LOTTERIES, PROBABILITIES, AND PERMISSIONS

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ABSTRACT: Thomas Kroedel argues that we can solve a version of the lottery paradox if we identify justified beliefs with permissible beliefs. Since permissions do not agglomerate, we might grant that someone could justifiably believe any ticket in a large and fair lottery is a loser without being permitted to believe that all the tickets will lose. I shall argue that Kroedel's solution fails. While permissions do not agglomerate, we would have too many permissions if we characterized justified belief as sufficiently probable belief. If we reject the idea that justified beliefs can be characterized as sufficiently probably beliefs, Kroedel's solution is otiose because the paradox can be dissolved at the outset.

KEYWORDS: epistemic obligation, evidence, justification, Thomas Kroedel, lottery paradox, probability

Thomas Kroedel[1] argues that if we assume that we are permitted to believe *p* iff we are justified in believing *p*, we can easily solve a version of Kyburg's[2] lottery paradox.[3] Here is the set up. You know that there is a large and fair lottery. Only one ticket can win and the odds of any ticket winning is the same as the odds of any other ticket winning. It seems to him that:

(1-J) For each ticket, you are justified in believing that it will lose.

The paradoxical conclusion that is supposed to follow from (1-J) is that:

(2-J) You are justified in believing that all the tickets will lose.

It seems that (2-J) is false. What to do?

Kroedel suggests that (1-J) and (2-J) are equivalent to:

(1-P) For each ticket, you are permitted to believe that it will lose.

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[1] Thomas Kroedel, "The Lottery Paradox, Epistemic Justification, and Permissibility," *Analysis*, 72, 1 (2012): 57-60.

[2] Henry Kyburg, *Probability and the Logic of Rational Belief* (Middletown: Wesleyan University Press, 1961).

[3] For arguments in support of the view that justified beliefs are permissibly held beliefs (but not beliefs you're obligated to have), see Clayton Littlejohn, Justification and the Truth-Connection (Cambridge: Cambridge University Press, 2012) and Mark Nelson, "We Have No Positive Epistemic Duties," *Mind* 119 (473): 83-102.

(2-P) You are permitted to believe that all the tickets will lose.

He says that (1-P) is ambiguous between a narrow- and wide-scope reading. On one reading, (1-P) is false. On one reading (1-P) is true, but it does not entail (2-P).

Let *p*, *q*, *r*, etc. be propositions about particular tickets losing. Let PB*p* be the sentence "It is permissible for you to believe *p*." Here is the first reading of (1-P), a narrow-scope reading:

(N1-P) PB*p* & PB*q* & PB*r*

Kroedel says that (N1-P) is true. He is right that (N1-P) does not entail (2-P) because permissions do not agglomerate. If permissions agglomerate, whenever you are permitted to ϕ and permitted to ψ, you would be permitted to ϕ and ψ. If, say, we were sharing a cake and you were permitted to take one half or take the other half, you would thereby be permitted to take the whole thing if permissions agglomerated. Since you can be permitted to take part without thereby being permitted to take it all, permissions do not agglomerate. Having two permissions and using one can thereby lead you to lose the other. Likewise, being permitted to believe *p* and being permitted to believe *q* does not mean that you are permitted to believe both *p* and *q* (or believe the conjunctive proposition that *p* and *q*).

Contrast (N1-P) reading with this wide-scope reading of (1-P):

(W1-P) P[B*p* & B*q* & B*r*]

While (W1-P) does entail (2-P), Kroedel argues that (W1-P) is false. He says that the problem with (W1-P) is that it is plausible that *if* you are permitted to hold several beliefs then you are thereby permitted to have a single belief that is the conjunction of those contents. It would be if this closure principle were true:

(CP) If P[B*p* & B*q* & B*r*], then PB[*p* & *q* & *r*]

It is, as he notes, highly implausible that you are permitted to believe the conjunctive proposition [*p* & (*q* & *r*)], so if the permission to believe both *p* and *q* (together) carries with it the permission to believe the conjunction (*p* & *q*), we have some reason to think that (W1-P) is false.

