Observability, Visualizability and the question of metaphysical neutrality

J.E. Wolff

This is an Author's Manuscript. The final publication is available at Springer via http://dx.doi.org/10.1007/s10701-015-9904-7

Abstract

Theories in fundamental physics are unlikely to be ontologically neutral, yet they may nonetheless fail to offer decisive empirical support for or against particular metaphysical positions. I illustrate this point by close examination of a particular objection raised by Wolfgang Pauli against Hermann Weyl. The exchange reveals that both parties to the dispute appeal to broader epistemological principles to defend their preferred metaphysical starting points. I suggest that this should make us hesitant to assume that in deriving metaphysical conclusions from physical theories we place our metaphysical theories on a purely empirical foundation. The metaphysics within a particular physical theory may well be the result of a priori assumptions in the background, not particular empirical findings.

Keywords: Weyl; Pauli; interpreting physics; continuum; field concept;

1 Introduction

Contemporary philosophers of physics tend to conceive of their task as that of interpreting physical theories (van Fraassen, 1991). To give such an interpretation today typically means to present an ontology for particular theories. More recently, with the return of metaphysics to philosophical respectability, this task has taken on a new importance, beyond debates in the philosophy of physics. The ontology of our best physical theory, goes the thought, might serve to inform our metaphysical view of the world as well. Thus informed, our metaphysics is no longer confined to purely armchair speculation about the world, but rests on a firm empirical, non-anthropomorphic foundation. While the exact role of physics for the development of metaphysics remains controversial, the direction of influence is clearly from physics to metaphysics, not vice versa.[1]

My goal in this paper is to challenge this mono-directional approach. I shall do so by examining a particular case in the history of science: an objection,

[1] This view has perhaps been most forcefully articulated by Ladyman and Ross (2007), but somewhat less polemical versions of this idea appear to be wide-spread among philosophers of physics, as well as some metaphysicians.

raised by Wolfgang Pauli, against Hermann Weyl's unified field theory. A close look at this example will show, I believe, that not only philosophical views in general, but indeed particular metaphysical commitments, can at times drive the development of physical theories. I choose Pauli as a case, since his official position was deeply anti-metaphysical. Accordingly it would be especially interesting to find that even staunch anti-metaphysicians are not free from metaphysical commitments. Weyl makes for a nice contrast, since his physical theorizing was very explicitly driven by a broader philosophical framework.

In Pauli's own words, his objection is a "physical-conceptual concern", targeting Weyl's use of values for electric field strength inside the electron. Pauli's concern ostensibly rested on the idea that the value for the field strength inside the electron would be in principle unmeasurable, since the smallest test-body for measuring the field strength is the electron itself. Pauli's criticism is interesting for a number of reasons. It was taken seriously and discussed both by Weyl and by Einstein, with interestingly different emphasis. It has also often (Mehra and Rechenberg, 1982b) been regarded as one of the sources of Heisenberg's notorious appeal to observability at the beginning of his breakthrough 1925 paper (Heisenberg, 1925). Finally, the appeal to observability, combined with what looks like a positivistic criterion for meaningfulness, suggests that Pauli commits himself to a form of positivism, which would turn the objection into an interesting example of the influence of philosophy on the development of physical theories.

In this paper I aim to reconstruct Pauli's objection, and to place it in a broader philosophical context. I argue that the main disagreement is about metaphysics, specifically about the question whether continua or discrete structures are more fundamental. This is so despite the fact that the arguments on both sides are couched in epistemological and, at times, semantic terms. As we shall see, the objection is important as an early instance of the divide between Anschaulichkeit (visualizability) and Beobachtbarkeit (observability), which would become the respective battle cries of defenders of wave mechanics and matrix mechanics. Understanding the metaphysical features of this challenge, through the lens of this early instance, will hence help to shed light on those later debates as well.

In section one I look in detail at the objection as posed by Pauli, and the responses it received by Einstein and Weyl. From this discussion it will emerge that while Pauli's objection seems quite weak because of its apparent reliance on positivistic criteria of meaningfulness, responses to it by Weyl and Einstein remain unpersuasive as well. In section two I try to shed light on this unsatisfying situation by looking at the relationships between the ostensibly epistemic notions of visualizability and observability, and the metaphysical commitments to continuum theories and discrete theories respectively. There I show that the disagreement between Weyl and Pauli was not merely methodological, but had a metaphysical undercurrent as well. After the discussion of this case study I return to the broader question of metaphysical neutrality. I argue that theories in physics do not present a picture of the world devoid of metaphysics. Instead they are the outcome of debates which include, among other considerations, metaphysical commitments. This should not stop philosophers from turning to physical theories for insight on metaphysical questions, but we need to be cautious in the use we make of the answers. A theory's ontology is not an independent, neutral piece of evidence for or against a particular metaphysical view.

2 The objection to Weyl

Weyl's treatise *Raum, Zeit, Materie* (1919)[2] is a unique and fascinating attempt to take the lessons of relativity theory beyond a theory of gravitation towards a unified field theory of both gravitational and electromagnetic forces. The goal is to understand not only gravitational, but also electromagnetic forces as features of a four-dimensional world-metric. Weyl's key idea is to demand a thoroughgoing adherence to pure infinitesimal geometry, and to purge field theory from all remnants of 'Ferngeometry' (literally: distance geometry), that is, from Euclidean assumptions. In particular Weyl suggested that in a purely infinitesimal geometry, not only the comparison of the *direction* of a vector is path-dependent, but also comparisons of *magnitude*, that is, length.

