Tr}\left\{\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right) \mathbf{A}\right\} \leq \nu\right) \geq 1 - \rho, \\
& \mathbf{F}_{i(q)} = \mathbf{F}_{i(q)}^H \succeq 0, \quad \text{rank}\left(\mathbf{F}_{i(q)}\right) = 1, \quad \forall i \in \mathcal{S}_l, q \in \mathcal{S}_b,
\end{aligned}
\tag{4.2}
$$

where ν is a slack variable and ρ is the probability of outage. The first and the second constraints in (4.2), respectively, ensure that the events $\text{SINR}_{i(q)} \geq \gamma_{i(q)}$ and $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \text{Tr}\left\{\left(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)}\right) \mathbf{A}\right\} \leq \nu$ hold with a minimum probability of $1 - \rho$ for every instantiation of the random matrices $\boldsymbol{\Delta}_{i(q)}$ and $\boldsymbol{\Delta}_{t(k)}$. Solving the optimization problem in (4.2) that involves probabilistic constraints is NP-hard, because the solutions should be feasible in the intersection of an infinite number of constraints. In the sequel, we overcome this problem by transforming the probabilistic constraints to more convenient and equivalent forms.

Lemma 4.1. *Let $\mathbf{X}$ be a $M \times M$ random matrix with independently distributed ZMC-SCG entries characterized as $[\mathbf{X}]_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$. Then, for any $\mathbf{L}$, $\mathbf{L} \in \mathbb{C}^{M \times M}$,*

$$
Tr(\mathbf{LX}) \sim \mathcal{N}\left(0, \||\mathbf{L}| \odot \boldsymbol{\Sigma}_{\mathbf{X}}\|_F^2\right),
\tag{4.3}
$$

where $|\mathbf{L}|$ is a real-valued $M \times M$ matrix with entries $[|\mathbf{L}|]_{ij} = |[\mathbf{L}]_{ij}|$, i.e., equal to

the absolute values of the entries of $\mathbf{L} \in \mathbb{C}^{M \times M}$, $\mathbf{\Sigma_X}$ is a real-valued $M \times M$ matrix with entries $[\mathbf{\Sigma_X}]_{ij} = \sigma_{ij}$ and $\odot$ defines the Hadamard product, i.e., the element-wise product of two matrices.

Proof. We can write $\mathrm{Tr}\,(\mathbf{LX}) = \left(\mathrm{vec}(\mathbf{L}^H)\right)^H \mathrm{vec}(\mathbf{X})$. Note that $\mathrm{Tr}\,(\mathbf{LX})$ is also a ZM-CSCG random variable, because it can be written as a weighted sum of independently distributed ZMCSCG random variables. Hence, the random variable $\mathrm{Tr}\,(\mathbf{LX})$ can be characterized as $\mathrm{Tr}\,(\mathbf{LX}) \sim \mathcal{CN}(0, \sigma_{\mathbf{LX}}^2)$. The variance $\sigma_{\mathbf{LX}}^2$ can be calculated as

$$
\begin{aligned}
\sigma_{\mathbf{LX}}^2 &= E\left[\left(\mathrm{vec}(\mathbf{L}^H)\right)^H \mathrm{vec}(\mathbf{X})\mathrm{vec}(\mathbf{X})^H\mathrm{vec}(\mathbf{L}^H)\right] \\
&= \left(\mathrm{vec}(\mathbf{L}^H)\right)^H E[\mathrm{vec}(\mathbf{X})\mathrm{vec}(\mathbf{X})^H]\mathrm{vec}(\mathbf{L}^H) \\
&= \left(\mathrm{vec}(\mathbf{L}^H)\right)^H \mathrm{diag}[\mathrm{vec}(\mathbf{\Sigma_X})]\mathrm{diag}[\mathrm{vec}(\mathbf{\Sigma_X})]\mathrm{vec}(\mathbf{L}^H) \\
&= \left(\mathrm{diag}[\mathrm{vec}(\mathbf{\Sigma_X})]\mathrm{vec}(\mathbf{L}^H)\right)^H \mathrm{diag}[\mathrm{vec}(\mathbf{\Sigma_X})]\mathrm{vec}(\mathbf{L}^H) \\
&= \||\mathbf{L}| \odot (\mathbf{\Sigma_X})\|_F^2
\end{aligned}
\tag{4.4}
$$

$\square$

Corollary 1: Let $U \sim \mathcal{N}(0,1)$ be a standard normal random variable. Then, the random variable $\mathrm{Tr}\,(\mathbf{LX})$ in (4.3) can be expressed as $\mathrm{Tr}\,(\mathbf{LX}) = \||\mathbf{L}| \odot \mathbf{\Sigma_X}\|_F U$.

In the sequel, we expand the event $\mathrm{SINR}_{i(q)} \geq \gamma_{i(q)}$ in the optimization problem (4.2) as

$$
\mathrm{Tr}\left(\left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)}\right)\mathbf{F}_{i(q)}\right) \geq \gamma_{i(q)} \sum_{j \in \mathcal{S}_l, j \neq i} \mathrm{Tr}\left(\left(\mathbf{R}_{i(q)}(q) + \mathbf{\Delta}_{i(q)}\right)\mathbf{F}_{j(q)}\right) + \gamma_{i(q)}(\xi_{i(q)} + \sigma_n^2),
\tag{4.5}
$$

and compactly rewrite it as

$$
\mathrm{Tr}\left(\mathbf{B}_{i(q)}\mathbf{\Delta}_{i(q)}\right) \leq \tau,
\tag{4.6}
$$

where $\mathbf{B}_{i(q)} = \gamma_{i(q)} \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{F}_{j(q)} - \mathbf{F}_{i(q)}$ and $\tau = -\mathrm{Tr}\left(\mathbf{B}_{i(q)}\mathbf{R}_{i(q)}(q)\right) - \gamma_{i(q)}\left(\xi_{i(q)} + \sigma_n^2\right)$.

Using Lemma 4.1 and *corollary 1*, (4.6) can be written as

$$
U \leq \frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \mathbf{\Sigma}_{\mathbf{\Delta}_{i(q)}}\right\|_F}.
\tag{4.7}
$$

Hence, the left-hand-side (LHS) of the first constraint in problem (4.2) is evaluated as

$$
\mathrm{Pr}(U \leq \frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \mathbf{\Sigma}_{\mathbf{\Delta}_{i(q)}}\right\|_F}) = \Phi(\frac{\tau}{\left\||\mathbf{B}_{i(q)}| \odot \mathbf{\Sigma}_{\mathbf{\Delta}_{i(q)}}\right\|_F}),
\tag{4.8}
$$

where $\Phi(u) = \Pr(U \leq u) = \frac{1}{2}[1 + \text{erf}(\frac{u}{\sqrt{2}})]$ is the cumulative distribution function (CDF) of a standard normal random variable U, and $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x exp(-t^2)\, dt$. Using (4.8), we can rewrite the LHS of the first constraint in problem (4.2) as

$$\Pr\left(\text{SINR}_{i(q)} \geq \gamma_{i(q)}\right) = \begin{cases} \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right), \tau > 0, \\ \frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{-\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right), \tau \leq 0. \end{cases} \tag{4.9}$$

To ensure that the first constraint in (4.2) is satisfied with an outage probability of no more than 50%, i.e., $\rho < 0.5$ for reliable communications purposes, we enforce (4.9) with $\tau > 0$. Notice that designs based on using (4.9) with $\tau \leq 0$ lead to $\rho > 0.5$. Hence, the first probabilistic constraint in (4.2) can be written as

$$\frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\tau}{\sqrt{2}\left\||\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right\|_F}\right) \geq 1 - \rho, \tag{4.10}$$

or equivalently as

$$\tau \geq c\left\|\text{vec}(|\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}})\right\|, \tag{4.11}$$

where $c \triangleq \sqrt{2}\text{erf}^{-1}(1 - 2\rho)$. Finally, using the Schur complement [106], we can write (4.11) in linear matrix inequality (LMI) form as

$$\begin{bmatrix} \frac{1}{c}\tau & \text{vec}^H\left(|\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right) \\ \text{vec}\left(|\mathbf{B}_{i(q)}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{i(q)}}\right) & \frac{1}{c}\tau\mathbf{I} \end{bmatrix} \succeq 0, \quad \forall i \in \mathcal{S}_l. \tag{4.12}$$

Similarly, we consider the second constraint in (4.2) and expand the event $\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\text{Tr}\{(\mathbf{R}_{t(k)}(q) + \boldsymbol{\Delta}_{t(k)})\mathbf{A}\} \leq \nu$ as

$$\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\text{Tr}\left(\mathbf{A}\boldsymbol{\Delta}_{t(k)}\right) \leq \nu - \sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\text{Tr}\left(\mathbf{A}\mathbf{R}_{t(k)}(q)\right). \tag{4.13}$$

It follows from Lemma 4.1 that the LHS of (4.13), which is sum of normally distributed random variables, is distributed as

$$\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\text{Tr}\left(\mathbf{A}\boldsymbol{\Delta}_{t(k)}\right) \sim \mathcal{N}(0, \sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\left\||\mathbf{A}|\odot\boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}}\right\|_F^2). \tag{4.14}$$

Hence, according to *corollary 1*, we can express (4.13) in terms of standard normal

variable U, as

$$U \leq \frac{\nu - b}{\sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\| |\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}} \right\|_F^2}}, \tag{4.15}$$

where $b = \sum_{t \in \mathcal{S}_l} \text{Tr} \left(\mathbf{A} \mathbf{R}_{t(k)}(q) \right)$. Consequently, the LHS of the second constraint in problem (4.2) can be expressed as

$$\Phi\left(\frac{\nu - b}{\sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\| |\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}} \right\|_F^2}}\right) = \begin{cases} \frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\nu - b}{\sqrt{2}a}\right), & \nu > b, \\ \frac{1}{2} - \frac{1}{2}\text{erf}\left(\frac{b - \nu}{\sqrt{2}a}\right), & \nu \leq b, \end{cases} \tag{4.16}$$

where $a = \sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\| |\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}} \right\|_F^2}$. For the same reason of ensuring a reliable communications as in (4.9), we enforce (4.16) with $v > b$. Hence the second probabilistic constraint in (4.2) can be substituted by

$$\frac{1}{2} + \frac{1}{2}\text{erf}\left(\frac{\nu - b}{\sqrt{2 \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\| |\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}} \right\|_F^2}}\right) \geq 1 - \rho, \tag{4.17}$$

or equivalently by

$$\nu - b \geq \beta \sqrt{\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left\| |\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{t(k)}} \right\|_F^2}, \tag{4.18}$$

where $\beta = \sqrt{2}\text{erf}^{-1}(1 - 2\rho)$. Let $\boldsymbol{\varrho}$ be defined as

$$\boldsymbol{\varrho} = \left[\text{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{1(1)}}) \cdots \text{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{U(q-1)}}), \text{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{1(q+1)}}) \cdots \right.$$
$$\left. \text{vec}^H(|\mathbf{A}| \odot \boldsymbol{\Sigma}_{\boldsymbol{\Delta}_{U(N)}})\right]^H. \tag{4.19}$$

Then (4.18) can be written as $\nu - b \geq \beta \|\boldsymbol{\varrho}\|$ and by applying the Schur complement is finally expressed in form as

$$\begin{bmatrix} \frac{1}{\beta}(\nu - b) & \boldsymbol{\varrho}^H \\ \boldsymbol{\varrho} & \frac{1}{\beta}(\nu - b)\mathbf{I} \end{bmatrix} \succeq 0, \quad \forall t \in \mathcal{S}_l. \tag{4.20}$$

Hence, the optimization problem in (4.2) with probabilistic constrains can be rewritten

with equivalent LMI constraints as

$$\min_{\mathbf{F}_{i(q)},\nu} \qquad \sum_{i\in\mathcal{S}_l}\mathrm{Tr}\left(\mathbf{F}_{i(q)}\right)+\nu \tag{4.21}$$

$$\text{subject to} \qquad \text{LMIs in} \quad (4.12) \quad \text{and} \quad (4.20),$$

$$\mathbf{F}_{i(q)}=\mathbf{F}_{i(q)}^{H}\succeq 0, \quad \mathrm{rank}\left(\mathbf{F}_{i(q)}\right)=1, \quad \forall i\in\mathcal{S}_l, q\in\mathcal{S}_b.$$

The optimization problem in (4.21) is a convex semidefinite programming (SDP) problem if the non-convex rank-one constraint is relaxed. The resulting SDP problem can be efficiently solved in $\mathbf{F}_{i(q)}=\mathbf{w}_{i(q)}\mathbf{w}_{i(q)}^{H}$ using the CVX [112]. In cases where the solution $\mathbf{F}_{i(q)}$ is not of rank-one, standard randomization techniques [119] can be applied to approximate $\mathbf{F}_{i(q)}$ by a rank-one matrix with sufficient accuracy. Finally, the optimal solution $\mathbf{w}_{i(q)}$ is determined as the principal eigenvector of the rank-one $\mathbf{F}_{i(q)}$ solution.

4.4 Implicit outage in worst-case setting

In this section, we establish a connection between the proposed probability-constrained stochastic optimization problem and the worst-case optimization problem in Chapter 3. In Chapter 3 it is assumed that the imperfections in CSI is bounded within a hyper-spherical region, hence the problem in (4.1) is expressed as

$$\min_{\mathbf{w}_{i(q)}} \quad \max_{\|\mathbf{\Delta}_{t(k)}\|_F\leq\delta_t} \sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^{H}\mathbf{w}_{i(q)}+\sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^{H}\left(\mathbf{R}_{t(k)}(q)+\mathbf{\Delta}_{t(k)}\right)\mathbf{w}_{i(q)}$$

$$\text{subject to} \min_{\|\mathbf{\Delta}_{i(q)}\|_F\leq\delta_{i(q)}} \mathrm{SINR}_i\geq\gamma_{i(q)}, \quad \forall i\in\mathcal{S}_l, q\in\mathcal{S}_b, \tag{4.22}$$

where δ_t and δ_i indicate the radii of the hyper-spheres corresponding to the uncertainties in the crosstalk and the local channel knowledge, respectively, at a given BS. For simplicity and without loss of generality, it is assumed $\delta_t=\delta_i=\delta$. The detailed solution can be found in Chapter 3.

In practical scenarios, the entries of $\mathbf{\Delta}_{t(k)}$ and $\mathbf{\Delta}_{i(q)}$ are unbounded random variables. Then, indeed their Frobenius norms, i.e., $\|\mathbf{\Delta}_{t(k)}\|_F$ and $\|\mathbf{\Delta}_{i(q)}\|_F$, become unbounded random variables. Hence, confining the CSI imperfections within a bounded uncertainty region in the worst-case approach would naturally imply that with a certain probability the uncertain CSI may fall outside of the considered uncertainty region. Thus, with certain outage probabilities the norm constraints in problem (4.22) may not hold in a realistic scenario and, hence, their corresponding optimal solutions may no longer be feasible. In this section, we find a metric that enables us to illustrate a link

between the worst case-based and probabilistically constrained robust designs. We provide an explicit relationship between the outage probability ρ and the uncertainty parameter δ and, therefore, provide a practical rule for choosing δ based on the QoS requirements.

