Is the UK an outlier?

An international comparison of upper secondary mathematics education

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Foreword

It is often said that the British education system is unusual in requiring or enabling so few of its young people to continue studying mathematics after the age of 16. If true this seems a matter of some significance, but looking into the matter we could find little systematic evidence either way. We therefore commissioned a study to gather evidence on the real position, both to support our own work and to inform the national debate about future directions for mathematics education.

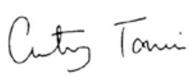
The research was carried out by Dr Jeremy Hodgen and David Pepper from King's College London; and Linda Sturman and Graham Ruddock from NFER. They have done an excellent job in tackling what turned out to be a complex question and we are grateful to them. The research addresses a number of questions about policy and participation in upper secondary mathematics education in 24 countries (mainly from the OECD), including the four countries of the UK. The results are summarised in this report and the associated country profiles that underpin it.

The findings are stark. In England, Wales and Northern Ireland fewer than one in five students study any mathematics after the age of 16 (Scotland does slightly better). In 18 of the 24 countries more than half of students in the age group study mathematics; in 14 of these, the participation rate is over 80%; and in eight of these *every* student studies mathematics. When it comes to the mathematics education of its upper secondary students the UK is out on a limb.

Moreover the situation is not static. Few of these other countries are satisfied with levels of achievement and participation in upper secondary mathematics, and most are devising policies and reforms aimed at increasing these levels.

Many questions arise from this research. Do these levels of participation and achievement meet the needs of the workforce? Do they adequately prepare young people for further and higher education? If not should A-level mathematics continue to provide the sole route for expansion or are alternative routes needed? And – given the critical role that primary schools play in providing all young people with the foundations for mathematical learning – is it acceptable that the majority of primary teachers do not study mathematics beyond GCSE?

We urge those involved with mathematics education, whether in policy or practice, to consider the new evidence provided in this report and its implications for post-16 mathematics education in the UK.



Anthony Tomei, Director, Nuffield Foundation

Key findings

This report examines mathematics education provision for 16-18/19 year-old (pre-university level) learners in 24 countries, including England, Scotland, Wales and Northern Ireland. The information presented comes from a survey carried out between April and July 2010.

For each country, we have identified the policy for mathematics education and the structure of provision. We have also set out the content and level of mathematics education available in each country, and compared participation rates.

This report provides a comparative analysis of the countries surveyed. A detailed summary for each individual country can be found in the accompanying series of country profiles, published online at **www.nuffieldfoundation.org**.

What is upper secondary education?

In the countries surveyed, upper secondary education is usually between two and four years in duration and intended for 16 to 18/19 year olds or for 15 to 17/18/19 year olds. It is non-compulsory in the vast majority of countries surveyed, although in some countries it is the norm for students to progress to this level even though it is optional. In the UK, this level of education is commonly referred to as "post-16". However for reasons of comparability, we refer to upper secondary throughout the report, unless in specific reference to UK education policy.

Mathematics education policy

Compulsory vs optional – see Table 2 (page 19)

Of the 24 countries included in the review, England, Scotland, Wales and Northern Ireland are four of only six which do not require compulsory participation in mathematics after the age of 16. The other two are Ireland and Australia (NSW). In New Zealand, participation becomes voluntary once a minimum standard has been reached. Significantly, these three countries all have strong cultural and historic links to the UK.

Mathematics is compulsory for all students in general education in 13 countries, and for all students in vocational education in nine countries.

Relation to other subjects – see table 3 (page 23)

Where mathematics is compulsory across general education, it is never the only compulsory subject. In addition to mathematics, the first language is almost always compulsory, a second language is usually compulsory and science is often compulsory.

In the UK, Ireland, the Netherlands and Spain, there is no compulsory subject or core of subjects that all upper secondary students must take.

However, in the Netherlands, Spain and other countries such as France and Germany, students' choices are largely limited to predetermined subject combinations and students in Ireland who wish to progress to higher education need to choose mathematics at upper secondary level. Therefore in practice, only in the UK is there no compulsion to study any particular subject at this level.

Participation rates

General mathematics – see table 6 column 2 (page 38)

England, Wales and Northern Ireland recorded lower levels of participation in upper secondary mathematics education than any other country surveyed. They are the only countries in the survey in which 20% or fewer of upper secondary students study mathematics.

Levels of participation are slightly higher in Scotland, but still below the level recorded in the majority of the 24 countries. Scotland is one of three countries in which 21-50% of students take mathematics. The other countries in this group are Hong Kong and Spain.

The remaining 18 countries have participation rates of over 50%. In eight of these countries, all students (95-100%) study mathematics. Unsurprisingly, these are the same eight countries in which mathematics is compulsory for all upper secondary students - the Czech Republic, Estonia, Finland, Japan, Korea, Russia, Sweden and Taiwan.

Advanced mathematics (equivalent to GCE) – see table 6 column 3 (page 38)

The countries with the highest rates of participation in advanced mathematics were Japan, Korea, New Zealand, Singapore and Taiwan. In these countries, over 31% of upper secondary students study advanced mathematics.

Eight countries, including Scotland, had participation rates in advanced mathematics of 16-30% of all upper secondary students.

England, Wales and Northern Ireland were amongst the eight countries with participation in advanced mathematics of 15% or lower. The others in this group were Germany, Ireland, Netherlands, Russia and Spain.

Content and level of upper secondary mathematics

See Tables 4 and 5 (pages 29 and 30)

In most countries, there is an option for extended study in further mathematics. This includes countries with Pathways or Baccalaureate options which emphasise breadth over specialisation. Countries vary in how the choice to study advanced mathematics is structured, but the relative freedom of choice of applied options in A-levels in the UK is unusual. England, Wales and Northern Ireland are also unusual in the proportion and variety of applied content within advanced mathematics.

The UK nations, Hong Kong, Ireland and Singapore are unusual in teaching mechanics within mathematics as well as in physics. Most countries are placing greater emphasis on uses and applications of mathematics, particularly in basic mathematics. In general, this is framed in terms of a modeling approach and involves some statistical problem solving.

Some countries provide mathematics at a level between 'basic' and 'advanced', which appears to provide progression for those relatively successful students who choose not to specialise in mathematics.

Drivers for take-up in mathematics

This was a subsidiary area of our study, however it was clear that a key driver for the take-up of mathematics is national policies that give the subject either an optional or a compulsory status.

Making mathematics compulsory is not a guarantee of positive student outcomes, but taking this approach to several subjects enables many countries to ensure their students receive a broad core curriculum at upper secondary level.

Where mathematics is optional, the requirement for a qualification in mathematics for entry to higher education, or indeed to specific vocations, was highlighted as an important factor, notably in Ireland.

The fact that mathematics is not required for many higher education courses, or that students do not intend to enter higher education, can be disincentives for take-up. There is also some evidence of students behaving strategically by not choosing mathematics, particularly advanced mathematics, because it is perceived as being more difficult than other subjects or one in which it is harder to achieve higher grades.

In countries where participation is higher in advanced mathematics, it generally follows that participation in *any* mathematics is also higher - at least in countries where upper secondary general education is not targeted to a relatively small elite.

Recommendations

We have used the evidence gathered from these 24 different countries to make a series of recommendations for consideration in the formation and implementation of policy, and for future research.

1. A review of post-16 mathematics policy in the UK

Given the relatively low participation in basic and advanced mathematics in the UK, there is a need for the Government to initiate a review of post-16 mathematics policy, specifically in relation to the status (compulsory or optional) of the subject. The review should take a broad view of the policy area and consider:

- The different types and level of mathematics that are or could be offered to students in post-16 education. This should be considered within the context of current and projected social and economic needs.
- Post-16 education as a whole, including vocational and general education, and taking into account the different destinations of students, from engineering to teaching and nursing.
- The status of other subjects in the post-16 curriculum, particularly English language (or other first language). The implication is that, as a minimum, the mathematics and English subject communities would need to pool their resources and cooperate closely.

2. Develop alternative models for post-16 mathematics

Potential models for post-16 mathematics should be researched and developed. Where appropriate, pilots should be conducted and evaluated. In the UK context, starting points for alternative models could include:

- The continued availability of non-compulsory options in mathematics but with a new requirement in mathematics for entry to higher education and a strengthened requirement for entry to relevant courses.
- Compulsory mathematics or numeracy at more basic levels, potentially delivered via existing qualifications such as Functional/Key Skills and Free Standing Mathematics Qualifications (FSMQs). The recently announced review of 14-19 vocational education is therefore welcome. In addition, students could be given the option of taking mathematics at more advanced levels, potentially through existing options such as Advanced Level FSMQ.

- The creation of an intermediate option or options between basic and advanced mathematics, aimed at those students who have already achieved an A*-C grade at GCSE. This would reflect the different career pathways of students and provide them with an appropriate option in mathematics at post-16 level.

3. Consideration of the resource implications of changes to post-16 mathematics

Policy discussions about post-16 maths need to consider the resource implications of implementing changes, including the impact on the phases of education *preceding* and *following* post-16. Stakeholders at these levels will need to be consulted, including the multiple awarding bodies operating across much of the UK, both to inform any policy changes and as a first step towards implementation.