Weyl's attempt was mathematically thorough, but by his own admission highly speculative. Unsurprisingly, the reception of the book in the physics community was critical. Very quickly the theory was found to be empirically inadequate, due to an objection raised by Einstein and formalized by Pauli.[3] According to Weyl's theory, the wavelengths of spectral lines of atoms should depend on the path they have taken, yet empirically the spectral lines of atoms of the same elements are found to have the same wavelengths. Weyl's response to this objection was to emphasize the ideal or mathematical character of this theory, which led some critics, including Pauli, to question whether the theory could be regarded as a physical theory at all. While this objection may have been enough for the wider physics community to dismiss Weyl's theory as too speculative, it didn't settle the matter for Weyl, or interestingly, Einstein himself, as Ryckman (2005) has demonstrated.

For the purposes of the rest of this paper, I will set this particular matter aside, although it should already serve as a hint that the emerging debate is conceptual and philosophical, rather than empirical. The objection I will be concerned with here was recognized by Pauli to be conceptual even when he first raised it. To see where it fits, we need to add one more detail to Weyl's theory.

An important component at least in the early versions of Weyl's unified field theory is the interpretation of seemingly particulate matter as "Energieknoten", as nodes of energy. To do so is important for overcoming the divide between matter and field. "The electron, which in earlier times one might have imag-

[2] The first edition appeared in 1918. Weyl continued to revise his arguments considerably in subsequent editions, largely in response to Einstein's objections. For a detailed reconstruction of the arguments leading to this revision process, see Ryckman (2005).

[3] For details of the objection, and how Weyl successively dealt with it, see Ryckman (2005).

ined as a substantial foreign body in the substance-less electromagnetic field, now appears, not at all sharply separated from the field, as a small neighborhood in which the field quantities and the electrical density take extremely high values."(Weyl, 1919, 172; my translation.)

Weyl here tries to pave the way for a purely field-theoretic physics by characterising particles, like electrons, as features of the field.[4] Pauli's objection pertains to this aspect of Weyl's theory. Towards the end of his paper *Merkurperihelbewegung und Strahlenablenkung in Weyls Gravitationstheorie* (Pauli, 1919), after raising a number of technical concerns for Weyl's view, Pauli adds a different sort of problem: "I should also like to mention a physical-conceptual concern. In Weyl's theory we constantly work with the field strength in the interior of the electron. For the physicist, however, this [the field-strength] is defined solely as the force on a test-body; since there is no smaller test-body than the electron itself, the concept of electric field strength inside a mathematical point would appear to be an empty meaningless fiction. One would like to hold on to the idea of admitting only quantities into physics which are in principle observable. Should we perhaps be on the wrong track altogether using continuum theories for the interior of electrons?"(Pauli, 1919, my translation)

On a quick reading, Pauli's objection appears to be driven by empiricism, specifically by the idea that what is not observable is suspicious in one way or another. Moreover, he seems to connect observability and meaningfulness, a doctrine that would become an important part of logical positivism and operationalism a few years later.[5] Is Pauli's argument just a form of positivism? A closer look reveals a more convoluted argument.

The idea that Pauli's objection must be driven by empiricism is suggested by the emphasis on observable quantities and the possibility of measurement. As a source for such an empiricist outlook, many commentators (Mehra and Rechenberg, 1982a) point to Ernst Mach, who was Pauli's godfather. But while Pauli was certainly proud of this connection, it is unclear that the objection pressed against Weyl can actually be motivated by a straightforward Machian empiricism.

First of all, since the electric field strength even outside the electron is not observable in the sense of being perceivable, the requirement is that quantities should be measurable, not that they should be perceptible. This suggests that the problem with quantities that are in principle unobservable is not analogous to the problem of, say, the small constituents of matter, which are inaccessible to human perception. Indeed, Pauli here in no way questions the existence of electrons, only the legitimacy of assuming that there is an electric field in the interior of the electron. Traditional empiricists tend to be as suspicious of electrons as of any attributions of quantities to the inside of electrons.[6]

[4] For a succinct technical discussion of this version of a field-theoretic conception of matter, see Scholz (2000). Weyl abandoned this field-theoretic conception of matter shortly afterwards, in favor of an agent theory of matter (see Sieroka (2010) for a detailed discussion).

[5] For operational definitions of meaning, as well as the physical inspiration for the view, see (Bridgman, 1927).

[6] Mach, notoriously, remained highly skeptical about the existence of atoms for most of his

Secondly, Pauli refers to the idea that only quantities which are in principle observable should be introduced into physics as though this is a longstanding, widely accepted principle, which one should 'hold on to' [festhalten]. Moreover, he also makes no attempt to further elaborate or justify this principle: he seems to assume that it should be obvious to anybody that the unobservability of the electric field inside the electron marks a problem for the theory. This suggests that Pauli does not take himself to be advancing a particularly radical form of empiricism, but to express a sentiment or idea commonly held in the physics community. The most likely explanation is, that Pauli, who had primarily been working on relativity theory at the time, took inspiration for his objection from Einstein's development of special relativity, which was typically taken to involve an appeal to a kind of observability principle. To judge the force of Pauli's objection to Weyl's theory, then, we should perhaps read it with Einstein's argument in view, and not as driven by traditional or positivistic empiricism.[7]

Even setting aside the question of how Pauli was inspired to raise the concern, certain features of the objection suggest that Pauli's objection depends on operationalist or positivistic criteria of meaningfulness, whether he intended it or not. Moreover, if Pauli's argument depends on an appeal to operationalism, he inherits the weaknesses of this view, whether he held it or not. In what way, then, does Pauli's argument seem to depend on operationalism?

To determine the strength (and direction) of the EM-field in any region of space, we need to send a test-body through that region, to see how its trajectory is affected by the field. The smallest test-body one might use to determine the strength of the EM-field in this way is the electron. Since an electron cannot be sent 'through' another electron to determine how much its path deviates due to the action of the field, there is no way we can determine the field-strength in the interior of the electron by the usual means. Read this way, Pauli's objection seems to point to a practical limitation: we do not have suitable instruments to determine the field strength in a point-particle. This is correct, but it does not seem to warrant the far-reaching conclusion Pauli wants to draw, namely that the field strength inside the electron is 'an empty, meaningless fiction'.