Lemma 4.2. *Let* $\mathbf{\Delta}$ *be a* $n \times n$ *random matrix with ZMCSCG entries defined as* $[\mathbf{\Delta}]_{ij} \sim \mathcal{CN}(0, \sigma^2)$. *Then* $Pr(\|\mathbf{\Delta}\|_F^2 \leq \delta^2) = 1 - \rho$, *where*

$$\delta = \sqrt{\frac{\sigma^2 \Psi_{\chi^2(2n^2)}^{-1}(1 - \rho)}{2}}, \tag{4.23}$$

$0 \leq \rho \leq 1$ *is the outage, i.e., the probability that* $\|\mathbf{\Delta}\|_F^2 > \delta^2$ *and* $\Psi_{\chi^2(2n^2)}^{-1}(.)$ *is the inverse CDF of a standard chi-square random variable with* $2n^2$ *degrees of freedom.*

Proof. We can write

$$\|\mathbf{\Delta}\|_F^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} |[\mathbf{\Delta}]_{ij}|^2, \tag{4.24}$$

where $|[\mathbf{\Delta}]_{ij}|^2 = \Re\{[\mathbf{\Delta}]_{ij}\}^2 + \Im\{[\mathbf{\Delta}]_{ij}\}^2$. Since $[\mathbf{\Delta}]_{ij}$ is ZMCSCG, then its real and imaginary parts can be expressed in terms of standard normal random variables $U_k \sim \mathcal{N}(0,1)$ as $\Re\{[\mathbf{\Delta}]_{ij}\} = \frac{\sigma}{\sqrt{2}}U_k$ and $\Im\{[\mathbf{\Delta}]_{ij}\} = \frac{\sigma}{\sqrt{2}}U_{k+1}$, respectively. Hence, (4.24) can be rewritten as

$$\|\mathbf{\Delta}\|_F^2 = \frac{\sigma^2}{2}Q, \tag{4.25}$$

where $Q = \sum_{k=1}^{2n^2} U_k^2$ is distributed as a standard chi-square random variable with $2n^2$ degrees of freedom, i.e., $Q \sim \chi^2(2n^2)$. Hence, $\Pr(\|\mathbf{\Delta}\|_F^2 \leq \delta^2) = \Pr(Q \leq \frac{2\delta^2}{\sigma^2}) = \Psi_{\chi^2(2n^2)}(\frac{2\delta^2}{\sigma^2})$, where $\Psi_{\chi^2(2n^2)}(\cdot)$ indicates the CDF of a standard chi-square random variable with $2n^2$ degrees of freedom. By setting $\Psi_{\chi^2(2n^2)}(\frac{2\delta^2}{\sigma^2}) = 1 - \rho$ and calculating δ in terms of the outage probability ρ, we obtain (4.23). (see Appendix A for more details) $\qquad\square$

4.5 Simulation results

In this section, computer simulations are carried out to verify the performance of our proposed approach compared to Bengtssons method [49], which is used for perfect CSI case, a conventional downlink beamforming and worst-case robust approach of Chapter 3. For the conventional beamforming, we remove the ICI-balancing term, i.e., $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^H (\mathbf{R}_{t(k)}(q) + \mathbf{\Delta}_{t(k)}) \mathbf{w}_{i(q)}$ in the proposed objective function in (4.1), to illustrate the effect of ICI balancing term on the total transmit power of BSs.

We adopted the same simulation parameters as Table 3.1 and carried out Monte-Carlo simulations with 6 antenna elements per sector BSs and over 30 independent user distributions. An example of one user distribution has been shown in Fig. 3.2, Chapter 3. In the following simulations, we have assumed that the entries of each one of the random matrices $\boldsymbol{\Delta}_{i(q)}$ and $\boldsymbol{\Delta}_{t(k)}$ have the same variances, i.e., $[\boldsymbol{\Delta}_{i(q)}]_{rd} \sim \mathcal{CN}(0, \sigma_i^2)$ and $[\boldsymbol{\Delta}_{t(k)}]_{rd} \sim \mathcal{CN}(0, \sigma_t^2), \forall r, d$, and furthermore $\sigma_i^2 = \sigma_t^2 = \sigma^2$.
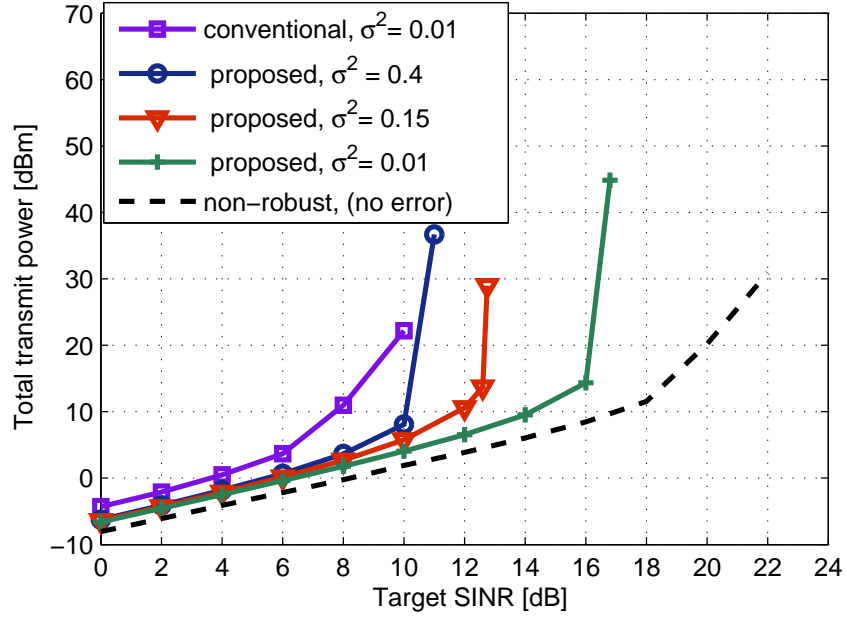
Figure 4.1: The proposed probabilistic approach with various variances of uncertainties at a fixed outage of $\rho = 0.3$.

Fig. 4.1 shows that at a fixed outage probability of $\rho = 0.3$, the total transmit power of 3 BSs increases as the variance, i.e., σ^2, of channel uncertainty increases, in the proposed probabilistic approach. In addition an increase in power level as a result of an increased variance of CSI error, i.e., σ^2, leads to an increase in interference power, which reduces the convergence speed of the algorithm as it requires more iterations to stabilize the network in an equilibrium. To further verify the proposed approach and to illustrate the impact of the ICI-balancing term on the total transmit power of BSs, we have also shown in Fig. 4.1 the result for chance-constraint conventional beamforming. A comparison of the results confirms the effectiveness of the proposed ICI-balancing term in significant reduction of the total transmit power at BSs and in achieving higher SINR targets with affordable sum-power levels at BSs. Furthermore from Fig. 4.1, the non-robust approach in [49] appears to be more power efficient than the proposed probabilistic design. This increase in transmit power in the proposed
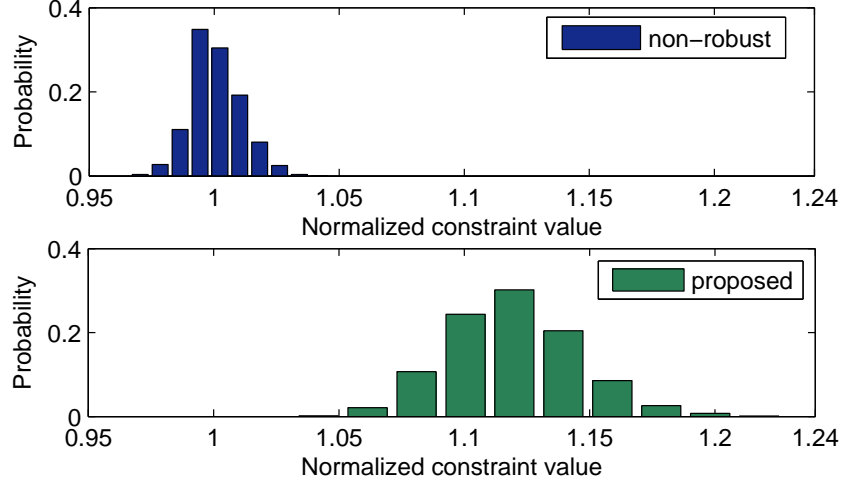
Figure 4.2: Histogram of normalized SINR constraints for $\sigma^2 = 0.01, \rho = 0.1$ and target SINR values of 10 dB.
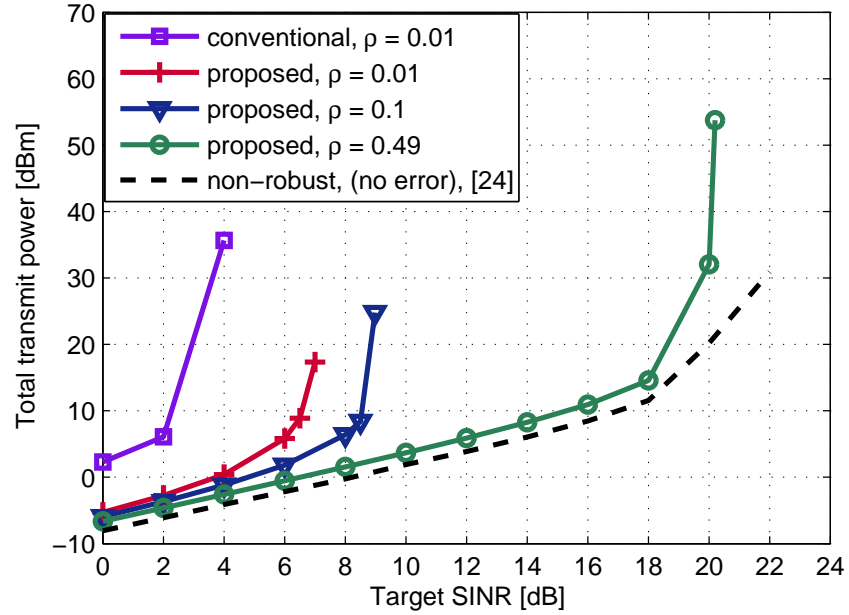
Figure 4.3: The proposed probabilistic approach with various outages at fix error variance of $\sigma^2 = 0.2$.

design is the price to be paid to achieve robustness against channel uncertainties. We have further shown the trade-off between the cost and the pay-off of our robust design scheme in Fig. 4.2, where we plot the histogram of the achievable normalized QoS using equation 3.28, as given previously in Chapter 3. As explained in Chapter 3, due to the normalization, a value greater than one for $\psi_{i(q)}$ corresponds to the satisfied QoS
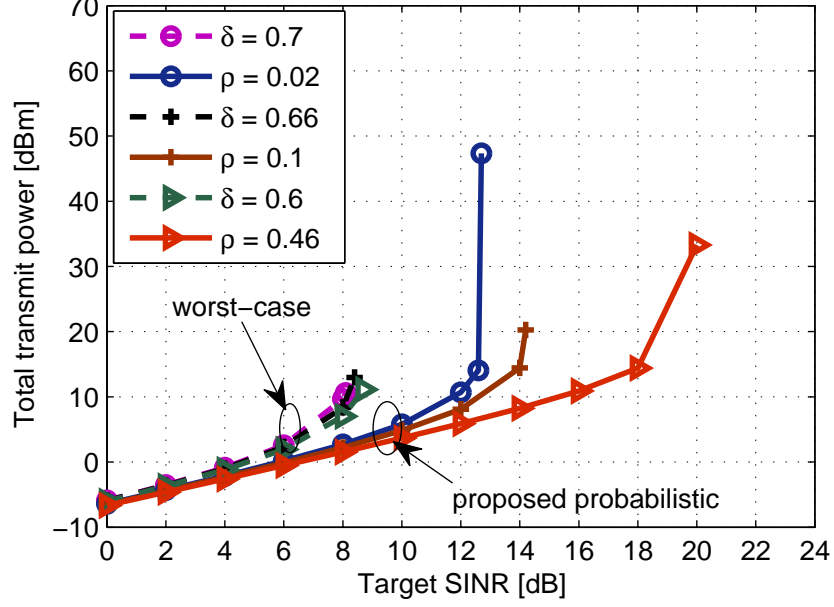
Figure 4.4: Comparison of the proposed approach with $\sigma^2 = 0.01$ with the worst-case approach at various outages.

constraints. As it can be seen in Fig. 4.2, when exposed to a scenario with channel estimation error, the non-robust scheme [49] fails to satisfy users SINR constraints in more than 50% of beamforming instances. This result reveals the fact that a design based on perfect CSI assumption can be quite sensitive to CSI errors in a realistic scenario.