4. Further research

Given the resource implications, further research should be commissioned in order to supplement the information on policy structures in this study. Particular attention should be given to the **content** of provision at basic and advanced levels and, most importantly, to the **impact** of policy change in other countries. This research should focus on a smaller number of countries, which should be a mix of near and far neighbours.

In considering policy in the UK, we should take a closer look at the dynamics of different policies and participation levels within its four nations. This research would help us account for school, teacher and student level variables and their relation to participation levels.

Introduction

Research questions

Participation in mathematics education when it becomes optional at the age of 16 has been a consistent subject for debate in the UK over the past decade. Yet this debate has not been informed by a clear understanding of upper secondary mathematics education in other countries. Indeed, this information appears to be absent from existing literature.

Our research, commissioned by the Nuffield Foundation, was undertaken to address this evidence gap. We were asked to investigate a series of questions:

Main research questions

1. What is the national policy for, and structure of, mathematics education provision for 16-18/19 year-old (pre-university level) learners?

2. What are the overall participation rates in mathematics study for 16-18 year-olds both as proportions of students and proportions of the age cohort?

3. What are the patterns of participation in terms of following different routes involving mathematics?

4. What is the content and level of the different kinds of provision? In particular, what might be deemed general mathematics and what aligned to specific pathways?

Additional research questions

5. What drives the pattern of take-up? How is it linked to the needs of higher education, employers and national policy objectives?

6. How is the picture changing over time?

Countries surveyed

Our survey includes a total of 24 countries, including the four UK nations. Sixteen of these (including the UK), are economically developed members of the Organisation for Economic Co-operation and Development (OECD), two are OECD accession countries and three are non-OECD countries from the Pacific Rim. We chose the three Pacific Rim countries because of their high performance in international surveys of attainment in mathematics (PISA and TIMSS).

For the three countries in which education policy is decided by sub-national jurisdictions, we focussed on New South Wales in Australia, British Columbia in Canada and Massachusetts in the USA. The focus on these single jurisdictions reflects the limited time available for the investigation. It also reflects our judgment that they are of particular interest in the UK context and that the necessary information was easily accessible. The full list of countries surveyed is as follows:

UK NATIONS	OECD COUNTRIES	OECD ACCESSION COUNTRIES	NON-OECD / PACIFIC RIM / HIGH ATTAINING
England Northern Ireland Scotland Wales	Australia (New South Wales) Canada (British Columbia) Czech Republic Finland France Germany Hungary Ireland Japan Korea Netherlands New Zealand Spain Sweden USA (Massachusetts)	Estonia Russian Federation	Hong Kong Singapore Taiwan

Methodology

The methodology responds to the apparent lack of international comparative literature on upper secondary mathematics education and, more specifically, the research questions set out above. The work was carried out between April and July 2010 in four stages.

Stage 1: Online searches

We began with a systematic online search including the INCA website (www.inca.org.uk), Eurydice website (www.eurydice.org) and the TIMSS 2006 Encyclopaedia. We also included ministry of education websites or those of the arms-length technical bodies (Boyle, 2008) responsible for curriculum and qualifications arrangements in each of the target countries.

We used a mixture of English-language sources, including international databases, policy literature, national data and curricula or syllabi documents. We wrote up results in the form of draft country profiles that addressed each of the six research questions.

Stage 2: International enquiries

For each of the countries, we asked a national contact from our international networks to review the relevant draft country profiles collected in Stage 1. Their role was to validate this information and to provide any important supplementary detail they could offer.

Stage 3: Final short descriptions for each country and a detailed annex

We developed the information gathered in Stages 1 and 2 into final country profiles, including full references. The 24 country profiles are available to download from **www.nuffieldfoundation.org.**

Stage 4: Overall summary and comparative analysis

In this final stage of the investigation we identified similarities and differences across the countries, particularly in comparison with the education systems in the four UK countries.

Caveats and limitations

Any international comparison comes with a caveat – different countries collect data in different ways, for different purposes. In addition, terms such as 'vocational' can have different meanings in different countries.

We are confident that in broad terms our findings represent a reliable and valid comparison of upper secondary mathematics in the UK and internationally. Nevertheless, there is a strong note of caution to the figures in this report. The statistics produced by each country are not absolutely comparable. They are produced on different bases and often over different time periods. Rarely are margins of error noted in the statistical sources. It is also important to note that few other countries have the level and detail of statistics available in the UK (Schnepf & Micklewright, 2006).

We have often calculated proportions of proportions thus increasing the margin of error. When making quantitative comparisons between countries (as in Tables 5 and 6), we have made judgements based on interval estimates rather than point estimates. We have indicated how these judgements have been reached in the notes to the tables.

Abbreviations

ISCED: International Standard Classification of Education

FSMQ: Free-Standing Mathematics Qualification

GCE: General Certificate of Education

GCSE: General Certificate of Secondary Education

HAVO: Hoger Algemeen Voortgezet Onderwijs
(Netherlands, Senior Secondary Education)

NCEA: National Certificate of Educational Achievement
(New Zealand, Upper Secondary)

VWO: Voorbereidend Wetenschappelijk Onderwijs
(Netherlands, Pre-university education)

Terminology

Upper secondary education: This term includes all forms of education for this age group, including school, college and employment-based options, but excluding tertiary education. In the UK, upper secondary education is commonly referred to as "post-16", but we chose not use this term for reasons of comparability between countries. For example, in several countries upper secondary begins at a younger age than 16. Furthermore, many countries require students to repeat a grade, meaning that unlike the UK, not all post-16 students are in upper secondary education.

General education: Indicates pre-university or, more broadly, pre-tertiary academic education as delivered by schools and colleges.

Vocational education: Vocational education and training (VET) within upper secondary education. This covers education programmes and pathways explicitly linked to particular employment sectors or occupations. In many countries, upper secondary vocational education and training is provided partially or wholly by schools and colleges, as a function of the balance between theoretical and practical learning.

Advanced mathematics: Studying mathematics at some level equivalent to GCE Mathematics. In practice, this consists of ISCED Level 3 study with a minimum content level of the pure modules of GCE AS Mathematics in England.

Basic mathematics: Anything at a lower level than advanced mathematics and may consist of anything above ISCED Level 1. In curriculum terms, this is usually described as equivalent to GCSE Mathematics in England, but with the addition of what is often described in terms of "use of mathematics", "modelling" or "applications" that may involve a small amount of ISCED Level 3 study.

Further mathematics: Extended study in advanced mathematics equivalent to at least the pure elements of GCE A2 mathematics and AS Further Mathematics in England.

Research findings

We have presented our findings in a series of six tables, each preceded by a short introduction and followed by an explanation of how the countries compare.

Table 1: Is upper secondary education compulsory?

Table 2: To what extent is mathematics a compulsory subject in upper secondary education?

Table 3: Which subjects are compulsory across upper secondary general or vocational education?

Tables 4: What level of mathematics is available in upper secondary general education?

Table 5: How are advanced mathematics choices structured in upper secondary general education?

Table 6: What are participation rates in upper secondary mathematics education?

More detailed information about each country is available online at **www.nuffieldfoundation.org**.

Is upper secondary education compulsory?

Table 1 shows the approximate age range and number of years in upper secondary education and whether it is compulsory. There are two major patterns for upper secondary education:

- Age 16 to 18/19 (two-three years in duration)
- Age 15 to 17/18/19 (three-four years in duration)

Hungary is distinct in having a wide age range from 14 to 18/19/20. Scotland has a short age range from 16 to 17/18.

Table 1 also shows that upper secondary education is optional in the vast majority of countries. However, it is compulsory in Germany, Hungary and the Netherlands and compulsory up to the age of 17 in Australia. In the Czech Republic, Estonia, Finland, Korea and Taiwan, it is the norm for students to progress to this level of education even though it is optional.

Overall, there is some diversity in the definition and nature of upper secondary education across the countries, but enough similarity to make broad comparisons meaningful.

TABLE 1: IS UPPER SECONDARY EDUCATION COMPULSORY?

	Upper secondary age range	No. of upper secondary years	Compulsory upper secondary
Australia (NSW)	16–18	2	○
Canada (BC)	16–19	3	
Czech Republic	15–16/17/18/19	4	○
Estonia	16–18/19	3	○
Finland	17–19/20	2–3	○
France	15–17/18/19	2-4	
Germany	15–18/19	3–4	●
Hong Kong	17–19	2	
Hungary	14–18/19/20	4–6	●
Ireland	15–17/18	2–3	
Japan	16–19	3	
Korea	15–17	2	○
Netherlands	14/15–17/18	2–5	●
New Zealand	15–18	3	
Russia	15–17	2	
Singapore	16–19	3	
Spain	16–18	2	
Sweden	16–19	3	
Taiwan	15–17	3	○
USA (Mass)	15–18	3	
England	16–18/19	2	
Scotland	16–17/18	1–2	
Wales	16–18/19	2	
Northern Ireland	16–18/19	2	

Note: The age ranges are approximate and indicate the approximate length of upper secondary education, which may vary according to student pathways.

KEY: ● Upper secondary education is compulsory
○ Although not compulsory, few (<10%) students do not continue into upper secondary education

Table 1 – how the different countries compare

Australia: It is compulsory for students to participate in education or training until they are 17 years old, which is equivalent to the first year of upper secondary education, unless they are moving into full-time work.