Whether 'field strength in the interior of the electron' is genuinely an empty, i.e. non-referring, term, will depend upon whether or not the field *has* a value inside the electron, not on whether we can detect it. Pauli seems to be justifying his controversial move by appeal to a form of operationalism: electric field strength is defined as the force exerted on a test body. So how can we speak of field strength in the absence of a suitable test body on which the force could be exerted? This appeal to a connection between the measurability and meaning, like later positivistic attempts, seems weak. After all, it seems we understand perfectly well what field strength in the interior of the electron means: if we had a test-body smaller than an electron, then, when sent through the interior

life.

[7]We should not be surprised to find that Pauli (1920) objects to field-theoretic conceptions of matter in general, on the grounds that something over and above the continuous field needs to be added in order to account for matter. Sieroka (2010) suggests that this, more than any technical problems, was seen as a problem for pure field-theoretic physics.

of the electron, it would be deflected from its trajectory by the field inside the electron by a certain amount. We may not have such a suitable test-body, but this hardly hampers our understanding of what it would be like to have one.[8]

Finally, pointing out a practical limitation hardly amounts to a 'conceptual-physical concern'; after all, our current abilities to measure field-strength may be limited to using test-bodies, but that does not preclude the development of indirect means of measuring the field-strength. Such means could then be employed to measure the field-strength inside the electron, circumventing even the operationalist scruples mentioned above. Weyl indeed responds to Pauli, in a letter of December 9th, 1919 along these lines: "To say that one could not measure the fields in the interior of the electron can only mean: differences inside the electron can never causally determine changes in the course of the world that would amount to immediately observable orders of magnitude. As soon as such effects occur, I can use them to "measure" these inner differences. But why shouldn't that happen!"(Hermann et al., 1979, 4; my translation)

Weyl here suggests that the difficulty with measuring the field strength inside the electron can only mean that any changes inside the electron, including changes in the field strength, will only have a very minor effect on the surroundings, and accordingly they will be difficult to detect. However, there is no reason to think that such an indirect measurement is not, in principle, possible. That means that even though the standard 'direct' measurement of the field, namely measuring the deflection of a test-particle, is not possible for the field inside the electron, it remains an open question whether there might not be other means by which to determine the field-strength inside the electron. So the field strength inside the electron is not in principle unmeasurable.

In his response, Weyl focuses on Pauli's objection primarily as a practical difficulty, since he does not seem to think that the conceptual concern poses a lasting problem. Moreover, he implicitly seems to grant Pauli that if the field strength inside the electron could not be made measurable by indirect means along the lines suggested, this would indeed constitute a serious problem for the theory. That is, it seems that Weyl is on board with the principle only to introduce observable quantities into physical theories, he merely thinks his own particular use field strength does not violate this principle. Observability is not restricted to direct measurement.

It seems as though Weyl has found an effective means to block the objection, but not one that Pauli or an empiricist sympathizer would find especially satisfying. It would indeed be difficult to establish that changes inside the electron do not occur, since the effects would be so small; so if Pauli needs to be able to show that there are none before his objection can even get off the ground, his chances to do so are slim. In this way Weyl's reply is certainly effective. On the other hand, it seems to ignore key features of Pauli's critique. Pauli is

[8]Strictly speaking, Weyl does not actually want to measure the field strength 'in the interior of the electron'. As we saw above, Weyl first re-conceptualizes the meaning of electron: it is not to be understood as a body, but as a region of the field. Accordingly, measuring the field strength in the interior of the electron really just means measuring the field strength in a very small region of the field, where those values are extremely high.

not merely concerned with the possibility of measuring any changes to the field strength inside the electron, he is concerned about the legitimacy of assuming that the field will have values of any sort inside the electron, or better: that the field strength inside the electron is well-defined. To say that it is unproblematic to assume that it takes definite values, provided only that the possibility of indirect measurement is not precluded, seems a little question-begging.

The reason that appeal to an electric field inside the electron might seem innocuous enough is that we think of a field as defined everywhere in space. If we are furthermore willing to follow Weyl's over-coming of the divide between matter and the field, by interpreting particulate matter as regions of the field, there seems to be no problem with the assumption that the field is defined everywhere 'inside' particulate matter as well.[9]

Pauli questions just this assumption, in effect pointing to an unresolved tension in the mathematical and physical definitions of an electric field. Mathematically, the field is assumed to be continuous, that is, it has definite values at every point. Physically the field strength is defined as the effect it has on a test-particle; at the same time charged particles are treated as the sources of fields. The usual physical understanding of fields and particles treats the two as distinct and as having different causal roles. Weyl's theory consciously blurs this distinction, but while the mathematical treatment seems clear enough, the physical interpretation is less clear. Pauli's claim that the field strength inside the electron is a meaningless fiction is exaggerated, since there is a clear mathematical meaning to it, but he is right to think that an argument needs to be given to assure us that to this mathematical meaning there in fact corresponds a physical meaning.

In this regard Pauli's objection is very much analogous to Einstein's. It is not enough to show the mathematical coherence of a quantity; for it to be physically respectable, it also has to be shown how we can relate it to measurements. Einstein's argument at the beginning of his paper on the electrodynamics of moving bodies seems to proceed from a similar observation: giving a mathematically coherent description of reality is one thing, but it needs to be accompanied by a physical interpretation (Einstein, 1905). It is in giving such a physical interpretation that hidden presuppositions of the theory may be exposed. In the case of Weyl's theory, the (not so hidden) presupposition is that there really is no fundamental difference between matter and fields, and that ultimately, continuous fields are more fundamental than particulate matter.

In light of this parallelism between Pauli's and Einstein's arguments, we should not be surprised to find that Pauli used the opportunity, in 1920, at the *86th Assembly of the Association of German Scientists and Physicians in Bad Nauheim* to pose the criticism directly to Einstein, asking for his opinion.