To investigate the power consumption of the proposed approach, we plot the total transmit power of BSs at a fixed statistical CSI uncertainty of $\sigma^2 = 0.2$ versus the required SINR threshold, with different outage probability constraints measured by ρ, in Fig. 4.3. From Fig. 4.3, it can be observed for a given SINR target the required transmit power increases as the outage probability decreases. For instance, the proposed method with $\rho = 0.1$ requires 4.87 dB more power than with $\rho = 0.49$ for SINR = 8 dB. This is due to the cost to be paid in terms of more power consumption to achieve the pay-off in terms of gaining more insurance level for robustness against channel uncertainties. When the outage constraint is becoming stricter (i.e., less values of ρ), the probability of non-outage, e.g., $1 - \rho$, which defines the probability of delivering the required quality of service to the users, increases, and therefore, more power is required to meet the requested targets by the users, as shown in the Fig. 4.3. Further, it can be observed that at $\rho = 0.1$ the proposed method cannot go beyond a critical SINR point of 8.96 dB with limited transmit power. It is noticed that higher SINR targets

Figure 4.5: Histogram of normalized SINR constraints for target SINR value of 8 dB.

at lower transmit power can be achieved at higher outage probabilities, i.e., ρ, hence
the critical SINR decreases with a smaller ρ value.
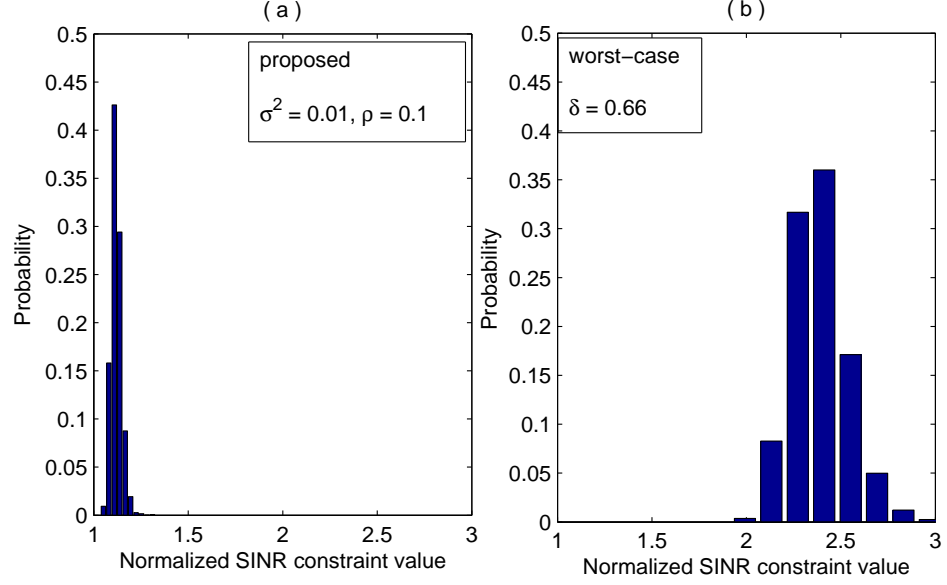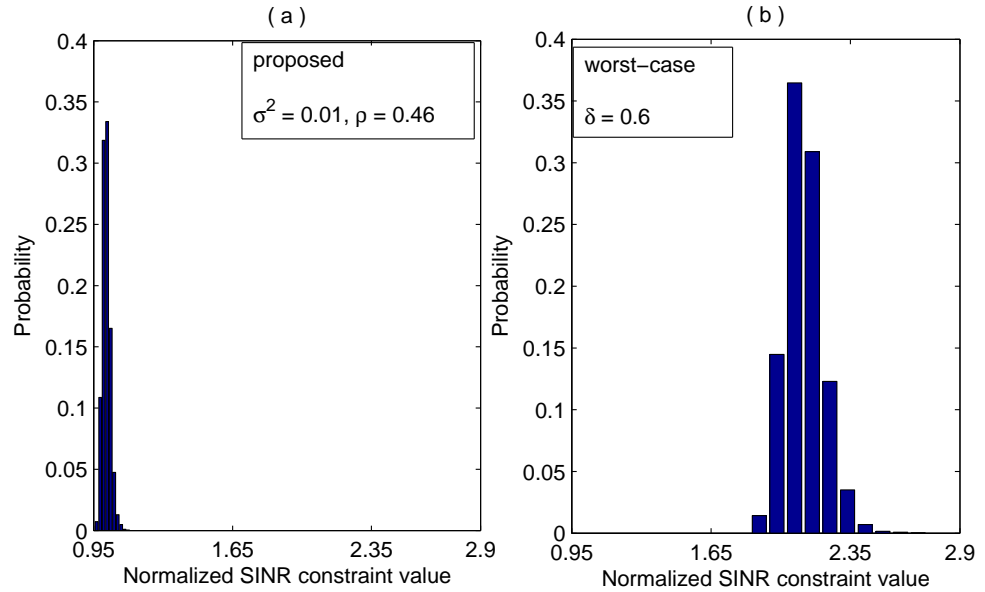

Figure 4.6: Histogram of normalized SINR constraints for target SINR value of 8 dB.

In Fig. 4.4, we have compared the performance of the proposed approach with that of the worst-case approach in Chapter 3 at various outage probabilities of 0.46, 0.1 and 0.02 and a fixed statistical CSI uncertainty of $\sigma^2 = 0.01$. Using (4.23), we calculate the deterministic upper bounds in the worst-case, i.e., conservative, approach in Chapter 3 as $\delta = 0.6$, 0.66 and 0.7, respectively, corresponding to $\rho = 0.46$, 0.1 and 0.02 in $\left\|\boldsymbol{\Delta}_{i(q)}\right\|_F \leq \delta$ and $\left\|\boldsymbol{\Delta}_{t(k)}\right\|_F \leq \delta$. A comparison of results in Fig. 4.4 shows that the proposed probabilistic approach is more power efficient than its conservative worst-case counterpart in Chapter 3. In particular, this superiority in being more power efficient becomes even more significant at higher SINR targets, i.e., SINR> 8 dB. Furthermore, Fig. 4.4 also confirms that the results for conservative cases are less sensitive to variations in outage values than the results for the probabilistic cases.

Figs. 4.5 and 4.6 demonstrate the corresponding histograms for the normalized SINR constraint at 10% and 46% outages, respectively, at a target SINR = 8 dB. Comparing 4.5(a) with 4.5(b) and 4.6(a) with 4.6(b), one can see that although, the worst-case approach in 4.5(b) and 4.6(b) fully satisfy the set SINR target, it consumes nearly 72% more power at SINR = 8 dB than the proposed approach, i.e., see Fig. 4.4. Furthermore, a comparison of 4.5(a) and 4.6(a) reveals that although, the proposed approach at 46% of outage ensures the satisfaction of SINR targets at above 95% of beamforming instants, it perfectly meets, i.e., well above 100%, the target SINR at 10% of outage.

4.6   Conclusions

We have proposed a probabilistic robust downlink beamforming approach to deliver users' desired SINR targets with certain adjustable outages. Users who are located within the adjacent cells of a cellular network communicate only with their own BSs and over a shared bandwidth. The proposed scheme is amenable to distributed implementation. This is due to accounting for the inter-BS coupling effect by minimizing the resulting inflicted aggregate ICI by each BS on the users of the other cells as an integral part of the proposed objective function of optimization. However, such an amenability to distributed implementation comes at the price of additional computational complexity at user terminals for estimating the incoming ICI from the other BSs in the adjacent cells and feeding it back to the local BSs. In this chapter, we have relaxed the rank one constraints and solved the proposed optimization problem using the SDP method. Interestingly, our simulations by CVX always generate exact rank one solutions, such that we have never needed to use an additional randomization process to approximate

the rank one solutions with an additional computational complexity. An interesting direction for future research is to attempt to prove analytically that the proposed optimization problem always generates rank one solutions by SDP approach. Simulation results confirm that not only does the proposed approach outperforms the conventional scheme, but it also shows a significantly superior power saving performance at higher SINR targets, when compared with its conservative worst-case design counterpart.

Chapter 5

Robust Downlink beamforming with imperfect CSI on both transceiver sides

In this chapter, we consider the problem of power-efficient transmit beamforming design at multi-antenna base stations (BSs) of a multi-cell network, when the channel state information (CSI) at both transceiver ends are imperfect. We introduce an optimization problem that accounts for robustness at, both, the constraints and the objective function. The robust constraints guarantee the quality of service (QoS) at mobile users (MUs) by ensuring that a set of signal-to-interference-plus-noise ratio (SINR) targets are met, despite the presence of erroneous CSI at both the BS and the MU sides. The robust objective function minimizes a linear combination of total transmit power at each BS and the overall inflicted interference power on the other users of the other cells under the worst-case of channel uncertainties. As the proposed problem is NP-hard, in general, we reformulate the problem into a semidefinite programming (SDP) with linear matrix inequality (LMI) constraints using the standard rank relaxation and the S-procedure. The simulation results confirm the effectiveness of the proposed robust beamforming design in terms of power efficiency at BSs and QoS guarantee at MUs, when compared with the conventional method in the presence of imperfect CSI.

5.1   Introduction

In a fully loaded cellular environment, cell-edge users experience considerably large inter-cell interference (ICI) causing severe degradation in system performance. Since, the adverse effect of ICI cannot be mitigated by increasing the transmission power, the

spectrum efficiency is compromised by employing orthogonal frequency reuse schemes in current cellular systems. Recently, cooperative multipoint (CoMP) transmission scheme, e.g., [51], [141], has been adopted by cloud-random access network (C-RAN) [142] to improve the spectrum efficiency of future multi-cell wireless networks. However, there are several problems, such as the provision of accountable channel state information, backhaul overhead and scheduling, that limits the scalability and effective application of this technology in practical networks.

In this chapter, we focus on the robustness of downlink coordinated beamforming design in the presence of imperfect CSI, both at the transmitter (CSIT), i.e., at the base station (BS), and at the mobile user (CSIR). The BS can capture CSIT by exploiting the channel reciprocity in time division duplex (TDD) systems or via a feedback mechanism initiated by the mobile user (MU) in frequency division duplex (FDD) systems. In both cases, however, CSIT can be contaminated by various sources of errors, such as estimation error, delay, e.g., due to high mobility, and quantization error, i.e., due to limited backhaul resources. Furthermore, the CSIR is also vulnerable to channel estimation error at the receiving side. As an erroneous CSIR affects the optimality of symbol detection at the receiving end, the assumption of perfect CSIR in designing the information beams at BSs may lead to unexpected results in practical scenarios.

Recently, multiple efforts have been made to design robust multi-cell beamformers. For instance, in [143], assuming second order statistical channel knowledge with imperfect CSIT and perfect CSIR, the authors design robust downlink beamforming vectors that minimize the total transmit power at BSs under both worst-case and probabilistic SINR constraints at user terminals. In [105], the authors focus on maximizing the weighted sum-rate as well as the minimum worst-case rate of a multi-cell network by adopting a bounded deterministic model for the uncertainty region and design centralized (fully cooperative) and distributed (limited cooperation) precoding algorithms at BSs. Most of the previous works on downlink beamforming designs assume imperfect CSIT, only, e.g., [144]. Most recently, in [145], robust downlink beamforming designs are studied with both imperfect CSIT and CSIR in the downlink of a single cell multicast scenario, comprising of a several groups of single antenna mobile terminals, to minimize the transmit power, subject to ensuring SINR targets in the worst-case of channel uncertainties. However, the approach in [145] is a single cell design and ignores the overall interference power on the other users of the other cells.

In this chapter, assuming instantaneous channel knowledge with imperfect CSIT and CSIR, we formulate a robust optimization problem that minimizes a linear combination of total transmitted power at each BS and the induced aggregate interference

power on the users of the other cells, in the worst-case. The aim is to guarantee a set of SINR targets at user terminals under the worst-case of channel uncertainties. Our approach differs from the existing ones in inclusion of a robust ICI controlling cost term in the objective function of the proposed optimization problem. This transmission strategy can increase the scalability of multi-cell networks with highly efficient re-usability of spectral resources across the adjacent cells. The robust SINR constraints account for the uncertainties in both CSIT and CSIR, which are assumed to be confined within a hyper-spherical set, and achieves a power-efficient and stable QoS guarantee in terms of the satisfaction of SINR targets at user terminals.

The remainder of this chapter is structured as follows. System model and problem formulation are given in Section 5.2. In section 5.3, we develop a tractable robust beamforming solution to the proposed problem. Simulation results are presented in Section 5.4. Finally, 5.5 concludes the chapter.

5.2   System Model and problem formulation

In this chapter we adopt a same system model as previous chapters and we assume that only a corrupted version of actual channel between a BS and a user is known and is modeled as

$$\mathbf{h}_{i(q)}(q) = \tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}, \tag{5.1}$$

where $\mathbf{h}_{i(q)}(q) \in \mathbb{C}^{M \times 1}$ is the actual complex channel vector between BS q and user $i(q)$, $\tilde{\mathbf{h}}_{i(q)}(q)$ is the corrupted version of the actual channel by an error $\mathbf{e}_{i(q)} \in \mathbb{C}^{M \times 1}$. The CSI error vector is modeled to be bound within a spherical region with radius ϵ, i.e., $\left\|\mathbf{e}_{i(q)}\right\|^2 \leq \epsilon^2$. Hence, the received signal at user i in cell q can be written as

$$y_{i(q)} = \left(\tilde{\mathbf{h}}_{i(q)}^H(q) + \mathbf{e}_{i(q)}^H\right) \mathbf{w}_{i(q)} s_{i(q)} + \sum_{j \in \mathcal{S}_l, j \neq i} \left(\tilde{\mathbf{h}}_{i(q)}^H(q) + \mathbf{e}_{i(q)}^H\right) \mathbf{w}_{j(q)} s_{j(q)} + v_{i(q)} + n_{i(q)},$$

$$\tag{5.2}$$

In detecting the received symbol $s_{i(q)}$, we assume that the receiver considers the estimated channel as the actual one and models the effect of channel estimation error as an additional Gaussian noise, statistically uncorrelated from noise and other

inter/intra-cell interferences at user $i(q)$. Expanding (5.2), we get

$$
y_{i(q)} = \underbrace{\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{w}_{i(q)}s_{i(q)}}_{\text{desired signal}} + \underbrace{\sum_{j\in\mathcal{S}_l,j\neq i}\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{w}_{j(q)}s_{j(q)}}_{\text{intra-cell interference}} +
$$
$$
\underbrace{\sum_{j\in\mathcal{S}_l,j\neq i}\mathbf{e}_{i(q)}^H\mathbf{w}_{j(q)}s_{j(q)} + \mathbf{e}_{i(q)}^H\mathbf{w}_{i(q)}s_{i(q)} + n_{i(q)}}_{\text{noise}} + \underbrace{v_{i(q)}}_{\text{inter-cell interference}} . \tag{5.3}
$$

Therefore the worst *effective* SINR at the input of an optimal detector, detecting the transmitted symbol $s_{i(q)}$, at user $i(q)$ is given by

$$
\text{eSINR}_{i(q)} = \frac{\left|\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{w}_{i(q)}\right|^2}{\displaystyle\sum_{\substack{j\in\mathcal{S}_l,\\j\neq i}}\left|\left(\tilde{\mathbf{h}}_{i(q)}^H(q)+\mathbf{e}_{i(q)}^H\right)\mathbf{w}_{j(q)}\right|^2 + \left|\mathbf{e}_{i(q)}^H\mathbf{w}_{i(q)}\right|^2 + \xi_{i(q)} + \sigma_n^2}, \tag{5.4}
$$

where $\xi_{i(q)} = \mathbb{E}\left(\left|v_{i(q)}\right|^2\right)$ is the total ICI power received by user $i(q)$. We have assumed a Gaussian model for the ICI and that each user $i(q) \in \mathcal{S}_l$ can estimate the arrived total ICI power $\xi_i(q)$, i.e., using the MMSE approach described in [127], and feed it back to its local BS. The BSs use the received information, i.e., $\xi_i(q)$, to design their beamforming vectors towards their corresponding users. Interested readers are referred to [127] for more details on ICI modeling. To design the downlink beamforming vectors at any given BS q, we propose a robust optimization problem given by

$$
\min_{\mathbf{w}_{i(q)}}\max_{\|\mathbf{e}_{t(k)}\|^2\leq\epsilon^2}\quad \sum_{i\in\mathcal{S}_l}\mathbf{w}_{i(q)}^H\mathbf{w}_{i(q)} + \sum_{k\in\mathcal{S}_b,k\neq q}\sum_{t\in\mathcal{S}_l}\sum_{i\in\mathcal{S}_l}\left|\left(\tilde{\mathbf{h}}_{t(k)}^H(q)+\mathbf{e}_{t(k)}^H\right)\mathbf{w}_{i(q)}\right|^2
$$
$$
\text{s.t.}\min_{\|\mathbf{e}_{i(q)}\|^2\leq\epsilon^2}\quad\frac{\left|\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{w}_{i(q)}\right|^2}{\displaystyle\sum_{\substack{j\in\mathcal{S}_l,\\j\neq i}}\left|\left(\tilde{\mathbf{h}}_{i(q)}^H(q)+\mathbf{e}_{i(q)}^H\right)\mathbf{w}_{j(q)}\right|^2 + \left|\mathbf{e}_{i(q)}^H\mathbf{w}_{i(q)}\right|^2 + \xi_{i(q)} + \sigma_n^2} \geq \gamma_{i(q)},
$$
$$
\forall i \in \mathcal{S}_l, q \in \mathcal{S}_b, \tag{5.5}
$$

where $\gamma_{i(q)}$ is the eSINR target required by an active local user $i(q)$. The first term of the objective function in (5.5) indicates the total signal power transmitted to locally active users in cell q, while the second term represents the overall interference power on the users of the other cells due to the transmissions to the locally active users within cell q. Note that $\tilde{\mathbf{h}}_{t(k)}(q) \in \mathbb{C}^{M\times1}$ is the corrupted cross channel vector between the BS of cell q and the user t of cell k and $\mathbf{e}_{t(k)} \in \mathbb{C}^{M\times1}$ is its associated error.