Czech Republic: Upper secondary education is not compulsory but over 90% of students continue in formal education.

Estonia: About 14% of students do not complete compulsory education but of those who do, over 99% proceed to upper secondary education.

Finland: Upper secondary education is not compulsory but over 90% of students continue in formal education.

Germany: Part-time education is compulsory until the age of 18 for those who do not attend a full-time school.

Korea: Education is optional from the age of 15. However, over 99% of students continue into upper secondary education.

Taiwan: Upper secondary education is not compulsory but 92% of 15-17 year olds were in upper secondary education in 2009.

To what extent is mathematics a compulsory subject in upper secondary education?

Table 2 details whether mathematics is compulsory in all of upper secondary general and vocational education or only within programmes that have set subject combinations.

- Mathematics is compulsory for all students in general education in over half the countries surveyed (thirteen), and compulsory for all students in vocational education in nine countries.
- It is compulsory for all students, whether they are in general or vocational education, in eight countries.
- Mathematics is compulsory for all general education students but **not** all vocational students in five countries.
- Only in France is mathematics compulsory for all vocational students but not all general education students. However, it is compulsory for most students in general education too.

- Mathematics is compulsory for **some** students in general and vocational education in three countries. These are the Netherlands, Spain and Singapore, where it is required in some programmes with set combinations of subjects.

- There are only six countries in which mathematics is entirely optional in upper secondary general education. These are the four UK countries, Australia and Ireland. In addition, in New Zealand, participation in mathematics is voluntary once a minimum standard has been reached.

It is significant that the only other two countries in which mathematics is entirely optional, Australia and Ireland, both have strong cultural and historic links with the UK, including within education.

New Zealand could also be included in this group, as although there is a requirement for some students to meet a minimum numeracy requirement, this usually happens by the age of 16 or 17, after which they can choose whether to continue with mathematics.

Although mathematics is optional in Ireland there is an expectation that many students will take mathematics because they need the subject for entry to university, and the participation rate is high in general education (as detailed later).

In the UK, Ireland and Australia, mathematics is only compulsory if students choose a vocational programme that requires it. In the UK and Ireland, this largely means the Application of Number award in the Key Skills or Functional Skills qualifications, as part of a wider vocational programme, although students who have already gained at least a C at GCSE may be exempted from this.

In addition, in England, Scotland, Wales and Northern Ireland, a small proportion of vocational students (e.g. those taking vocational programmes in subjects such as engineering) study some advanced mathematics.

TABLE 2: TO WHAT EXTENT IS MATHEMATICS A COMPULSORY SUBJECT IN UPPER SECONDARY EDUCATION?

	All of general education	Some of general education	None of general education	All of vocational education	Some of vocational education
Australia (NSW)			●		●
Canada (BC)	●				●
Czech Republic	●			●	
Estonia	●			●	
Finland	●			●	
France		●		●	
Germany	●				●
Hong Kong	●				●
Hungary	●				●
Ireland			●		●
Japan	●			●	
Korea	●			●	
Netherlands		●			●
New Zealand		○			○
Russia	●			●	
Singapore		●			●
Spain		●			●
Sweden	●			●	
Taiwan	●			●	
USA (Mass)	●				●
England			●		●
Scotland			●		●
Wales			●		●
Northern Ireland			●		●

KEY: ● Mathematics is compulsory
○ Mathematics is voluntary once a minimum level is reached.

Table 2 – how the different countries compare

Australia: Students in vocational education may need to study additional mathematics, either in general bridging courses or vocationally-specific topics.

Canada (British Columbia): General education is the major pathway at upper secondary level and mathematics is compulsory. There was a lack of information on vocational education but mathematics is likely to be compulsory when needed for particular vocations.

Czech Republic: Mathematics is compulsory, except for a small number of students in conservatoires. All students can choose whether to take an examination in the subject. However, there are plans to make mathematics optional at upper secondary level.

Estonia: Mathematics is compulsory across general and vocational education. In general education, students can choose whether to take a basic or advanced syllabus. If they take the advanced syllabus, they can choose whether to take the related examination. The introduction of an examination for the basic syllabus is being considered.

Finland: Mathematics is compulsory across general and vocational education. In general education, students can choose whether to take a basic or advanced syllabus and whether to take the related examinations.

France: All general and vocational students take some mathematics except in the General Baccalaureate, where students taking the Literary series (rather than the Economic and Social series or the Science series) can opt out of mathematics.

Germany: Mathematics forms part of core curricula in general education (in Gymnasium) and technical vocational education (in Fachoberschulen), and students can choose additional mathematics. Mathematics is compulsory in occupational vocational education (such as in Berufsschule).

Hong Kong: Under the system being phased in from 2009 to 2011, mathematics is compulsory in general education. Previously it was simply needed for entrance to university. There was a lack of information on vocational education but it is likely to be compulsory when needed for particular vocations.

Hungary: Mathematics is only compulsory in general education and when needed for vocations. Different options are available depending on the type of school and qualification.

Ireland: Mathematics is not compulsory but in general education, students choose a range of subjects and they almost universally choose mathematics.

Japan: Mathematics is compulsory across upper secondary education and several different options are available.

Korea: Students who entered upper secondary education from 2009 onwards must take mathematics. Those in general education must take at least three of the seven mathematics subjects and those in vocational education must take at least two of them.

Netherlands: In general education mathematics is compulsory except for students beginning the Culture and Society subject combination of HAVO from 2007. Mathematics is not compulsory across vocational education but it is compulsory in many options within VMBO.

New Zealand: Once students have completed literacy and numeracy to a minimum basic level, usually at the age of 16/17, mathematics is no longer compulsory. Thereafter, they choose subjects in which to specialise, usually from age 16/17 to 18.

Russia: Mathematics is compulsory in upper secondary education, except in one type of vocational school attended by a small proportion of students.

Spain: Mathematics is not compulsory for upper secondary students, except as part of some specialised general or vocational programmes.

Singapore: Mathematics is not compulsory in general education but students are required to take 'contrasting subjects'. Mathematics is only compulsory as part of some vocational programmes.

Sweden: Mathematics is compulsory across all upper secondary pathways.

Taiwan: Mathematics is compulsory across general and vocational education.

USA (Massachusetts): Mathematics is compulsory in general education, which constitutes the vast majority of provision at this level. Approximately 2% of students enter vocational education instead, which is essentially designed for adults. In these cases, mathematics is not always compulsory.

UK (England, Scotland, Wales and Northern Ireland): Students have a free choice of subjects and mathematics is only compulsory in some vocational programmes - often as remedial provision. By remedial provision, we mean students repeat and retake the same pre-16 mathematics they have already studied.

Which subjects are compulsory across general or vocational education?

Table 3 shows which other subjects are compulsory in all of general or vocational education, representing a core curriculum for all students. Mathematics is never the **only** compulsory subject in upper secondary education. In countries where mathematics is compulsory, the first language is always compulsory, a second language is usually compulsory and science is often compulsory. Australia is unusual in that the first language is compulsory but mathematics is not.

The absence of an explicit requirement for students to take a particular subject or core of subjects at upper secondary level in the UK, Ireland and New Zealand contrasts with the 18 other countries. Although in New Zealand students must take literacy and numeracy, this requirement relates to Level 1 of the NCEA, which is equivalent to GCSE and is usually taken at the age of 15 or 16. Thus students in the UK and Ireland are unusual in having a free choice of upper secondary subjects. However, students in Ireland who apply for higher education generally need to take mathematics in order to compete for a limited number of places. **So in practice, only in the UK is there no compulsion to study any particular subject at upper secondary level.**

Upper secondary vocational mathematics education in the UK countries is overwhelmingly remedial and aimed at those students who have not attained a C at GCSE. Whilst there may be a few exceptions (such as the Engineering Diploma), very little emphasis is placed on the application or use of mathematics in vocational learning. This is in contrast to the type of provision offered in other countries.

TABLE 3: WHICH SUBJECTS ARE COMPULSORY ACROSS UPPER SECONDARY GENERAL OR VOCATIONAL EDUCATION?

	First language is compulsory	Mathematics is compulsory	Second language is compulsory	Science is compulsory	One or more other subjects is compulsory
Australia (NSW)	●				
Canada (BC)	●	●	●	●	●
Czech Republic	●	●	●		
Estonia	●	●	●	●	●
Finland	●	●	●	●	●
France		●	●		
Germany	●	●	●	●	
Hong Kong	●	●	●		●
Hungary	●	●	●	●	●
Ireland					
Japan	●	●		●	●
Korea	●	●	●	●	●
Netherlands	●		●		●
New Zealand					
Russia	●	●	●	●	●
Singapore		●			●
Spain	●		●		●
Sweden	●	●	●	●	●
Taiwan	●	●	●	●	●
USA (Mass)	●	●	●	●	●
England					
Scotland					
Wales					
Northern Ireland					

This table provides information on subjects that are compulsory across upper secondary general eduction or across vocational education. The notes for each country (overleaf) provide details.