[9]Indeed, Weyl's argument for treating apparently particulate matter in this fashion draws heuristically on the treatment of matter in theories like hydrodynamics. "In phenomenological theories in which we abstract from the particulate structure of matter, we think of the energy stored in electrons, atoms and so forth as continuously distributed across the body; we only have to take it into account by introducing the energy-momentum tensor–relative to a coordinate system–as energy density into the rest mass μ_0." (Weyl, 1919, 174) My translation.

Einstein's response at the meeting was 'diplomatic', as Mehra and Rechenberg (1982b, 279/80) observe, but we know that Einstein was unpersuaded. He saw Pauli's objection as a threat to all continuum theories. In a letter to Born, from January 1920, that is, before the Bad Nauheim assembly, Einstein wrote: "Pauli's objection is directed not only against Weyl's, but also against anyone else's continuum theory. Even against one which treated the electron as a singularity. I believe now, as before, that one has to look for redundancy in determination by using differential equations so that the solutions themselves no longer have the character of a continuum. But how?" (Born et al., 1916-1955, 21)

Pauli's objection, then, is not so much a particular empiricist concern about an aspect of Weyl's theory, it is really an attack on the legitimacy of a fundamental assumption of field theory. The mathematical understanding of a field as defined everywhere in space(-time) is in need of physical interpretation, and what's worse, the usual physical interpretation of the field is not applicable in the limit. Einstein clearly sees this, but admits to being at a loss for how to start from a continuum theory, while arriving at a theory that does justice to the discrete structure of matter. Einstein's reason for his reluctance to give up the idea that nature is fundamentally continuous is clear as well, from the same letter: "I myself do not believe that the solution to the quanta has to be found by giving up the continuum. Similarly, it could be assumed that one could arrive at general relativity by giving up the coordinate system. In principle, the continuum could possibly be dispensed with. But how could the relative movement of n points be described without the continuum?"(Born et al., 1916-1955, 21)

It is clear from these brief remarks (which occur in a letter that otherwise touches on everything from personal matters to world politics) that Einstein, despite his own criticisms of Weyl's theory, agrees with Weyl about the fundamentally continuous character of physical theories. This Einstein hopes to maintain even in the face of the emerging quantum physics. Mechanics and kinematics would still be described by differential equations, and the world should accordingly be treated as a continuum.

It is natural to assume that Pauli criticized Weyl at least not only out of empiricist sentiments, but also because he felt that quantum physics demanded a more radical departure from physics than Einstein and others were prepared to admit. It is perhaps no surprise, then, that the by far most sympathetic reaction to Pauli's objection came from Max Born, who complimented Pauli in a letter of December 23, 1919 on his paper: "I was especially intrigued by your remark at the end, where you say that you believe the application of continuum theories to the interior of electrons to be meaningless, because it involves in principle unobservable things. It is exactly this line of thought that I have been pursuing myself for quite some time now, although without much positive success; namely that the solution to all the quantum difficulties must be sought from a principled point of view: one must not transfer the concepts of space and time as a four-dimensional continuum from the macroscopic world of experience to the world of atoms; the latter apparently demands a different kind of number-manifold as an adequate image." ((Hermann et al., 1979, 10), my translation.)

Born here connects explicitly the looming difficulty of quantum physics with the worries about continuum theories. Notice also that Born does not cast the conflict as one between field theories and particle theories. Instead he sees it as a conflict about the nature of space-time: is it continuous or discrete? We can now see how the fault line that was to rupture in 1926/27 during the dispute over matrix and wave mechanics begins to emerge. On the one hand we have Born and Pauli as defenders of a discretizing approach to physics, although as of now lacking a mathematical framework within which to express their perspective. On the other hand we have Einstein and Weyl, who have the mathematical tools available, but are challenged for focusing exclusively on field-theories, which fit these mathematical tools. Neither side, however, is able to present conclusive arguments. In 1920, field theory had the upper hand, simply because there was no viable alternative for doing mechanics, not because the philosophical arguments in its favor were better. The reasons thinkers like Einstein and Weyl favored it, whereas Born and Pauli rejected it, were metaphysical, however. The latter two thought that a discrete space-time was metaphysically possible, whereas Einstein and Weyl were skeptical. Of these four, only Weyl had put forward a thoroughgoing philosophical argument to support his position, as we shall see in the next section.

We find ourselves in an odd position, then. On the one hand, Pauli's objection seems to be weak in virtue of relying on poorly defended epistemic principles, on the other hand Einstein's defense of continuum theories does not seem especially compelling either, since it seems to be the result of an inability to conceive of an alternative. In the next section I will attempt to explain this dissatisfying situation by taking a closer look at the philosophical frameworks between which this debate was played out.

3 Conflicting epistemic principles and their metaphysical correlates

It has often been remarked that philosophical arguments and theories were deeply involved in the revolutionary climate in the theoretical physics in the first half of the 20th century.[10] In the German context, two strands of philosophy were especially relevant: Neo-Kantianism and logical positivism. Representatives of both sides, notably Ernst Cassirer and Moritz Schlick, paid close attention to the new developments in physics, and soon began to draw philosophical lessons from them. Both sides sought to find confirmation for their respective epistemological positions in the development of relativity theory. At the same time, physicists responded to the crises in fundamental physics (as well as in the foundations of mathematics) by turning to philosophy for heuristic or

[10] On the different strands of philosophy, their engagement with physics, and with each other, see (Friedman, 1999).

at least rhetorical help. For the purposes of this paper, two concepts, which emerge on opposite sides of this conflict, play a central role: Anschaulichkeit, or visualizability, and Beobachtbarkeit, or observability. Let's look at these notions in turn.