Kroedel thinks that to solve the lottery paradox, we should say that beliefs concerning lottery propositions can be justified but there is a limit as to how many such beliefs we can permissibly form. He thinks that there is a reading of (1-P) that is true, (N1-P) and urges us to reject (2-P). Why should we accept (N1-P) or (1-J)? His suggestion is that the high probability of a proposition is sufficient for the permissibility of believing it (forthcoming: 3). What is wrong with (2-P)? The problem cannot be that if you were permitted to believe that all the tickets will lose you will thereby believe something you know is false. The lottery might not

have a guaranteed winner and (2-P) still seems false. Perhaps the reason that (2-P) is false is just that the probability that all the tickets will lose is too low. It might seem that Kroedel's solution should work given the following principle linking justification to probability:

(PJ) PB*p* iff the probability of *p* on your evidence is sufficiently high.

While (PJ) would (if true) explain why (1-P) is true and (2-P) is not, I think Kroedel has to reject (PJ). If he does that, his solution becomes otiose. We can dissolve the paradox by denying (1-J) and (N1-P) rather than worrying about whether these false claims entail further false claims.

To see this, notice that since Kroedel is committed to (N1-P) and denying (2-P), he is committed to the following claim:

(*) For a lottery with n tickets (assuming that n is suitably large number), you are not permitted to form n beliefs that represent tickets in the lottery as losers but for some number m such that $n > m > 0$, you are permitted to form m beliefs that represent tickets in the lottery as losers.

Suppose you had a ticket for a lottery with 1,000,000 tickets. You know that this ticket, ticket #1, is not terribly likely to win. Let us suppose that in a lottery this size, you are permitted to believe that at least one ticket will lose. You decide to make use of this permission and believe ticket #1 will lose. You know (or should know!) that forming this belief does not change the probability that ticket #2 will lose. Since you were permitted to believe it before believing ticket #1 will lose and its probability remains unchanged when you add that first belief to your belief set, you can add the belief that ticket #2 will lose without being compelled to abandon your belief that ticket #1 will lose. (It is not as if adding the belief that ticket #2 will lose to your belief set forces you to lower the probability that ticket #1 will lose.) Given (PJ), it seems you can permissibly believe both that ticket #1 and ticket #2 will lose. We can apply the same reasoning again and you can permissibly add the belief that ticket #3 will lose without having to abandon previously formed beliefs about losing tickets. Repeat. At some point, (PJ) says that you can add some number of beliefs about losing lottery tickets greater than m without impermissibly adding beliefs to your stock of beliefs. At this point, (*) says that you formed more beliefs than you are permitted to form. You cannot consistently endorse (PJ) and (*).

If Kroedel denies (*), he has to either accept (2-P) or reject (N1-P). If he does that, he has to abandon his proposed solution. If instead he retains (*) and denies (PJ), he has to deny one of the following:

(High) PB*p* if the probability of *p* on your evidence is sufficiently high.

(Low) ~PB*p* if the probability of *p* on your evidence is sufficiently low.

If he denies (Low), it is not clear why we should reject (2-P). If he denies (High), it is not clear what motivation there is for (N1-P). If forced to choose, it seems rather obvious that he should deny (High) rather than (Low). If he rejects (High), he can dispense with the lottery paradox much more quickly. If it is possible for the probability of *p* to be sufficiently high on your evidence and for you to be obligated to refrain from believing *p*, I cannot see what would be wrong with saying that your obligation is to refrain from believing lottery propositions. The only thing they have going for them from the epistemic point of view is their high probability.

There is a common objection to (High) and to (1-P). Many people think that you cannot know that a ticket in a lottery with 1,000,000 tickets will lose. Suppose this is so and suppose that you know that you cannot know that the ticket you have is a loser. If you believe that your ticket is a loser and know that you cannot know that your ticket is a loser, this is how you see things:

(1) This ticket will lose, but I do not know that it will.

If you believed such a thing, you would be deeply irrational. While you cannot justifiably believe (1), the probability of (1) on your evidence, however, is quite high. So, (High) says that it is permissible to believe (1). So, (High) is mistaken. If (High) is rejected, this should take some of the sting out of denying (1-J) and (N1-P).