Table 3 – how the different countries compare

Australia (New South Wales): English is the only subject required for the Higher School Certificate, which is the upper secondary school qualification.

Canada (British Columbia): In general education, the compulsory subjects are: language arts, science, mathematics and social studies.

Czech Republic: Czech language, mathematics and a foreign language are compulsory. However, from 2011 mathematics will no longer be compulsory and students will choose whether to take one of either mathematics or civics and one of either social science or informatics.

Estonia: There is a compulsory core of subjects across general and vocational education. The extent to which they are externally assessed varies. Only the general education examination for mother tongue language is compulsory.

Finland: There is a compulsory core of subjects in general and vocational education. The extent to which they are externally assessed varies. Only the general education examination for mother tongue language is compulsory.

France: Compulsory subjects vary according to pathways, programmes and year groups in upper secondary education. In terms of universality, mathematics is second only to a foreign language, which is a requirement throughout upper secondary education (all Baccalaureates, BEP and CAP), whilst mathematics is optional for students on the Literary strand of the General Baccalaureate.

Germany: German language, mathematics and a foreign language are core subjects in general education and technical vocational education. There is no compulsory core of subjects in occupational vocational education.

Hong Kong: In the new system currently being introduced, mathematics is only compulsory in general education, whereas previously mathematics was not compulsory. There will be four core subjects and all students will be examined in them: Chinese, English, Mathematics and Liberal Subjects.

Hungary: In general education, examinations are compulsory in: mathematics, Hungarian language and literature, a second language, and history.

Ireland: Students in upper secondary education choose from a wide range of subjects. Many choose to study mathematics, reflecting the high level of competition for university places.

Japan: The compulsory subjects in upper secondary education are: Japanese language; geography and history; civics; mathematics; sciences; health and physical education; art or music; and home economics or living skills.

Korea: Students must choose subjects from each of five 'liberal arts' areas: humanities and social sciences; science and technology; arts and physical education; foreign languages; and general studies.

Netherlands: Compulsory subjects vary according to students' chosen pathways but Dutch, English, Social Studies and PE are compulsory across upper secondary general education (VWO and HAVO) and upper secondary initial vocational education (VMBO).

New Zealand: Students taking the over-arching upper secondary qualification must take some credits in numeracy and literacy at a more basic level but this is usually completed by the age of 16 or 17. They can then choose subjects in which to specialise (which may include mathematics) at a higher level, usually at ages 16/17-18.

Russia: In general education, the compulsory subjects are: Russian; literature; mathematics; social sciences; natural sciences; and physical training. However, the only compulsory examinations are in Russian and mathematics. Students choose three or more other subjects in which to be examined.

Singapore: Students on the pre-university pathway usually choose four subjects in addition to the compulsory 'subjects' of 'General Paper', 'Mother Tongue Language' and 'Project Work'. One of the four chosen subjects must be a contrasting subject; for example, arts students could take mathematics. There are no similar requirements for vocational education.

Spain: Bachillerato students take a common core of subjects that are compulsory for all students (language and literature, a foreign language, philosophy, history, religion or independent study and physical education). They also take subjects that are compulsory within their Bachillerato type and optional subjects of their choosing.

Sweden: All upper secondary pathways include the same eight core subjects: Swedish; English; mathematics; civics; religion; science; physical education and health; and art.

Taiwan: In addition to mathematics, the following subjects are compulsory: Chinese; English; science; social studies and courses from the 'arts field' (music or fine art); the 'life field' (living technology, home economics, computer science); and health and physical education.

USA (Massachusetts): All upper secondary high school students study seven areas: mathematics; science and technology; social studies; English language arts; world languages; the arts; and health.

UK (England, Scotland, Wales and Northern Ireland): There is a free choice of subjects in upper secondary education, whether general or vocational. However, a large minority of upper secondary students take qualifications in English (communication) or mathematics (number) at ISCED Level 2 or 1 in order to progress. In Scotland, compared to the other UK countries, students in general education choose to study more subjects and this choice is from a narrower range of subjects.

What is the content and structure of basic and advanced mathematics in general education?

In **Table 4**, we compare the content offered in both basic and advanced mathematics, noting the availability of both applied/applications of mathematics and further mathematics. In **Table 5**, we compare the ways in which students' options for specialising in advanced mathematics are structured.

The content and structure of basic mathematics

In most countries surveyed, some basic mathematics is widely available in upper secondary education. Most countries also have, or are in the process of including, units focused on mathematical literacy or more general uses of mathematics (see **Table 3**). These appear to be aimed at a wider group of students than those traditionally studying advanced levels of mathematics. We have not been able to examine the content of these options in detail. However, the policy intentions appear to envisage similar applied content to that available within Level 2 and to some extent Level 1 of the Free Standing Mathematics Qualifications (FSMQs) in England and Wales.

The content and structure of advanced mathematics

In Advanced Mathematics, all countries surveyed include a broadly similar range of pure mathematics topics covering functions, calculus, trigonometry and geometry. Although **Table 4** shows there are some differences within this, for example some countries, such as Sweden, do not teach complex numbers or limits (see also Mullis et al, 2009). Whilst there is significant overlap in content, curricular strands are described in different terms reflecting cultural issues as well as more or less emphasis on applications and uses of mathematics. For example, in the Czech Republic, the subject is called "Mathematics and its Applications" and five curriculum areas are described as Logic and Proof, Number and Variable, Working with Data, Combinatorial Analysis, Probability, Functional Relations and Dependence, and Geometry.

One trend, partially driven by international surveys, is towards more similarity between the mathematics curricula in different countries (Brown, 1998). However, it is likely that this broad trend obscures some significant differences. We also note that the *intended* curriculum as described at national or state level may be different from the *implemented* curriculum in schools and classrooms. In the 2008 TIMSS Advanced Survey, for example, the figure for implemented instructional hours in the Russian Federation for advanced mathematics was lower than the intended, or recommended, figure (Mullis et al, 2008, p.45).

Another factor is the balance between school and higher education mathematics. In Scotland, for example, where students tend to enter higher education earlier and where higher education courses tend to be longer, the mathematical content in schools is less advanced than that in England, Wales and Northern Ireland. Similarly, countries with a broader 16-19 curriculum tend to teach less advanced mathematics. Extrapolating from 11-16 mathematics, it is clear that different countries place different emphases on different aspects of the curriculum. For example, the Pacific Rim countries tend to emphasise algebraic manipulation (Askew et al, 2010) and the UK is unusual in placing little emphasis on 3D or geometric constructions (Hoyles et al, 2002).

Applied Mathematics

Most countries include some statistics content in mathematics, although in most cases this appears to be a traditional course focused on procedures and non-computer methods. New Zealand is a notable exception. The UK countries, Hong Kong and Singapore are relatively unusual in offering mechanics within mathematics. In most countries it is taught in physics only. In addition, several countries offer a discrete mathematics option, similar to decision mathematics in England. This is often aimed at Business or IT Pathways and may be included as a relatively 'easy' option within advanced mathematics.

Further mathematics

In many countries, students have the option of further mathematics, or the extended study of mathematics roughly equivalent to the content of A level mathematics and A level further mathematics in England, Wales and Northern Ireland. Whilst a detailed examination of the curriculum is beyond the scope of this study, the curriculum time allocated to this option (in the Russian Federation, for example) suggests that content may be beyond that available in England. However, this intended time allocation may not be widely implemented and this extended study may only be available to a small elite of students. Further Mathematics study may be severely limited by the associated costs and the subject knowledge of mathematics teachers as was the case in England prior to the establishment of the Further Mathematics Network, now known as the Further Mathematics Support Programme (www.fmnetwork.org.uk).

The structure of choice and specialisation

Almost all countries give students a choice about whether to specialise in advanced mathematics. This choice is structured in three different ways (see **Table 5**):

Pathways: A model in which school subject choices are grouped together in a coherent way according to students' intended HE or employment destinations. This is usually a small number of groupings, such as the Netherland's four options, however Sweden's strong Pathways structure consists of 17 National Programmes. We have classified most Baccalaureate programmes in this category. Many of the Pathways models are structured around a hierarchical set of mathematics options with different Pathways allowing different amounts of mathematical study. The Pathways model tends to provide more breadth of subjects studied.

Free choice: Students are largely free to choose subject combinations for advanced study. Again this may take a variety of forms. In some countries (e.g. Czech Republic), students study a broad range of subjects, but may choose a smaller subset to be examined in. Others (e.g. Japan, Singapore) allow free choice but certain combinations of subjects are "recommended". Countries vary in the extent to which they encourage breadth of subjects studied. The revised curriculum in Singapore requires students to take a "contrasting subject" to encourage breadth. England, Northern Ireland and Wales are unusual in the narrow range of subjects studied by individual students.

Limited choice: Estonia and Finland are unusual in offering a limited choice of just two options for mathematics within general education: Basic and Advanced in Finland; and Narrow and Extended in Estonia. Of the two, Finland gives slightly more choice by also offering "commercial mathematics", and a further mathematical models course aimed at students intending to take mathematics, science or engineering options.

TABLE 4: WHAT LEVEL OF MATHEMATICS IS AVAILABLE IN UPPER SECONDARY GENERAL EDUCATION?