3.1 Anschaulichkeit and continuity

Anschaulichkeit, which I will translate as "visualizability" for the remainder of the paper, is a difficult term in German. On the one hand, it has a fairly innocuous, ordinary meaning: to present something in an anschaulich way is to make it concrete, more easily accessible, or vivid. In this sense to be anschaulich usually has a positive connotation. It may be valued as a feature of a presentation, including the presentation of scientific theories, but it is certainly not a requirement for scientific theorizing. On the other hand, Anschaulichkeit has had a technical connotation, at least in academic circles, going back to Kant's distinction between Anschauung and Begriff (concept). Anschauung, as one of the types of representation we have, is singular and immediate, according to Kant, whereas concepts are general and mediated representations. In order to form judgments we need both Anschauungen and concepts.[11] Since both the ordinary and the technical notions play a role in the debates in physics, I will say a few more things about the Kantian notion.

A Kantian-inspired meaning of Anschaulichkeit as a feature of representations would suggest that Anschaulichkeit is not merely a nice, though optional feature of representations, but a necessary one, since without it we cannot form proper judgments; our concepts remain empty. Moreover, the details of Kant's view on Anschauung suggest fairly specific constraints on whether or not something can count as anschaulich. What can be given in the Anschauung must be spatio-temporal for Kant, since it has to conform to the forms of Anschauung: space and time. Anschaulichkeit, thus understood, would seem to place a particular kind of constraint on theorizing, including in particular scientific theorizing: everything referred to in physical theorizing has to be spatio-temporal in form.

When physicists like Weyl and Einstein make reference to Anschauung and Anschaulichkeit, then, we should be careful to determine whether the notion employed is closer to the technical, Kantian one, or closer to the ordinary notion. In Weyl's case it is fairly clear that he employs Anschaulichkeit in a technical sense, although not in exactly the Kantian sense. Kant's original conception was widely regarded to have been challenged by the development of relativity theory, since for Kant, space was necessarily Euclidean. A fairly common strategy to preserve a Kantian outlook, in light of the development of non-Euclidean geometry and its eventual application to physical space, was to suggest that

[11] In the literature on Kant, Anschauung and anschaulich are more commonly translated as intuition and intuitive. While those terms are suitable as translations of Kant's notions, I nonetheless prefer to translate Anschaulichkeit as visualizability for the purposes of this paper, since within the debate I'm looking at, the term is not used exclusively in its Kantian sense.

Kant had not been wrong about the role space (and time) play in our cognition; he had just been mistaken about the amount of structure inherent in that space.

Weyl's approach in *Raum, Zeit, Materie*, although from a Husserlian rather than a strictly Kantian outlook, follows this strategy.[12] Weyl thinks that the manifold used in physical theories must first be constructed from purely infinitesimal elements. In this way, Weyl weakens the Kantian assumptions: not only is space not Euclidean, it is not even given as a homogenous whole. At the same time, however, Weyl heightens the role of space-time and its geometry for physical theorizing. It is from the geometrical features of the space-time manifold that he aims to arrive at far-reaching conclusions about the forms of the equations describing the fundamental forces of physics. Moreover, the idealistic feature of Kant's approach to space and time as forms of intuitions is preserved in Weyl's construction, with a phenomenological twist. The space-time manifold of physics is not simply out there, given to us; instead it is constructed, bit by bit, out of infinitesimal homogenous neighborhoods. Weyl believes that his own construction (added in the third edition) proceeds with "in full naturalness, visualizability [Anschaulichkeit], and necessity" (Weyl, 1919, vi).

Weyl's infinitesimal geometry is a response to a deeper problem with the mathematical concept of the continuum, which had occupied him earlier in *Das Kontinuum* (Weyl, 1918). There Weyl had contrasted the visualizable (anschauliches) continuum with the mathematical continuum. The latter, represented by the real numbers, fails to be an adequate way of capturing the continuous character of the anschauliches continuum we find in phenomenal time, Weyl argues, because the real numbers are thought of as point-like, whereas the anschauliches continuum is one in which points "flow into" one another.[13] Even without going into the technical details of the Weyl's conception of the mathematical continuum,[14] it is clear that he finds that our access to a genuine continuum is given in Anschauung, not through mathematical means, since the latter are inevitably 'discrete'. Anschaulichkeit, then, is a guide to continuity.

To understand Weyl's concern here, we need to take into account his commitment to the idea that mathematics should not only provide us with consistent tools but that it should also be, as John L. Bell (2000) writes "reasonable". The foundations of mathematics, as Weyl makes clear even in the introduction to *Das Kontinuum*, have to be philosophical, and in particular they have to be satisfactory from the epistemological viewpoint we in fact occupy. Part of that viewpoint is our anschaulich conception of the continuum, as we experience it in phenomenal time.

If Anschaulichkeit is meant to evoke this kind of neo-Kantian/Husserlian ap-

[12]For a detailed reconstruction of Weyl's approach, as well as its Husserlian roots, see once again Ryckman (2005), especially chapters five and six.

[13]"The conception of the process [of experienced temporal development] as consisting of points, and hence as disintegrating into points, turns out to be mistaken. We are missing precisely what makes for continuity, [namely] the flow from point to point." (Weyl, 1918, 69-70; my translation)

[14]It is important to note that Weyl's technical answer to this problem kept changing in this period; see Scholz (2000). For a comparison of Weyl's anschauliches continuum and Brouwer's intuitive continuum, see van Atten et al. (2002).

proach, we can see why it leads Weyl to a continuum approach to physics, but at the same time it is also clear that taking this route towards the continuum comes with huge philosophical commitments. After all, the 'Weltgeometrie" (world geometry) thus constructed remains the result of a particular construction procedure, motivated ultimately by epistemological concerns about the permissible of comparisons of magnitude at a distance. This is a far cry from the usual assumption that the space-time manifold is the arena within which physical events take place. Weyl was fully aware of this consequence: "Where according to the traditional point of view we had empty space as the form, as a framework in which matter of a solid reality constitutes itself, and as the stage on which the real physical events—which are the changes of this matter—take place, now the entire physical world instead has become form, which gains its contents from realms quite different from those of nature [physis]."(Weyl, 1919, 263; my translation)