5.3 Min-Max Robust Optimization

In order to ensure robustness, the feasibility region of the optimal solutions to the optimization problem in (5.5) should fall in the intersection of an infinite number of SINR constraints, making the numerical solution intractable. In this section, we propose a tractable solution to the problem in (5.5) by applying semidefinite relaxation and rewriting the constraints in more convenient forms. Introducing a new matrix variable as $\mathbf{W}_{i(q)} = \mathbf{w}_{i(q)}\mathbf{w}_{i(q)}^H \in \mathbb{C}^{M \times M}$ and using the slack variable ν, we can reformulate the problem in (5.5) as

$$\min_{\mathbf{W}_{i(q)}, \nu} \quad \sum_{i \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_{i(q)}\right) + \nu \tag{5.6a}$$

$$\mathrm{s.t.} \quad \min_{\mathbf{e}_{i(q)}} \quad \mathrm{eSINR}_{i(q)} \geq \gamma_{i(q)}, \tag{5.6b}$$

$$\left\|\mathbf{e}_{i(q)}\right\|^2 \leq \epsilon^2, \quad \forall i \in \mathcal{S}_l \tag{5.6c}$$

$$\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \left|\left(\tilde{\mathbf{h}}_{t(k)}^H(q) + \mathbf{e}_{t(k)}^H\right)\mathbf{w}_{i(q)}\right|^2 \leq \nu \tag{5.6d}$$

$$\left\|\mathbf{e}_{t(k)}\right\|^2 \leq \epsilon^2, \quad \forall t \in \mathcal{S}_l, \tag{5.6e}$$

$$\mathbf{W}_{i(q)} = \mathbf{W}_{i(q)}^H \succeq 0, \tag{5.6f}$$

$$\mathrm{rank}\left(\mathbf{W}_{i(q)}\right) = 1, \quad \forall i \in \mathcal{S}_l. \tag{5.6g}$$

The constraint (5.6b) can be expanded as follows

$$\min_{\mathbf{e}_{i(q)}} \frac{\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{W}_{i(q)}\tilde{\mathbf{h}}_{i(q)}(q)}{\sum_{\substack{j \in \mathcal{S}_l, \\ j \neq i}} \left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right)^H \mathbf{W}_{j(q)}\left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right) + \mathbf{e}_{i(q)}^H \mathbf{W}_{i(q)}\mathbf{e}_{i(q)} + \xi_{i(q)} + \sigma_n^2} \geq \gamma_{i(q)},$$

$$\tag{5.7}$$

and rewritten as

$$\tau_{i(q)} \geq \max_{\mathbf{e}_{i(q)}} \quad \left[\left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right)^H \mathbf{B}_{j(q)}\left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right) + \left(\mathbf{e}_{i(q)}^H \mathbf{W}_{i(q)}\mathbf{e}_{i(q)}\right)\right], \tag{5.8}$$

where $\mathbf{B}_{j(q)} = \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{W}_{j(q)}$ and

$$\tau_{i(q)} = \frac{\tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{W}_{i(q)}\tilde{\mathbf{h}}_{i(q)}(q)}{\gamma_{i(q)}} - \xi_{i(q)} - \sigma_n^2. \tag{5.9}$$

The right-hand side of (5.8) is the upper-bound of sum of two non-negative terms. Defining the auxiliary scalar variables of $\omega_{i(q)}$ and $\kappa_{i(q)}$, we decompose (5.8) into three constraints, as

$$\omega_{i(q)} \geq \left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right)^H \mathbf{B}_{j(q)}\left(\tilde{\mathbf{h}}_{i(q)}(q) + \mathbf{e}_{i(q)}\right), \tag{5.10}$$

$$\kappa_{i(q)} \geq \left(\mathbf{e}_{i(q)}^H \mathbf{W}_{i(q)}\mathbf{e}_{i(q)}\right), \tag{5.11}$$

$$\tau_{i(q)} \geq \omega_{i(q)} + \kappa_{i(q)}. \tag{5.12}$$

Note that the constraints (5.10), (5.11) and (5.12) involve infinite number of inequalities, due to the existence of variable $\mathbf{e}_{i(q)}$, and hence, make the solution to the optimization problem in (5.6) intractable. To overcome this problem, we convert the set of intractable robust constraints into tractable ones using the S-procedure.

Lemma 5.1. *(S-procedure [106]) Let*

$$f_i(\mathbf{e}) = \mathbf{e}^H \mathbf{A}_i \mathbf{e} + 2\Re\left\{\mathbf{e}^H \mathbf{r}_i\right\} + c_i, \quad for \quad i = 0, 1, \tag{5.13}$$

where $\mathbf{A}_i \in \mathbb{H}^{M \times M}, \mathbf{r}_i \in \mathbb{C}^{M \times 1}$ and $c_i \in \mathbb{R}$. Suppose that there exists an $\hat{\mathbf{e}} \in \mathbb{C}^{M \times 1}$ such that $f_1(\hat{\mathbf{e}}) \leq 0$. Then the following two statements are equivalent:

1. $f_0(\mathbf{e}) \geq 0$ and $f_1(\mathbf{e}) \leq 0$ are satisfied for all $\mathbf{e} \in \mathbb{C}^{M \times 1}$

2. There exists a $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{Q}_0 & \mathbf{r}_0 \\ \mathbf{r}_0^H & c_0 \end{bmatrix} + \lambda \begin{bmatrix} \mathbf{Q}_1 & \mathbf{r}_1 \\ \mathbf{r}_1^H & c_1 \end{bmatrix} \succeq \mathbf{0}. \tag{5.14}$$

Using the constraints in (5.10) and (5.6c) and after some algebra, we form

$$f_0(\mathbf{e}_{i(q)}) = -\mathbf{e}_{i(q)}^H \mathbf{B}_{j(q)} \mathbf{e}_{i(q)} - 2\Re\left\{\mathbf{e}_{i(q)}^H \mathbf{B}_{j(q)} \tilde{\mathbf{h}}_{i(q)}(q)\right\} - \tilde{\mathbf{h}}_{i(q)}^H(q)\mathbf{B}_{j(q)}\tilde{\mathbf{h}}_{i(q)}(q) + \omega_{i(q)} \geq 0,$$

$$f_1(\mathbf{e}_{i(q)}) = \mathbf{e}_{i(q)}^H \mathbf{I}\mathbf{e}_{i(q)} - \epsilon^2 \leq 0. \tag{5.15}$$

Notice that, since $\epsilon^2 \geq 0$, there exists a point $\hat{\mathbf{e}}_{i(q)}$ such that $f_1(\hat{\mathbf{e}}_{i(q)}) \leq 0$. Hence, according to Lemma 5.1, there exists $\alpha_{i(q)} \geq 0$, such that the equivalent LMI for the set of constraints in (5.10) and (5.6c) can be formed as

$$\begin{bmatrix} -\mathbf{B}_{j(q)} + \alpha_{i(q)}\mathbf{I} & -\mathbf{B}_{j(q)}\tilde{\mathbf{h}}_{i(q)}(q) \\ \left(-\mathbf{B}_{j(q)}\tilde{\mathbf{h}}_{i(q)}(q)\right)^H & t_{i(q)} \end{bmatrix} \succeq \mathbf{0}, \tag{5.16}$$

where $t_{i(q)} = \omega_{i(q)} - \tilde{\mathbf{h}}_{i(q)}^{H}(q)\mathbf{B}_{j(q)}\tilde{\mathbf{h}}_{i(q)}(q) - \alpha_{i(q)}\epsilon^2$. Following the same approach, we can write the equivalent LMI for the constraints (5.11) and (5.6c) as

$$\begin{bmatrix} -\mathbf{W}_{i(q)} + \mu_{i(q)}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \kappa_{i(q)} - \mu_{i(q)}\epsilon^2 \end{bmatrix} \succeq \mathbf{0}, \tag{5.17}$$

where $\mu_{i(q)} \geq 0$ is the auxiliary variable of S-Procedure. Furthermore, the constraint (5.6d) can be rewritten as

$$\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left(\tilde{\mathbf{h}}_{t(k)}^{H}(q) + \mathbf{e}_{t(k)}^{H} \right) \mathbf{A}_{i(q)} \left(\tilde{\mathbf{h}}_{t(k)}(q) + \mathbf{e}_{t(k)} \right) \leq \nu, \tag{5.18}$$

where $\mathbf{A}_{i(q)} = \sum_{i \in \mathcal{S}_l} \mathbf{W}_{i(q)}$. Defining

$$\begin{aligned} g_0(\mathbf{e}_{t(k)}) &= -\mathbf{e}_{t(k)}^{H}\mathbf{A}_{i(q)}\mathbf{e}_{t(k)} - 2\Re\left\{ \mathbf{e}_{t(k)}^{H}\mathbf{A}_{i(q)}\tilde{\mathbf{h}}_{t(k)}(q) \right\} - \tilde{\mathbf{h}}_{t(k)}^{H}(q)\mathbf{A}_{i(q)}\tilde{\mathbf{h}}_{t(k)}(q) + \nu \geq 0, \\ g_1(\mathbf{e}_{t(k)}) &= \mathbf{e}_{t(k)}^{H}\mathbf{I}\mathbf{e}_{t(k)} - \epsilon^2 \leq 0, \end{aligned} \tag{5.19}$$

and applying Lemma 5.1, we can form the equivalent LMI for the set of constraints in (5.6d) and (5.6e) as

$$\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \begin{bmatrix} -\mathbf{A}_{i(q)} + \lambda_{t(k)}\mathbf{I} & -\mathbf{A}_{i(q)}\tilde{\mathbf{h}}_{t(k)}(q) \\ \left(-\mathbf{A}_{i(q)}\tilde{\mathbf{h}}_{t(k)}(q) \right)^{H} & d_{t(k)} \end{bmatrix} \succeq \mathbf{0}, \tag{5.20}$$

where $d_{t(k)} = -\left(\tilde{\mathbf{h}}_{t(k)}^{H}(q)\mathbf{A}_{i(q)}\tilde{\mathbf{h}}_{t(k)}(q) \right) - \lambda_{t(k)}\epsilon^2 + \nu$ and $\lambda_{t(k)}$ are the auxiliary variables of S-Procedure. Finally, the original optimization problem in (5.5) is reformulated with equivalent LMI constraints as

$$\begin{aligned} \min_{\mathbf{W}_{i(q)},\nu} \quad & \sum_{i \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_{i(q)} \right) + \nu \\ \text{subject to} \quad & (5.12),(5.16),(5.17) \text{ and } (5.20), \\ & \mathbf{W}_{i(q)} = \mathbf{W}_{i(q)}^{H} \succeq 0, \quad \mathrm{rank}\left(\mathbf{W}_{i(q)} \right) = 1, \\ & \forall i \in \mathcal{S}_l, \forall t \in \mathcal{S}_l, \forall k \in \mathcal{S}_b. \end{aligned} \tag{5.21}$$

Algorithm 5.1 Gaussian Randomization Procedure for BS q

1: Given: A number of randomization iterations R, the optimal solutions $\mathbf{W}^*_{i(q)}, \forall i \in$ $\mathcal{S}_l$ to problem (5.21);

2: **for** $l = 1 : R$ **do**

3:    Generate random vectors $\mathbf{w}^{(l)}_{i(q)} \sim \mathcal{CN}\left(0, \mathbf{W}^*_{i(q)}\right), \forall i$;

4:    Set beam directions $\mathbf{u}^{(l)}_{i(q)} = \mathbf{w}^{(l)}_{i(q)}/\|\mathbf{w}^{(l)}_{i(q)}\|, \forall i$;

5:    Using $\mathbf{W}_{i(q)} = u_{i(q)}\mathbf{u}^{(l)}_{i(q)}\left(\mathbf{u}^{(l)}_{i(q)}\right)^H, \forall i$, in (5.21), where $\sqrt{u_{i(q)}} = \|\mathbf{w}^{(l)}_{i(q)}\|$, solve

$$\min_{u_{i(q)} \geq 0, \forall i} \sum_{i \in \mathcal{S}_l} u_{i(q)} + \nu \tag{5.22}$$

$$\text{subject to:} \quad \text{constraints in (5.21).}$$

6:    Let $\left\{u^{(l)}_{i(q)}\right\}_{i \in \mathcal{S}_l}$ and $F^{(l)}$ be the set of feasible solutions to and the optimum objective value of (5.22), respectively;

7: **end for**

8: Let $\mathbb{L}$ be the set of indexes l for which (5.22) is feasible;

9: $l^* = \arg\left\{\min_{l \in \mathbb{L}} F^{(l)}\right\}$;

10: **return** $\mathbf{w}^*_{i(q)} = \sqrt{u^{(l^*)}_{i(q)}}\mathbf{u}^{(l^*)}_{i(q)}, \forall i$.

The problem in (5.21) is convex if the rank one constraints are relaxed. Hence, ignoring the rank one constraint, one can solve the optimization problem in (5.21), e.g., using the SeDuMi solver [111], and find the optimum solutions, $\mathbf{W}^*_{i(q)}$. If the rank of an optimum solution is one, then the corresponding direction of the optimum beamforming vector can be found as the eigenvector associated with the nonzero eigenvalue of the optimum solution. Then, the square root of the nonzero eigenvalue corresponds to the Euclidean norm of the optimum beamforming vector. Otherwise the randomization technique, i.e., the Gaussian randomization procedure [144], detailed in pseudo-code in Algorithm 1 for the problem in (5.21), is used to find the rank one approximations to the optimal solutions $\mathbf{W}^*_{i(q)}$. Notice that using an adequate number of randomization iterations R in Algorithm 1, the gap between the approximation and the exact optimal solutions can be sufficiently reduced.