		BASIC		ADVANCED			
	Is mathematics compulsory?	Pure Mathematics	Applications of Mathematics	Pure Mathematics	Mechanics	Statistics	Further Mathematics
Australia (NSW)		●	●	●	●	●	●
Canada (BC)	●	●	●	●		●	
Czech Republic	●	●	●	●		●	
Estonia	●	●	○	●		○	○
Finland	●	●	●	●		●	
France		●	●	●		○	●
Germany	●	●	○	●		●	
Hong Kong	●	●	●	●	●	●	●
Hungary	●	●	●	●		●	●
Ireland		●	●	●	●	●	
Japan	●	●	●	●		○	●
Korea	●	●	●	●		●	●
Netherlands		●	●	●		●	
New Zealand		●	●	●		●	
Russian	●	●	●	●			●
Singapore		○	○	●	●	●	●
Spain		●	●	●		●	–
Sweden	●	●	●	●		●	●
Taiwan	●	●	○	●		●	●
USA (Mass)	●	●	○	●		●	○
England		○	○	●	●	●	●
Scotland		●	●	●	○	○	
Wales		○	○	●	●	●	●
Northern Ireland		○	○	●	●	●	●

KEY: ● Fully available

○ Available, but not universally or with very limited take-up

– Insufficiant data available

TABLE 5: HOW ARE ADVANCED MATHEMATICS CHOICES STRUCTURED IN UPPER SECONDARY GENERAL EDUCATION?

	HOW DO STUDENTS CHOOSE TO SPECIALISE IN ADVANCED MATHEMATICS?			HOW IS THE ADVANCED MATHEMATICS CURRICULUM ORGANISED?	
	Pathways	Free choice	No or limited choice	All / most hierarchically structured	Alternative options are available
Australia (NSW)		●		●	
Canada (BC)	●			●	
Czech Republic		●		●	
Estonia			●	●	
Finland			●	●	●
France	●			●	
Germany	●			●	
Hong Kong		●		●	
Hungary		●		●	
Ireland		●		●	
Japan	○	○		●	
Korea	○	○		□	●
Netherlands	●			●	●
New Zealand		●		●	●
Russian Fed.		●		●	
Singapore	○	○		●	
Spain	●			□	●
Sweden	●			●	●
Taiwan	●			●	
USA (Mass)		●		●	
England		●		□	●
Scotland		●		●	●
Wales		●		□	●
Northern Ireland		●		□	●

KEY: ○ No formal pathways, but particular subject combinations recommended to students intending to follow particular HE routes.

□ Around 50% of the advanced mathematics curriculum is hierarchically structured.

Tables 4 and 5 – how the different countries compare

Australia (New South Wales): Five mathematics options are available: General Mathematics; Mathematics; Mathematics Extension 1; Mathematics Extension 2; and Life Skills Mathematics (for students with special needs and taken by fewer than 2% of those in General Education). General Mathematics is a non-calculus course covering broadly similar mathematics content to GCSE but with considerable more focus on use, application and modeling. The specialist options of Mathematics, Mathematics Extension 1 and Mathematics Extension 2 have a more traditional pure and applied focus than the General Mathematics option.

Canada (British Columbia): We do not have information on current content. The new curriculum being phased in from 2010 has three pathways: Apprenticeship and Workplace Mathematics; Foundations of Mathematics at basic level; and Pre-Calculus at advanced level.

Czech Republic: In general education (Gymnázium), "Mathematics and its Applications" is taught through five curriculum areas: Logic and Proof; Number and Variable; Working with Data, Combinatorial Analysis, Probability; Functional Relations and Dependence; and Geometry. In fact, applications is only explicitly mentioned in the third and fourth of these strands. In addition, students are expected to develop EU competencies, including mathematics. However, neither mathematics nor quantity is explicitly mentioned in curricular documents, although risk is. In vocational education, basic level courses in applications of mathematics are available. Mechanics is taught within science. The choice to take advanced mathematics is likely to be influenced by the type of school, with limited availability outside the Gymnázium.

Estonia: Students choose between a narrow and extended syllabus for mathematics. The narrow syllabus comprises nine hierarchically organised compulsory courses, focusing on pure mathematics, and the extended syllabus builds on that. Students in vocational education take the first five courses as in general education. Pathways-related applied options are also available, often statistics.

Finland: Students choose from two options in mathematics: Advanced and Basic, both of which include statistics. Basic is mostly at Level 2, but includes a small amount of content equivalent to Level 3 in England. Basic includes application options tailored to particular pathways. Advanced level allows some pure specialisation.

France: In general education, the Baccalaureate includes a broad range of subjects. The emphasis is on pure mathematics and mechanics is taught in science. Students can choose to specialise in mathematics and the extent of content coverage in pure mathematics is similar to that in England, Wales and Northern Ireland. Reforms to be phased in from 2010/11 include greater emphasis on probability (statistics), scientific research methods and exploration.

Germany: In the Gymnasium, all students take a basic mathematics course. In addition, students can opt to take general or advanced mathematics. The general course consists of around 3 hours of teaching each week, and the an advanced course between 5 and 6 hours. Germany has a federal system and curriculum content varies to some extent between Länder. For example, the Hessen curriculum makes explicit links to subjects (such as Physics). In contrast, the Baden-Württemberg curriculum appears to be a traditional pure mathematics course with some statistics. Vocational courses involving mathematics will be tailored towards the individual requirements of the associated profession, and can include advanced mathematics for subjects such as engineering.

Hungary: Students may study mathematics at different levels. Based on the 2000 curriculum documents, the grammar school (Baccalaureate) curriculum is a largely pure curriculum with some traditional statistics. The vocational and technical curriculum places more emphasis on modelling, although the content is largely a subset of the grammar school curriculum.

Hong Kong: Previously, mathematics education in Hong Kong largely reflected historic links with the UK (and England in particular). Under new reforms introduced in 2009, all students will take some mathematics with extensions available.

Ireland: Students can take Foundation, Ordinary or Higher following a largely pure programme. Traditional applied mathematics (mechanics) is available as an option, but take up is low.

Japan: Mathematics is currently being restructured into six courses: Mathematics I, II and III; Mathematics A and B; and Application of Mathematics. Mathematics I is compulsory. The other courses can form a mathematics programme of either a Science, Mathematics and Medicine HE route or a Social Science and Humanities HE route. More flexible programmes are also available. The emphasis is on pure mathematics, although some statistics is now studied by all students. Application of mathematics is related to mathematical literacy and appears not to be aimed at the science, mathematics and medicine route. Mechanics and discrete mathematics are not studied.

Korea: The courses offered are: Practical Mathematics; Mathematics I; Mathematics II; and a choice of one from three applied options: Differentiation and Integration; Probability and Statistics; and Discrete Mathematics.

Netherlands: Students in general education take one of two routes: either pre-university education (VWO) or senior secondary general education (HAVO). In each route they can choose from one of four courses: science and technology (NT); science and health (NG); economics and society (EM); and culture and society (CM).

Mathematics is a compulsory component in all courses except CM within the HAVO route.

New Zealand: Three options are available in addition to standard mathematics: accounting; mathematics with calculus; and mathematics with statistics.

Russian Federation: For secondary (complete) general education there are two different levels of study: Base (four hours per week) and Advanced (6-12 hours per week). The curriculum is largely pure mathematics. Schools have some freedom in practice to set their curriculum.

Singapore: Students study for awards (Cambridge International) A-levels similar to English GCEs with a similar curriculum content. Reforms introduced in 2006 offer mathematics specialisation at three levels: H1 (equivalent to AS); H2 (A-level mathematics); and H3 (equivalent to a double award). Mathematics can be studied in a Knowledge & Inquiry A-level. Vocational programmes include subjects such as engineering, which require an advanced understanding of mathematics.

Spain: Pure and applied mathematics are available within the Bachillerato (general education) depending on the pathway followed: Pure Mathematics in Science and Technology pathway and Mathematics applied to either Social or to Health Sciences. Additionally, some students taking a combined course may also take some mathematics. There was insufficient data on the availability of Further Mathematics.

Sweden: Mathematics is organised into seven courses: Mathematics A-E, which build on each other; Discrete Mathematics; and a Mathematics Extension course. Mathematics A is a core subject course and is included in all programmes. Other Mathematics courses are included with the 17 national programmes as appropriate to the programme of study. The content is broad - covering pure mathematics and statistics.

Taiwan: In Senior High School, the mathematics curriculum is largely pure with some statistics. Students take Basic Mathematics and Mathematics I, which includes some advanced mathematics but no calculus. Some students then choose to study calculus in Mathematics II, which is targeted at students aiming to study Science, Engineering and Medicine programmes at university. Senior Vocational School mathematics places more emphasis on real-life mathematical skills. Four vocationally orientated options are available, two of which include calculus and other advanced topics.

USA (Massachusetts): The curriculum is described in terms of learning standards for each grade. So, Grades 11 and 12 (ages 16-18) are described in terms of a small set of very specific advanced level study items including complex numbers, proof by induction, axiomatic Euclidean and non-Euclidean geometric systems, fractals, conics, optimisation problems and linear, quadratic, and exponential regression. It seems likely that most Grade 11 and 12 students do not study any of these topics, although specific Massachusetts data are not available. Figures for the USA as a whole (2005) indicate that, whilst 100% of high school students studied some mathematics and 70% studied Algebra II, only 14% studied calculus and 8% trigonometry. Given the State's performance in TIMSS relative the US generally, it is likely that in Massachusetts participation is higher.