Not everyone might be willing to take on far reaching philosophical commitments for the purpose of motivating a research program in physics. Without it, however, the appeal to Anschaulichkeit loses much of its force. One might try to build a connection between Anschaulichkeit and space and time without commitment to the wider implications of Kant's view, by connecting the technical and the ordinary notions of Anschaulichkeit. If we assume that a physical process, entity or quantity is located in space and time, it is easy for us to imagine what that process (or entity, or quantity) is like, because Kant got something right about our psychological make-up: when we try to imagine physical things, we depict them spatio-temporally. Our imagination is a visualization, and our visualization is spatial. Because of that, we find it easier to grasp presentations of theories which involve concrete, i.e. spatio-temporal examples.

Defenses along these lines drew heavy attacks from Pauli, especially later on, and seem to have persuaded him that Anschaulichkeit was fundamentally bound up with old-school, classical physics, which he believed had been shown to be inadequate for dealing with quantum mechanics. With reference to the Uncertainty Relation he writes: "On the other hand it has appeared that there can be an essential limitation to the *simultaneous* use of two or more classical concepts or visualisable (anschauliche) images, on account of the finiteness of the quantum of action."(Pauli, 1994, 98) Anschaulich has here become synonymous with 'classical', and hence inadequate for the quantum context. Moreover, the connection between Anschaulichkeit and continuity has been reduced to a mere desire to depict physical processes and entities in classical terms. This suggests that appeal to Anschaulichkeit requires a thoroughgoing philosophical defense; appeal to ordinary comprehensibility will not suffice to persuade critics.

3.2 Beobachtbarkeit and discretizing

The connection between Anschaulichkeit and continuum theories may be controversial, but at least within a certain framework—transcendental or phenomenological idealism—it is clear how it might go. The connection between Beobachtbarkeit and discretizing is much less clear.

A first way of connecting the two is to point to the apparent discreteness of matter as an entry point. By 1919, the experimental evidence spoke strongly in favor of an understanding of electrons as particles and for an atomistic conception of matter. Pauli and other friends of maintaining a discrete approach to the particulate structure of matter might have tried to appeal to observability as a way of saying: the experimental evidence suggests that electrons and atoms are real. So you cannot just eliminate them from your theories by going for a complete field-theoretical approach.

This strategy has two important limitations. First, philosophical empiricism is not helpful in serving as a background program for the defense of this position, for reasons already hinted at above: empiricists tend to be suspicious of the epistemic respectability of small constituents of matter as well. Second, the appearance of particularity was not denied by the field-theorists, it was just interpreted, by Weyl at least, as emergent from the field, not as a difference in ontological kind. The same field-quantities which were present 'outside' the electron were also found 'within' the electron. These limitations might explain why Pauli made no attempt to appeal directly to the apparent particulate character of matter.

The second way of connecting discretizing and observability was put forward sharply quite some time later, by Max Born, on the occasion of his Nobel Lecture in 1954. Born's explanation is bound up with a particular understanding of the measurement problem, but given the divide between the two camps, it can perhaps be enlisted as a potential argument for Pauli's position as well:

"As a mathematical tool the concept of a real number represented by a nonterminating decimal fraction is exceptionally important and fruitful. As the measure of a physical quantity it is nonsense. If π is taken to the 20th or the 25th place of decimals, two numbers are obtained which are indistinguishable from each other and the true value of π by any measurement. According to the heuristic principle used by Einstein in the theory of relativity, and by Heisenberg in the quantum theory, concepts which correspond to no conceivable observation should be eliminated from physics."(Born, 1955, 264)

Here we find a much more direct connection between observability and discretizing. Not only does the empirical evidence happen to show that matter is discrete; we are in fact systematically limited to discrete structures for representing nature, simply because our measurements at most yield rational numbers, not real numbers. This limitation does not hold for pure mathematics, but it does apply to physics. The continuum, beloved by those doing differential geometry, was always an idealization that went beyond the actually measurable.

Born's response, unlike the response considered above, links observability to discretizing through quantities and measurement, rather than entities, which allows for a much more direct comparison with the competing field-theoretic approach. Field-theory was motivated by the idea that the appropriate tools for physics were differential equations, which required a differentiable manifold. As a result, the construction of this manifold from as few assumptions as possible was desirable. Exaggerating a bit one might say, in Weyl's hands this idea takes the form of deriving physics from the geometrical requirements of working with differential equations. When Pauli first raised his objection to Weyl, and Born enthusiastically embraced it, the mathematical tools for a mechanics not based on differential equations still had to be developed. But as Born had noted even then: quantum physics might require the development of just such tools. At the time he was considering approaches that introduce a discretized space-time structure, but a few years later he would become instrumental to the development of matrix mechanics. When Born gave his Nobel lecture, he could look back on this achievement as a way of providing a much needed alternative to the continuum approach.

Nonetheless, even Born's response remains unsatisfying. The reason is that Born tries to dissolve an essentially metaphysical debate into a methodological one. Metaphysically there are two possibilities: either the world is continuous, or it is discrete. Born is right to point out that we face practical limitations in approaching this world: our measurements only yield rational numbers. But this does not yet speak to the question whether the world is continuous or discrete. Furthermore, the availability of two different types of mathematical description, one on which a continuum is assumed, and physical processes are assumed to be continuous, and one on which no continuum is assumed and physical processes can be discrete, raises the question which of them correctly represents the world. To assign one representation to mathematics, while claiming the other for physics, does not resolve the issue, and it raises the question why mathematics should be suitable for describing the physical world at all.