5.4   Simulation results

5.4.1   simulation set up

The numerical results of the computer simulation will be presented in this section to asses the performance of the proposed beamforming design against the conventional robust and the non-robust approaches. Please see appendix B for the non-robust approach formulation stages.

We consider a 3-cell cellular network, where two simultaneously active users per cell are randomly scheduled within their 3 adjacent sectors to reflect a severe impact of ICI on the network. Fig. 3.2, chapter 3 illustrates an example of one user distribution with 6 users. To obtain the channel model between any BS q and any user i in cell p, we use a similar model as in [146, 147], as

$$\mathbf{h}_{i(p)}^{T}(q) = \mathbf{h}_{w}^{T}\mathbf{R}_{i(p)}^{1/2}(q), \tag{5.23}$$

where $\mathbf{h}_{w} \in \mathbb{C}^{M\times 1}$ is a ZMCSCG random vector with unit variance entries, $\mathbf{R}_{i(p)}(q) \in \mathbb{C}^{M\times M}$ is the spatial covariance matrix of user $i(p)$ as seen by BS q. The (m,n)th element of the spatial covariance matrix $\mathbf{R}_{i(p)}(q)$ in (5.23) is given by equation (3.27), in Section 3.4.1. Monte-Carlo simulations are carried out with 6 antenna elements per sector BSs and over 30 independent random user distributions and 1000 channel realizations per distribution. The simulation parameters used in the following chapter are the same as the previous chapters. Table 3.1 summarizes these parameters.

5.4.2   Performance evaluation

To illustrate the advantage of the proposed robust design, histograms of the distribution of SINR satisfaction are shown in Fig. 5.1. We define $\zeta_{i(q)}$ as the normalized SINR, i.e., the ratio of the achievable SINR to the target one, given by

$$\zeta_{i(q)} = \frac{\left|\tilde{\mathbf{h}}_{i(q)}^{H}(q)\mathbf{w}_{i(q)}\right|^{2}}{\gamma_{i(q)}\left(\displaystyle\sum_{\substack{j\in\mathcal{S}_{l},\\ j\neq i}}\left|\left(\tilde{\mathbf{h}}_{i(q)}^{H}(q)+\mathbf{e}_{i(q)}^{H}\right)\mathbf{w}_{j(q)}\right|^{2}+\left|\mathbf{e}_{i(q)}^{H}\mathbf{w}_{i(q)}\right|^{2}+\xi_{i(q)}+\sigma_{n}^{2}\right)}, \forall i \in \mathcal{S}_{l}. \tag{5.24}$$

A comparison of results in Figs. 5.1 (a) and (b) confirms the effectiveness of the proposed robust design against channel uncertainties. As seen in Fig. 5.1 (a), the non-robust design fails to meet the SINR targets in more than 50% of occasions in

the presence of channel imperfection, whereas, Fig. 5.1 (b) confirms that the proposed robust design scheme satisfies the SINR constraints above the set targets in all of the trials, despite the presence of the same channel uncertainties.
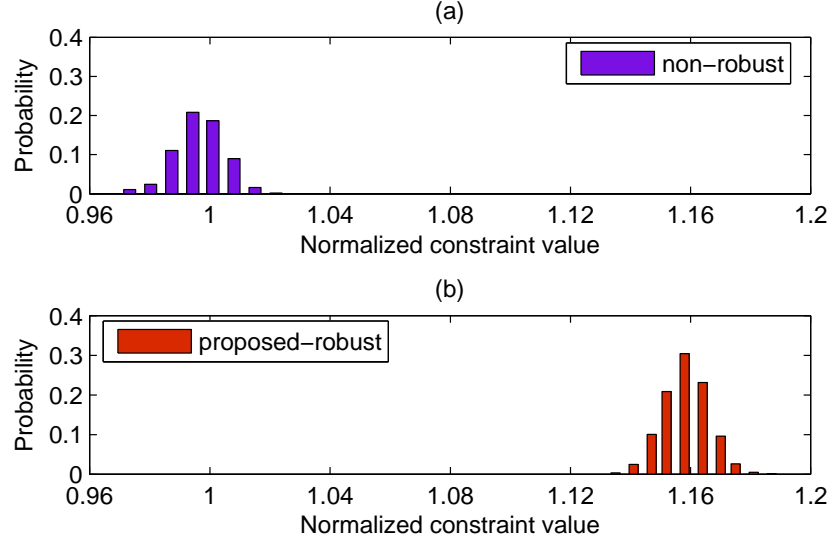

Figure 5.1: Histogram of normalized QoS constraints for $\epsilon^2 = 0.01$ and target SINR values of 10 dB.

Fig. 5.2 shows the total transmit power versus various target SINR levels for a robust conventional, proposed robust and non-robust designs (see appendix B for the optimization problem with perfect CSI, i.e., non-robust) in the presence of different channel uncertainties. These results confirm that robustness comes at the price of increased transmit power. Furthermore, as the uncertainty level increases, more transmit power is required to meet the same SINR targets. Results in Fig. 5.2 also confirm that the dynamic range of SINR targets supported by the design at affordable transmit power levels diminishes fast as the severity of channel imperfections grows. The non-robust appears to be more power efficient but, as can be observed from Fig. 5.1, the additional required transmit power of the proposed method compared to the non-robust method is the cost of the guaranteed QoS performance, in terms of SINR, against the channel uncertainty. In order to further illustrate the benefits of the proposed robust approach, we compare the performance of our scheme against the robust conventional scheme, where there is no control on the induced inter-cell interference, i.e., in the absence of the ICI controlling term of $\sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \left| \left(\tilde{\mathbf{h}}_{t(k)}^H(q) + \mathbf{e}_{t(k)}^H \right) \mathbf{w}_{i(q)} \right|^2$ of the proposed objective function in (5.5), on the total transmit power of BSs. As indicated in Fig. 5.2 a comparison of results between the robust conventional at a fixed CSI uncertainty level of $\epsilon^2 = 0.01$ and the proposed robust design shows that the robust

conventional scheme fails to operate efficiently in terms of power consumption beyond 4 dB of SINR target. This is due to the fact that in the robust conventional design, there is no control on inflicted ICI and as each BS strives to satisfy a certain SINR for its own local users, it, inevitably, degrades the neighboring ones, who reciprocally respond in the same manners. Whereas the second utility of the objective function of the proposed optimization problem in (5.5) controls the inflicted interference by the BSs and stabilizes the egoistic dynamic of the robust conventional network in an equilibrium point, agreed by all BSs. Hence the proposed scheme and its ICI-balancing term offers a significant reduction of the total transmit power at BSs and achieves higher SINR targets with affordable sum-power levels at BSs.
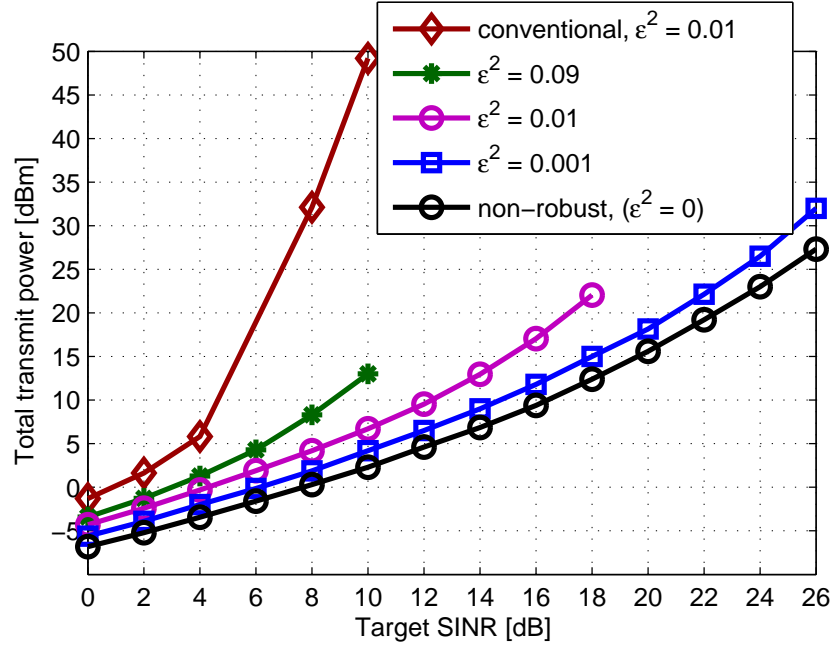
Figure 5.2: Minimum transmit power required versus the target SINR

As shown in Fig. 5.2 there exists a limit on the maximum achievable SINR and the simulation under given scenarios may not yield feasible solutions during each simulation. The optimization problem becomes infeasible beyond this limit. The feasibility rate versus the target SINR, i.e., the percentage of channel realizations for which the different schemes under consideration yield feasible solutions is shown in Fig. 5.3. We observe that the percentage of feasibility runs for the proposed robust approach decreases exponentially as the CSI uncertainty together with the target SINR increases. Moreover the non-robust approach has a 100% feasibility rate as shown in Fig. 5.3. But when the method is tested with channel estimation errors it fails to satisfy the needed

Figure 5.3: Feasibility rate versus SINR target

SINR in more than 50% of the beamforming cases, i.e., see the histograms in Fig. 5.1. Furthermore, as confirmed by the extracted results in Fig. 5.3, the proposed robust approach shows an increased feasibility rate as compared to the robust conventional approach.

The low feasibility in robust designs can be explained in the following way: a perfect CSI would correspond to a single point in space, where a particular user is located. When an error is present the b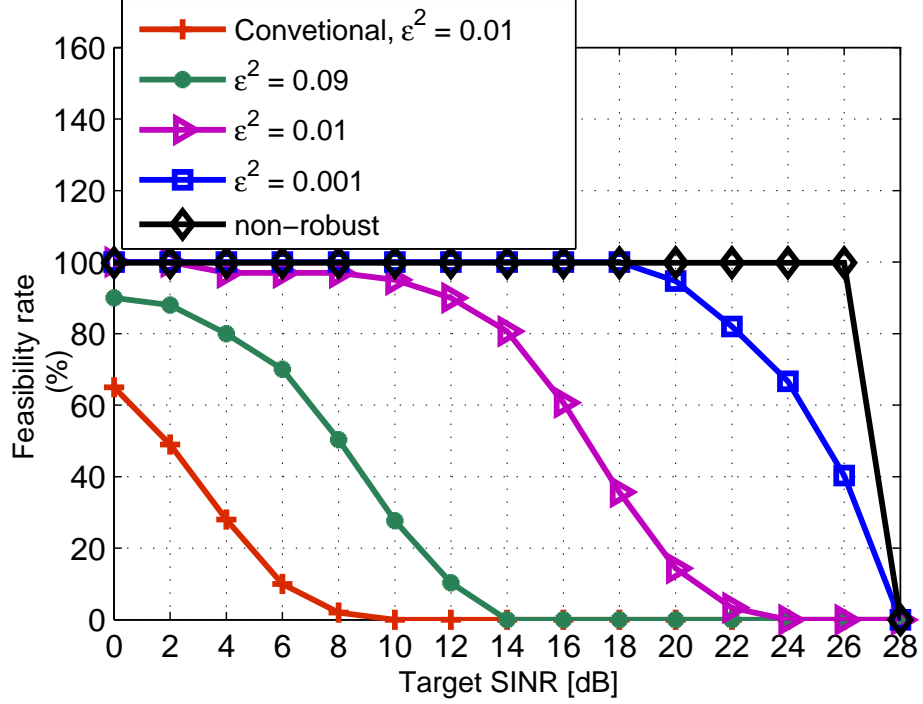eamforming has to be done for a wider region of space, i.e. the bigger the error radius is, the wider the beams become and eventually they start to overlap over the different users causing interference to them. This in turn results in higher power and if this higher power can not solve the case, it iteratively goes up to infinity, which results in infeasibility. A possible solution for this is the investigation of the problem with larger antenna array systems with more than 8 elements, which can form narrower beams and steer them in space with greater precision. They can also form more complex patterns that can better avoid this overlapping.

In a real situation whenever the system finds that the instant solution is going to infinity in terms of transmit power (is infeasible in terms of convex optimization), scheduling should be used, especially when trying to serve two users per cell at a time in the same frequency. Thus it is not surprising why one of the currently proposed methods for CoMP is Coordinated Scheduling/Coordinated Beamforming, i.e., a com-

bination of these two. The topic of scheduling is beyond the scope of this thesis but could be considered as an extension to this work for the future research.

5.5   conclusion

We investigate robust transmit beamforming design under the assumption of that both CSIT and CSIR are imperfect. The proposed robust design minimizes a combination of the total transmit power required by each BS, to deliver to its locally active users a certain set of desired SINR levels, and the resulting inflicted inter-cell interference on the users of the other cells. As the originally proposed problem is naturally non-convex and intractable due to the presence of robust constraints, we employ the S-procedure and the semidefinite relaxation method [119] to derive an equivalent formulation which is numerically tractable and convex. It is shown that when accounting for the CSI errors, power-efficient feasible solutions can be achieved for certain sets of SINR targets. Simulation results reveal that the proposed robust design with imperfect CSI offers superior results compared to a robust conventional method in terms of minimized transmit power. We also observed that the achievable SINR targets at user terminals shrinks with an increase in error bound, i.e., the radius of the hyper-spherical uncertainty set. Furthermore, increasing the error bound leads to increased transmit power at base stations.

Chapter 6

Conclusions and future work

This thesis focuses on the optimization of the system level performance of multiple antenna cellular wireless communication systems that employ beamforming to provide a reliable and robust transmission against channel imperfections. Beamforming is one of the popular techniques to exploit the benefits of multiple antenna systems with the requirement of perfect CSI. However, only imperfect CSI is available in real scenarios, which leads to significant performance degradation, and consequently posing challenges in system analysis and signal design. It motivates to exploit a robust transmit beamforming against errors in CSI. Our proposed techniques provide robustness against various types of channel uncertainty. Commonly, there are two methods of deterministic (or worst-case) and stochastic modeling of imperfect CSI. The system performance of the proposed optimization problem is evaluated under deterministic as well as stochastic channel estimation error. The objective of the proposed design is to minimize the transmit power by individual BSs and the resulting inter-cell interference jointly and in a robust manner, while still guarantee a QoS requirement for all active user terminals. For the different scenarios, the optimal solution of the problem was investigated and low complexity efficient algorithms were proposed. Simulation results provide a substantially improved robustness against imperfect knowledge of the wireless channel with respect to their classical non-robust counterparts, by means of maintaining the required QoS for all active user terminals.

This chapter summarizes the findings of previous chapters and outlines possible future research directions.

6.1   Thesis summary

The introductory chapter outlined the motivation and the contributions of this thesis.

6.1.1   Summary of Chapter 2

In this chapter, we investigated the literature survey of topics required for understanding the novel contributions of this thesis. In addition, some mathematical preliminary topics such as the concepts of Convex Optimization and semidefinite programming were reviewed. Furthermore, an optimization problem to calculate transmit beamformers for multiple active users in a single-cell scenario and some different approaches to solve the optimization problem were outlined.