The high-stakes examination for entry to higher education is the SAT, which tests mathematical reasoning together with some (optional) mathematics content. From the perspective of the USA as a whole, Common Core State Standards, including for mathematics, have recently been developed. Most states, including Massachusetts, have adopted them. The Standards place more emphasis on modeling, statistics and use of mathematics.

England: Students take GCSE examination at the end of compulsory schooling (age 16) then opt for a narrow range of subjects post-16 (normally 3-4). Many students who do not achieve a grade C currently retake GCSE. In some centres (mainly Further Education Colleges rather than schools), Free-Standing Mathematics Qualifications or "Use of Mathematics" qualifications are available at Level 1, 2 and 3, although numbers taking this qualification are relatively small (in 2008 this was approximately 3000 at Level 1, under 500 at Level 2 and 2000 at Level 3). In practice, the majority of those studying mathematics either take an academic specialist route or take what is effectively remedial mathematics.

Students specialising in mathematics post-16 take GCE A-level, a modular examination with a hierarchically structured pure mathematics core and a structured choice of applied modules in statistics (up to 6 hierarchical options), mechanics (up to four hierarchical options) and decision (up to two hierarchical options). Students may take AS Mathematics (equivalent to one year single subject), A2 (equivalent to two year single subject), or AS/A2 Further Mathematics (equivalent to one or two years additional single subject). Students' applied choices appear to be limited by schools and colleges' capacity to teach the options rather than their HE or employment options. Some schools are beginning to offer AS mathematics pre-16 but the numbers are not known.

Scotland: Students take a Standard Grade examination at the end of compulsory schooling (age 16) then opt for a range of subjects post-16 (up to 5 or 6). Students achieving Foundation or General level at Standard Grade, may then opt to take Intermediate Grade 1 or 2 respectively. Standard and Intermediate Grade qualifications are intended to lead to more advanced study, and approximately 9% of those in upper secondary general education progress from Intermediate 2 to Higher Grade. The Higher and Advanced Higher mathematics courses focus on pure mathematics, though a small number of students taking Applied Mathematics at Advanced Higher choose to specialise in either mechanics or statistics. Higher Grade roughly equates to AS mathematics in England, Wales and Northern Ireland and Advanced Higher Grade to A2 mathematics.

Northern Ireland: The system is very similar to that in England. A consultation exercise is underway relating to a simplification of GCE mathematics qualifications and the introduction of two applied routes: Use of Mathematics and Use of Statistics. Some students may take the FSMQs available from awarding bodies in England and Wales.

Wales: The system is similar to that in England.

What are participation rates in upper secondary mathematics education?

How many students study mathematics in upper secondary education?

Table 6 shows participation rates in the countries surveyed. It is important to bear in mind that the nature of upper secondary education varies across the countries in several ways. These include the age at which upper secondary education begins and ends, the balance between general and vocational pathways, and the subjects, if any, that comprise the compulsory curriculum.

Proportion of students studying mathematics

Table 6 shows that 18 countries have participation rates of over 50% of students in upper secondary education. Eight of these, the Czech Republic, Estonia, Finland, Japan, Korea, Russia, Sweden and Taiwan, have rates of participation of between 95-100% of students. These are the same countries that make mathematics compulsory at this level.

England, Wales and Northern Ireland have lower levels of participation than any other country surveyed. They are the only countries in which 20% or fewer of upper secondary students take mathematics. Levels of participation are slightly higher in Scotland, but still below the level recorded in the majority of the 24 countries. Scotland is one of three countries in which 21-50% of students take mathematics. The other countries in this group are Hong Kong and Spain. In these countries with participation below 50%, mathematics is not compulsory. It is either chosen freely or as part of a more elite pathway. We note that Hong Kong is in the process of introducing compulsory mathematics to general education in upper secondary.

Studying mathematics to an advanced level

Japan, Korea, New Zealand, Singapore and Taiwan recorded high levels of participation in advanced mathematics (over 30% of upper secondary students). For the four Pacific Rim countries, a significant factor is likely to be the relatively high attainment of pupils in lower secondary. In New Zealand, the option of mathematics with statistics appears to be highly attractive to students.

Australia (NSW), Estonia, Finland, France, Hong Kong, Sweden, USA (Mass) and Scotland recorded medium levels of participation in advanced mathematics (16-30% of upper secondary students).

Germany, Ireland, Netherlands, Russia Spain, England, Wales and Northern Ireland recorded low levels of participation in advanced mathematics (up to 15% of upper secondary students).

Countries with high participation in any mathematics do not necessarily have high participation in advanced mathematics as well. However, countries with high participation in advanced mathematics also have relatively high participation in any mathematics.

England, Wales and Northern Ireland's lower participation rates in advanced mathematics are in keeping with the overall number studying any mathematics. This contrasts with Russia, where all students, except those in some vocational schools, study some mathematics, but just over 1% of upper secondary students study advanced mathematics. Thus, in Russia, advanced mathematics is available for a small elite, whilst a relatively basic level of provision is made for the vast majority of students.

Trends in participation

Longitudinal data on upper secondary participation was only available in some of the countries surveyed. In some countries the data indicates that participation has been falling in both mathematics and advanced mathematics (Australia, Estonia and Ireland). In Korea, whilst participation in mathematics in general has been increasing, in advanced mathematics it has been decreasing. Students appear to have acted strategically in opting to take mathematics exams at more basic levels. By contrast, upper secondary participation in mathematics at any level has been increasing in the USA (Massachusetts). There has also been an increase at advanced level in England, Wales and Northern Ireland, with numbers exceeding those seen prior to the sharp decline in 2001/2. Whilst this increasing trend is encouraging, it should be noted that this increase in absolute numbers reflects the growing population of young people as well as the growing number of students participating in upper secondary education. When considered as a proportion of either the cohort or those in upper secondary, the level of participation has equaled but not gone beyond 2001 levels and is lower than in 1996.

Factors affecting participation

There was limited information in the country profiles for factors affecting participation. Evidence from wider sources in England suggests that students' attainment and attitudes are strongly inter-related (Matthews & Pepper, 2007; Brown et al., 2007), and that students cease studying mathematics because they find it boring, abstract and difficult (e.g. Osborne et al., 1997; Mendick, 2006). This clearly has implications for participation in mathematics.

Concerns expressed around the world about mathematics education suggest that these issues are not unique to the UK. Making mathematics compulsory at upper secondary level would certainly increase participation rates but not necessarily achievement or participation beyond that. Our survey suggests that the requirement for a qualification in mathematics for entry to higher education is likely to be an important factor and Ireland is one notable example of this. Similarly, where upper secondary mathematics is not a requirement for higher education, students may behave strategically, particularly in opting out of advanced mathematics. In the UK, many courses do not require a qualification in mathematics for entry and some courses in numerate subjects (such as accountancy, chemistry and economics) only require a minimum C at GCSE (Osmon, 2009).

It seems likely that a crucial factor in the low participation rates in the UK is the role of GCSE Mathematics (or Ordinary Grade in Scotland) as an endpoint. As a result, a significant proportion of students, destined for numerate or mathematical professions including nursing and primary teaching, will cease to study mathematics some time before the end of their general education. There is good evidence that effective primary teaching requires strong mathematical understanding (e.g. Askew et al, 1997; Hill et al, 2005). Yet our data indicates that in the UK, the majority of primary teachers will not study any mathematics from the age of 16 until the age of 22 or more, when they commence teacher training.

TABLE 6: WHAT ARE PARTICIPATION RATES IN UPPER SECONDARY MATHEMATICS EDUCATION?

	Not studying mathematics	Studying any mathematics	Studying advanced mathematics
Australia (NSW)	Some	Many	Medium
Canada (BC)	Few	Most	-
Czech Republic	Negligible	All	-
Estonia	Negligible	All	Medium
Finland	Negligible	All	Medium
France	Few	Most	Medium
Germany	Few	Most	Low
Hong Kong	Many	Some	Medium
Hungary	Few	Most	-
Ireland	Some	Most	Low
Japan	Negligible	All	High
Korea	Negligible	All	High
Netherlands	Some	Many	Low
New Zealand	Some	Many	High
Russia	Negligible	All	Low
Singapore	Some	Many	High
Spain	Many	Some	Low
Sweden	Negligible	All	Medium
Taiwan	Negligible	All	High
USA (Mass)	Few	Most	Medium
England	Most	Few	Low
Scotland	Many	Some	Medium
Wales	Most	Few	Low
Northern Ireland	Most	Few	Low

Key to categories: **Any mathematics**:

0-5%	Negligible
6-20%	Few
21-50%	Some
51-80%	Many
81-94%	Most
95-100%	All

Advanced mathematics:

0-15%	Low
16-30%	Medium
31%-100%	High

Note: The base for the percentages is the number of students in upper secondary education or training.
Data on participation in advanced mathematics was insufficient in Canada (British Columbia), Czech Republic and Hungary. We have used different categories for advanced mathematics (Low/Medium/high). This reflects the different patterns of participation at this level.