It echoes Pauli's complaint to Weyl, however. For the mathematician, the continuum may be fundamental and meaningful, but for a physicist it is not. Notice that here, as in Pauli's objection, continuum theories can never be physically significant, because real numbers can never be 'reached' through measurement, as it were. Given that all previous mechanics had proceeded on the basis of differential equations, we would have to conclude that the quantities of classical mechanics were never physically significant. This price might seem too high a price to pay. Notice also that this result can be reached even without the details of quantum mechanics. After all, Pauli came close to it in 1920. So it seems that if this argument is enlisted in favor of discretizing, it is just as a priori as Weyl's argument for the continuum.

Indeed, one might note a certain irony in Born's rejection of real numbers as representations of physical quantities. For as we've seen in the previous section, Weyl himself was critical of the use of real numbers to represent phenomenal and 'objective' time. While Born's criticism makes out real numbers as 'too continuous', and hence unmeasurable, Weyl takes them to be insufficiently continuous, because they fail to capture an anschauliches continuum. This suggests, I think, that the dispute is about far more than just mathematical tools or methodologies. It is a dispute, on the one hand, about the epistemological foundations of mathematics, and on the other hand about the nature of the physical world: is it continuous or discrete?

3.3 Epistemic constraints and metaphysical conclusions

In the previous two subsections I've connected Anschaulichkeit and continuity, as well as observability and discretizing. We found, however, that even where we can motivate the connection between the epistemic notions and their metaphysical companions,[15] the arguments for favoring one over the other remain weak. This is not especially surprising.

Anschaulichkeit and Beobachtbarkeit are both epistemic notions, and more specifically concepts which describe our epistemic possibilities. The question is whether a particular entity can be visualized (or given in intuition, if the more technical Kantian meaning is intended) for the case of Anschaulichkeit, and whether it can be observed (in the sense of being accessible to measurement). Invoking epistemic possibilities to judge the legitimacy of scientific theories, or of the introduction of particular entities into scientific theories, is problematic for two reasons.

First, it is not easy to say, precisely, what the limits of our cognitive capacities are. If the notion of Anschaulichkeit invoked is the ordinary one, then we have little guidance as to what counts as anschaulich beyond the affirmation by other reasoners that a particular presentation was felt, by them, to be anschaulich. Anschaulichkeit, in a non-technical sense, has not clear application criterion. By contrast, the technical Kantian imposes a clear criterion, but as we have seen before, its applicability to physics requires a more heavy duty philosophical defense.

Observability confronts a similar problem. If observability is restricted to an empiricists' understanding of it, according to which only what we have actually observed, and perhaps what is accessible to unaided sense perception counts as observable, then the limits of observability are clearly determined. But it is difficult to see this limitation as a legitimate constraint on scientific theorizing, which always seems to reach beyond what has been observed and aims to extend our epistemic reach by means of instruments. If, by contrast, we count as observable whatever may come within the reach of our theories and instruments, the criterion would reflect scientific practice, but it is open ended, unlike the empiricists' criterion. There is no way of telling whether something that is currently not accessible by instruments might not become so in the future.

There is, however, a more fundamental problem with these two notions. Epistemic possibilities are always somewhat anthropocentric, that is, they are imposed in part by our particular cognitive make-up. If we assume that our theories are an attempt to describe the world, and not merely the world within the limits of our cognitive capacities, we should not set up a priori constraints on the form our physical theorizing takes on the basis of actual or perceived limitations of our cognitive capacities. That is, just because we, as human cognizers, prefer theories we can visualize, and that visualization requires depicting

[15] In Kant's own work, these epistemic notions of course have semantic companions: intuitions and concepts as two distinct types of representations. I do not mean to invoke this semantic distinction here, and I do not think it mirrors the metaphysical distinction between the continuous and the discrete in a straightforward way.

them in space and time, does not tell us that the physical world we are trying to describe in our theories is similarly restricted to space and time, or must be visualizable. Similarly, the fact that a particular entity is outside of our observational capacities is not by itself a reason to think that the entity does not exist. Inferences from epistemic constraints to metaphysical conclusions are unconvincing,[16] and that holds equally true for Anschaulichkeit and Beobachtbarkeit, unless one is prepared to adopt full-blown transcendental idealism. Transcendental or phenomenological idealism avoid this difficulty by denying, ultimately, that the world described by our theories is independent of the world as experience by us. Accordingly constraints on our experience of the world can become substantive constraints on the correct formulation of the theory.

Realists, however, need to look for a different kind of philosophical justification for their choice of mathematical framework. Since mathematics, when used in physical theories, is supposed to be representative of physical reality *independent* of our experience of such reality, the choice of acceptable mathematical frameworks involves metaphysical commitments about what the world is like, fundamentally. If the world is fundamentally continuous, there is nothing objectionable in using field theories to describe it, even if our epistemic access to continua is restricted. If the world is fundamentally discrete, we need to use mathematical tools which do not depend on continua, not because of our epistemic limitations, but because to use such tools would involve metaphysical commitments that are not 'held up' by the world. To use a continuum approach would, at the very least, amount to over-representation, and arguably to a misleading representation.

4 Conclusion

Where does all of this leave contemporary philosophers of physics aiming to derive metaphysics from physical theories?

It might be tempting to conclude, as many physics secretly may have, that this is a good way to lay the old dispute over particles and fields to rest. At the end of the day, one might say, it boils down to a preference for certain types of mathematical tools. I think the lesson at least philosophers might learn is a different one. With the exception of Weyl, none of the physicists whose views I've discussed here had a unified philosophical theory in the background as a driving force for their physical theories. Yet, each of them had certain metaphysical proclivities, which led them to be open to certain physical possibilities, while denying others. As a result the physical theories developed and defended by these different thinkers look quite different, and seem to suggest different ontologies. But as we sit down to interpret these theories for the purposes of arriving at a philosophically defensible metaphysics, we should bear in mind that they were not conceived of in a metaphysical vacuum. At least some of the

[16] This should of course not suggest that epistemic constraints have no role to play in theorizing, either in physics or philosophy. As the Weyl-Pauli exchange shows, they have important heuristic functions.

metaphysics we are trying to read off our physical theories may have been put in there from the start.