One reason to think that you cannot justifiably believe lottery propositions is that you cannot justifiably believe what you know you cannot know.[4] Lottery propositions are known unknowns. There is a further reason to think that it would be better to solve the lottery paradox by denying (1-J) and (N1-P) than to accept these and reject (2-J). It seems you cannot have proper warrant to assert:

(2) Your ticket will lose.

You might explain why it would be improper to assert (2) given only knowledge of the odds that the ticket will lose on the grounds that you cannot know (2) and cannot have warrant to assert what you do not know.[5] For various reasons, some object to this suggestion on the grounds that there are propositions that the speaker does not know that the speaker does have sufficient warrant to

[4] Alexander Bird, "Justified Judging," *Philosophy and Phenomenological Research* 74 (2007): 81-110; Jonathan Sutton, "Stick to What You Know," *Nous* 39 (2005): 359-96.

[5] Timothy Williamson, *Knowledge and its Limits* (Oxford: Oxford University Press, 2000).

assert. (Maybe you do not know that the building you see on the hillside is a barn, but if you do not know this just because you are in fake barn country, it is not obvious that you do not have sufficient warrant to assert that the building is a barn.) Suppose that knowledge is not necessary for warranted assertion. If knowledge is not necessary for warranted assertion, what is? Douven[6] argues that reasonable belief is necessary and sufficient for warranted assertion. Given Douven's account, we have to reject (N1-P) and (1-J) to explain why you do not have sufficient warrant to assert (2). You cannot properly assert (2) because you cannot justifiably believe (2). Thus, (1-J) is mistaken. Paradox dissolved.

Someone might say that there is an alternative approach available to Kroedel, one on which we modify (High) as follows:

> (High*) PBp if the probability of *p* on your evidence and what you permissibly believe is sufficiently high.

Armed with this, he might argue that as you add more and more lottery beliefs to your set of beliefs, there will come a point at which the probability that some ticket is a loser will be too low for you to add another belief to your belief set. The thought seems to be that it would be impermissible to add a belief to your belief set because the probability that the remaining tickets will lose *on your beliefs* falls further and further the more beliefs you form. Eventually, it becomes impermissible to add more.

For this move to work, we would have to say that whether one has sufficient propositional justification to believe a lottery proposition depends upon the probability of that proposition conditional on what you justifiably believe and on what your evidence is. The oddity of this response, however, seems to be that the minimally rational agent knows that the probability of each ticket turning out to be a loser is the same and remains invariant however many lottery beliefs the subject forms. Yet, (High*) suggests that the reason that you don't have the permission to believe some lottery proposition is that the probability of some ticket turning out to be a loser has dropped below some threshold.

Someone might instead suggest that the problem has to do with the agglomeration of risk. With each belief comes additional risk and with each additional risk taken, you get closer and closer to a level of risk taking that is unacceptable.

The problem with this reformulation of the solution is that it succumbs to a version of the wrong kind of reasons problem. We have already established that

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[6] Igor Douven, "Assertion, Knowledge, and Rational Credibility," *Philosophical Review* 115 (2006): 449-85.

the additional lottery beliefs you form are no riskier than other beliefs you have formed. For Kroedel's solution to work, we have to assume that some beliefs in the set of lottery beliefs are permissibly held. While adding more and more beliefs means that the probability of the set of lottery beliefs will include a falsehood increases, the fact that adding a belief to a set increases the probability that the set contains a falsehood is not the sort of reason that counts against adding that particular belief to the set. Adding the preface-like belief to your belief set (e.g., that something you believe is mistaken) guarantees that the set of beliefs you have contains a falsehood, but *that* fact does not constitute a decisive reason to remain agnostic about your own fallibility or to believe that you are not in error in any of your beliefs.

If (High) should be abandoned, (High*) is not a suitable alternative. If neither (High) nor (High*) is acceptable, it is difficult to see how a solution along the lines proposed by Kroedel could work. If no modification of (High) or (High*) is acceptable, we can reject (High) and (High*). Having done that, there is no problem left for us to solve.