Table 6 – how the different countries compare

Australia NSW (2008 data): Of those taking the High School Certificate (HSC), 81% study mathematics. This is equivalent to approximately 68% of the cohort in education and training. However, this includes some students who are older or younger than this cohort and does not include students in technical and further education and who may study some (non-advanced) mathematics. These two factors would respectively overestimate and underestimate the actual level of participation but no further data is available. Advanced Mathematics is taken by 32% of students taking the HSC, which is approximately 27% of the education and training cohort.

Canada BC (2010 data): All high school students take mathematics as part of the graduation programme. There is specific mathematics provision for vocational students but participation rates are unclear. Since the majority of students are likely to be on the high school programme, perhaps as many as 80%, it seems fair to conclude that, as a minimum, most students take mathematics and few do not. There was insufficient data for advanced mathematics participation.

Czech Republic: The information presented in the table is based on policies rather than data and therefore assumes the implementation broadly reflects the policy of compulsory mathematics.

Estonia (2010 data): All students in general and vocational education study the narrow syllabus in mathematics but can opt to take the extended syllabus and the national examination. Each year, approximately 25% of the education and training cohort takes the exam (which is a little under 40% of general education students including a small number of vocational students). We assume that students who choose the extended syllabus take the examination and therefore participation in advanced mathematics is medium.

Finland (2010 data): All students in general and vocational education study mathematics. Those in general education can choose from a basic syllabus and an advanced syllabus. About 40% of them choose the advanced syllabus, representing about 20% of upper secondary students. They can also choose whether to take the related examinations. In vocational education, mathematics is assessed as part of the qualification. About 85% of upper secondary students are actually examined in mathematics.

France (2009 data): In vocational education, mathematics is compulsory in all programmes. In general education, mathematics is only optional in the Literature series of the General Baccalaureate. Half of the 65% of students who take the Baccalaureate take the General Baccalaureate, representing one third of the education and training cohort. Literature is one of three series in the General Baccalaureate. No data were available on the proportion of General Baccalaureate students taking the Literature series. Crudely assuming a third take each of the three series, then just over 10% of the education and training cohort can opt out of mathematics. Thus most students will take some mathematics.

Data for participation in advanced mathematics were not available. However, given that many students on the General and Technological Baccalaureate, and possibly some students on other pathways too, will take advanced mathematics, participation in advanced mathematics is estimated to be at a medium level.

Germany (2007/08 data): Upper secondary students in general education and 'technical' vocational education are required to take mathematics. Around 90% of vocational students take some basic mathematics, so roughly 93% of the education and training cohort take basic mathematics.

Around 25% of students in general education take advanced mathematics. Some students of vocational subjects, e.g. engineering, also study advanced mathematics, although data is not available. We estimate between 2.5% and 10% of students in vocational education study advanced mathematics, and therefore the proportion of the entire education and training cohort studying advanced mathematics is between 8% and 14%.

Hong Kong (2010 data): Approximately one third of the age cohort is in upper secondary general education but all of these students will take mathematics under the new system being introduced in 2009-11. Information for vocational education was not available but at least one third of the age cohort, and more of the education and training cohort, will do some mathematics. Approximately one third of the students in general education are expected to take advanced mathematics under the new system being implemented in 2009-11. This is just over 10% of the age cohort and a slightly higher percentage of the education and training cohort (but still under 15%).

Hungary (2007/8 data): Of those in upper secondary general education, 77% took mathematics as part of the Baccalaureate. No data on vocational students taking mathematics was available. However, in excess of 77% upper secondary students took mathematics. For the purposes of the table this is therefore rounded to 81% or more and categorised as most. One third (35%) of students are in Gimnazium schools. Some of these students would be expected to take a more advanced level of mathematics but there was insufficient data on the numbers.

Ireland (2008 data): Mathematics is almost universally taken by upper secondary general education students. No data on vocational education were available and the information in the table should be read in this context. Of the students in general education taking mathematics, 12% take higher level, 16% take foundation level and 72% take ordinary level. Since most upper secondary students take mathematics, the actual percentage of the education and training cohort taking these options will be only slightly lower.

Japan (2005 data): Mathematics is compulsory for all students in upper secondary education. Eighty-five percent of upper secondary students took some advanced mathematics (through the Mathematics II and Mathematics B options).

Korea (2009 data): Mathematics is effectively compulsory for all students in upper secondary education. Over 90% of upper secondary students took some advanced mathematics through options in Mathematics I, although this contains no calculus and is not quite equivalent to the AS Pure curriculum. Of the age cohort, 54% take the more advanced Mathematics II option. This represents 57% of the education and training cohort. However, only 21.5% of students studying Mathematics II choose to be examined in it for the College Scholastic Aptitude Test (CSAT), with the remainder largely choosing to be examined in content relating only to Mathematics I.

Netherlands (2008 data): Of the final year initial vocational (VMBO) students, 53% had chosen the theoretical option, which includes compulsory mathematics as part of sector-specific programmes. This represents 25% of upper secondary students. Further data for other VMBO students were not available. However, the data show that at least 47% of upper secondary students took mathematics as part of a general education pathway and 25% took mathematics as part of a vocational pathway. Thus at least 72% of upper secondary students, and perhaps substantially more, take mathematics. Twelve percent of final year general education students took advanced mathematics (though the Science and Technology option available in the HAVO and VWO pathways). This represents 6% of upper secondary students.

New Zealand (2009 data): All students are required to take mathematics for the NCEA. However, this is Level 1 numeracy equivalent to GCSE level and it is completed by about 85% of 16 year old students. A few students take accountancy as an alternative or additional subject. Eighty three percent of students in Years 12 and 13 (ages 17 and 18), take at least basic mathematics (although the number of students drops significantly across upper secondary - there were about 17% fewer students in Year 13 than in Year 12). About 41% of students in Year 12 will take advanced mathematics (with statistics or calculus options). Within the smaller Year 13 cohort, 49% take advanced mathematics and 66% take at least basic mathematics. Around half of these students take mathematics with statistics rather than mathematics with calculus.

Russia (2008 data): Almost all upper secondary students take some mathematics. Of students in upper secondary general education, 1.4% take Advanced Mathematics. As a percentage of the cohort, this will be fewer than 1%.

Singapore (2008/9 data): A detailed breakdown of student entries was not available for the vocational routes, so the proportions in Table 6 are estimates. Not all students in upper secondary education take mathematics, although many of the vocational routes include mathematics and some such as engineering, include advanced mathematics. Those studying A-level mathematics make up 28% of the age cohort, 31% of the education and training cohort, and over 80% of the total studying A-level.

Spain (2009 data): Fifty-five percent of students registered for the university entrance exams at the end of upper secondary education were registered for the Baccillerato option in Science and Technology or in Social Sciences. Both options include mathematics as a major subject. Twenty-six percent of the students were registered for the Health Sciences option and 3% for the Combined option. In both cases, mathematics is optional. Thus the proportion of these students taking mathematics is between 55% and 85%. This, however, does not take account of vocational education beyond the Baccillerato, which appears to account to a similar proportion of provision and where mathematics is not compulsory.

Twenty-two percent of upper secondary students were registered for Science and Technology, which focuses on pure mathematics. Since a little less than half those in education or training take the Baccillerato, about 10% take advanced mathematics. The Bachillerato qualification is intended to prepare students for general or vocational higher education. We lack information on any vocational provision outside of the Bachillerato.

Sweden (2007/8 data): All upper secondary students take a core of mathematics in Mathematics A. Fifty-two percent do Mathematics B (Arts, Natural Science, Social Science and Technology students). Twenty-one percent do some advanced mathematics in Mathematics C (Natural Science and Technology students). Fourteen percent do advanced mathematics in Mathematics D (Natural Science students). Mathematics C is judged to be equivalent to AS Pure mathematics.

Taiwan (2009/10 data): All upper secondary students take mathematics. Forty percent of the education and training cohort were in Senior High Schools, where Basic Mathematics and Mathematics I are compulsory. Data on students taking Mathematics II were not available but participation is likely to at least reflect the proportion (47%) of those studying programmes of Natural Sciences, Engineering and Medicine at university. Therefore around 19% of the total education and training cohort are studying advanced mathematics in Senior High School.

Students in Senior Vocational Schools entered for Mathematics B and C, which are the advanced mathematics options available to them, represented a further 48% of the education and training cohort. A further small proportion were entered for Mathematics B in 5-year Junior Colleges (around 2% of the education and training cohort). In total, at least 70% of the education and training cohort took some advanced mathematics at upper secondary level. This figure may be as high as 85% given that the Senior High School figure is a minimum estimate. This is at least 64% of the age cohort of 15-17 year olds.

USA Mass (2008 data): All upper secondary students study mathematics for the High School Diploma (84% of the age cohort). The participation rate of 14% for advanced mathematics is based on the proportion of students studying calculus in 2005 data for High Schools across the USA (14%). Massachusetts' performance is significantly higher than the US average. Hence, advanced mathematics participation is likely to be somewhat higher than 15%. Note, however, that the participation in advanced mathematics in the US generally is low.