The debate over Pauli's objection, and more broadly the debate between continuum and discretizing approaches, suggests that sometimes, metaphysics will decide which course in physics to pursue. Moreover, these background metaphysical views are not defended by appeal to the physical theories they spawn; instead the physicists involved in the debate aimed to defend them by appeal to epistemological principles. Metaphysics goes *into* the decision which theory to pursue, it is not the outcome of a particular theory.

If metaphysical disputes are part of theory construction, this should change our understanding of what it means to engage with physical theories for the purpose of developing metaphysical views. Physical theories cannot simply be appealed to as correctives for our naive or anthropocentric intuitions, since they are themselves the outcome of debates which invoke metaphysical preferences, even if not always by this name. Indeed, sometimes they are defended on principles that are themselves anthropocentric, as when observability is appealed to as a criterion for what is physically acceptable. As a result physical theories do not provide metaphysically 'neutral' support for particular metaphysical viewpoints.

Moreover, the metaphysical preferences that went into theory-building may not be visible as particular ontological commitments, but may instead be built into the very mathematics used to construct the supposedly neutral 'formalism' we set out to interpret. Which mathematical tools are permissible to begin with was very much the center of debate between Pauli and Weyl, and also between defenders of matrix mechanics and wave mechanics later on. If this is so, we should be skeptical that we can draw metaphysical lessons from the interpretations of physical theories without also taking into account, for example questions in the foundations of mathematics, which were relevant to the construction of the theory in the first place.

Even though Weyl's theory in *Raum, Zeit, Materie* may be unusual, I do not think the observations made here are unique to this case. As I've hinted throughout the paper, the very same concerns, and indeed some of the same protagonists, are involved in the debate over the foundations of quantum mechanics a few years later. So at the very least it might be worth keeping in mind these debates as we try to interpret contemporary physical theories like quantum field theory and quantum gravity. Here too we find not only that clear ontological interpretations might be lacking, but also that the very mathematics used to express the theories in the first place remains contested.[17]

None of this is to say that we should no longer aim to interpret physical theories, or that our metaphysics should not be in touch with our best physics. But it suggests that the influence between metaphysics and physics is not monodirectional, from physics to metaphysics.

[17]I have in mind here, for example, the debate between defenders of Lagrangian QFT and axiomatic QFT, for discussion see Wallace (2011).

References

John L. Bell. Hermann Weyl on intuition and the continuum. *Philosophica Mathematica*, 8(3):259–273, 2000.

Max Born. The statistical interpretation of quantum mechanics. In Nobel Foundation, editor, *Nobel Lectures: Physics 1942-64*, Nobel Lecture. Elsevier Publishing Company, 1955.

Max Born, Irene Born, Werner Heisenberg, and Bertrand Russell, editors. *The Born-Einstein Letters*. MacMillan, 1916-1955.

Percy Bridgman. *The Logic of Modern Physics*. MacMillan, 1927.

Albert Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, pages 891–921, 1905.

Michael Friedman. *Reconsidering Logical Positivism*. Cambridge University Press, 1999.

Werner Heisenberg. Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. *Zeitschrift für Physik*, XXXIII, June 1925.

A. Hermann, K. v. Meyenn, and V.F. Weisskopf, editors. *Wolfgang Pauli: Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg u.a.*, volume I - 1919-1929. Springer, 1979.

James Ladyman and Don Ross. *Every thing must go*. Oxford University Press, 2007.

Jagdish Mehra and Helmut Rechenberg. *The Historical Development of Quantum Theory, Vol.1*. Springer, 1982a.

Jagdish Mehra and Helmut Rechenberg. *The Historical Development of Quantum Theory, Vol.2*. Springer, 1982b.

Wolfgang Pauli. Merkurperihelbewegung und Strahlenablenkung in Weyls Gravitationstheorie. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 21:742–750, 1919.

Wolfgang Pauli. Relativitätstheorie. In A. Sommerfeld, editor, *Enzyklopädie der mathematischen Wissenschaften*, volume V, pages 539–775. Teubner, 1920.

Wolfgang Pauli. *Writings on Physics and Philosophy*, chapter Space, Time and Causality in Modern Physics. Springer, 1994.

Thomas Ryckman. *The Reign of Relativity - Philosophy in Physics 1915-1925*. Oxford Studies in Philosophy of Science. Oxford University Press, 2005.

Erhard Scholz. Hermann Weyl on the concept of continuum. In V. Hendricks, S.A. Pedersen, and Jørgensen, K.F., editors, *Proof Theory - History and Philosophical Significance*, pages 195–217. Kluwer Academic Publishers, Dordrecht, 2000.

Norman Sieroka. Geometrization versus transcendent matter: A systematic historiography of theories of matter following Weyl. *The British Journal for the Philosophy of Science*, 61:769–802, 2010.

Mark van Atten, Dirk van Dalen, and Richard Tieszen. Brouwer and Weyl: The Phenomenology and Mathematics of the Intuitive Continuum. *Philosophica Mathematica*, 10(3):203–226, 2002.

Bas van Fraassen. *Quantum Mechanics*. Clarendon Press, Oxford, 1991.

David Wallace. Taking particle physics seriously: A critique of the algebraic approach to quantum field theory. *Studies in the History and Philosophy of Modern Physics*, 42:116–125, 2011.

Hermann Weyl. *Das Kontinuum - Kritische Untersuchungen über die Grundlagen der Analysis*. Metzger und Wittig, Leipzig, 1918.

Hermann Weyl. *Raum, Zeit, Materie*. Julius Springer, Berlin, third edition, 1919.