6.1.2   Summary of Chapter 3

In this chapter, we proposed a worst-case based robust approach for solving the problem of multi-cell downlink beamforming using second order covariance-based CSI. A spherical uncertainty set to model imperfections in the second-order statistical channel knowledge between the BSs and the users was adopted. A robust optimization problem was formulated to minimize a linear combination of total transmitted power at each BS and the induced aggregate interference power on the users of the other cells. The aim was to maintain local users SINR demands in a power efficient way, under the worst-case of channel uncertainties. Furthermore, by introducing the second utility function in the objective function of the proposed optimization problem, we controlled the inter-cell interference and brought into balance and stabilized the multi-cell network in order to provide the desired levels of quality of services for the users who were located within the adjacent cells and communicated with their corresponding BSs over a shared bandwidth.

In our problem formulations and their derived solutions, we avoided the coarse approximations used in the previous methods. The exact reformulations of worst-case QoS and inter-cell interference constraints using Lagrange duality were derived. The resulting problem was then converted into a SDP problem that is convex under standard rank relaxation. The simulation results revealed that an increase in the uncertainty region of the CSI led to increased power consumption at the BSs, and also adversely affected the range of quality of service in terms of limited SINR targets achievable at affordable levels of power consumption at the BSs. The effectiveness of the proposed approach in terms of transmitted power in comparison to the conventional technique was also verified through simulations. Moreover, it was shown that the proposed decentralized scheme in which each BS locally designed its own beamforming vectors without relying on data or downlink CSI of links from other BSs to the users, closely followed the coordinated beamforming schemes in terms of the total transmitted

power for lower SINR targets.

6.1.3 Summary of Chapter 4

In this chapter, in order to provide less conservative solutions in favor of improved resource-efficient design, the aforementioned multi-cell downlink beamforming optimization problem proposed in chapter 3 was optimally solved in a closed form using the probabilistic constraints. We considered an analytical approach to solve the problem by directly characterizing the statistical behavior of the random matrix to model the imperfections in second order statistical CSI at the transmitter. Additionally, a relationship between the Frobenius norm of the random matrix, modeling the radius of a hyper-spherical uncertainty region in the worst-case approach, and the outage parameter controlling the probability of the satisfaction of QoS requirement at users in the chance-constraint approach were developed. This relation revealed and quantified the implicit outage in the worst-case approach, i.e., due to the fact that the uncertainties in practical scenarios are statistically unbounded, and helped to compare it with the chance-constraint approach fairly. The performance of the proposed chance-constraint robust method was compared with the robust method based on worst-case performance optimization as well as non-robust and conventional chance-constraint methods in terms of the total transmit power of the BSs. Simulation results confirmed that not only the proposed robust chance-constraint method outperforms the conventional chance-constraint scheme, but it also showed a significantly superior power saving performance at higher SINR targets, when compared with its conservative worst-case design counterpart.

6.1.4 Summary of Chapter 5

In this chapter a bounded deterministic model for the error in instantaneous CSI was assumed, and a new SINR criterion considering imperfect channel knowledge at both transceivers sides was adopted. The proposed robust design minimized a combination of the power allocation among the signals to be transmitted to the users, and the resulting inflicted inter-cell interference on the users of the other cells, while a certain predefined quality of service per user was guaranteed. The formulated problem was within the framework of convex optimization that employed S-procedure and the semidefinite relaxation methods, which enabled us to efficiently find the solution to the robust optimization problem. The simulation results showed that when accounting for the CSI

errors, power-efficient feasible solutions can be achieved for certain sets of SINR targets. We proved that the proposed design improves the performance achieved by the non-robust design in terms of satisfying the users' QoS. In particular, it was shown that the proposed robust technique demands less power than the robust conventional methods while still guaranteeing QoS. Furthermore, the results confirmed that increasing the error bounds shrinks the range of achievable SINR targets at user terminals.

6.2   Future research directions

There is an endless road of possible improvements and generalizations to the results of this thesis. Some extensions have intentionally been left out to make the thesis coherent, while other limitations were necessary to achieve analytical tractability or to avoid making assumptions that would affect the generality. However, several ideas for future work have been conceived in the process of writing this thesis:

6.2.1   Rate maximization under power constraint

The focus of this thesis is on robustness and energy efficiency. The objective function that was defined in this thesis, was aimed to minimize a linear combination of two utility functions, characterizing each BS's weighted sum of transmitted power to the intra-cell users and its resulting weighted sum of interference power inflicted upon the users of the other cells. The constraints of all optimization problems introduced in this thesis is on users' signal-to-interference-plus-noise ratios (SINRs). In other words, beamforming schemes proposed in this thesis ensure all users' quality of services above requirement levels with minimum total transmit power. A possible extension for the work in this thesis is to maximize the effective sum rate under transmit-power and backhaul-power constraints.

6.2.2   Multi-antenna users

An assumption used to develop beamforming schemes in this thesis is that user termi-nals are equipped with single antenna. When user terminals and base stations both have multiple antennas, there are more degree of freedom to effectively control in-terference. However, transmit and receive beamforming should be jointly designed. A question arising here is whether global optimality can be achieved by iteratively optimizing transmit and receive beamforming. Complexity and signalling overhead

are expected to significantly increase. Therefore, practical solutions to the optimal beamforming and trade-off between optimality and complexity are open problems for research.

6.2.3 Rank-one solution

In this thesis, we have relaxed the rank one constraints and solved the proposed optimization problems using the SDP method. Interestingly, our simulations by CVX always generate exact rank one solutions, such that we have never needed to use an additional randomization process to approximate the rank one solutions with an additional computational complexity. An interesting direction for future research is to attempt to prove analytically that the proposed optimization problem always generates rank one solutions by SDP approach.

6.2.4 Beamforming scheduling

The robust downlink beamforming problems presented in this thesis can be infeasible depending upon the number of transmit antennas, the number of users, channel conditions, value of noise power and the required thresholds for the QoS constraints. In this case, however, some kind of admission control can be introduced that selects a number of users from the complete set of users and tries to solve the robust beamforming problem only for the selected users. Note that the admission control techniques for the non-robust conventional downlink beamforming have already been presented in [148–151] where the instantaneous CSI is used.

Appendix A

6.3 Chi-square distribution with σ^2 for a complex standard random variable

If $Z_1, ..., Z_k$ are independent standard normal random variables, then the sum of their squares,

$$Q = \sum_{i=1}^{k} Z_i^2, \quad Z_i \sim \mathcal{N}(0, 1), \tag{6.1}$$

is distributed according to the Chi-squared distribution with k degrees of freedom. This is usually denoted as

$$Q \sim \mathcal{X}^2(k). \tag{6.2}$$

Now consider $X_i \sim \mathcal{N}(0, \sigma^2)$ then

$$\sigma^2 Q = \sum_{i=1}^{k} X_i^2. \tag{6.3}$$

Therefore the cumulative distribution function (CDF) can be written as

$$Pr\left(\sigma^2 Q \leq d\right) = Pr\left(Q \leq \frac{d}{\sigma^2}\right) \tag{6.4}$$

$$= CDF_{\mathcal{X}^2(k)}\left(\frac{d}{\sigma^2}\right) \cong \Psi\left(\frac{d}{\sigma^2}\right), \tag{6.5}$$

$$\tag{6.6}$$

where $\Psi\left(\frac{d}{\sigma^2}\right)$ is the CDF of standard Chi-square calculated at $\frac{d}{\sigma^2}$.

Now consider the Chi-square when $Z_i \sim \mathcal{CN}(0,1)$, i.e., complex standard random variable (ZMCSCG)

$$Z_i = Z_i^R + jZ_i^I, \quad \text{where} \quad Z_i^R, Z_i^I \sim \mathcal{N}(0, \frac{1}{2}) \tag{6.7}$$

let $|Z_i| = Z_i^{R^2} + Z_i^{I^2}$ then $\acute{Q} = \sum\limits_{i=1}^{2k} y_i^2$, where $y_i^2 \sim \mathcal{N}(0, \frac{1}{2})$.

y_i replaces both Z_i^R, Z_i^I random variables; hence there are 2k random variables in total and $\acute{Q} \sim \mathcal{X}^2(2k)$.

In terms of standard Chi-square :

$$y_i = \sqrt{\frac{1}{2}}X_i, \quad X_i \sim \mathcal{N}(0,1)$$

$$\acute{Q}_S = 2\acute{Q} = \sum\limits_{i=1}^{2k} X_i^2 \quad \Rightarrow \quad \acute{Q} = \frac{1}{2}\acute{Q}_S$$

$$Pr(\frac{1}{2}\acute{Q}_S \leq d^2) = Pr(\acute{Q}_S \leq 2d^2). \tag{6.8}$$

Therefore

$$CDF_{\mathcal{X}^2(2k)}\left(2d^2\right) = \Psi\left(2d^2\right). \tag{6.9}$$

Similarly when $Z_i \sim \mathcal{CN}\left(0, \sigma^2\right)$, then

$$CDF_{\mathcal{X}^2(2k)}\left(\frac{2d^2}{\sigma^2}\right) = Pr(\sigma^2\acute{Q}_S \leq 2d^2). \tag{6.10}$$

Let $\mathbf{\Delta}$ be a $n \times n$ random matrix with ZMCSCG entries defined as $[\mathbf{\Delta}]_{ij} \sim \mathcal{CN}(0, \sigma^2)$. Denote $[\mathbf{\Delta}]_{ij} = d_{ij}$, we can write

$$\|\mathbf{\Delta}\|_F^2 = \sum\limits_{i=1}^{n}\sum\limits_{i=1}^{n} |[\mathbf{\Delta}]_{ij}| = \sum\limits_{i=1}^{n}\sum\limits_{i=1}^{n} |d_{ij}|^2, \tag{6.11}$$

where $|d_{ij}|^2 = \Re\{d_{ij}\}^2 + \Im\{d_{ij}\}^2$ and $\Re\{d_{ij}\}^2, \Im\{d_{ij}\}^2 \sim \mathcal{N}\left(0, \frac{\sigma^2}{2}\right)$. Hence the real and imaginary parts can be expressed in terms of standard normal random variables

$U \sim N(0,1)$ as $\Re\{d_{ij}\} = \frac{\sigma^2}{\sqrt{2}} U$ and $\Im\{d_{ij}\} = \frac{\sigma^2}{\sqrt{2}} U$ respectively. Therefore,

$$
\begin{aligned}
\|\mathbf{\Delta}\|_F^2 &= \frac{\sigma^2}{2} \sum_{i=1}^{2n^2} U_i^2 \\
&= \frac{\sigma^2}{2} Q, \quad U_i \sim (0,1)
\end{aligned}
\tag{6.12}
$$

where $Q \sim \mathcal{X}^2(2n^2)$, i.e., standard Chi-square random variable with $2n^2$ degrees of freedom. Hence,

$$
\begin{aligned}
\Pr\left(\|\mathbf{\Delta}\|_F^2 \le \delta^2\right) &= \Pr\left(\frac{\sigma^2}{2} Q \le \delta^2\right) = \tag{6.13} \\
\Pr\left(Q \le \frac{2\delta^2}{\sigma^2}\right) &= \Psi_{\mathcal{X}^2(2n^2)}\left(\frac{2\delta^2}{\sigma^2}\right). \tag{6.14}
\end{aligned}
$$

By setting $\Psi_{\mathcal{X}^2(2n^2)}\left(\frac{2\delta^2}{\sigma^2}\right) = 1 - \rho$ we can find a relation between the outage ρ and δ as

$$
\begin{aligned}
\frac{2\delta^2}{\sigma^2} &= \Psi_{\mathcal{X}^2(2n^2)}^{-1}(1-\rho) \tag{6.15} \\
\Rightarrow \delta &= \sqrt{\frac{\sigma^2 \Psi_{\mathcal{X}^2(2n^2)}^{-1}(1-\rho)}{2}}, \tag{6.16}
\end{aligned}
$$

where $\Psi_{\mathcal{X}^2(2n^2)}^{-1}(\cdot)$ is the inverse CDF of a standard chi-square random variable with $2n^2$ degrees of freedom.

Appendix B

6.4 Beamforming strategy with perfect CSI

We assume the channel vectors $\mathbf{h}_{i(q)}(q)$ and $\mathbf{h}_{t(k)}(q)$ are perfectly known at both transceiver sides for the proposed optimization problem in chapter 5, and introduce the following optimization problem for the non-robust design of beamforming vectors as

$$
\min_{\mathbf{W}_{i(q)}} \quad \sum_{i \in \mathcal{S}_l} \mathbf{w}_{i(q)}^{H} \mathbf{w}_{i(q)} + \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \sum_{i \in \mathcal{S}_l} \left| \mathbf{h}_{t(k)}^{H}(q) \mathbf{w}_{i(q)} \right|^2
$$

$$
\text{s.t.} \quad \frac{\left| \mathbf{h}_{i(q)}^{H}(q) \mathbf{w}_{i(q)} \right|^2}{\sum_{j \in \mathcal{S}_l, j \neq i} \left| \mathbf{h}_{i(q)}^{H}(q) \mathbf{w}_{j(q)} \right|^2 + \xi_{i(q)} + \sigma^2} \geq \gamma_{i(q)}, \quad \forall i \in \mathcal{S}_l. \tag{6.17}
$$

The constraints involve quadratic nonconvex functions of variables. However, it can be modified into the SDP standard formulation. This can be done by changing the vector variables $\mathbf{w}_{i(q)}$ into matrix variables $\mathbf{W}_{i(q)}$. Let us define $\mathbf{W}_{i(q)} = \mathbf{w}_{i(q)} \mathbf{w}_{i(q)}^{H}$, therefore, using the following conditions

$$
\mathbf{w}_{i(q)}^{H} \mathbf{w}_{i(q)} = \text{Tr}\left(\mathbf{w}_{i(q)} \mathbf{w}_{i(q)}^{H} \right) = \text{Tr}\left(\mathbf{W}_{i(q)} \right), \tag{6.18}
$$

$$
\sum_{i \in \mathcal{S}_l} \mathbf{W}_{i(q)} = \overline{\mathbf{W}}, \tag{6.19}
$$

problem (6.17) can be represented as

$$\min_{\mathbf{W}_{i(q)}} \quad \sum_{i \in \mathcal{S}_l} \mathrm{Tr}\left(\mathbf{W}_{i(q)}\right) + \sum_{k \in \mathcal{S}_b, k \neq q} \sum_{t \in \mathcal{S}_l} \left(\overline{\mathbf{W}}_{i(q)} \mathbf{h}_{t(k)}(q) \mathbf{h}_{t(k)}^H(q)\right)$$

$$\text{s.t.} \quad \mathrm{Tr}\left(\mathbf{V}_{i(q)} \mathbf{h}_{i(q)}(q) \mathbf{h}_{i(q)}^H(q)\right) \geq \xi_{i(q)} + \sigma^2,$$

$$\mathbf{W}_{i(q)} \succeq 0,$$

$$\mathrm{rank}(\mathbf{W}_{i(q)}) = 1, \forall i \in \mathcal{S}_l$$

where

$$\mathbf{V}_{i(q)} = \frac{\mathbf{W}_{i(q)}}{\gamma_{i(q)}} - \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{W}_{j(q)}. \tag{6.20}$$

Problem (6.20) can be solved by the SeDuMi solver [111], to find $\mathbf{W}_{i(q)}$. However, to obtain the optimal beamforming vectors $\mathbf{w}_{i(q)}, \forall i \in \mathcal{S}_l$, we are only interested in $\mathbf{W}_{i(q)}$ solutions of (6.20) that are of Rank 1.[111]

References

[1] "3GPP long term evolution (LTE) ICIC and eICIC." [Online]. Available: http://4g-lteworld.blogspot.kr/2012/06/icic-and-eicic.html

[2] G. Americas, "MIMO and smart antennas for 3G and 4G wireless systems - practical aspects and deployment considerations," May 2010. [Online]. Available: http://www.3gamericas.org

[3] Mobiltel, "LTE downlink received levels and throughput predictions," 2012. [Online]. Available: www.mobiltel.bg

[4] F. Dietrich, *Robust Signal Processing for Wireless Communications.* Springer, 2008.