England, Wales, Northern Ireland: About 9% of A level exams sat were in Mathematics and about 1% were in Further Mathematics (2009 data). However, this underestimates the actual proportion of students taking mathematics at A level because students generally enter for examinations in several subjects (usually three but this varies). The best measure uses data that matches candidates to examination subjects. An analysis of matched candidate data for A level participation in mathematics was only readily available for England, only for 2007 and not compared to overall participation in education and training.

The following figures for England are therefore approximate but based on matched candidate data. Forty-three percent of the age cohort took A levels. One quarter (25%) of those taking A level took mathematics (AS or A2). Thus approximately 11% of the age cohort took advanced mathematics. Approximately 80% of the age cohort are in education and training. Thus about 13% of the education and training cohort took mathematics at A level. This may slightly underestimate current participation levels, which (based on examinations data) appear to have improved in more recent years. See also Matthews & Pepper (2007) and Noyes (2009).

Similar approximate calculations for Wales and Northern Ireland give advanced mathematics participation rates of 11% and 15%, respectively. Thus, whilst we have judged Northern Ireland's participation rate to be low, we note that it lies on the threshold between low and medium.

A small proportion of students at upper secondary level take qualifications in mathematics other than A Levels - Key Skills for example. However, even including these students, the proportion of students studying basic mathematics in upper secondary is still under 20%.

Furthermore, a relatively large proportion of "post-16" students are retaking GCSE mathematics. We have not included these students in the participation figures for reasons of comparability. This is because many countries require students to repeat a grade (`grade retention'), meaning that unlike the UK, not all post-16 students are in upper secondary education.

Scotland (2008 data): In S5 (Secondary 5th year; 16-17 year olds), approximately 26% of the cohort in education or training took basic mathematics and around 23% took advanced mathematics. In S6, 4% took basic mathematics and 14% took advanced mathematics. Many students enter higher education after S5 and so we use the S5 figures for the judgement included in the table.

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Appendix B: Example country profile

As outlined in the methodology section (page 11), we have compiled a country profile for each of the 24 countries surveyed. These have been reviewed and amended by national experts in the relevant country. The profiles are published as an electronic document, which is available to download from www.nuffieldfoundation.org.

A example country profile, for Taiwan, is reproduced here to give an indication of the format and content.

Country profile for Taiwan

1. What is the national policy for, and structure of, mathematics education provision for 16-18/19 year-old (pre-university level) learners?

Education is non-compulsory from 16-19 in Taiwan. Students typically graduate from Junior High School at about 15 years old and then enter a three-year programme in a Senior High School or a Senior Vocational School. Some students may choose Five-Year Junior College.[1]

The curriculum structure of mathematics education in Taiwan is divided into two major categories. One category is for senior high school students or students who study in vocational school but want to pursue academic research in the future. In the 1st and 2nd year, these students study Basic Mathematics which is compulsory. In the 3rd year, students study Mathematics I and may choose Mathematics II depending on their interests.[2] The other category is for senior vocational school students, and has four types of mathematics courses to fit different needs. The vocational mathematics curriculum can also be chosen in the first three years in Five-Year Junior College.[3] The mathematics courses available are shown in the table below.

1 http://english.moe.gov.tw/ct.asp?xItem=4133&CtNode=2003&mp=1

2 http://www.edu.tw/high-school/content.aspx?site_content_sn=8411

3 http://vtedu.ntust.edu.tw/front/bin/ptlist.phtml?Category=7

SCHOOL	COURSE	DETAIL
SENIOR HIGH SCHOOL	Basic Mathematics	All students must take Basic Mathematics in the 1st and 2nd year.
	Mathematics I	For all students. This is a full-year course for students who intend to enter programmes of Social Sciences, Humanity, Law, Education and Arts at university, and a one-semester course for students who intend to enter programmes of Natural Sciences, Engineering and Medicine at university.
	Mathematics II	This is studied in the last semester for students who intend to enter programmes of Natural Sciences, Engineering and Medicine at university.
SENIOR VOCATIONAL SCHOOL	Mathematics A (8 hours/ 3 years)	For students who study Home Economics.
	Mathematics B (12 hours/ 3 years)	For students who study Commerce and Management, Agriculture and Food Science, Marine Technology and Fishery, Hospitality, Design, and Foreign Language.
	Mathematics C (16 hours/ 3 years)	For students who study Engineering and Architecture.
	Mathematics S (4-6 hours/ 3 years)	For students who study Art.

2. What are the overall participation rates in mathematics study for 16-18 year-olds both as proportions of students and proportions of the age cohort?

In 2009, the number of 15-17 year olds in Taiwan was 967,141.[4] Of the age cohort, 92% were in upper secondary education (about 38% were in senior high school and about 54% were in the vocational school system including the first three years of the Five-Year Junior College).[5]

The table below shows the number of mathematics entries for each category and version studied in 2009/2010. It also shows these entries as percentages of the total student cohort and of the total age cohort.

4 http://www.edu.tw/statistics/content.aspx?site_content_sn=8869, table 28.

5 http://english.moe.gov.tw/public/Attachment/04201511971.xls

SCHOOL AND COURSE		TOTAL ENTRIES[6] (1)	APPROX SIZE OF AGE 15-17 COHORT[7] (2)	APPROX SIZE OF COHORT IN EDUCATION[8] (3)	TOTAL ENTRIES AS % OF AGE COHORT [= (1)/(2)]	TOTAL ENTRIES AS % OF COHORT IN EDUCATION [=(1)/(3)]
Senior High School [9]		363,479	967,141	893,182	37.6%	40.7%
Senior Vocational School	Mathematics A	41,478			4.3%	4.6%
	Mathematics B	270,512			28.0%	30.3%
	Mathematics C	160,933			16.6%	18.0%
	Mathematics S	6,243			0.6%	0.7%
Five-year Junior College (the first three years only)[10]	Mathematics A	26,610			2.8%	3.0%
	Mathematics B	14,164			1.5%	1.6%
	Mathematics C	4,557			0.5%	0.5%
	Mathematics S	1,138			0.1%	0.1%

3. What are the patterns of participation in terms of following different routes involving mathematics?

Mathematics is a compulsory subject for all students whether they study in senior high schools or in senior vocational schools. Students must take the related mathematics route for the programme they study in order to gain the senior high school or senior vocational school qualification.[11]

4. What is the content and level of the different kinds of provision? In particular, what might be deemed general mathematics and what aligned to specific pathways?

The mathematics provided to Senior High School students emphasises pure mathematics with fundamental probability and statistics, and is geared for study in university. Basic Mathematics is studied by the 1st and 2nd year students while Mathematics I and II are advanced and are studied by the 3rd year students. Mathematics I is about advanced statistics, matrix, and inequality. Mathematics II is

6 http://www.edu.tw/statistics/publication_list.aspx

7 http://www.edu.tw/statistics/content.aspx?site_content_sn=8869, table 28.

8 http://www.edu.tw/statistics/content.aspx?site_content_sn=8869, table 27, age 15~17, number of students in senior high and vocational groups.

9 Proportion who go on to study Natural Sciences, Engineering and Medicine at university was 47% in 2009/10: http://english.moe.gov.tw/public/Data/052817225871.xls

10 http://www.edu.tw/statistics/content.aspx?site_content_sn= 21549

11 http://www.edu.tw/high-school/content.aspx?site_content_sn=8411 and http://vtedu.ntust.edu.tw/front/bin/ptlist.phtml?Category=7

about calculus. Mathematics II is especially for equipping those students who would like to enter Science, Engineering and Medicine programmes at university.[12]

In comparison with Senior High School Mathematics, the mathematics for Senior Vocational School is more oriented to real life mathematical skills. Each course has been designed with a different focus and weight to fit the vocational needs and the programme arrangement.[13] Mathematics S and A are the lowest two levels. They share the same topics including linear equations, functions, vectors, inequality, progression and series, probability, and statistics. However, Mathematics A is deeper than Mathematics S. As well as the fundamental topics in Mathematics S and A, Mathematics B has conic section and calculus. Mathematics C is the highest level and has the content of Mathematics B besides the topic of complex number.

5. What drives the pattern of take-up? How is it linked to the needs of HE, employers and national policy objectives?

Since courses of mathematics are compulsory for all programmes and the curricula have been developed to fit the needs of students for coping with different vocations or future studies, students have no choice but to follow the relevant mathematics curriculum. [14] Moreover, for those students who want to enter universities, Basic Mathematics, Mathematics I, and Mathematics II are a necessity for passing the university entrance examination.

6. How is the picture changing over time?

The Ministry of Education has plans to make upper secondary compulsory.[15] This is still being debated, but mathematics education would change dramatically if the length of compulsory education were to be extended to 12 years.

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12 http://www.edu.tw/high-school/content.aspx?site_content_sn=8411

13 http://vtedu.ntust.edu.tw/front/bin/ptlist.phtml?Category=7

14 http://www.edu.tw/high-school/content.aspx?site_content_sn=8411 and http://vtedu.ntust.edu.tw/front/bin/ptlist.phtml?Category=7

15 http://english.moe.gov.tw/ct.asp?xItem=11701&ctNode=814&mp=1

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