[5] T. A. Le, S. Nasseri, A. Zarrebin-Esfahani, A. Mills, and M. R. Nakhai, "Power-efficient downlink transmission in multicell networks with limited wireless backhaul," *IEEE Wireless Communications Magazine, Special Issue on Technologies for Green Radio Communication Networks*, vol. 18, no. 5, pp. 82–88, Oct. 2011.

[6] Huawei Technologies Co., Ltd., "Improving energy efficiency, lower CO_2 emission and TCO," 2011. [Online]. Available: http://www.huawei.com/en/static/hw-076768.pdf

[7] C. Han, T. Harrold, S. Armour, I. Krikidis, S. Videv, P. M. Grant, H. Haas, J. S. Thompson, I. Ku, C.-X. Wang, T. A. Le, M. R. Nakhai, J. Zhang, and L. Hanzo, "Green radio: Radio techniques to enable energy-efficient wireless networks," *IEEE Wireless Communications Magazine*, vol. 49, no. 6, pp. 46–54, Jun. 2011.

[8] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313– 1324, Oct. 1998.

[9] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[10] M. R. Nakhai and A. K. Yousafzai, "Interference alignment with cyclic unidirectional cooperation in multicell networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2012)*, Jun. 2012, pp. 2250 – 2254.

[11] K. Karakayali, G. J. Foschini, and R. A. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Wireless Communications*, vol. 13, no. 4, pp. 56–61, Aug. 2006.

[12] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.

[13] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 4, pp. 1451–1461, Apr. 2010.

[14] A. Tolli, H. Pennanen, and P. Komulainen, "Decentralized minimum power multi-cell beamforming with limited backhaul signaling," *IEEE Trans. Wireless. Commun*, vol. 2, no. 5, pp. 570–580, Feb. 2011.

[15] Y. Huang, G. Zheng, M. Bengtsson, K.-K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming with limited intercell coordination," *IEEE Trans. signal processing*, vol. 59, no. 2, pp. 728–738, Feb. 2011.

[16] R. Mudumbai, D. R. Brown, U. Madhow, and H. V. Poor, "Distributed transmit beamforming: Challenges and recent progress," *IEEE Commun. Magazine*, vol. 47, no. 2, pp. 102– 110, Feb. 2009.

[17] M. Bengtsson and B. Ottersten, *Optimal and Suboptimal Transmit Beamforming*. in: Handbook of Antennas in Wireless Communications/ [ed] Lal C. Godara, CRC Press, 2001.

[18] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for multiuser cognitive radios using worst-case performance optimization," *in Proc. Asilomar Conference on Signals, Systems and Computers, (Asilomar)*, pp. 378–382, Oct. 2008.

[19] G. Zheng, K.-K. Wong, and T.-S. Ng, "Robust linear MIMO in the downlink: A worst-case optimization with ellipsoidal uncertainty regions," *EURASIP J. Adv. Signal Process*, vol. 2008, no. 609028, pp. 1–15, 2008.

[20] M. B. Shenouda and T. N. Davidson, "Convex conic formulations of robust down-link precoder designs with quality of service constraints," *IEEE Journal Selected Topics in Signal Processing*, vol. 1, no. 4, pp. 714–724, 2007.

[21] Y. W. Huang, D. P. Palomar, and S. Z. Zhang, "Lorentz-positive maps and quadratic matrix inequalities with applications to robust MISO transmit beam-forming," *IEEE Transaction on Signal Processing*, vol. 61, no. 5, pp. 1121 – 1130, Mar. 2013.

[22] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Processing*, vol. 51, no. 2, pp. 313–324, Feb. 2003.

[23] K. Y. Wang, A. M. C. So, T. H. Chang, W. K. Ma, and C. Y. Chi, "Outage constrained robust transmit optimization for multiuser MISO down-links: Tractable approximations by conic optimization," available online at http://arxiv.org/abs/1108.0982.

[24] B. k Chalise, shahram Shahbazpanahi, A. Czylwik, and A. B. Gersham, "Robust downlink beamforming based on outage probability specifications," *IEEE Trans. Wirelss Commun*, vol. 6, no. 10, pp. 3498–3503, Oct. 2007.

[25] M. B. Shenouda and T. N. Davidson, "Probabilistically-constrained approaches to the design of the multiple antenna downlink," *in Proc. 42nd Asilomar Conference on Signals, Systems and Computers*, pp. 1120–1124, Oct. 2008.

[26] P. jung chung, H. Du, and J. Gondzio, "A probabilistic constraint approach for robust beamforming with imperfect channel information," *IEEE Trans. signal processing*, vol. 59, no. 6, pp. 2773–2782, 2011.

[27] C. Shen, T. H. Chang, K. Y. Wang, Z. Qui, and C. Y. Chi, "Distributed robust multicell coordinated beamforming with imperfect CSI: An ADMM approach," *IEEE Transaction on signal processing*, vol. 60, no. 6, pp. 2988–3003, Jun. 2011.

[28] K. Y. Wang, T. H. Chang, W. K. Ma, A. M. C. So, and C. Y. Chi, "Probabilistic SINR constrained robust transmit beamforming: a bernstein type inequality based conservative approach," *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP'11*, pp. 3080–3083, May 2011.

[29] C. Farsakh and J. A. Nossek, "Spatial covariance based downlink beamforming in an SDMA mobile radio system," *IEEE Trans. Communications*, vol. 46, no. 11, pp. 1497–1506, Nov. 1998.

[30] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1437– 1450, Oct. 1998.

[31] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Communications*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.

[32] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[33] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in *Proc. IEEE Veh. Technol. Conf.*, May 1999, pp. 851–856.

[34] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via Conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161– 176, Jan. 2006.

[35] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.

[36] S. Nasseri, T. A. Le, and M. R. Nakhai, "Robust and power efficient interference management in downlink multicell networks," in *IEEE 24th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2013)*, Sep 2013, pp. 1092–1096.

[37] S. Nasseri and M. R. Nakhai, "Robust interference management via outage-constrained downlink beamforming in multicell networks," *IEEE Global Communications Conference, GLOBECOM'13*, pp. 3492–3497, Dec. 2013.

[38] ——, "Chance constrained robust downlink beamforming in multi-cell networks," *IET Communications*, under review.

[39] B. Hassibi and B. H. Hochwald, "How much training is needed in multiple-antenna wireless links," *IEEE Trans. Inform. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[40] S. Zhou and G. B. Giannakis, "How accurate channel prediction needs to be for transmit beamforming with adaptive modulation over rayleigh MIMO channels," *IEEE Trans. Wireless Communications*, vol. 3, no. 4, pp. 1285–1294, Jul. 2004.

[41] N. Jindal, "Mimo broadcast channels with finite-rate feedback," *IEEE Trans. Information Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.

[42] S. Nasseri and M. R. Nakhai, "Min-Max robust transmit beamforming for power efficient quality of service guarantee," *IEEE Global Communications Conference, GLOBECOM'14*, pp. 3448–3453, Dec. 2014.

[43] B. Walke, *Mobile Radio Networks: Networking, Protocols and Traffic Performance.* John Wiley and Sons, 2001.

[44] G. Jongren and M. S. B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Transactions on Information Theory*, vol. 48, no. 3, pp. 611–627, Mar 2002.

[45] S. Ohmori, Y. Yamao, and N. Nakajima, "The future generations of mobile communications based on broadband access technologies," *IEEE Communications Magazine*, vol. 38, no. 12, pp. 134–142, Dec. 2000.

[46] K. Sheikh, D. Gesbert, D. Gore, and A. Paulraj, "Smart antennas for broadband wireless access networks," *IEEE Communications Magazine*, vol. 37, no. 11, pp. 100–105, Nov. 1999.

[47] A. Lozano, F. R. Farrokhi, and R. A. Valenzuela, "Lifting the limits on high-speed wireless data access using antenna arrays," *IEEE Communication Magazine*, vol. 39, no. 9, pp. 156–162, Sep. 2001.

[48] T. OHIRA, "Adaptive array antenna beamforming architectures as viewed by a microwave circuit designer," in *Microwave Conference Asia-Pacific 2000*, 2000, pp. 828–833.

[49] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using Semidefinite optimization," *Proc. 37th Annual Allerton Conference on Communication, Control, and Computing*, pp. 987–996, 1999.

[50] T. A. Le and M. R. Nakhai, "Coordinated beamforming using semidefinite programming," *IEEE International Conference on Communications, ICC*, pp. 1–6, Jun. 2012.

[51] ——, "An iterative algorithm for downlink multi-cell beam-forming," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2011)*, Dec. 2011, pp. 1–6.

[52] C. Shen, K.-Y. Wang, T.-H. Chang, Z. Qiu, and C.-Y. Chi, "Worst-case SINR constrained robust coordinated beamforming for multicell wireless systems," *IEEE International Conference on Communications, ICC*, pp. 1–5, Jun. 2011.

[53] H. Cox, R. M. Zeskind, and M. M. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 35, no. 10, pp. 1365–1376, Oct. 1987.

[54] J. R. Fonollosai, J. Goldberg1, and G. Vdzquezf, "Downlink beamforming for cellular mobile communications," *First IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, pp. 197–200, Apr.

[55] P. Zetterberg and B. Ottersten, "The spectrum efficiency of a base station antenna array system for spatially selective transmission," *IEEE Trans. on Vehicular Technology*, vol. 44, no. 3, pp. 651–660, Aug. 1995.

[56] T. Haynes, "A primer on digital beamforming," *Spectrum Signal Processing*, 1998.

[57] J. Liberti and T. S. Rappaport, *Smart Antennas for Wireless Communications: Is-95 and Third Generation CDMA Applications.* Prentice-Hall, 1999.

[58] 3GPP, "Third generation partnership project, technical report group, radio access network, spatial channel model for multiple input multiple output (MIMO) simulations, 25.996 (release 10)." [Online]. Available: http://www.3gpp.org/ftp/Specs/html-info/25996.htm

[59] L. Hanzo and T. Keller, *OFDM and MC-CDMA A Primer.* John Wiley & Sons Inc., 2006.

[60] S. Isam and I. Darwazeh, "Robust channel estimation for spectrally efficient FDM system," *19th International Conference on Telecommunications (ICT 2012)*, pp. 1–6, Apr. 2012.

[61] J. Zhang, S. Chen, X. Mu, and L. Hanzo, "Joint channel estimation and multiuser detection for SDMA/OFDM based on dual repeated weighted boosting search," *IEEE Trans. Veh. Technol.*, vol. 60, no. 7, pp. 3265–3275, Sep. 2011.

[62] S. Vigneshwaran, N. Sundararajan, and P. Saratchandran, "Direction of arrival (DoA) estimation under array sensor failures using a minimal resource allocation neural network," *IEEE Trans. Antennas and Propagation*, vol. 55, no. 2, pp. 334–343, Feb. 2007.

[63] M. Jansson, B. Goransson, and B. Ottersten, "A subspace method for direction of arrival estimation of uncorrelated emitter signals," *IEEE Trans. Signal Processing*, vol. 47, no. 4, pp. 945–956, Apr. 1999.

[64] D. Astely, L. Swindlehurst, and B. Ottersten, "Spatial signature estimation for uniform linear arrays with unknown receiver gains and phases," *IEEE Trans. Signal Processing*, vol. 47, no. 8, pp. 2128–2138, Aug. 1999.

[65] T. Trump and B. Ottersten, "Estimation of normal direction of arrival and angular spreed using an array of sensors," *Elsevier Signal Processing*, vol. 50, no. 1-2, pp. 57–69, Apr. 1996.

[66] M. SAWAHASHI, Y. KISHIYAMA, A. MORIMOTO, D. NISHIKAWA, and M. TANNO, "Coordinated multipoint transmission/reception techniques for LTE-advanced," *IEEE Trans. Wireless Communication.*, vol. 17, no. 3, pp. 26–34, Jun. 2010.

[67] F. Capozzi, G. Piro, L. A. Grieco, G. Boggia, and P. Camarda, "Downlink packet scheduling in LTE cellular networks: Key design issues and a survey," *in IEEE Communications Surveys and Tutorials*, vol. 15, no. 2, pp. 678–700, Jun. 2012.

[68] V. Jungnickel, T. Wirth, M. Schellmann, T. Haustein, and W. Zirwas, "Synchronization of cooperative base stations," *IEEE International Symposium on Wireless Communication Systems, ISWCS'08*, pp. 329–334, Oct. 2008.

[69] A. Alexiou and F. Boccardi, "Coordination and cooperation for next generation wireless systems overhead signalling requirements and cross layer considerations," *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP'09*, pp. 3613–3616, Apr. 2009.

[70] 3GPP, "Lte; evolved universal terrestrial radio access network (e-utran): X2 general aspects and principles," *in TSG RAN TS 36.420 V9.0.0*, 2009.

[71] B. L. Ng, J. S. Evans, S. V. Hanly, and D. Aktas, "Distributed downlink beamforming with cooperative base stations," *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5491–5499, Dec. 2008.

[72] O. Simeone, O. Somekh, H. Poor, and S. Shamai, "Distributed MIMO in multi-cell wireless systems via finite-capacity links," *3rd IEEE International Symposium on Communication, Control, and Signal Processing*, pp. 203–206, Mar. 2008.

[73] P. Marsch and G. Fettweis, "A framework for optimizing the downlink performance of distributed antenna systems under a constrained backhaul," *European Wireless Conference*, Apr. 2007.

[74] A. Papadogiannis, E. Hardouin, and D. Gesbert, "Decentralising multicell cooperative processing: A novel robust framework," *EURASIP Journal on Wireless Communications and Networking*, vol. 2009, Aug. 2009, article ID 890685, 10 pages.

[75] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans on Information Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[76] M. Biguesh and A. B. Gershman, "Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals," *IEEE Trans. Signal Processing*, vol. 54, no. 3, pp. 884–893, Mar. 2006.

[77] D. J. Love, R. W. H. Jr, W. Santipach, and M. L. Honig, "What is the value of limited feedback for MIMO channels?" *IEEE Communications Magazine*, vol. 42, no. 10, pp. 54–59, Oct. 2004.

[78] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming: From receive to transmit and network designs," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 62–75, May 2010.

[79] H. Cox and R. Pitre, "Robust DMR and multi-rate adaptive beamforming," *Conference Record of the Thirty-First Asilomar Conference on Signals, Systems and Computers*, vol. 1, pp. 920–924, Nov. 1997.

[80] D. J. Love, R. W. H. Jr, V. K. N. Lau, , D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wireless communications," *IEEE Journal on Selected Areas In Communications*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[81] D. J. Love and R. W. H. Jr, "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE TRANS. Information Theory*, vol. 51, no. 8, pp. 2967–2976, Aug. 2005.

[82] C. B. Chae, D. Mazzarese, N. Jindal, and R. W. H. Jr, "Coordinated beamforming with limited feedback in the MIMO broadcast channel," *IEEE Journal on Selected Areas In Communications*, vol. 26, no. 8, pp. 1505–1515, Oct. 2008.

[83] E. Dahlman, S. Parkvall, and J. Skold, *4G LTE/LTE-Advanced for Mobile Broadband.* Elsevier, 2011.

[84] S. P. Alex and L. M. A. Jalloul, "Performance evaluation of MIMO in ieee802.16e/WiMAX," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 2, pp. 181–190, Apr. 2008.

[85] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," *in IEEE 49th Vehicular Technology Conference, 1999,*, vol. 1, pp. 851–856, Jul. 1999.

[86] E. Karipidis, N. D. Sidiropoulos, and Z.-Q. Luo, "Quality of service and Max-Min fair transmit beamforming to multiple cochannel multicast groups," *IEEE Trans. Signal Processing*, vol. 56, no. 3, pp. 1268–1279, Mar. 2008.

[87] N. Bornhorst and M. Pesavento, "An iterative convex approximation approach for transmit beamforming in multi-group multicasting," *IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications, SPAWC'11*, pp. 426–430, Jun. 2011.

[88] S. A. Kassam and H. V. Poor, "Robust techniques for signal processing: A survey," *Proceedings Of The IEEE,*, vol. 73, no. 3, pp. 433–482, Mar. 1985.

[89] J. Wang and D. P. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Processing*, vol. 58, no. 8, pp. 3086–3100, Aug. 2009.

[90] ——, "Robust MMSE precoding in mimo channels with pre-fixed receivers," *IEEE Trans. Signal Processing*, vol. 58, no. 11, pp. 5802–5818, Nov. 2010.

[91] A. Abdel-Samad, A. B. Gershman, and T. N. Davidson, "Robust transmit beamforming based on imperfect channel feedback," *IEEE Trans. Signal Processing*, vol. 54, no. 5, pp. 596–1609, May 2004.

[92] M. B. Shenouda and T. N. Davidson, "Convex formulations of robust downlink precoder designs with quality of service constraints," *IEEE Journal of Sel Topics In Signal Process*, vol. 1, no. 4, pp. 714–724,, Dec. 2007.

[93] N. Vucic and H. Boche, "Robust QoS-constrained optimization of downlink multiuser MISO systems," *IEEE Trans. Signal processing*, vol. 57, no. 2, pp. 714–725, Feb. 2009.

[94] ——, "Robust transceiver optimization in downlink multiuser MIMO systems," *IEEE Trans. Signal Processing*, vol. 57, no. 9, pp. 3576–3587, Sep. 2009.

[95] M. B. Shenouda and T. N. Davidson, "On the design of linear transceivers for multiuser systems with channel uncertainty," *Journal of sel. area in commun*, vol. 26, no. 6, pp. 1015–1024, Aug. 2008.

[96] ——, "Nonlinear and linear broadcasting with QoS requirements: Tractable approaches for bounded channel uncertainties," *IEEE Trans. Signal Processing*, vol. 57, no. 5, pp. 1936–1947, May 2009.

[97] A. Wiesel, Y. C. Elda, and S. S. (Shitz), "Optimization of the mimo compound capacity," *IEEE Trans. Wireless Communications*, vol. 6, no. 3, pp. 1094–1101, Mar. 2007.

[98] A. Prkopa, *Stochastic Programming (Mathematics and Its Applications)*. Kluwer Academic Publishers, 1995.

[99] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. Springer, 1997.

[100] S. A. Vorobyov, Y. C. Eldar, A. Nemirovski, and A. B. Gershman, "Probability constrained approach to estimation of random gaussian parameters," *In IEEE Computational Advances in Multi-Sensor Adaptive Processing,CAMSAP'05*, pp. 101–104, Dec. 2007.

[101] Y. Rong, S. A. Vorobyov, and A. B. Gershmnn, "Robust linear receiver design for multi-access space-time block coded MIMO systems using stochastic optimization," *in IEEE Workshop Statistical Signal Processing*, pp. 65–70., Jul. 2005.

[102] S. A. Vorobyov, "Robust CDMA multiuser detectors: Probability-constrained versus the worst-case based design," *IEEE Signal Processing Letters*, pp. 273–276, 2008.

[103] S. A. Vorobyov, H. Chen, and A. B. Gershman, "On the relationship between robust minimum variance beamformers with probabilistic and worst-case distortionless response constraints," *IEEE Trans. Signal Processing*, vol. 56, no. 11, pp. 5719–5724, Nov. 2008.

[104] K. Y. Wang, T. H. Chang, W.-K. Ma, and C.-Y. Chi, "A semidefinite relaxation based conservative approach to robust transmit beamforming with probabilistic SINR constraints," *18th European Signal Processing Conference, EUSIPCO'10*, Aug. 2010.

[105] A. Tajer, N. Prasad, and X. Wang, "Robust linear precoder design for multi-cell downlink transmission," *IEEE Trans. signal processing*, vol. 59, no. 1, pp. 235–251, Jan. 2011.

[106] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[107] H. Hindi, "A tutorial on convex optimization," *Proc. American Control Conference*, vol. 4, pp. 3252–3265, Jul. 2004.

[108] Z. Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1426–1438, Aug. 2006.

[109] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, "Linear matrix inequalities in system and control theory," *studies in Applied Mathematics Society for Industrial and Applied Mathematics (SIAM)*, vol. 15, 1994.

[110] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Review*, vol. 38, no. 1, pp. 49–95, Mar. 1996.

[111] J. F. Sturm, *Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones*. Optimization Methods and Software, 1999, software availabe at http://sedumi.mcmaster.ca/.

[112] M. Grant and S. Boyd, *cvx Users' Guide for cvx version 1.2 (build 711)*, 2009, software availabe at http://www.stanford.edu/~boyd/cvx/download.html.

[113] D. Peaucelle, D. Henrion, Y. Labit, and K. Taitz, "User's guide for sedumi interface 1.04," *Laboratory for Analysis and Architecture of Systems*, Sep 2002.

[114] P. GAHINET, A. NEMIROVSKI, A. J. LAUB, and M. CHILALI, "LMI control toolbox user's guide," *The Mathworks Partner Series*, 1995.

[115] L. E. GHAOUI, F. OUSTRY, and M. AITRAMI, "A cone complementarity linearization algorithm for static ouput-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1171–1176, 1997.

[116] T. IWASAKI, "LPV system analysis with quadratic separator for uncertain implicit systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 8, pp. 1195–1208, Aug. 2001.

[117] D. PEAUCELLE and D. ARZELIER, "An efficient numerical solution for H_2 static output feedback synthesis," *proceedings of European Control Conference*, pp. 3800–3805, 2001.

[118] F. LEIBFRITZ, "An LMI-based algorithm for designing suboptimal static H_2/H_∞ output feedback controllers," *SIAM Journal on Control and Optimization*, vol. 39, no. 6, pp. 1711–1735, 2001.

[119] Z. Q. Luo, W. K. Ma, A. M. C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE signal processing. Magazine*, vol. 27, no. 3, pp. 20–34, May 2010.

[120] A. B. Gershman, N. D. Sidiropoulos, Shahhazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 62–75, May 2010.

[121] H. Dahrouj and W. Yu, "Multicell interference mitigation with joint beamforming and common message decoding," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2264–2273, Aug. 2011.

[122] O. Somekh, B. M.Zaidel, and S. S. Shitz, "Sum rate characterization of joint multiple cell-site processing," *IEEE Trans. Inf. Theory*, vol. 53, pp. 4473–4497, Dec. 2007.

[123] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE JSAC*, vol. 21, no. 5, pp. 802–811, Jun. 2003.

[124] S. Jing, D. N.C.Tse, J. B.Soriaga, J. Hou, J. E.Smee, and R. Padovani, "Multicell downlink capacity with coordinated processing," in *EURASIP J. wirel. Commun. Netw.*, Jan. 2008.

[125] D. Gesbert, S. Hanly, H. Huang, S. S. Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE JSAC*, vol. 28, no. 9, pp. 1380–1408, Dec. 2010.

[126] M. Bengtsson, "Robust and constrained downlink beamforming," *in Proc. Euro. Sig. Proc. Conf. (EUSIPCO)*, pp. 1433–1436, Sep. 2000.

[127] F. Penna, S. Stanczak, Z. Ren, and P. Fertl, "MMSE interference estimation in LTE networks," *in 2014 IEEE International Conference on Communications, ICC*, pp. 4548–4552, Jun. 2014.

[128] T. A. Le and M. R. Nakhai, "A decentralized downlink beamforming algorithm for multicell processing," in *Proc. IEEE Global Telecommun. Conf. (GLOBCOM 2012)*, Dec. 2012, pp. 3924–3929.

[129] A. Papadogiannis, H. J. Bang, D. Gesbert, and E. Hardouin, "Efficient selective feedback design for multicell cooperative networks," *IEEE Trans. vehicular technology*, vol. 60, no. 1, pp. 196–205, Jan. 2011.

[130] C. Botella, G. Pinero, A. Gonzalez, and M. D. Diego, "Coordianted in a multi-cell multi-antenna multi-user W-CDMA system: A beamforming approach," *IEEE Trans. Wireless Communication*, vol. 7, no. 11, pp. 4479–4485, Nov. 2011.

[131] M. V. Clark, T. M. W. III, L. J. Greenstein, A. J. Rustako, V. Erceg, and R. S. Roman, "Distributed versus centralized antenna arrays in boardband wireless networks," *Proc. IEEE Veh. Technol. Conf.*, pp. 33–37, May 2001.

[132] E. Bjrnson and B. Ottersten, "On the principles of multicell precoding with centralized and distributed cooperation," *Internation Conference on WCSP*, pp. 1–5, Dec. 2009.

[133] A. Pascual-Iserte, D. P. Palomar, A. I. Prez-Neira, and M. ngel Lagunas, "A robust maximin approach for MIMO communications with imperfect channel state information based on convex optimization," *IEEE Trans. signal processing*, vol. 54, no. 1, pp. 346–360, Jan. 2006.

[134] S.-J. Kim, A. Magnani, A. Mutapcic, and S. P. Boyd, "Robust beamforming via worst-case sinr maximization," *IEEE Trans. Signal Processing*, vol. 56, no. 4, pp. 1539–1547, Apr. 2008.

[135] C. Shen, T. H. Changy, K. Y. Wangz, Z. Qiu, and C. Y. Chi, "Chance coordinated robust beamforming for multi cell coordinated downlink," in *Proc. IEEE Global Telecommun. Conf. (Globecom 2012)*, Dec. 2012, pp. 4957–4962.

[136] M. B. Shenouda and T. N. Davidson, "Outage-based designs for multiuser transceivers," *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP'09*, pp. 2389–2392, Apr. 2009.

[137] N. Vucic and H. Boche, "A tractable method for chance-constrained power control in downlink multiuser MISO systems with channel uncertainty," *IEEE Signal Processing letters*, pp. 346–349, May 2009.

[138] W. W. L. Li, Y. J. A. Zhang, A. M. C. So, and M. Z. Win, "Slow adaptive OFDMA systems through chance constrained programming," *IEEE Trans. Signal Processing*, vol. 58, pp. 3858–3869, Jul. 2008.

[139] A. M. C. So and Y. J. A. Zhang, "Distributionally robust slow adaptive OFDMA with soft QoS via linear programming," *IEEE Journal on Selected Areas in Communications*, vol. 31, pp. 947–958, May 2013.

[140] V. Lau, F. Zhang, and Y. Cui, "Low complexity delay-constrained beamforming for multi-user MIMO systems with imperfect CSIT," *IEEE Trans. Signal Processing*, vol. 61, pp. 4090–4099, Aug. 2013.

[141] A. K. Yousafzai and M. R. Nakhai, "Block QR decomposition and near-optimal ordering in intercell cooperative MIMO-OFDM," *IET Communications*, vol. 4, no. 12, pp. 1452–1462, Aug. 2010.

[142] C. Chen, *C-RAN: the Road Towards Green Radio Access Network*. China Mobile Research Institute, 2012.

[143] D. Ponukumati, F. Gao, and M. Bode, "Robust multicell downlink beamforming based on second-order statistics of channel state information," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2011)*, 2011, pp. 1–5.

[144] A. Shaverdian and M. R. Nakhai, "Robust distributed beamforming with interference coordination in downlink cellular networks," *IEEE Trans. on Communications*, vol. 62, no. 7, pp. 2411–2421, Jun. 2014.

[145] Q. Gong, Z. Chen, and G. Wei, "Downlink multicasting beamforming with imperfect CSI on both transceiver sides," in *proc. IEEE 23th International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC 2012)*, Sep. 2012, pp. 1472–1477.

[146] A. Papadogiannis, H. J. Bang, D. Gesbert, and E. Hardouin, "Efficient selective feedback design for multicell cooperative networks," *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 196–205, Jan. 2011.

[147] C. Botella, G. Pinero, A. Gonzalez, and M. D. Diego, "Coordianted in a multi-cell multi-antenna multi-user W-CDMA system: A beamforming approach," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4479–4485, Nov. 2008.

[148] M. Butussi and M. Bengtsson, "Low complexity admission in downlink beamforming," *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP'06*, vol. 4, p. IV, May 2006.

[149] E. Matskani, N. D. Sidiropoulos, and L. Tassiulas, "On multicast beamforming and admission control for UMTS-LTE," *in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP'08*, pp. 2361–2364, Apr. 2008.

[150] E. Matskani, N. D. Sidiropoulos, Z. Q. Luo, and L. Tassiulas, "Convex approximation techniques for joint multiuser downlink beamforming and admission control," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2682–2693, Jul. 2008.

[151] ——, "Efficient batch and adaptive approximation algorithms for joint multicast beamforming and admission control," *IEEE Transactions on Signal Processing*, vol. 57, no. 12, pp. 4882–4894, Dec. 2009.